

The Subleading Isgur-Wise Form Factor $\chi_3(v \cdot v')$
to Order α_s in QCD Sum Rules

Matthias Neubert
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

Zoltan Ligeti and Yosef Nir
Weizmann Institute of Science
Physics Department, Rehovot 76100, Israel

We calculate the contributions arising at order α_s in the QCD sum rule for the spin-symmetry violating universal function $\chi_3(v \cdot v')$, which appears at order $1/m_Q$ in the heavy quark expansion of meson form factors. In particular, we derive the two-loop perturbative contribution to the sum rule. Over the kinematic range accessible in $B \rightarrow D^{(*)} \ell \nu$ decays, we find that $\chi_3(v \cdot v')$ does not exceed the level of $\sim 1\%$, indicating that power corrections induced by the chromo-magnetic operator in the heavy quark expansion are small.

(submitted to Physical Review D)

hep-ph/9212266 15 Dec 1992

I. INTRODUCTION

In the heavy quark effective theory (HQET), the hadronic matrix elements describing the semileptonic decays $M(v) \rightarrow M'(v') \ell \nu$, where M and M' are pseudoscalar or vector mesons containing a heavy quark, can be systematically expanded in inverse powers of the heavy quark masses [1–5]. The coefficients in this expansion are m_Q -independent, universal functions of the kinematic variable $y = v \cdot v'$. These so-called Isgur-Wise form factors characterize the properties of the cloud of light quarks and gluons surrounding the heavy quarks, which act as static color sources. At leading order, a single function $\xi(y)$ suffices to parameterize all matrix elements [6]. This is expressed in the compact trace formula [5,7]

$$\langle M'(v') | J(0) | M(v) \rangle = -\xi(y) \text{tr} \{ \overline{\mathcal{M}}'(v') \Gamma \mathcal{M}(v) \}, \quad (1)$$

which is a consequence of a spin-flavor symmetry that QCD reveals in the limit $m_Q \rightarrow \infty$ [6,8]. Here $J = \bar{h}'(v') \Gamma h(v)$ is an arbitrary heavy quark current, $h(v)$ describes a heavy quark with velocity v , and

$$\mathcal{M}(v) = \sqrt{m_M} P_+ \begin{cases} -\gamma_5 & ; \text{pseudoscalar meson} \\ \not{\epsilon} & ; \text{vector meson} \end{cases} \quad (2)$$

is a spin wave function that describes correctly the transformation properties (under boosts and heavy quark spin rotations) of the meson states in the effective theory. $P_+ = \frac{1}{2}(1 + \not{v})$ is an on-shell projection operator, and ϵ denotes the polarization of the vector meson. Vector current conservation implies that the Isgur-Wise function $\xi(y)$ is normalized to unity at zero recoil, where the velocities of the initial and final meson are the same, and the maximum momentum is transferred to the lepton pair [6].

At order $1/m_Q$, the spin-flavor symmetry is explicitly broken by the presence of higher dimension operators in both the effective currents and the effective Lagrangian of HQET. Some of these power corrections, which violate the spin symmetry, arise from the chromo-magnetic interaction between the spin of the heavy quark and the gluon field. The corresponding term in the effective Lagrangian is [9]

$$\mathcal{L}_{\text{mag}} = \frac{Z}{2m_Q} \mathcal{O}_{\text{mag}}, \quad \mathcal{O}_{\text{mag}} = \frac{g_s}{2} \bar{h}(v) \sigma_{\alpha\beta} G^{\alpha\beta} h(v), \quad (3)$$

where Z is a renormalization factor. The effects of an insertion of this operator can be parameterized by a tensor form factor $\chi^{\alpha\beta}$ [10]:

$$\langle M'(v') | i \int dz T \{ J(0), \mathcal{O}_{\text{mag}}(z) \} | M(v) \rangle = -\bar{\Lambda} \text{tr} \{ \chi^{\alpha\beta}(v, v') \overline{\mathcal{M}}'(v') \Gamma P_+ i \sigma_{\alpha\beta} \mathcal{M}(v) \}. \quad (4)$$

The mass parameter $\bar{\Lambda}$ sets the canonical scale for power corrections in HQET. In the $m_Q \rightarrow \infty$ limit, it measures the finite mass difference between a heavy meson and the heavy quark that it contains [11]. By factoring out this parameter, $\chi^{\alpha\beta}(v, v')$ becomes dimensionless. The most general decomposition of this form factor involves two real, scalar functions $\chi_2(y)$ and $\chi_3(y)$ defined by [10]

$$\chi^{\alpha\beta}(v, v') = (v'^\alpha \gamma^\beta - v'^\beta \gamma^\alpha) \chi_2(y) - 2i \sigma^{\alpha\beta} \chi_3(y). \quad (5)$$

Irrespective of the structure of the current J , the form factor $\chi_3(y)$ appears always in the following combination with $\xi(y)$:

$$\xi(y) + 2Z\bar{\Lambda}\left(\frac{d_M}{m_Q} + \frac{d_{M'}}{m_{Q'}}\right)\chi_3(y), \quad (6)$$

where $d_P = 3$ for a pseudoscalar and $d_V = -1$ for a vector meson. It thus effectively renormalizes the leading Isgur-Wise function, preserving its normalization at $y = 1$ since $\chi_3(1) = 0$ according to Luke's theorem [10]. Eq. (6) shows that knowledge of $\chi_3(y)$ is needed if one wants to relate processes which are connected by the spin symmetry, such as $B \rightarrow D \ell \nu$ and $B \rightarrow D^* \ell \nu$.

