SLAC–PUB–6025
December 1992
(T/E)

BEYOND THE TAU: OTHER DIRECTIONS IN TAU PHYSICS*

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ABSTRACT

This paper calls attention to four topics in tau lepton physics which are outside our present areas of tau physics research: $\tau^+\tau^-$ atoms, τ^- nucleus atoms, photoproduction of τ's, and heavy ion production of τ's.

Paper presented at the
Second Workshop on Tau Lepton Physics
The Ohio State University, Columbus, Ohio
September 8 – 11, 1992

* This work was supported by the Department of Energy, contract DE–AC03–76SF00515.

A. Introduction

This paper is based on a talk delivered at the *Second Workshop on Tau Lepton Physics* held at The Ohio State University, September 8–11, 1992. In that talk I called attention to four out-of-the-way topics in tau physics: $\tau^+\tau^-$ atoms, τ^-– nucleus atoms, photoproduction of τ's, and heavy ion production of τ's; and these are the areas covered in this paper. Two other topics from that talk will not be discussed in this paper: future searches for heavy leptons and speculations on missing modes in tau decay (Perl 1992).

B. The $\tau^-\tau^+$ Atom

B.1 Introduction

In the early years of the discovery of the τ there was some discussion of the physics of an atom that would consist of the Coulombic bound state of a τ^+ and a τ^- (Moffat 1975, Avilez *et al.* 1978, Avilez *et al.* 1979), an entity analagous to the e^+e^- atom positronium (Rich 1978). The $\tau^+\tau^-$ atom can be made in e^+e^- annihilation just below τ pair threshold.

$$e^+ + e^- \rightarrow \tau^+\tau^- \text{ atom} \quad , \tag{1}$$

hence the tau-charm factory offers the best route for making these atoms as discussed in Sec. B.5.

B.2 Static Properties

The energy levels of the $\tau^+\tau^-$ atom are shown in Fig. 1 where the atomic spectroscopy notation

$$n^{2S+1}L_J$$

is used. Here n is the principle quantum number; S is the total spin quantum number and is 0 or 1, L is the orbital angular momentum quantum number with L = S, P, D ...for L = 0, 1, 2 ..., and J is the total angular momentum quantum number. Ignoring fine structure, the energy levels are given by

$$E_n = -\frac{m_\tau c^2 \alpha^2}{4n^2} = -\frac{23.7 \text{ keV}}{n^2} \tag{2}$$

The 4 in the denominator comes from the usual 2 in the denominator and and $m_{reduced}(\tau^+\tau^- \text{ atom}) = m_\tau/2$ in the numerator. I use $m_\tau = 1777$ MeV/c^2.

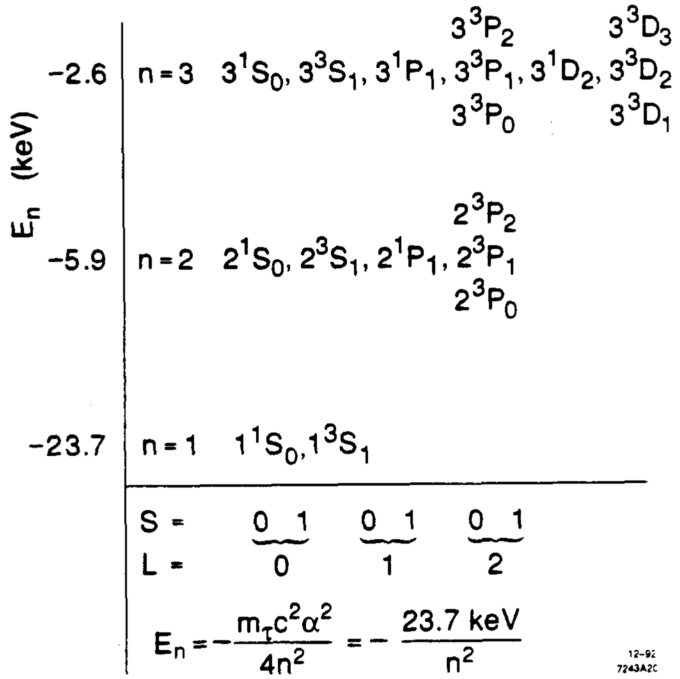

Figure 1

The Bohr radius is given by

$$a_0 = \frac{2\hbar^2}{m_\tau e^2} = 3.04 \times 10^{-12} \text{ cm} \quad , \tag{3}$$

which is three orders of magnitude smaller than the Bohr radius for hydrogen of 5.29×10^{-9} cm.

