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The Landau-Pomeranchuk-Migdal Effect and Suppression of Beamstrahlung and Bremsstrahlung in Linear Colliders

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ABSTRACT

It is well known that beamstrahlung and bremsstrahlung take place over a finite formation zone distance. If something disturbs the electron during this time, the emission can be suppressed. In this paper, we examine the Landau-Pomeranchuk-Migdal (LPM) effect and other LPM-like effects, such as the longitudinal density (dielectric) suppression and EM field suppression of beamstrahlungand bremsstrahlungin e^+e^- linear colliders. We show that while the LPM effect and the density effect are not sufficient in suppressing these radiations, the strong EM field of the opposing beam does help to suppress bremsstrahlung.

1. Introduction

One of the most important issues in the design of future e^+e^- colliders is the effect of the beam-beam interaction on the physics environment. The single-pass nature of linear colliders necessitates the need for colliding tiny, intense bunches of electrons and positrons in order to achieve the required high luminosity. In this circumstance, these bunches interact strongly with one another, producing large numbers of hard photons, a phenomenon called beamstrahlung,[1] in addition to the conventional bremsstrahlung radiation process. These photons potentially create troublesome backgrounds for experiments on e^+e^- annihilation, and it is highly desirable if they can be suppressed in some way.

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Bremsstrahlung and beamstrahlung have usually been treated in the context of single particle having an *incoherent* point interaction with another particle or a *coherent* interaction with a collection of target particles, respectively. However, the uncertainty principle puts a lower limit on the size of the interaction; because of the small (especially in the longitudinal direction) momentum transfer between the particles, the interaction must take place in a finite area. For bremsstrahlung in a nuclear medium, this finite size leads to departures from the Bethe-Heitler formula. This paper will consider the effects that this finite size has on beamstrahlung and bremsstrahlung in e^+e^- colliders.

In contrast to bremsstrahlung, beamstrahlung occurs in the situation where the scattering amplitudes between the radiating particle and the target particles within the characteristic length add coherently. Typically for the beam-beam collision in linear colliders there can be over a million target particles involved within the coherence length. The process can therefore be well described in a semi-classical calculation where the target particles are replaced by their collective EM fields.

High energy e^+e^- beams generally follow Gaussian distributions in the three spatial dimensions. In the weak disruption limit, where particle motions are para-axial, it is possible to integrate the radiation process over this volume and derive relation which depend only on averaged, global beam parameters.[2] The overall beamstrahlung intensity is then described by a global *beamstrahlung parameter*,

$$\Upsilon = \gamma \frac{\langle B \rangle}{B_c} = \frac{5}{6} \frac{r_e^2 \gamma N}{\alpha \sigma_z (\sigma_x + \sigma_y)} \quad , \tag{1.1}$$

where $\langle B \rangle$ is the mean electromagnetic field strength of the beam, $B_c = m^2/e \simeq 4.4 \times 10^{13}$ Gauss is the Schwinger critical field, N is the total number of particles in a bunch, $\sigma_x, \sigma_y, \sigma_z$ are the nominal sizes of the Gaussian beam, $\gamma = E_e/m$ is the Lorentz factor of the radiating particle, r_e is the classical electron radius, and α is the fine structure constant. Roughly speaking, for $\Upsilon < 1$, the beamstrahlung spectrum scales as $x^{-2/3}$ for $x \lesssim \Upsilon$, where $x \equiv E_\gamma/E_e$ is the fractional energy of the radiated photon; and decreases exponentially for $x \gtrsim \Upsilon$. When $\Upsilon \gtrsim 1$, the spectrum scales as $x^{-2/3}$ for the entire range of $0 \leq x \leq 1$.

Also relevant to our following discussions is the average particle density of the colliding beams (in the e^+e^- center-of-mass frame). For tri-gaussian distributions, the beam density is

$$n_b = \frac{N}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z} \quad . \tag{1.2}$$

To provide a framework for discussion, we will consider here three examples of linear colliders, SLC, a 500 GeV on 500 GeV intermediate collider, and a 5 TeV on 5 TeV super collider. The parameters for the machines are shown in Table 1. Section 2 of this paper will introduce the scale lengths in bremsstrahlung and beamstrahlung. In section 3 we review the physics of the LPM effect and the density suppression in a nuclear medium. Section 4 discusses these two effects in the e^+e^- linear collider environment. We show that both are ineffective in suppressing beamstrahlung and bremsstrahlung. In section 5 we turn to yet another LPM-like mechanism, magnetic suppression. It is shown that magnetic effects can in principle suppress bremsstrahlung effectively. However, they are still not sufficient to suppress beamstrahlung. A conclusion is given in Section 6.