Being hadronic form factors, the universal functions in HQET can only be investigated using nonperturbative methods. QCD sum rules have become very popular for this purpose. They have been reformulated in the context of the effective theory and have been applied to the study of meson decay constants and the Isgur-Wise functions both in leading and next-to-leading order in the $1/m_Q$ expansion [12–21]. In particular, it has been shown that very simple predictions for the spin-symmetry violating form factors are obtained when terms of order α_s are neglected, namely [17]

$$\begin{aligned} \chi_2(y) &= 0, \\ \chi_3(y) &\propto \langle \bar{q} g_s \sigma_{\alpha\beta} G^{\alpha\beta} q \rangle [1 - \xi(y)]. \end{aligned} \quad (7)$$

In this approach $\chi_3(y)$ is proportional to the mixed quark-gluon condensate, and it was estimated that $\chi_3(y) \sim 1\%$ for large recoil ($y \sim 1.5$). In a recent work we have refined the prediction for $\chi_2(y)$ by including contributions of order α_s in the sum rule analysis [20]. We found that these are as important as the contribution of the mixed condensate in (7). It is, therefore, worthwhile to include such effects also in the analysis of $\chi_3(y)$. This is the purpose of this article.

II. DERIVATION OF THE SUM RULE

The QCD sum rule analysis of the functions $\chi_2(y)$ and $\chi_3(y)$ is very similar. We shall, therefore, only briefly sketch the general procedure and refer for details to Refs. [17,20]. Our starting point is the correlator

$$\begin{aligned} &\int dx dx' dz e^{i(k' \cdot x' - k \cdot x)} \langle 0 | T \left\{ [\bar{q} \bar{\Gamma}_{M'} h']_{x'}, J(0), \mathcal{O}_{\text{mag}}(z), [\bar{h} \Gamma_M q]_x \right\} | 0 \rangle \\ &= \Xi_2(\omega, \omega', y) \text{tr} \left\{ (v'^\alpha \gamma^\beta - v'^\beta \gamma^\alpha) \bar{\Gamma}_{M'} P'_+ \Gamma P_+ i \sigma_{\alpha\beta} P_+ \Gamma_M \right\} \\ &+ \Xi_3(\omega, \omega', y) \text{tr} \left\{ 2 \sigma^{\alpha\beta} \bar{\Gamma}_{M'} P'_+ \Gamma P_+ \sigma_{\alpha\beta} P_+ \Gamma_M \right\}, \end{aligned} \quad (8)$$

where $P'_+ = \frac{1}{2}(1 + \not{v}')$, and we omit the velocity labels in h and h' for simplicity. The heavy-light currents interpolate pseudoscalar or vector mesons, depending on the choice $\Gamma_M = -\gamma_5$ or $\Gamma_M = \gamma_\mu - v_\mu$, respectively. The external momenta k and k' in (8) are the “residual” off-shell momenta of the heavy quarks. Due to the phase redefinition of the effective heavy quark fields in HQET, they are related to the total momenta P and P' by $k = P - m_Q v$ and $k' = P' - m_{Q'} v'$ [3].

The coefficient functions Ξ_i are analytic in $\omega = 2v \cdot k$ and $\omega' = 2v' \cdot k'$, with discontinuities for positive values of these variables. They can be saturated by intermediate states which couple to the heavy-light currents. In particular, there is a double-pole contribution from the ground-state mesons M and M' . To leading order in the $1/m_Q$ expansion the pole position is at $\omega = \omega' = 2\bar{\Lambda}$. In the case of Ξ_2 , the residue of the pole is proportional to the universal function $\chi_2(y)$. For Ξ_3 the situation is more complicated, however, since insertions of the chromo-magnetic operator not only renormalize the leading Isgur-Wise function, but also the coupling of the heavy mesons to the interpolating heavy-light currents (*i.e.*, the meson decay constants) and the physical meson masses, which define the position of the pole.¹ The correct expression for the pole contribution to Ξ_3 is [17]

$$\begin{aligned} \Xi_3^{\text{pole}}(\omega, \omega', y) &= \frac{F^2}{(\omega - 2\bar{\Lambda} + i\epsilon)(\omega' - 2\bar{\Lambda} + i\epsilon)} \\ &\times \left\{ \bar{\Lambda} \chi_3(y) + G_2 \xi(y) + \frac{\bar{\Lambda} \delta\Lambda_2 \xi(y)}{(\omega - 2\bar{\Lambda} + i\epsilon)} \right\}. \end{aligned} \quad (9)$$

Here F is the analog of the meson decay constant in the effective theory ($F \sim f_M \sqrt{m_M}$), G_2 is the spin-symmetry violating correction to it, and $\delta\Lambda_2$ denotes the spin-symmetry violating mass shift of the meson masses at order $1/m_Q$. More precisely, these quantities are defined by [16]

$$\begin{aligned} m_M - m_Q &= \bar{\Lambda} \left\{ 1 + \frac{d_M}{m_Q} \delta\Lambda_2 + \dots \right\}, \\ \langle 0 | j(0) | M(v) \rangle &= \frac{iF}{2} \text{tr} \{ \Gamma \mathcal{M}(v) \}, \\ \langle 0 | i \int dz T \{ j(0), \mathcal{O}_{\text{mag}}(z) \} | M(v) \rangle &= \frac{iF}{2} G_2 \text{tr} \{ 2\sigma^{\alpha\beta} \Gamma P_+ \sigma_{\alpha\beta} \mathcal{M}(v) \}, \end{aligned} \quad (10)$$

where the ellipses represent spin-symmetry conserving or higher order power corrections, and $j = \bar{q} \Gamma h(v)$. In terms of the vector–pseudoscalar mass splitting, the parameter $\delta\Lambda_2$ is given by $m_V^2 - m_P^2 = -8\bar{\Lambda} \delta\Lambda_2$.

