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Direct Semi-Exclusive Production of Energetic Mesons*

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ABSTRACT

The formation of color-singlet hadrons in hard-scattering processes is governed by process-independent distribution amplitudes. Such amplitudes may be measured in a class of processes we call *semi-exclusive*, where one hadron is formed directly in hard scattering and the other QCD partons hadronize in jets. These processes can be identified experimentally by requiring a large angular or rapidity gap around a directly produced meson. We present cross-sections for such processes, and show that experimental results, especially the ratio of charged to neutral K production, are highly sensitive to the hadronic distribution amplitude. We discuss methods and prospects for thus gaining information about hadronic wavefunctions at present and future colliders.

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1. INTRODUCTION

The analysis of exclusive hadronic processes in Quantum Chromodynamics (QCD) depends on knowledge of the underlying distribution amplitudes [1] which relate hadronic amplitudes to perturbatively calculable QCD processes. While some progress has been made in the theoretical extraction of these non-perturbative quantities from QCD sum rules [2], quantitative experimental tests of the resulting models are difficult to construct.

In this paper we propose a new method by which the distribution amplitudes may be measured in some detail: analysis of "semi-exclusive" processes in which a single-hadron state recoils against an inclusive state. We show how these processes can be used to shed light on the meson distribution amplitudes; future colliders, such as a high-luminosity B-factory, will be able to precisely examine the leading-twist portion of the distribution amplitude.

This paper is organized as follows. Section 2 introduces distribution amplitudes and the conventions we will use. Section 3 is devoted to the computation of one such process, $e^+e^- \to K^-X$, where the isolated K^- is produced directly rather than in the hadronization process. Section 4 extends the result of Section 3 to other similar processes and proposes predictions which are insensitive to soft physics. Section 5 derives results for semi-exclusive production of heavy $Q\bar{q}$ mesons in Z^0 decays. Finally, Sec. 6 summarizes our work and presents conclusions.

2. DISTRIBUTION AMPLITUDES

In exclusive hadronic processes where a "hard" scale $Q \gg \Lambda_{\rm QCD}$ can be identified, the overall amplitude can be factorized as [1]

$$\mathcal{M}_{\rm hadronic} = \int [{\rm d}x][{\rm d}y] \phi_f^*(y;Q) T_{\rm hard}(x,y;Q) \phi_i(x;Q), \tag{1}$$

where

ϕ_f, ϕ_i are non-perturbative distribution amplitudes for the final- and initial-state hadrons;

$T_{\rm hard}$ is a hard scattering amplitude at the parton level, calculable in perturbative QCD (pQCD);

Q is the *factorization scale*, such that soft subprocesses with momentum scale $< Q$ are absorbed into the distribution amplitudes, while processes with momenta $> Q$ are considered as corrections to $T_{\rm hard}$; and

x, y are the longitudinal momentum fractions carried by parton constituents, collinear up to scale Q, within the hadrons.

Here the distribution amplitude $\phi_f(x;Q)$ is the probability amplitude for partons collinear up to scale Q and carrying momenta $x_j p$ to combine into a hadron of momentum p. This is the fundamental physical quantity which determines the QCD structure and coherence properties of the hadron. Due to QCD evolution, it is mildly dependent on Q [1]; we shall generally ignore this dependence. