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The Electric and Magnetic Form Factors of the Neutron*

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ABSTRACT

We derive the electric and magnetic form factors of the neutron in the framework of a relativistic constituent quark model. Our parameter free prediction agrees well with a recent, accurate measurement. The relativistic features of the model and the specific form of the wave function are essential for the result. Comparisons are made to other models based on VMD, PQCD and QCD sum rules.

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A recent measurement [1] of the neutron electromagnetic form factors, $G_{En}(Q^2)$ and $G_{Mn}(Q^2)$, greatly increased the Q^2 range of previous data [2] and has significantly smaller errors. For the first time it is therefore possible to distinguish theoretical models with respect to experimental data from the neutron form factors. In the low Q^2 region, vector meson dominance (VMD) models [3] are traditionally used to make predictions for the form factor. For sufficiently high momentum transfer perturbative QCD (PQCD) [4] predicts the Q^2 dependence of the form factors. To describe the behavior at intermediate values of Q^2 the parameterization of Ref. [5] uses the VMD form at low Q^2, constrained by PQCD results at high Q^2. There are additional models which predict the neutron form factors. Reference [6] describes a relativistic constituent quark model which is similar to our approach. QCD sum rules are used in Ref. [7] to fix the parameters of the soft quark functions for calculating the form factors. None of these theoretical models are in good agreements with the data for both form factors.

We recently investigated the predictive power of a relativistic constituent quark model formulated on the light-front [8,9]. It provides a simple model wherein we have overall an excellent and consistent picture of the magnetic moments and the semileptonic decays of the baryon octet. The parameters of the model have been fixed in Ref. [9] so that we have a parameter free prediction of the neutron electromagnetic form factors.

The light-front dynamic is a convenient scheme for dealing with a relativistic system. If we introduce the light-front variables $p^\pm \equiv p^0 \pm p^3$, the Einstein mass relation $p_\mu p^\mu = m^2$ is linear in p^- and linear in p^+, in contrast to the quadratic form in p^0 and $\vec{p}$ in the usual dynamical scheme. A consequence is a single solution of the mass shell relation in terms of p^-, in contrast to two solutions for p^0:

$$p^- = (p_\perp^2 + m^2)/p^+ \ , \quad p^0 = \pm\sqrt{\vec{p}^{\,2} + m^2}\ . \tag{1}$$

The quadratic relation of p^- and $p_\perp \equiv (p^1, p^2)$ in the above Equation resembles the nonrelativistic scheme [10], and the variable p^+ plays the role of "mass" in this nonrelativistic analogy. It is therefore a good idea to introduce relative variables like the Jacobi momenta when dealing with several particles. As in the nonrelativistic scheme such variables allow us to decouple the center of mass motion from the internal dynamics. The light-front scheme shows another attractive feature that it has in common with the infinite momentum technique [11]. In terms of the old fashioned perturbation theory, the diagrams with quarks created out of or annihilated into the vacuum do not contribute. The usual qqq quark structure is therefore conserved as in the nonrelativistic theory. It is, however, harder to get the hadron states to be eigenfunctions of the spin operator [12].

The light-front formalism is specified by the invariant hypersurface $x^+ = x^0 + x^3 =$ constant. The following notation is used: The four-vector is given by $x = (x^+, x^-, x_\perp)$, where $x^\pm = x^0 \pm x^3$ and $x_\perp = (x^1, x^2)$. Light-front vectors are denoted by an arrow $\vec{x} = (x^+, x_\perp)$, and they are covariant under kinematic Lorentz transformations [13]. The three momenta $\vec{p}_i$ of the quarks can be transformed to the total and relative momenta to facilitate the separation of the center of mass motion [14]:

$$
\begin{aligned}
\vec{P} &= \vec{p}_1 + \vec{p}_2 + \vec{p}_3, \\
\xi &= \frac{p_1^+}{p_1^+ + p_2^+}, \qquad \eta = \frac{p_1^+ + p_2^+}{P^+}, \\
k_\perp &= (1-\xi)p_{1\perp} - \xi p_{2\perp}, \\
K_\perp &= (1-\eta)(p_{1\perp} + p_{2\perp}) - \eta p_{3\perp}.
\end{aligned} \tag{2}
$$