For not too small, negative values of ω and ω' , the coefficient function Ξ_3 can be approximated as a perturbative series in α_s , supplemented by the leading power corrections in $1/\omega$ and $1/\omega'$, which are proportional to vacuum expectation values of local quark-gluon operators, the so-called condensates [22]. This is how nonperturbative corrections are incorporated in this approach. The idea of QCD sum rules is to match this theoretical representation of Ξ_3 to the phenomenological pole contribution given in (9). To this end, one first writes the theoretical expression in terms of a double dispersion integral,

$$\Xi_3^{\text{th}}(\omega, \omega', y) = \int d\nu d\nu' \frac{\rho_3^{\text{th}}(\nu, \nu', y)}{(\nu - \omega - i\epsilon)(\nu' - \omega' - i\epsilon)} + \text{subtractions}, \quad (11)$$

and performs a Borel transformation in ω and ω' . This yields an exponential damping factor, which suppresses the contamination from higher resonance states and eliminates

¹There are no such additional terms for Ξ_2 because of the peculiar trace structure associated with this coefficient function.

possible subtraction terms. Because of the flavor symmetry it is natural to set the Borel parameters associated with ω and ω' equal: $\tau = \tau' = 2T$. One then introduces new variables $\omega_{\pm} = \frac{1}{2}(\nu \pm \nu')$ and integrates over ω_- . At this stage, one employs quark-hadron duality to argue that the remaining integral over the “diagonal” variable ω_+ above a threshold ω_0 is dual to the contribution of higher resonance states [15,19]. One thus equates the integral up to ω_0 to the Borel transform of the pole contribution in (9). This gives the QCD sum rule

$$\left\{ \bar{\Lambda} \chi_3(y) + \left[G_2 - \frac{\bar{\Lambda} \delta \Lambda_2}{2T} \right] \xi(y) \right\} F^2 e^{-2\bar{\Lambda}/T} = \int_0^{\omega_0} d\omega_+ e^{-\omega_+/T} \tilde{\rho}_3^{\text{th}}(\omega_+, y) \equiv K(T, \omega_0, y). \quad (12)$$

The effective spectral density $\tilde{\rho}_3^{\text{th}}$ arises after integration of the double spectral density over ω_- . Note that for each contribution to it the dependence on ω_+ is known on dimensional grounds. It thus suffices to calculate directly the Borel transform of the individual contributions to Ξ_3^{th} , corresponding to the limit $\omega_0 \rightarrow \infty$ in (12). The ω_0 -dependence can be recovered at the end of the calculation.

When terms of order α_s are neglected, contributions to the sum rule for Ξ_3 can only be proportional to condensates involving the gluon field, since there is no way to contract the gluon contained in \mathcal{O}_{mag} . The leading power correction of this type is represented by the diagram shown in Fig. 1(d). It is proportional to the mixed quark-gluon condensate and, as shown in Ref. [17], leads to (7). Here we are interested in the additional contributions arising at order α_s . They are shown in Fig. 1(a)-(c). Besides a two-loop perturbative contribution, one encounters further nonperturbative corrections proportional to the quark and the gluon condensate.

Let us first present the result for the nonperturbative power corrections. We find

$$K_{\text{cond}}(T, \omega_0, y) = \frac{\alpha_s \langle \bar{q}q \rangle T}{6\pi} [2 - r(y)] \delta_0\left(\frac{\omega_0}{T}\right) + \frac{\langle \alpha_s GG \rangle}{96\pi} \left(\frac{2}{y+1}\right) - \frac{\langle \bar{q} g_s \sigma_{\alpha\beta} G^{\alpha\beta} q \rangle}{48T}, \quad (13)$$

where

$$r(y) = \frac{1}{\sqrt{y^2 - 1}} \ln(y + \sqrt{y^2 - 1}),$$

$$\delta_n(x) = \frac{1}{\Gamma(n+1)} \int_0^x dz z^n e^{-z}. \quad (14)$$

We think it is safe to truncate the series of power corrections after the mixed condensate, since effects of higher dimension condensates are expected to be very small. The next-order power corrections would come from four-quark operators, which would contribute at the level of $|\langle \bar{q}q \rangle|/T^3 \sim 1 - 5\%$ as compared to the quark condensate.