The $n = 1, 2$ wave functions are given by

$$\psi_{n\ell} = R_{n\ell}(r) Y_{\ell m}(\theta, \phi) \tag{4a}$$

where $Y_{\ell m}$ is a normalized spherical harmonic and

$$\begin{aligned} R_{10} &= \frac{1}{a_0^{3/2}}\, 2\, e^{-r/a_0} \\ R_{20} &= \frac{1}{a_0^{3/2}} \frac{1}{\sqrt{2}} \left(1 - \frac{r}{2a_0}\right) e^{-r/2a_0} \\ R_{21} &= \frac{1}{a_0^{3/2}} \frac{1}{\sqrt{6}} \frac{r}{2a_0} e^{-r/2a_0} \end{aligned} \tag{4b}$$

B.3 Charge Conjugation Rules for Production and Decay

Charge conjugation, C, imposes selection rules on the production and decay of the $\tau^+\tau^-$ atom

$$C\psi(\tau^+\tau^- \text{ atom}, \ n, S, L) = (-1)^{S+L}\psi(\tau^+\tau^- \text{ atom}, \ n, S, L) \tag{5}$$

and for a state of N photons

$$C \ \psi(N \text{ photons}) = (-1)^N \ \psi(N \text{ photons}) \tag{6}$$

Therefore in production

$$e^+ + e^- \rightarrow \gamma_{virtual} \rightarrow \tau^+\tau^- \text{ atom} \tag{7a}$$

the atom must be produced in a state with

$$S + L = \text{ odd number} \tag{7b}$$

The decay

$$\tau^+\tau^- \text{ atom} \rightarrow \gamma + \gamma \tag{8a}$$

requires

$$S + L = \text{ even number} \tag{8b}$$

and the decay

$$\tau^+\tau^- \text{ atom} \rightarrow \gamma + \gamma + \gamma \tag{9a}$$

requires

$$S + L = \text{ odd number} \tag{9b}$$

B.4 Decay channels of the $\tau^+\tau^-$ atom

Next I discuss the decay of the $\tau^+\tau^-$ atom. There are two classes of decay channel. In the first class the τ^+ or τ^- decay through the weak interaction in the normal way and the atomic state disappears. The decay width is

$$\Gamma(\text{atom}, \ \tau \text{ decay}) = 2\hbar/\tau_{lifetime} = 4.4 \times 10^{-3} \text{ eV} \tag{10a}$$

where the 2 occurs because the decay of either τ breaks up the atomic state. I have used the τ lifetime (Trischuk 1992) of

$$T_\tau = (2.96 \pm 0.03) \times 10^{-13} \text{ s} \tag{10b}$$

In the second class of decay channels the τ^+ and τ^- annihilate. The annihilation requires that the atomic wave function $\psi(\mathrm{r})$ be unequal to 0 at r = 0

$$\psi(0) \neq 0$$

Here r is the distance between the τ^+ and τ^-. Therefore in lowest order annihilation only occurs in L = 0 states, that is, S states. This is illustrated in Eq. 4b. There are five annihilation channels:

$$\tau^+\tau^- \text{ atom} \rightarrow \gamma + \gamma \quad (11a)$$
$$\tau^+\tau^- \text{atom} \rightarrow \gamma + \gamma + \gamma \quad (11b)$$
$$\tau^+\tau^- \text{atom} \rightarrow e^+ + e^- \quad (11c)$$
$$\tau^+\tau^- \text{atom} \rightarrow \mu^+ + \mu^- \quad (11d)$$
$$\tau^+\tau^- \text{atom} \rightarrow \text{hadrons} \quad (11e)$$

The annihilation channel

$$\tau^+\tau^- \text{ atom} \rightarrow \gamma + \gamma \quad (12a)$$

is even under charge conjugation, therefore

$$\text{atomic state } = n\ {}^1S_0 \quad (12b)$$

The decay width is

$$\Gamma(\text{atom} \rightarrow 2\gamma) = \frac{\alpha^5 m_\tau c^2}{2n^3} = \frac{1.8 \times 10^{-2}\ \text{eV}}{n^3} \quad (12c)$$

The four other annihilation channels have odd charge conjugation, therefore

$$\text{atomic state} = n\ {}^3S_1 \quad (13)$$

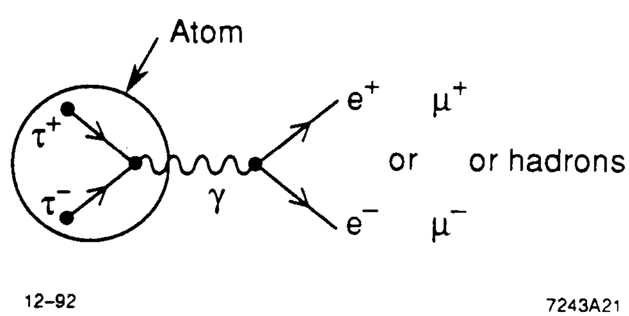

Figure 2

The channel

$$\tau^+\tau^-\text{atom} \rightarrow \gamma + \gamma + \gamma \tag{14a}$$

has the width

$$\begin{aligned}\Gamma\ (\text{atom} \rightarrow 3\gamma) &= \frac{2(\pi^2 - 9)\alpha^6 m_\tau c^2}{9\pi n^3} \\ &= \frac{1.7 \times 10^{-5}\ \text{eV}}{n^3}\end{aligned} \tag{14b}$$