Note that the four-vectors are not conserved, i.e., $p_1 + p_2 + p_3 \neq P$. In the light-front dynamics the Hamiltonian takes the form

$$
H = \frac{P_\perp^2 + M^2}{2P^+}, \tag{3}
$$

where M is the mass operator with the interaction term W

$$
\begin{aligned}
M &= M_0 + W, \\
M_0^2 &= \frac{K_\perp^2}{\eta(1-\eta)} + \frac{M_3^2}{\eta} + \frac{m^2}{1-\eta}, \\
M_3^2 &= \frac{k_\perp^2 + m^2}{\xi(1-\xi)},
\end{aligned} \tag{4}
$$

with m being the mass of the constituent quarks. To get a clearer picture of M_0 we transform to k_3 and K_3 by

$$\xi = \frac{E_1 + k_3}{E_1 + E_2}, \quad \eta = \frac{E_{12} + K_3}{E_{12} + E_3},$$
$$E_{1/2} = (\mathbf{k}^2 + m_{1/2}^2)^{1/2}, E_3 = (\mathbf{K}^2 + m_3^2)^{1/2},$$
$$E_{12} = (\mathbf{K}^2 + M_3^2)^{1/2}, \tag{5}$$

where $\mathbf{k} = (k_1, k_2, k_3)$, and $\mathbf{K} = (K_1, K_2, K_3)$. The expression for the mass operator is now simply

$$M_0 = E_{12} + E_3, \quad M_3 = E_1 + E_2. \tag{6}$$

The diagrammatic approach to light-front theory is well known [15,16]. It provides in principal a framework for a systematic treatment of higher-order gluon exchange. In this work we limit ourselves to the tree graph. Since we set $Q^+ = 0$ we can preserve the correct qqq structure of the vertex. All relevant matrix elements that we investigate are related to

$$\left\langle \vec{p}' \left| \bar{q}\gamma^+ q \right| \vec{p} \right\rangle \sqrt{P'^+ P^+} \equiv M^+, \tag{7}$$

where the state $|\vec{p}\rangle \equiv |p\rangle / \sqrt{p^+}$ is normalized according to

$$\langle \vec{p}' | \vec{p} \rangle = \delta(\vec{p}' - \vec{p}). \tag{8}$$

The matrix element M^+ can be written in terms of wave functions as [9]:

$$M^+ = 3\frac{N_c}{(2\pi)^6} \int d^3k d^3K \left(\frac{E_3' E_{12}' M_0}{E_3 E_{12} M_0'} \right)^{1/2}$$
$$\times \Psi^\dagger(\mathbf{k}', \mathbf{K}') \Psi(\mathbf{k}, \mathbf{K}), \tag{9}$$

where $K_\perp' = K_\perp + \eta Q_\perp$, and N_c being the number of colors.

The electromagnetic current matrix element for the transition $n \to n'\gamma$ can be written in terms of two form factors taking into account current and parity conservation:

$$\langle n', \lambda' p' \, | J^\mu | \, n, \lambda p \rangle =$$
$$\bar{u}_{\lambda'}(p') \left[F_1(Q^2) \gamma^\mu + \frac{F_2(Q^2)}{2M_n} i\sigma^{\mu\nu} Q_\nu \right] u_\lambda(p) \tag{10}$$

with momentum transfer $Q = p' - p$, and the current $J^\mu = e\bar{q}\gamma^\mu q$. In order to use Eq. (9) we express the form factors in terms of the + component of the current:

$$F_1(Q^2) = \left\langle n', \uparrow \left| J^+ \right| n, \uparrow \right\rangle,$$
$$Q_\perp F_2(Q^2) = -2M_n \left\langle n', \uparrow \left| J^+ \right| n, \downarrow \right\rangle. \tag{11}$$

For $Q^2 = 0$ the form factors F_1 and F_2 are respectively equal to the charge and the anomalous magnetic moment in units e and e/M_N. The Sachs form factors are defined as $G_M = F_1 + F_2$ and $G_E = F_1 - \tau F_2$ with $\tau = Q^2/4M_n^2$.