Table 1. Parameters for the 3 linear colliders discussed in the text.

Parameter	SLC	NLC	SuperLC
E_e[GeV]	50	500	5000
N [10^{10}]	4	1.3	0.4
σ_x [nm]	2000	425	26.5
σ_y [nm]	2000	2	0.1
σ_z [μm]	1000	100	15
n_b [cm^{-3}]	6.4×10^{17}	9.7×10^{21}	6.4×10^{24}
Υ	0.001	0.27	70

2. Length Scales in Bremsstrahlung and Beamstrahlung

The classical diagram for bremsstrahlung is presented in Figure 1a; an electron emits a photon, conserving momentum by exchanging a virtual photon with a nearby nucleus. When the electron is of high enough momentum, the longitudinal momentum carried by the virtual photon becomes very small,

$$q_{\parallel} = p_e - p_e' - k = \sqrt{E_e^2 - m^2} - \sqrt{E_e'^2 - m^2} - E_\gamma \quad , \tag{2.1}$$

where p_e, p_e', E_e, and E_e' are the electron momentum and energy before and after the interaction, respectively, and E_γ is the photon energy. For high energy electrons this simplifies to

$$q_{\parallel} \sim \frac{m^2 E_\gamma}{2E_e(E_e - E_\gamma)} = \frac{m}{2\gamma}\frac{x}{1-x} \equiv \frac{m}{2\gamma}u \quad , \tag{2.2}$$

where $x = E_\gamma/E_e$ is the fractional energy of the radiated photon. This momentum transfer can be very small. Then, by the uncertainty principle, the virtual photon exchange distance, or the *formation length*, l_f,

$$l_f \sim \frac{\hbar}{q_{\|}} = 2\gamma \lambda_c \frac{1}{u} \quad . \tag{2.3}$$

For example, for a 25 GeV electron emitting a 100 MeV photon, $q_{\|}$ is only 0.03 eV/c and the formation length l_f is 2μm long. The expression for the formation length is unchanged in the case of e^+e^- scattering.

In the infrared limit, the transverse momentum transfer is essentially absorbed by the final state electron, so the out-going angle of the radiating electron is

$$\theta_{br} \sim \frac{m}{E_e} = \frac{1}{\gamma} \quad . \tag{2.4}$$

This angle is independent of the energy of the radiated photon.

In contrast to the bremsstrahlung process, beamstrahlung occurs due to the bending of the particle trajectory by the external classical EM fields. The overlapping wave-functions between the initial state and the final state contribute maximally to the radiation within a *coherence length.* In terms of Υ, the coherence length is

$$l_c \approx \frac{\gamma \lambda_c}{\Upsilon} \begin{cases} \left(\Upsilon/u\right)^{1/2} \quad , & \Upsilon/u \ll 1 \quad ; \\ \\ \left(\Upsilon/u\right)^{1/3} \quad , & \Upsilon/u \gg 1 \quad ; \end{cases} \tag{2.5}$$

Because of the nature of the beamstrahlung spectrum, the part of the spectrum that we are interested will always satisfy the condition $x \ll \Upsilon$. Therefore we will be dealing with the regime where $\Upsilon/u \gg 1$ is always satisfied. So from now on we will simply put

$$l_c \approx \frac{\gamma \lambda_c}{u^{1/3} \Upsilon^{2/3}} \quad . \tag{2.6}$$

The radius of curvature of electron's classical trajectory is $\rho = \gamma^2 \lambda_c / \Upsilon$. Thus the corresponding bending angle for the final state electron is

$$\theta_{be} = \frac{l_c}{\rho} = \frac{1}{\gamma} \left(\frac{\Upsilon}{u}\right)^{1/3} \quad . \tag{2.7}$$

We see that the bending angle for radiating a low energy beamstrahlung photon is substantially larger than the typical $\theta_{br} \sim 1/\gamma$ found in bremsstrahlung. The angle is increased due to the large transverse momentum imparted by the electromagnetic field.