The two channels, Fig. 2,

$$\tau^+\tau^-\ \text{atom} \rightarrow e^+ + e^- \tag{15a}$$

$$\tau^+\tau^-\ \text{atom} \rightarrow \mu^+ + \mu^- \tag{15b}$$

have the same width

$$\begin{aligned}\Gamma(\text{atom} \rightarrow e^+e^-) = \Gamma(\text{atom} \rightarrow \mu^+\mu^-) &= \frac{\alpha^5 m_\tau c^2}{6n^3} \\ &= \frac{6.1 \times 10^{-3}\ \text{eV}}{n^3}\end{aligned} \tag{15c}$$

when we neglect the masses of the e and μ. Finally there is the channel, Fig. 2,

$$\tau^+\tau^-\ \text{atom} \rightarrow \text{hadrons} \tag{16a}$$

The width cannot be calculated from first principles, however from colliding beams e^+e^- annihilation data at $E_{tot} \sim 2\,m_\tau$ we know

$$\sigma(e^+ + e^- \rightarrow \text{hadrons}) \approx 2\sigma(e^+ + e^- \rightarrow \mu^+ + \mu^-) \tag{16b}$$

Therefore

$$\Gamma(\text{atom} \rightarrow \text{hadrons}) \approx 2\ \Gamma_{\mu\mu} \tag{16c}$$

Collecting all this together, for $n\ {}^1S_0$ states

$$\begin{aligned}\Gamma_{tot}(n\ {}^1S_0) =& \Gamma(\text{atom},\ \tau\ \text{decay}) + \Gamma(\text{atom} \rightarrow 2\gamma) \\ =& \left(4.4 \times 10^{-3} + \frac{3.7 \times 10^{-2}}{n^3}\right) \text{eV}\end{aligned} \tag{17}$$

For the $n\ {}^3S_1$ states we can neglect $\Gamma(\text{atom} \rightarrow 3\gamma)$, Eq. 14b, and set

$$\begin{aligned}\Gamma_{tot}(n\ {}^3S_1) &\approx \Gamma(\text{atom}, \tau\ \text{decay}) + 4\Gamma(\text{atom} \rightarrow e^+e^-) \\ &\approx \left(4.4 \times 10^{-3} + \frac{2.44 \times 10^{-2}}{n^3}\right)\ \text{eV}\end{aligned} \tag{18}$$

Table I gives the widths and lifetimes for various S states.

Table I. Widths and lifetimes of $^3S_1$ states of the $\tau^+\tau^-$ atom due to τ decay and $\tau^+\tau^-$ annihilations.

n	Width (eV)	Lifetimes (s)
1	29×10^{-3}	2.3×10^{-14}
2	7.5×10^{-3}	8.8×10^{-14}
3	5.3×10^{-3}	12×10^{-14}
4	4.8×10^{-3}	14×10^{-14}

I remind the reader that in addition to the decays which destroy the $\tau^+\tau^-$ atom there are electromagnetic decays within the atom from an upper level to a lower level (Sec. B6)

$$\psi(\tau^+\tau^- \text{ atom}, n') \rightarrow \psi(\tau^+\tau^- \text{ atom}, n) + \gamma \quad , \quad n' > n \tag{19}$$

B.5 Production of the $\tau^+\tau^-$ Atom

As noted in Sec. B.1 the production process

$$e^+ + e^- \rightarrow \gamma_{virtual} \rightarrow \tau^+\tau^- \text{ atom} \tag{20}$$

requires S + L = odd number. Furthermore, the produced state must have $\psi(0) \neq 0$ and hence L = 0. Therefore, S = 1 and the produced state must be $n\ ^3S_1$.