The calculation of the perturbative contribution is more cumbersome. Two-loop diagrams such as that depicted in Fig. 1(a) were considered in detail in Ref. [21]. We use dimensional regularization and the integral representations discussed in this reference. After Borel transformation we find

$$K_{\text{pert}}(T, \infty, y) = 4N_c C_F g_s^2 \Gamma(D-1) \frac{(2T)^{2D-4}}{(4\pi)^D}$$

$$\times \int_0^1 d\lambda \lambda^{1-D} \int_{\lambda}^{\infty} du_1 \int_{1/\lambda}^{\infty} du_2 \frac{(u_1 u_2 - 1)^{D/2-2}}{[u_1 + 2(y+1)(u_2 - 1)]^{D-1}}, \quad (15)$$

where $C_F = (N_c^2 - 1)/2N_c$, and D is the dimension of space-time. For $D = 4$, the integrand diverges as $\lambda \rightarrow 0$. To regulate the integral, we assume $D < 2$ and use a triple integration by parts in λ to obtain an expression which can be analytically continued to the vicinity of $D = 4$. Next we set $D = 4 + 2\epsilon$, expand in ϵ , write the result as an integral over ω_+ , and introduce back the continuum threshold. This gives

$$K_{\text{pert}}(T, \omega_0, y) = -\frac{\alpha_s}{48\pi^3} \left(\frac{2}{y+1}\right)^2 \int_0^{\omega_0} d\omega_+ \omega_+^3 e^{-\omega_+/T} \quad (16)$$

$$\times \left\{ \frac{1}{\epsilon} + 2\gamma_E - 2\ln 4\pi + 4\ln \omega_+ - yr(y) - 2\ln \frac{y+1}{2} - \frac{23}{6} + \mathcal{O}(\epsilon) \right\}.$$

On first sight, the appearance of a divergence at order α_s seems surprising. Since the leading contribution to the spin-symmetry violating form factors is of order g_s , one would expect divergences to appear at order g_s^3 . In fact, the one-loop renormalization group equations which control the running of the hadronic quantities $\chi_3(y)$ and $\delta\Lambda_2$ are homogeneous equations, confirming this assertion. But G_2 satisfies the inhomogeneous equation [16]

$$\left\{ \mu \frac{\partial}{\partial \mu} + \frac{3\alpha_s}{2\pi} \right\} G_2(\mu) = \frac{2\alpha_s}{9\pi} \bar{\Lambda}, \quad (17)$$

which shows that divergences arise at order α_s . At this order, the renormalization of the sum rule is thus accomplished by a renormalization of the ‘‘bare’’ parameter G_2 in (12). In the $\overline{\text{MS}}$ subtraction scheme, one defines a renormalized parameter $G_2(\mu)$ by

$$G_2(\mu) = G_2^{\text{bare}} + \frac{\alpha_s}{9\pi} \bar{\Lambda} \left(\frac{1}{\epsilon} + \gamma_E - \ln \frac{4\pi}{\mu^2} \right) + \mathcal{O}(g_s^3). \quad (18)$$

Hence a counterterm proportional to $\bar{\Lambda} \xi(y)$ has to be added to the bracket on the left-hand side of the sum rule (12). To evaluate its effect on the right-hand side, we note that in D dimensions [17]

$$\bar{\Lambda} \xi(y) F^2 e^{-2\bar{\Lambda}/T} = \frac{3}{16\pi^2} \left(\frac{2}{y+1}\right)^2 \int_0^{\omega_0} d\omega_+ \omega_+^3 e^{-\omega_+/T} \quad (19)$$

$$\times \left\{ 1 + \epsilon \left[\gamma_E - \ln 4\pi + 2\ln \omega_+ - \ln \frac{y+1}{2} - 2 \right] \right\} + \mathcal{O}(g_s, \epsilon^2).$$

From (12), (16), (18) and (19) it is seen that indeed the $1/\epsilon$ pole cancels upon renormalization of G_2 . We find

$$\left\{ \bar{\Lambda} \chi_3(y) + \left[G_2(\mu) - \frac{\bar{\Lambda} \delta\Lambda_2}{2T} \right] \xi(y) \right\} F^2 e^{-2\bar{\Lambda}/T}$$

$$= \frac{\alpha_s}{48\pi^3} \left(\frac{2}{y+1}\right)^2 \int_0^{\omega_0} d\omega_+ \omega_+^3 e^{-\omega_+/T} \left\{ 2\ln \frac{\mu}{\omega_+} + \frac{17}{6} + \left[yr(y) - 1 + \ln \frac{y+1}{2} \right] \right\}$$

$$+ K_{\text{cond}}(T, \omega_0, y). \quad (20)$$

In this expression, all hadronic parameters are defined at the scale μ . However, we have only made the μ -dependence of $G_2(\mu)$ explicit since this is what we are sensitive to in our analysis.

According to Luke's theorem, the universal function $\chi_3(y)$ vanishes at zero recoil [10]. Evaluating (20) for $y = 1$, we thus obtain a sum rule for $G_2(\mu)$ and $\delta\Lambda_2$. It reads

$$\left[G_2(\mu) - \frac{\bar{\Lambda} \delta\Lambda_2}{2T} \right] F^2 e^{-2\bar{\Lambda}/T} = \frac{\alpha_s}{24\pi^3} \int_0^{\omega_0} d\omega_+ \omega_+^3 e^{-\omega_+/T} \left\{ \ln \frac{\mu}{\omega_+} + \frac{17}{12} \right\} + K_{\text{cond}}(T, \omega_0, 1), \quad (21)$$

where we have used that $r(1) = 1$. Precisely this sum rule has been derived previously, starting from a two-current correlator, in Ref. [16]. This provides a nontrivial check of our calculation. Using the fact that $\xi(y) = [2/(y+1)]^2 + \mathcal{O}(g_s)$ according to (19), we find that the μ -dependent terms cancel out when we eliminate $G_2(\mu)$ and $\delta\Lambda_2$ from the sum rule for $\chi_3(y)$.