Since the center of mass motion can be separated from the internal motion, the wave function Ψ is a function of the relative momenta $\mathbf{k}$ and $\mathbf{K}$. The product $\Psi = \Phi\chi\phi$ with $\Phi =$ flavor, $\chi =$ spin, and $\phi =$ momentum distribution, is a symmetric function. The neutron wave function is given by:

$$\Psi = \frac{1}{\sqrt{3}}\left(ddu\chi^{\lambda 3} + \text{permutation}\right)\phi, \tag{12}$$

with

$$\chi_\uparrow^{\lambda 3} = \frac{1}{\sqrt{6}}(\downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow - 2\uparrow\uparrow\downarrow),$$
$$\chi_\downarrow^{\lambda 3} = \frac{1}{\sqrt{6}}(2\downarrow\downarrow\uparrow - \downarrow\uparrow\downarrow - \uparrow\downarrow\downarrow). \tag{13}$$

Since the wave function Ψ must be an eigenfunction of j^2 (j being the total spin of the neutron) and the longitudinal component j_3, the spins $\uparrow$ and $\downarrow$ have to be rotated by the Melosh transform [12,6]. The S-state orbital function $\phi(M_0)$ is chosen to be

$$\phi(M_0) = \frac{N}{(M_0^2 + \alpha^2)^n}, \tag{14}$$

with α and n being phenomenological parameters and N being the normalization given by:

$$\frac{N_c}{(2\pi)^6}\int d^3k d^3K \phi^2 = 1. \tag{15}$$

The form factors are calculated by inserting Eq. (12) into Eq. (9). The result is rather lengthy and the explicit expressions are given in Ref. [8]. The exponent n is fitted to the proton form factor G_{Mp} giving $n = 3.5$. The constituent quark mass m and the length scale parameter α are fitted to the proton magnetic moment and the weak neutron decay, which results in $m = 0.263$ GeV and $\alpha = 0.607$ GeV.

Figures 1 and 2 show the magnetic and electric form factors of the neutron respectively. The figures give the deviation from the dipol fit $G_D = (1+Q^2/M_V^2)^{-2}$ with $M_V = 0.84$ GeV. Only experimental data from SLAC NE11 [1] are given since previous data do not distinguish between the various theoretical predictions. The present calculation (solid curves) is in very good agreement for both form factors. There is only a slight deviation for the magnetic form factor around 2 GeV^2. To show that the specific form of the wave function in Eq. (14) is essential for the result we compare the result with the commonly used exponential wave function $\phi(M_0) = N \exp(-M_0^2/2\alpha^2)$. We also fixed the parameters by fitting other electroweak nucleon properties [9], and get $m = 0.267$ GeV and $\alpha = 0.56$ GeV. The dashed line shows a rapid decrease for G_{Mn} at already 1 GeV^2, which indicates that the exponential wave function is not useful at that energy range. In the nonrelativistic limit, $\alpha/m \to 0$, the form factors fall far below the dipol fit for any reasonable value of α and m [17]. The relativistic treatment is therefore important, which is a fact also observed for the pion [18]. The VMD model (dash-dotted curves) from Höhler [3] agrees with the G_{En} data, but overestimates G_{Mn}. The model from Gari and Krümpelmann [5] (dash-double-dotted curves) predicts $F_{1n} = 0$. It is therefore in very poor agreement with G_{En}, and in addition underestimates G_{Mn}. The QCD sum rule predictions from Radyushkin [7] (dotted curves) agrees for G_{En} and underestimates G_{Mn}, approaching G_{Mn} for high Q^2. The QCD sum rule is not valid in the infrared region $Q^2 < 1$ GeV^2 due to singular power corrections at $Q = 0$ [19]. It is possible that the use of the new SLAC NE11 data to adjust free parameters may improve the other models.