The production cross section for the process in Eq. 20 is

$$\sigma(e^+e^- \rightarrow \tau^+\tau^- \text{ atom}) = \frac{3\pi(\hbar c)^2}{4m_\tau^2} \frac{\Gamma_{ee}\,\Gamma_{tot}}{(E_{tot} - 2m_\tau)^2 + \Gamma_{tot}^2/4} \tag{21}$$

Here Γ_{ee} means $\Gamma(\text{atom} \rightarrow e^+e^-)$ and is given by Eq. 15c. Γ_{tot} is given by Eq. 18. Thus the production cross section is given by the Breit-Wigner equation with full width at half-height of Γ_{tot} and peak cross section

$$\sigma(e^+e^- \rightarrow \tau^+\tau^- \text{ atom, peak}) = \frac{3\pi(\hbar c)^2}{m_\tau^2} \frac{\Gamma_{ee}}{\Gamma_{tot}} \tag{22}$$

As an example consider $\tau^+\tau^-$ atom production into the ground state $1\ ^3S_1$. Then

$$\Gamma_{ee} = 6.1 \times 10^{-3} \text{ eV} \tag{23a}$$
$$\Gamma_{tot} \approx 2.9 \times 10^{-2} \text{ eV} \tag{23b}$$
$$\Gamma_{ee}/\Gamma_{tot} = 0.21 \tag{23c}$$

and

$$\sigma(e^+e^- \to \tau^+\tau^- \text{ atom, peak}) \approx 2.4 \times 10^{-28} \text{ cm}^2 \tag{24}$$

This is a large cross section, but the energy spread of the e^+ and e^- beams, ΔE, is much larger than Γ_{tot}. Thus in a tau-charm factory we expect

$$\Delta E \sim 1 \text{ MeV} \tag{F.25}$$

and the effective cross section is

$$\sigma(e^+e^- \to \tau^+\tau^- \text{ atom, effective}) \sim 2.4 \times 10^{-28} \text{ cm} \times \frac{2.9 \times 10^{-2}}{10^6} \sim 10^{-35} \text{ cm}^2 \tag{26}$$

Therefore for a tau-charm factory luminosity of 10^{33} cm^{-2} s^{-1} we expect

$$\tau^+\tau^- \text{ atoms produced per sec.} \sim 10^{-2} \tag{27}$$

B.6 Detecting $\tau^+\tau^-$ Atoms?

Equation 27 shows that $\tau^+\tau^-$ atoms can be produced at a reasonable rate at a tau-charm factory. However, we don't know how to detect $\tau^+\tau^-$ atoms in the ground state. One difficulty, Table I, is the small width, 2.9×10^{-2} eV, compared to the 1 MeV energy spread of the beams. The other difficulty is the short lifetime, 2.3×10^{-14} s.

Another approach discussed by Moffat (1975) and Avilez *et al.* (1979) is to look for atoms produced in an excited state and look for photons produced in the transition to the ground state. First, suppose $\tau^+\tau^-$ atoms are produced in the $2\,^3S_1$ state. This is a metastable state and will decay by annihilation

$$\tau^+\tau^- \text{ atom} \to e^+ + e^-,\ \mu^+ + \mu^-,\ \text{hadrons} \tag{28}$$

before it decays to an $n = 1,\ S$ state of the atom. The next possibility is to produce the $\tau^+\tau^-$ atom in the $3\,^3S_1$ state (Avilez *et al.* 1979) and look for the x-ray photon emitted in the transition

$$3\,^3S_1 \to 2\,^3P_J + \gamma\ ,\ E_\gamma = 3.3 \text{ kev} \tag{29}$$

where J = 0, 1, 2. E_γ is the energy of the x-ray.

The width for $\tau^+\tau^-$ atom decay from the atomic state a to the atomic state b is

$$\Gamma_{ab} = \frac{4e^2 w^2 \hbar}{m_\tau \ c^3} f_{ab} \tag{30}$$

where

$$E_\gamma = \hbar w$$

and f_{ab} is the oscillator strength, a number of order 0.1 or less. For our purpose it is useful to rewrite Eq. 30 as

$$\Gamma_{ab} = \frac{4\alpha E_\gamma^2}{m_\tau c^2} f_{ab} \tag{31}$$

and to use Eq. 2 to obtain

$$\Gamma_{ab} = \frac{\alpha^5 m_\tau c^2}{4} \left[\frac{1}{n_b^2} - \frac{1}{n_a^2}\right]^2 f_{ab} \tag{32}$$

If a is an $n\ ^3S_1$ state, then comparing Eq. 32 with Eq. 15c and then Eq. 18

$$\Gamma_{ab} < \Gamma_{tot}(n\ ^3S_1) \tag{33}$$

Hence in $\tau^+\tau^-$ atoms an $n\ ^3S_1$ state is more likely to decay by annihilation than make an x-ray transition to a lower atomic state.