Before we present our final result, there is one more effect which has to be taken into account, namely a spin-symmetry violating correction to the continuum threshold ω_0 . Since the chromo-magnetic interaction changes the masses of the ground-state mesons [cf. (10)], it also changes the masses of higher resonance states. Expanding the physical threshold as

$$\omega_{\text{phys}} = \omega_0 \left\{ 1 + \frac{d_M}{m_Q} \delta\omega_2 + \dots \right\}, \quad (22)$$

we expect $\delta\omega_2 \simeq \delta\Lambda_2 \simeq -0.12$ GeV, where the numerical value follows from the observed mass splitting between B^* and B . The appearance of such a shift is also in the spirit of QCD sum rules. Assume that we had constructed a sum rule for the combination of form factors given in (6). Then the presence of $1/m_Q$ corrections would affect the stability of the sum rule for the Isgur-Wise function $\xi(y)$ alone. The parameters that guaranteed optimal stability for $\xi(y)$ would have to be readjusted by an amount of order $1/m_Q$ in order to provide optimal stability of the sum rule for the combination (6). From such a self-consistent analysis, the parameters $\delta\Lambda_2$ and $\delta\omega_2$ have been determined in Ref. [16]. One finds the vector-pseudoscalar mass splitting in excellent agreement with experiment, and $\delta\omega_2 = -(0.10 \pm 0.02)$ GeV, which is just what we expected on physical grounds.

The contribution of $\delta\omega_2$ to the sum rule for $\chi_3(y)$ has been calculated in Ref. [17]. Including this term, we now present our final result:

$$\begin{aligned} \chi_3(y) \bar{\Lambda} F^2 e^{-2\bar{\Lambda}/T} &= \frac{\alpha_s T^4}{8\pi^3} \left(\frac{2}{y+1} \right)^2 \left[y r(y) - 1 + \ln \frac{y+1}{2} \right] \delta_3 \left(\frac{\omega_0}{T} \right) \\ &+ \frac{3 \delta\omega_2}{32\pi^2} \omega_0^3 e^{-\omega_0/T} \left[\left(\frac{2}{y+1} \right)^2 - \xi(y) \right] \\ &+ \frac{\alpha_s \langle \bar{q}q \rangle T}{6\pi} \left[2 - r(y) - \xi(y) \right] \delta_0 \left(\frac{\omega_0}{T} \right) \\ &+ \frac{\langle \alpha_s GG \rangle}{96\pi} \left[\frac{2}{y+1} - \xi(y) \right] - \frac{\langle \bar{q} g_s \sigma_{\alpha\beta} G^{\alpha\beta} q \rangle}{48T} \left[1 - \xi(y) \right]. \end{aligned} \quad (23)$$

It explicitly exhibits the fact that $\chi_3(1) = 0$.

III. NUMERICAL ANALYSIS

Let us now turn to the evaluation of the sum rule (23). For the QCD parameters we take the standard values

$$\begin{aligned}
\langle \bar{q}q \rangle &= -(0.23 \text{ GeV})^3, \\
\langle \alpha_s GG \rangle &= 0.04 \text{ GeV}^4, \\
\langle \bar{q} g_s \sigma_{\alpha\beta} G^{\alpha\beta} q \rangle &= m_0^2 \langle \bar{q}q \rangle, \quad m_0^2 = 0.8 \text{ GeV}^2.
\end{aligned} \tag{24}$$

Furthermore, we use $\delta\omega_2 = -0.1 \text{ GeV}$ from above, and $\alpha_s/\pi = 0.1$ corresponding to the scale $\mu = 2\bar{\Lambda} \simeq 1 \text{ GeV}$, which is appropriate for evaluating radiative corrections in the effective theory [15]. The sensitivity of our results to changes in these parameters will be discussed below. The dependence of the left-hand side of (23) on $\bar{\Lambda}$ and F can be eliminated by using a QCD sum rule for these parameters, too. It reads [16]

$$\bar{\Lambda} F^2 e^{-2\bar{\Lambda}/T} = \frac{9T^4}{8\pi^2} \delta_3\left(\frac{\omega_0}{T}\right) - \frac{\langle \bar{q} g_s \sigma_{\alpha\beta} G^{\alpha\beta} q \rangle}{4T}. \tag{25}$$

Similarly, we calculate the Isgur-Wise function from $\xi(y) = F(T, \omega_0, y)/F(T, \omega_0, 1)$, where [15]

$$F(T, \omega_0, y) = \frac{3T^3}{4\pi^2} \left(\frac{2}{y+1}\right)^2 \delta_2\left(\frac{\omega_0}{T}\right) - \langle \bar{q}q \rangle + \frac{(2y+1)}{3} \frac{\langle \bar{q} g_s \sigma_{\alpha\beta} G^{\alpha\beta} q \rangle}{4T^2}. \tag{26}$$

Combining (23), (25) and (26), we obtain $\chi_3(y)$ as a function of ω_0 and T . These parameters can be determined from the analysis of a QCD sum rule for the correlator of two heavy-light currents in the effective theory [16,18]. One finds good stability for $\omega_0 = 2.0 \pm 0.3 \text{ GeV}$, and the consistency of the theoretical calculation requires that the Borel parameter be in the range $0.6 < T < 1.0 \text{ GeV}$. It supports the self-consistency of the approach that, as shown in Fig. 2, we find stability of the sum rule (23) in the same region of parameter space. Note that it is in fact the $\delta\omega_2$ -term that stabilizes the sum rule. Without it there were no plateau.