3. The LPM Effect and the Density Suppression Effect

In the early 1950's, a group of Russian theorists, led by Landau, Pomeranchuk, Migdal and Feinberg began looking at bremsstrahlung in more detail. They realized that, because of the low longitudinal momentum transfer between the nucleus and the electron, bremsstrahlung is not instantaneous, but occurs over a finite formation zone. During this time, external influences can perturb the electron and suppress the photon emission. When this happens, the traditional Bethe-Heitler formula fails. This can occur in a number of places, most notably in crystals; we shall consider here two examples, due to multiple scattering and due to the dielectric constant of the medium.

Initially, Landau and Pomeranchuk studied suppression by multiple scattering with semiclassical arguments.[6] Later, Migdal presented a fully quantum treatment based on scattering theory[7] Since the LPM theory is unfamiliar to many physicists, we will present here a brief semiclassical derivation, following an article by Feinberg and Pomeranchuk.[8]

The LPM effect appears when one considers that the electron must be undisturbed while it traverses this distance. One factor that can disturb the electron and disrupt the bremsstrahlung is multiple Coulomb scattering. Semiclassically, if the electron multiple scatters by an angle θ_{ns}, greater than the angle made by the bremsstrahlung photon, $\theta_{br} \sim 1/\gamma$, then the bremsstrahlung is suppressed.

The average multiple scattering angle in a nuclear medium is

$$\langle \theta_{ns}^2 \rangle = \left(\frac{E_s}{E_e}\right)^2 \frac{l}{X_0} \quad , \tag{3.1}$$

where $E_s = \sqrt{4\pi/\alpha} \cdot m = 21$ MeV is the characteristic energy, l is the scatterer thickness, and X_0 is the radiation length. The LPM effect becomes important when θ_{ns} is larger than θ_{br}. This occurs for $(E_s/E_e)\sqrt{l/X_0} > m/E_e$. For a fixed electron energy, suppression becomes significant for photon energies below a certain value, given by

$$x < \frac{E_e}{E_{\text{LPM}}} \quad , \tag{3.2}$$

where all of the constants have been lumped into E_{LPM}, given by E_{LPM} [eV] $= m^4 X_0 / c\hbar E_s^2 = 7.6 \times 10^{12} X_0$ [cm], about 2.6 TeV in uranium and 4.2 TeV in lead. For example, suppression becomes significant for 250 MeV photons from a 25 GeV electron in uranium. For beamstrahlung, of course, these formulae must be modified.

Finding the magnitude of the suppression is more involved. For low energy photons, the photon spectrum is proportional to $E_\gamma^{-1/2}$, in contrast to the $1/E_\gamma$ Bethe-Heitler spectrum. To go further, Migdal applied scattering theory to the density of wave states to derive detailed formulae. Also of interest is a combined energy-angular distribution; unfortunately the angular aspect of the LPM effect has yet to be worked out.

An analogous effect occurs for pair creation by a high energy photon. As Fig'1. shows, the two processes are closely related. In pair creation, the LPM energy threshold is determined by the lepton with the lower energy. Because of this, the pair creation suppression begins at much higher energies than bremsstrahlung suppression.

Although the LPM effect reduces the divergence of the low energy photon production cross section, it does not eliminate it, since dN/dE_γ still grows as $E_\gamma^{-1/2}$. At low photon energies, another effect removes the divergence. There, the phase shift due to the medium ($\sqrt{\epsilon}k$, where k is the photon wave number) can become significant. In the infrared limit, the contributions to the photon amplitude, $\exp\{i(k \cdot x - \omega t)\}$, from different parts of the electron path through the formation zone can interfere, and photon emission is suppressed.[9,10] This is sometimes known as the longitudinal density effect, and it is related to the dE/dz (transverse) density effect discussed by Fermi. The density effect is significant for photon energies less than $\gamma\omega_p$, where ω_p is the plasma frequency of the medium. For a given material, this occurs at a fixed x, and the suppression factor is[11]

$$F_p = \left(1 + \frac{n r_e \lambda_p^2}{\pi x^2}\right)^{-1} \quad , \tag{3.3}$$

where n is the electron density, and λ_p is the plasma wavelength of the medium. The density effect becomes important for $x = 10^{-4}$ in lead, for example. Below these energies, dN/dE_γ goes as E_γ^2, removing the divergence.