For example, in the specific case of Eq. 29

$$f_{ab} = 0.42 \tag{34}$$

from Table 4[5] of Condon and Shortley (1959). Hence from Eq. 32

$$\Gamma(3\ ^3S_1 \rightarrow 2\ ^3P_J) = 7.4 \times 10^{-6} \text{ eV} \tag{35}$$

and from Table 1

$$\Gamma_{tot}(3\ ^3S_1) = 5.3 \times 10^{-3} \tag{36}$$

Dividing Eq. 35 by Eq. 36

$$\frac{\text{Probability } (3\ ^3S_1 \rightarrow 2\ ^3P_J)}{\text{Probability } (3\ ^3S_1 \text{ annihilation})} = 1.4 \times 10^{-3} \tag{37}$$

Therefore, if we made $\tau^+\tau^-$ atoms in the $3\ ^3S_1$ state only 1.4×10^{-3} of them will make an x-ray transition before decaying. Furthermore, the production rate in Eq. 27

is reduced because for the 3 $^3S_1$ state

$$\sigma(e^+e^- \rightarrow \tau^+\tau^- \text{ atom, effective}) \sim 3 \times 10^{-37} \tag{38}$$

For a luminosity of 10^{33} cm^{-2} s^{-1}, there will be about 4×10^{-7} transitions per second of the form

$$3\ ^3S_1 \rightarrow 2\ ^3P_J + \gamma \ ,$$

a rate much too small to detect.

Finally, as pointed out by Avilez *et al.* (1979) the transition

$$2\ ^3P_J \rightarrow 1\ ^3S_1 + \gamma \ , \ E_\gamma = 17.8 \text{ keV} \tag{39}$$

has the much more favorable ratio

$$\frac{\text{Probability } (2\ ^3P_J \rightarrow 1\ ^3S_1)}{\text{Probability } (2\ ^3P_J \text{ annihilation})} = 0.16 \tag{40}$$

But 2 ^3P_J states cannot be produced directly by

$$e^+ + e^- \rightarrow \tau^+\tau^- \text{ atom}$$

as discussed in Sec. B5.

Summarizing, with a tau-charm factory we can make $\tau^+\tau^-$ atoms but we don't know how to detect their production. Beyond that problem, is the yet deeper question of what physics we can do with $\tau^+\tau^-$ atoms.

C. The τ^-–Nucleus Atom

C.1 Static Properties

The τ^-–Nucleus atom in analogy to the μ^-–Nucleus atom consists of a τ^- and $Z-1$ e^-'s around a nucleus of charge Z and atomic number A. In the $\tau - N$ atom the reduced mass of the τ is

$$m = \frac{m_\tau m_N}{m_\tau + m_N} \tag{41}$$

and ignoring the fine structure and effects of the non-zero nuclear radius, the n^{th} energy level is

$$\begin{aligned} E_n &= -\frac{m_\tau c^2 \alpha^2 Z^2}{2n^2} \left(\frac{m_N}{m_\tau + m_N} \right) \\ &= -\frac{47.4 Z^2}{n^2} \left(\frac{m_N}{m_\tau + m_N} \right) \text{ keV} \end{aligned} \tag{42}$$

The Bohr radius is given by

$$a_0 = \frac{\hbar^2}{m_\tau e^2}\left(\frac{m_\tau + m_N}{m_N}\right)$$
$$= 1.52 \times 10^{-12}\left(\frac{m_\tau + m_N}{m_N}\right) \tag{43}$$

The average value of the radius of the τ^- orbit is

$$\bar{r} = \frac{a_0}{2Z}\left[3n^2 - \ell(\ell+1)\right] \tag{44}$$

ignoring the effect of the non-zero nuclear radius. Thus for $Z \gtrsim 4$ and small n, $\bar{r}$ is of the order of 10^{-13} cm or less. Then particularly for S states, the τ^- is inside the nucleus part of the time. This effect reduces the magnitude of E_n. This is illustrated in Table II taken from Strobel and Wills (1983) who limit their calculations to $Z \leq 12$.

Table II. Energy levels of the 1S and 2P states of a τ^- nucleus atom in keV. E_p is for a point nucleus and E_{ex} is for an extended size nucleus. The proton is always taken as a point. These calculations are from Strobel and Wills (1983) and are corrected for the τ mass of 1777 MeV/c^2.

Nucleus	1S		2P	
	E_p	E_{ex}	E_p	E_{ex}
$^1_1$H	–16.3	–16.3	–4.1	–4.1
$^4_2$He	–128	–118	–32	–32
$^9_4$Be	–625	–474	–156	–155
$^{24}_{12}$Mg	–6310	–2940	–1580	–1460

C.2 Atomic Transitions

Table III, also from Strobel and Wills (1983) gives the energy of the emitted x-ray and the lifetime for the transition

$$2P \rightarrow 1S + \gamma \tag{45}$$

We see that the lifetime of the 2P–1S transition is much shorter than the τ lifetime of 3.0 $\times 10^{-13}$ s, Eq. 10b. Therefore, once the τ^- is in the 2P state, the τ^- will make the transition to the 1S state before it decays. Of course, the experimental question is how to get the τ^- into that state or other low lying states.