Over the kinematic range accessible in semileptonic $B \rightarrow D^{(*)}\ell\nu$ decays, we show in Fig. 3(a) the range of predictions for $\chi_3(y)$ obtained for $1.7 < \omega_0 < 2.3 \text{ GeV}$ and $0.7 < T < 1.2 \text{ GeV}$. From this we estimate a relative uncertainty of $\sim \pm 25\%$, which is mainly due to the uncertainty in the continuum threshold. It is apparent that the form factor is small, not exceeding the level of 1%.²

Finally, we show in Fig. 3(b) the contributions of the individual terms in the sum rule (23). Due to the large negative contribution proportional to the quark condensate, the terms of order α_s , which we have calculated in this paper, cancel each other to a large extent. As a consequence, our final result for $\chi_3(y)$ is not very different from that obtained neglecting these terms [17]. This is, however, an accident. For instance, the order- α_s corrections would enhance the sum rule prediction by a factor of two if the $\langle \bar{q}q \rangle$ -term had the opposite sign. From this figure one can also deduce how changes in the values of the vacuum condensates would affect the numerical results. As long as one stays within the standard limits, the sensitivity to such changes is in fact rather small. For instance, working with the larger value $\langle \bar{q}q \rangle = -(0.26 \text{ GeV})^3$, or varying m_0^2 between 0.6 and 1.0 GeV^2 , changes $\chi_3(y)$ by no more than $\pm 0.15\%$.

²When comparing our result to the function $\chi_3^{\text{ren}}(y)$ shown in Fig. 2 of Ref. [17], one has to include a renormalization factor, which is approximately given by $\alpha_s^{-1/3} \simeq 1.5$.

In conclusion, we have presented the complete order- α_s QCD sum rule analysis of the subleading Isgur-Wise function $\chi_3(y)$, including in particular the two-loop perturbative contribution. We find that over the kinematic region accessible in semileptonic B decays this form factor is small, typically of the order of 1%. When combined with our previous analysis [20], which predicted similarly small values for the universal function $\chi_2(y)$, these results strongly indicate that power corrections in the heavy quark expansion which are induced by the chromo-magnetic interaction between the gluon field and the heavy quark spin are small.

ACKNOWLEDGMENTS

It is a pleasure to thank Michael Peskin for helpful discussions. M.N. gratefully acknowledges financial support from the BASF Aktiengesellschaft and from the German National Scholarship Foundation. Y.N. is an incumbent of the Ruth E. Recu Career Development chair, and is supported in part by the Israel Commission for Basic Research and by the Minerva Foundation. This work was also supported by the Department of Energy, contract DE-AC03-76SF00515.

REFERENCES

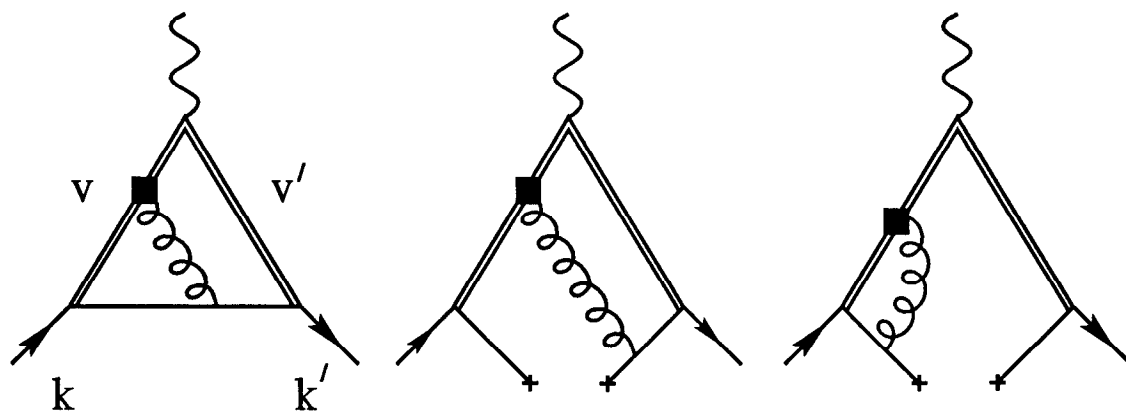
- [1] E. Eichten and B. Hill, Phys. Lett. B **234**, 511 (1990); **243**, 427 (1990).
- [2] B. Grinstein, Nucl. Phys. **B339**, 253 (1990).
- [3] H. Georgi, Phys. Lett. B **240**, 447 (1990).
- [4] T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. **B368**, 204 (1992).
- [5] A.F. Falk, H. Georgi, B. Grinstein, and M.B. Wise, Nucl. Phys. **B343**, 1 (1990).
- [6] N. Isgur and M.B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).
- [7] J.D. Bjorken, *Proceedings of the 18th SLAC Summer Institute on Particle Physics*, pp. 167, Stanford, California, July 1990, edited by J.F. Hawthorne (SLAC, Stanford, 1991).
- [8] M.B. Voloshin and M.A. Shifman, Yad. Fiz. **45**, 463 (1987) [Sov. J. Nucl. Phys. **45**, 292 (1987)]; **47**, 801 (1988) [**47**, 511 (1988)].
- [9] A.F. Falk, B. Grinstein, and M.E. Luke, Nucl. Phys. **B357**, 185 (1991).
- [10] M.E. Luke, Phys. Lett. B **252**, 447 (1990).
- [11] A.F. Falk, M. Neubert, and M.E. Luke, SLAC preprint SLAC-PUB-5771 (1992), to appear in Nucl. Phys. **B**.
- [12] M. Neubert, V. Rieckert, B. Stech, and Q.P. Xu, in *Heavy Flavours*, edited by A.J. Buras and M. Lindner, Advanced Series on Directions in High Energy Physics (World Scientific, Singapore, 1992).
- [13] A.V. Radyushkin, Phys. Lett. B **271**, 218 (1991).
- [14] D.J. Broadhurst and A.G. Grozin, Phys. Lett. B **274**, 421 (1992).
- [15] M. Neubert, Phys. Rev. D **45**, 2451 (1992).
- [16] M. Neubert, Phys. Rev. D **46**, 1076 (1992).
- [17] M. Neubert, Phys. Rev. D **46**, 3914 (1992).
- [18] E. Bagan, P. Ball, V.M. Braun, and H.G. Dosch, Phys. Lett. B **278**, 457 (1992); E. Bagan, P. Ball, and P. Gosdzinsky, Heidelberg preprint HD-THEP-92-40 (1992).
- [19] B. Blok and M. Shifman, Santa Barbara preprint NSF-ITP-92-100 (1992).
- [20] M. Neubert, Z. Ligeti, and Y. Nir, SLAC preprint SLAC-PUB-5915 (1992).
- [21] M. Neubert, SLAC preprint SLAC-PUB-5992 (1992).
- [22] M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979).