4. LPM and Density Effects in Beam-Beam Interaction

Although the above concepts remain unchanged for e^+e^- linear colliders, most of the details require modification. First, the multiple scattering formulae are modified. Second, the two interacting particles have equal masses, and so divide the momentum transfer equally, halving many of the relevant angles. Third, the relevant formation length changes due to the presence of the electromagnetic field carried by the beam.

There are a number of ways to compute the multiple scattering effect. We will start with the Bhabha cross section. For small angle scattering,

$$\frac{d\sigma}{d\theta} \approx \frac{8\pi r_e^2}{\gamma^2\theta^3} \quad , \qquad \theta \ll 1 \quad , \tag{4.1}$$

and the average scattering angle is

$$\langle\theta^2\rangle \approx \frac{\int_0^\infty \theta^2 (d\sigma/d\theta) \cdot d\theta}{\int_0^\infty (d\sigma/d\theta) \cdot d\theta} \quad . \tag{4.2}$$

These integrals are singular; to remove the singularity a minimum momentum cutoff is needed. This is given by the finite size of the beams: $\theta_{min} = \Delta p_{min}/p$. Since $\Delta p_{min} = \hbar/\sigma_x$, $\theta_{min} = \hbar/E_e\sigma_x = \lambda_c/\gamma\sigma_x$. Here, we are assuming that the beams are flat, so that $\sigma_x \gg \sigma_y$. Then, for a single scatter $\langle\theta^2\rangle = 2\theta^2_{min}\log(1/\theta_{min})$. The total number of scatters is given by $N_s = n_b\sigma_{tot}l$ where n_b is the beam particle density and σ_{ms} is the total integrated cross section, $4\pi(r_e/\gamma\theta_{min})^2$, and l is the applicable length. Then, the total scattering angle is

$$\Theta^2_{ms} = \frac{8\pi r_e^2 n_b l}{\gamma^2}\log\frac{\gamma\sigma_x}{\lambda_c} \quad . \tag{4.3}$$

One way to decide when LPM suppression is important is to calculate a length scale for it. For bremsstrahlung, this length is the distance over which the multiple scattering has accumulated an angle of the order $\theta_{br} = 1/\gamma$. Inserting this condition into Eq.(4.3), we get

$$l_{\text{LPM}}(\text{BR}) = \frac{1}{8\pi r_e^2 n_b}\log^{-1}\frac{\gamma\sigma_x}{\lambda_c} \quad . \tag{4.4}$$

As the beam density rises, this distance gets shorter, indicating that the LPM effect appears at shorter emission length scales. From Table 1 we find that $l_{\text{LPM}}(\text{BR}) \simeq 3\times 10^4, 1.9$, and 0.003cm, in SLC, NLC, and SuperLC, respectively. E_{LPM} decreases rapidly with energy. Nevertheless, all these lengths are larger than their corresponding bunch lengths, σ_z's, rendering the LPM effect ineffective in linear collider beam-beam interactions.

For beamstrahlung, the distance required to cumulate an angle of θ_{be} through multiple scattering is easily calculated:

$$l_{\text{LPM}}(\text{BE}) = l_{\text{LPM}}(\text{BR})\cdot\left(\frac{\Upsilon}{u}\right)^{2/3} \quad . \tag{4.5}$$

This length scale is photon-energy dependent. But as we discussed in Sec. 2, the condition $\Upsilon/u > 1$ is generally satisfied in linear colliders. Thus $l_{\text{LPM}}(\text{BE}) > l_{\text{LPM}}(\text{BR}) > \sigma_z$ for all three machines, and we also conclude that the LPM effect cannot suppress beamstrahlung.

The density effect depends on the electron plasma frequency. In colliders, the electrons are relativistic, and hence have increased mass, so, in the center-of-mass frame (also the lab frame), the plasma frequency becomes

$$\omega_p^2 = \frac{4\pi n_b e^2}{\gamma m} = \frac{4\pi c^2 r_e n_b}{\gamma} \quad . \tag{4.6}$$

The appropriate length scale is when $k\cdot x - \omega t$ becomes comparable to 1 for $l = ct$.