Table III. Transition energy and lifetime for $2P \to 1S$ in a τ^-– nucleus atom. From Strobel and Wills (1983) corrected for τ mass of 1777 MeV/c^2.

Nucleus	$E(2P \to 1S)$ keV	Lifetime $(2P \to 1S)$ s
$^1_1$H	12.2	5.0×10^{-14}
$^4_2$He	86	2.1×10^{-15}
$^9_4$Be	319	2.3×10^{-16}
$^{24}_{12}$Mg	1480	2.6×10^{-17}

C.3 τ^- Capture in the Nucleus

An interesting result of the τ^- orbit passing through the nucleus is that the τ^- can interact with the protons in the nucleus

$$\tau^- + p \to \nu_\tau + n \tag{46}$$

in analogy to e^- and μ^- capture. Ching and Oset (1991) have studied the process for heavy nuclei where the capture rate is greatest. They find for $^{208}_{82}Pb$ the following captive rates

$$\begin{aligned}
\Gamma(\tau \text{ capture from 1S}) &= 2.5 \times 10^9 \text{ s}^{-1} \\
\Gamma(\tau \text{ capture from 2S}) &= 2.3 \times 10^9 \text{ s}^{-1} \\
\Gamma(\tau \text{ capture from 2P}) &= 5.2 \times 10^9 \text{ s}^{-1}
\end{aligned} \tag{47}$$

However from Eq. 10b

$$\Gamma(\tau \text{ decay}) = 1/T_\tau = 3.4 \times 10^{12}\ s^{-1}$$

Therefore, even in the best case in Eq. 47 there is only a 10^{-3} chance that a τ will be captured with $\tau^- + p \to \nu_\tau + n$ compared to the chance that the τ^- decays.

Morley (1992) has given an interesting discussion of the $\tau^- - U$ atom. He discusses in some detail the process of the τ^- slowing down in solid uranium, the τ^- being captured in a high atomic orbit, and then cascading down to a low orbit.

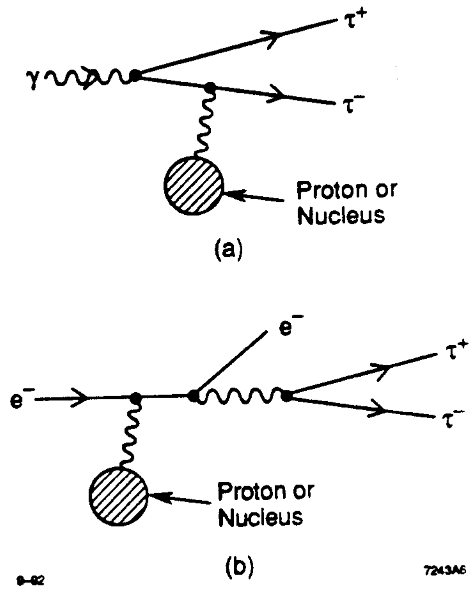

Figure 3

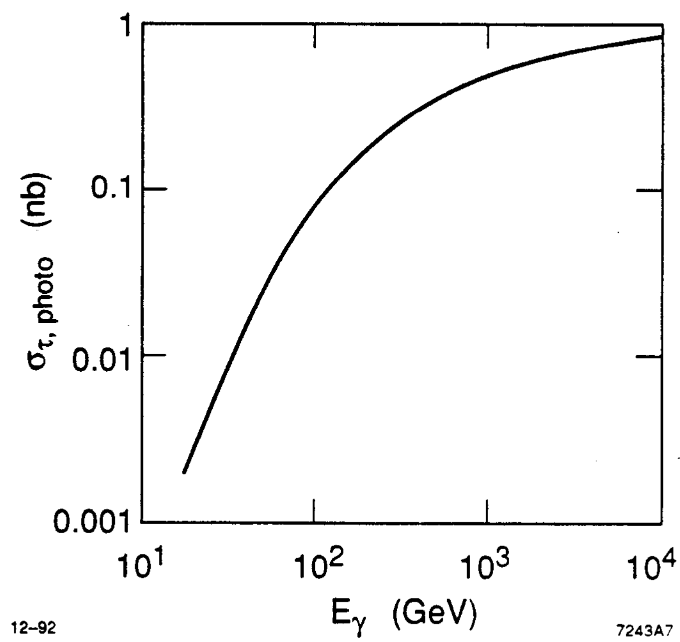

Figure 4

D. Photoproduction of τ's

τ pairs can be produced by photoproduction Tsai (1979)

$$\gamma + N \rightarrow \tau^+ + \tau^- + N' \tag{48}$$

as shown in Fig. 3a and by electroproduction (virtual photoproduction)

$$e^- + N \rightarrow e^- + \tau^+ + \tau^- + N' \tag{49}$$

as shown in Fig. 3b. Here N is a target proton or nucleus and N' is the final hadronic state. The cross section, $\sigma_{\tau,photo}$, for a proton target is given in Fig. 4 as a function of energy.