We conclude that the precise measurement of form factors at intermediate energies gives valuable constraints on theoretical models. We showed that a model, that is in excellent agreement with the electroweak properties of the baryon octet, gives a parameter free prediction of the nucleon form factors, which is in good agreement with recent experimental data. The relativistic features of the model and the specific form of the wave function are essential for the good result.

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nonrelativistic limit for low Q^2 and $G_{Mn} = \mu_n(\sqrt{2}Q/3m)^{1/2}(2Q^2/(\alpha^2 + 9m^2))^{-3.5}$ for high Q^2.

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FIGURES

FIG. 1. The magnetic form factor of the neutron compared with the dipol fit, $G_{Mn}/\mu_n G_D$. The experimental data are taken from Ref. [1] with statistical and systematical errors. Solid line, our calculation with pole type wave functions; dashed line, our calculation with a harmonic oscillator type wave function; dash-dotted line, VMD model from Höhler [3]; dash-double-dotted line, Gari-Krümpelmann model [5]; dotted line, QCD sum rule prediction by Radyushkin [7].

FIG. 2. The electric form factor of the neutron compared with the dipol fit, G_{En}^2/G_D^2. The data and curves are marked the same as in Fig. 1.

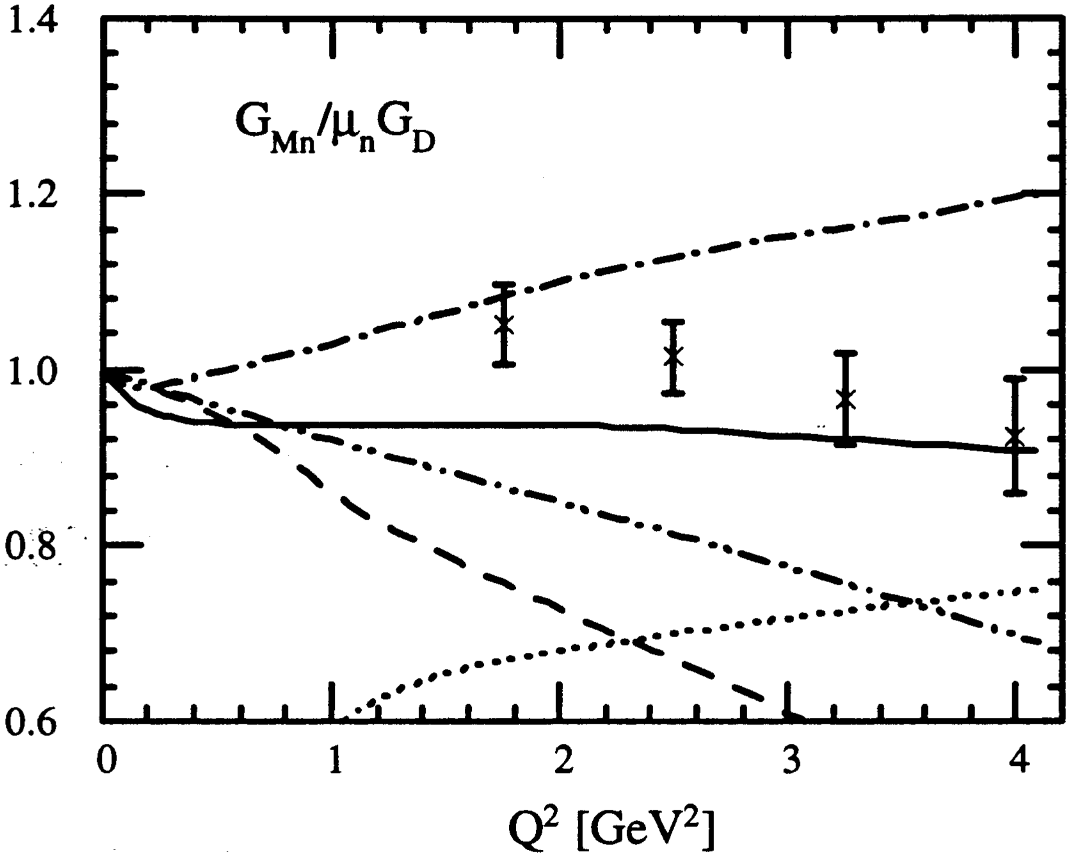

Fig. 1

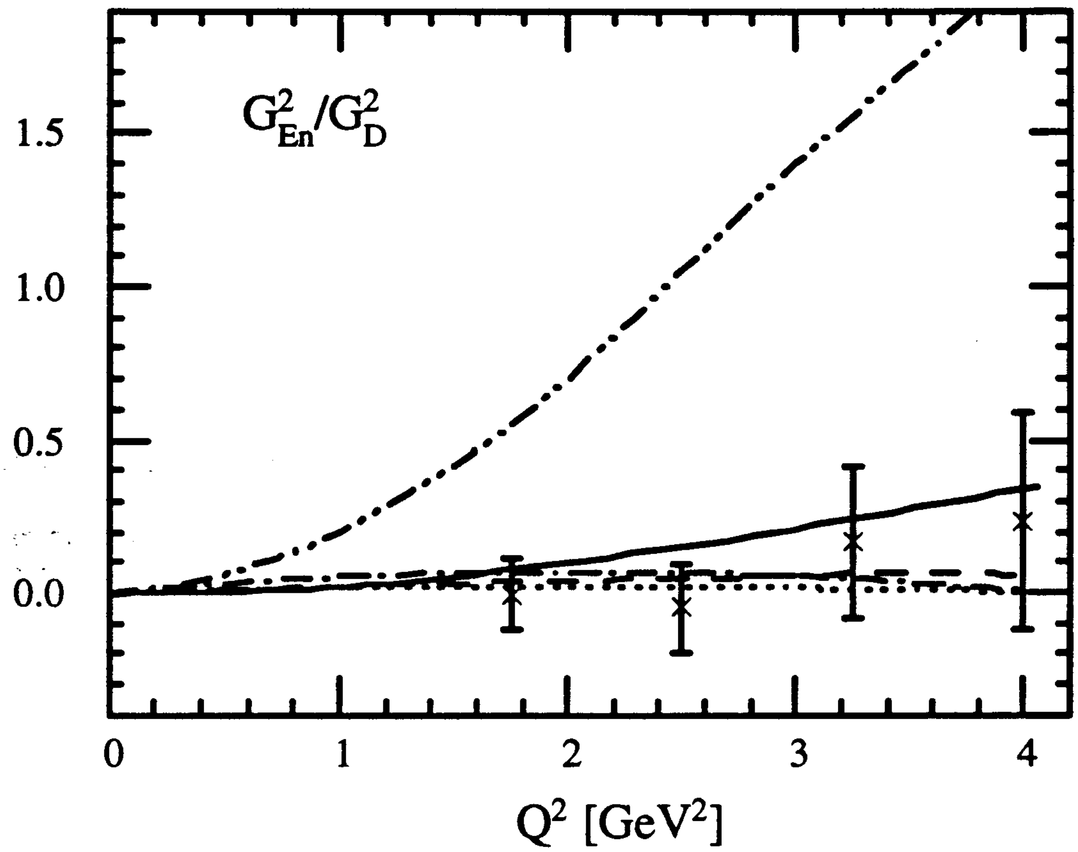

Fig. 2