This occurs when $(1-\sqrt{\epsilon})\omega l/c = 1$, or giving rise to a density-effect length

$$l_d \simeq \frac{2c\omega}{\omega_p^2} = \frac{1}{2\pi}\frac{\gamma^2 x}{\lambda_c r_e n_b} \quad , \qquad \omega \gg \omega_p \quad . \tag{4.7}$$

When the formation length or the coherence length is longer than this density length, then the density effect comes into play. Since $l_d \sim E_\gamma$, whereas $l_f \sim 1/x$ for small x, the density effect always cuts off the low energy photon spectrum. In the 3 cases that we study, the plasma frequencies are $9.2\times10^{-5}, 3.6\times10^{-3}$, and 2.9×10^{-2} eV, respectively.

If the bunch is infinitely long, then the density effect would apply for any x which satisfies the condition $l_d \lesssim l_f$, or

$$x \lesssim \frac{\hbar\omega_p}{mc^2} = \left[\frac{4\pi{\lambda_c}^2 r_e n_b}{\gamma}\right]^{1/2} \quad . \tag{4.8}$$

So in the absence of other suppression effects, the beam density in principle could affect the spectrum up to an energy of $xE_e \lesssim \gamma\omega_p \simeq 9.2$ eV, 3.6 keV, and 0.29 MeV, respectively. However, Eq.(4.7) shows that these energies require bunch lengths of 430, 110, and 130 m! Instead, since the bunch lengths are much shorter than these values, one should equate l_d with σ_z to find the threshold x:

$$x_d \lesssim \frac{2\pi\lambda_c r_e n_b \sigma_z}{\gamma^2} \quad . \tag{4.9}$$

This gives density suppression thresholds of $2.1\times10^{-5}, 3.2\times10^{-3}$, and 3.2×10^{-2} eV, respectively. All these suppressions are very small considering the very hard spectrums anticipated in linear colliders.

In addition to the changes in dielectric constant due to the real electrons and positrons, there will also be some change in the vacuum polarization due to virtual electron positron pairs in the presence of an external magnetic field.[10] This effect should mainly be important at very high Υ's.

5. Magnetic Suppression

In addition to the sideways kick due to multiple scatterings, which is a incoherent process, there is also the coherent bending of the trajectory. As discussed earlier, this is the same source that give rise to beamstrahlung. The distance associated with the bending of an angle $\sim 1/\gamma$ is

$$l_0(\mathrm{BE}) = \frac{\rho}{\gamma} = \frac{\gamma\lambda_c}{\Upsilon} \quad . \tag{5.1}$$

For SLC, NLC, and SuperLC, $l_0 \simeq 38, 1.4$, and 0.054 μm, respectively. These values are about 2 to 3 orders of magnitude smaller than the corresponding bunch lengths.

By comparing l_0 with l_f, we find that bremsstrahlung is suppressed for any

$$x_0(\mathrm{BR}) \lesssim \frac{2\Upsilon}{1+2\Upsilon} \quad . \tag{5.2}$$

For $\Upsilon \ll 1$, the spectrum is suppressed roughly up to twice the Υ parameter; whereas for $\Upsilon \gg 1$, essentially the entire spectrum of bremsstrahlung will be suppressed! Thus for bremsstrahlung the dominant source of perturbation comes from the collective EM fields in the beam.

This is in fact not a new effect. Earlier Baier, Katkov, and Strakhovenko[12] studied the suppression of bremsstrahlung in e^+e^- collision due to the presence of a transverse magnetic field. The only difference here is that the coherent bending is due to the collective classical field of the oncoming beam particles, which is locally transverse to the beam propagation. In effect, this puts an upper limit on the electron pathlength that can contribute to bremsstrahlung.[13]

It is also natural to wonder if the angles induced by bremsstrahlung can perturb the beamstrahlung process. It can be shown that for an angular increase of $\theta \sim 1/\gamma$ through bremsstrahlung, it takes about 2 orders of magnitude longer in distance than l_{LPM}. It is evident that the bremsstrahlung cannot suppress beamstrahlung.

6. Conclusions

We have examined the applicability of the LPM effect and density suppression to colliding beam bremsstrahlung and beamstrahlung. We conclude that neither will have measurable effects in SLC and in future colliders. Instead, we show that the collective EM fields in the beam is effective in suppressing bremsstrahlung. In the example of NLC, we find the suppression extends up to $x \sim 0.35$, which is quite significant. As for beamstrahlung, there is no comparable mechanism, as the beam field is exactly the same source that gives rise to the radiation.

We note that a similar effect should in principle also apply to the conversion of photons into e^+e^- pairs, although it would only occur at significantly higher energies.

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