As an example, suppose that at SLAC one photoproduces τ pairs with a photon beam of maximum energy 40 GeV and intensity 10^{12} γ/s. Then in a 1 radiation length hydrogen target using an average cross section of 3×10^{-36} cm^{-2}, the τ pair production rate would be

$$\tau \text{ pairs}/s \sim 100 \tag{50}$$

Thus in a one month run of effective length 10^6 s one could produce 10^8 τ pairs.

There has been very little discussion of the physics that might be done with photoproduced τ pairs. Tsai (1992) has suggested that a ν_τ, $\bar{\nu}_\tau$ beam could be made this way.

It is useful to remember that in τ pair photoproduction the basic process is

$$\gamma + \gamma_{virtual} \rightarrow \tau^+ + \tau^- \tag{51}$$

in contrast to production by e^+e^- annihilation where the basic process is

$$\gamma_{virtual} \rightarrow \tau^+ + \tau^- \tag{52}$$

In the next section on the proposal for production of τ pairs in heavy ion collisions the basic process is

$$\gamma_{virtual} + \gamma_{virtual} \rightarrow \tau^+ + \tau^- \tag{53}$$

Therefore some of the goals of heavy ion tau physics may be applicable to photoproduction τ physics.

Returning to the first topic in this paper, $\tau^+\tau^-$ atoms, consider

$$\gamma + N \rightarrow \tau^+\tau^- \text{ atom} + N' \tag{54}$$

Olsen (1986) has discussed the relativistic production of positronium

$$\gamma + N \rightarrow e^+e^- \text{ atom} + N' \tag{55}$$

He shows that at high energy there is the crude relationship

$$\sigma(\gamma + N \rightarrow \ell^+\ell^- \text{ atom} + N') \sim \alpha^3 \, \sigma(\gamma + N \rightarrow \ell^+ + \ell^- + N') \tag{56}$$

The α^3 comes from a_0^{-3} (Eq. 3) involved in the phase space factor for the atom relative to the phase space factor for the unbound pair. Applying Eq. 56 to the unbound τ pair cross section in Fig. 4 we see that the cross section for photoproduction of a $\tau^+\tau^-$ atom is in the range of 10^{-39} to 10^{-41} cm^2, much too small to use.

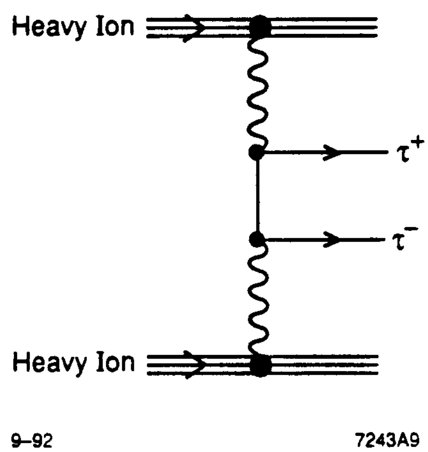

Figure 5

E. τ Pair Production in Heavy Ion Collisions

There have been a number of papers on the production of μ pairs and τ pairs in relativistic collisions of heavy ions (Bottcher and Strayer 1990, del Aquila *et al.* 1991, Almeida *et al.* 1991, Amaglobeli *et al.* 1991). The overall process is

$$\text{ion} + \text{ion} \rightarrow \tau^+ + \tau^- + \text{ion} + \text{ion} \tag{57}$$

as shown in Fig. 5. And the basic process is given in Eq. 53.