FIGURES

FIG. 1. Diagrams contributing to the sum rule for the universal form factor $\chi_3(v \cdot v')$: two-loop perturbative contribution (a), and nonperturbative contributions proportional to the quark condensate (b), the gluon condensate (c), and the mixed condensate (d). Heavy quark propagators are drawn as double lines. The square represents the chromo-magnetic operator.

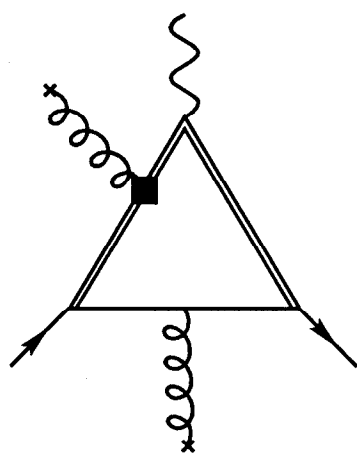
FIG. 2. Analysis of the stability region for the sum rule (23): The form factor $\chi_3(y)$ is shown for $y = 1.5$ as a function of the Borel parameter. From top to bottom, the solid curves refer to $\omega_0 = 1.7, 2.0,$ and 2.3 GeV. The dashes lines are obtained by neglecting the contribution proportional to $\delta\omega_2$.

FIG. 3. (a) Prediction for the form factor $\chi_3(v \cdot v')$ in the stability region $1.7 < \omega_0 < 2.3$ GeV and $0.7 < T < 1.2$ GeV. (b) Individual contributions to $\chi_3(v \cdot v')$ for $T = 0.8$ GeV and $\omega_0 = 2.0$ GeV: total (solid), mixed condensate (dashed-dotted), gluon condensate (wide dots), quark condensate (dashes). The perturbative contribution and the $\delta\omega_2$ -term are indistinguishable in this figure and are both represented by the narrow dots.

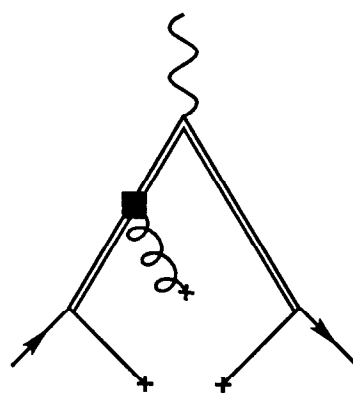


(a)

(b)



(c)



(d)