At sufficiently high energies the cross section will be of the order of

$$\sigma_0(\text{coherent}) = \frac{\alpha^4 (\hbar c)^2 Z^4}{m_\tau^2 c^4} \tag{58}$$

The charge Ze at each ion$-\gamma-$ion vertex entering the amplitude as Ze. At lower energies the momentum transfer to the ions becomes large and the process has an incoherent cross section of the order of

$$\sigma_0(\text{incoherent}) = \frac{\alpha^4 (\hbar c)^2 Z^2}{m_\tau^2 c^2} \tag{59}$$

Bottcher and Strayer (1990) have studied the production cross section when the ion is ${}^{197}_{79}A_u$. First consider $Au + Au$ at the LHC with 7.5 TeV per proton which is 3.0 TeV per nucleon. Extrapolating the Bottcher and Strayer calculation

$$\sigma(3.0 \text{ TeV/nucleon}) \approx 40\sigma(\text{coherent}) \approx 0.5 \text{ mb} \tag{60}$$

On the other hand, at a RHIC energy of 0.25 TeV per proton which is 0.1 TeV per nucleon, they obtain

$$\begin{aligned} \sigma(0.1 \text{ TeV/nucleon}) &\approx 0.2\sigma_0 \text{ (coherent} \\ &\approx 2.8 \times 10^{-3} \text{ mb} \end{aligned} \tag{61}$$

This is still larger than

$$\sigma_0(\text{incoherent}) = 2.0 \times 10^{-5} \text{ mb} \tag{62}$$

hence there is still some coherence at 0.1 TeV/nucleon. As another example del Aguila *et al.* (1991), consider the ${}^{20}_{82}P_b$ ion. For the LHC they find a cross section of 1 mb, similar to Eq. 60.

If we take the proposed LHC heavy ion luminosity as 10^{28} cm^{-2} s^{-1}, a 1 mb cross section for 10^7 s/year gives a yield of 10^8 τ pairs per year, comparable to a tau-charm factory. Can these pairs be used to do τ physics? This has been partially discussed by del Aguila *et al.* (1991). They point out that most of the τ pair events will be clean with the ions themselves proceeding along the beam pipe and no additional particles produced. But I think there is a problem in non$-\tau$ events contaminating the data sample, since the cross section for non$-\tau$ events is so much larger. It may be that the only clean samples are the old faithful

$$\tau^+ + \tau^- \rightarrow e^\pm + \mu^\mp + \text{missing energy} \tag{63}$$

events.

There have been two suggestions for the tau physics that might be done with τ pairs produced in heavy ion collisions. The suggestion of del Aguila *et al.* (1991) is that one can measure the anomalous magnetic moment of the τ.

$$\mu_\tau(\text{anom}) = a_\tau \frac{e\hbar}{2m_\tau c} \tag{64a}$$

$$a_\tau = \frac{\alpha}{2\pi} + \sum_{n>1} c_n \alpha^n \tag{64b}$$

to about 1%. And one can also look for unconventional behavior of the $\tau - \gamma - \tau$ vertex such as an electric dipole moment.

Amaglobeli *et al.* (1991) have suggested using high rate τ production to look for the unconventional decay

$$\tau^- \rightarrow \mu^- + \mu^+ + \mu^- \tag{65}$$

A τ pair event with one such decay would stand out in the data sample. It would have 4 or 6 charged particles, with 3 of the particles being μ's whose invariant mass is the τ mass.

F. Acknowledgement

This paper is based on the work of those authors given in the references. I am very grateful to them for this work.

References

Almeida L D *et al.* 1991 *Phys. Rev.* **D44** 118

Amaglobeli N S *et al.* 1991 *Massive Neutrinos, Tests of Fundamental Symmetries* (Editions Frontiéres, Gif-sur-Yvette) ed J Tran Thanh Van p 335

Avilez C *et al.* 1978 *Lett. Nuovo Cimento* **21** 301

Avilez C *et al.* 1979 *Phys. Rev.* **D19** 2214

Bottcher C and Strayer M R 1990 *J. Phys. G: Nucl. Part. Phys.* **16** 975

Ching C H and Oset E 1991 Phys. Lett. B **259** 239

Condon E V and Shortley G H 1959 The Theory of Atomic Spectra (Cambridge Univ. Press, Cambridge) p 136

del Aguila F *et al.*1991 *Phys. Lett.* **B271** 256

Moffat J W 1975 *Phys. Rev. Lett.* **35** 1605

Morley P D 1992 Univ. of Texas-Austin Preprint-92-0080

Olsen H A 1986 Phys. Rev. **D33** 2033

Perl M L 1992 *Proc. Trieste Workshop on Search for New Elementary Particles: Status and Prospects* (World Scientific) eds L Beers, G Herten, M L Perl, to be published

Rich A 1981 *Rev. Mod. Phys.* **53** 127

Strobel G L and Wills E L 1983 Phys. Rev. **D28** 2191

Trischuk W 1992 *Proc. Second Workshop on Tau Lepton Physics* (The Ohio State Univ., Columbus) ed K K Gan, to be published, and CERN PPE 92-190

Tsai Y S 1979 SLAC-PUB-2356

Tsai Y S 1992 private communication