Fig.1

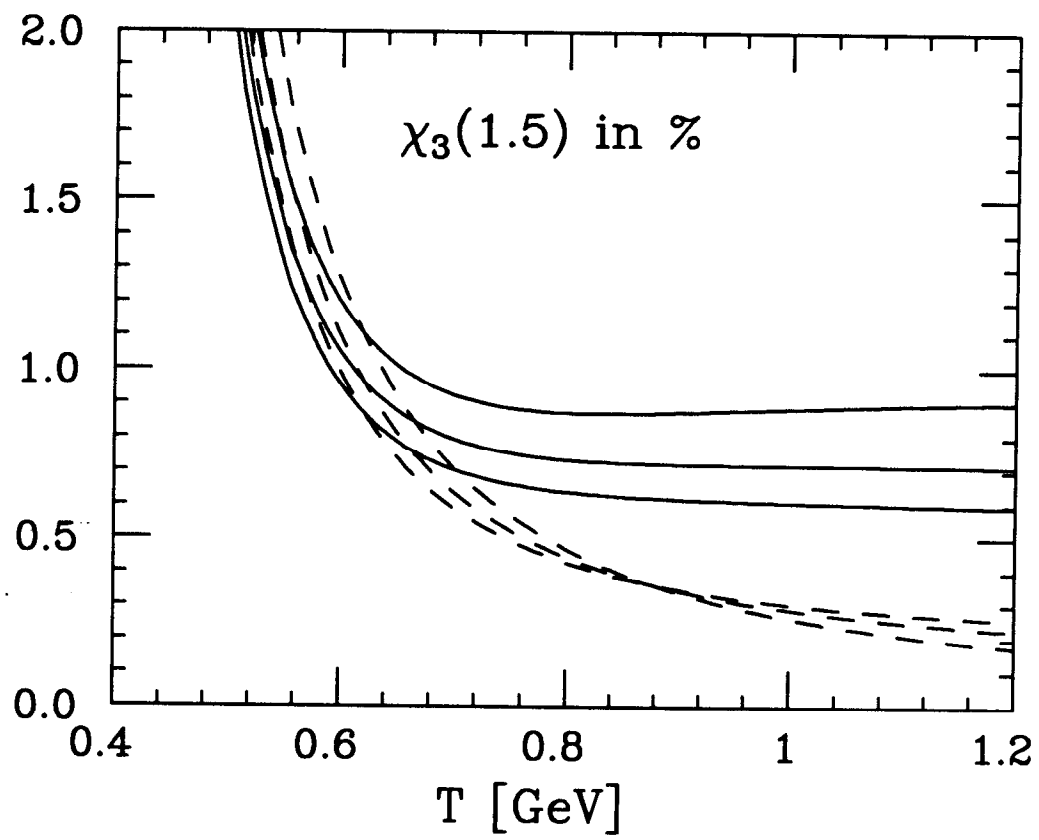


Fig.2

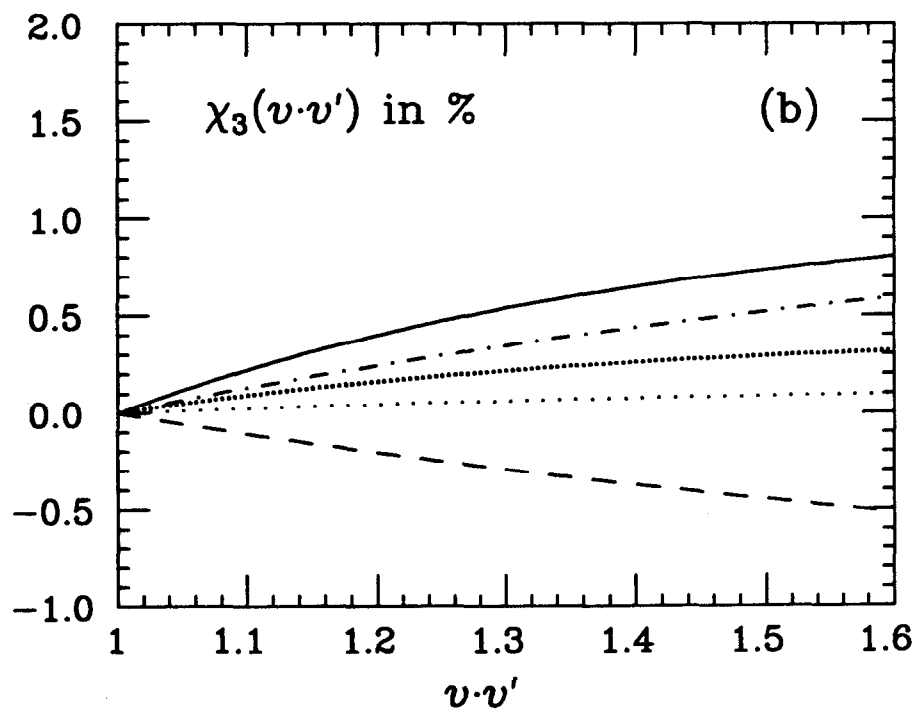
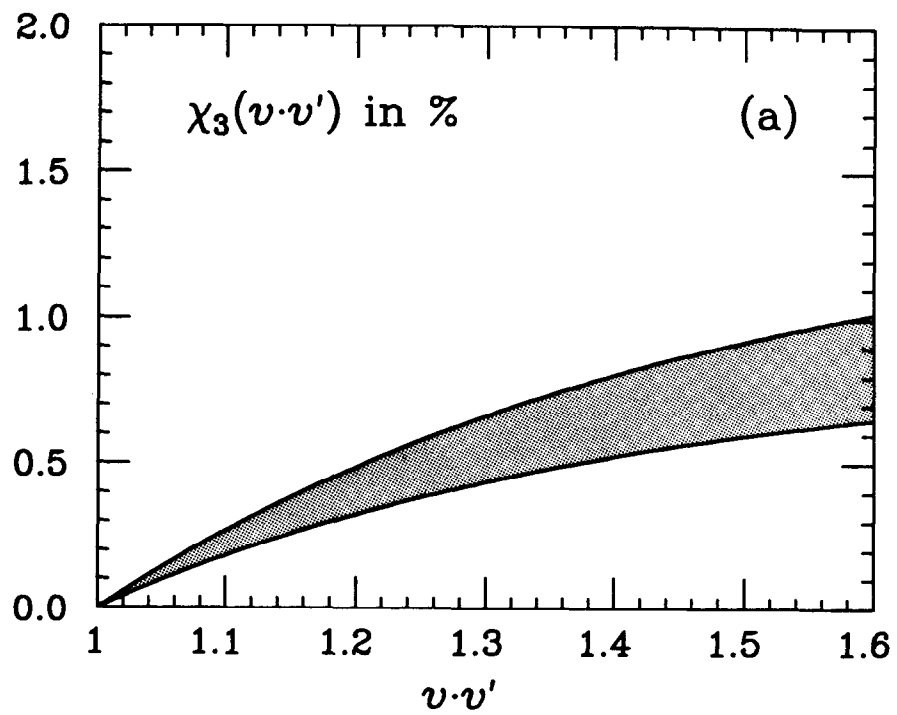


Fig.3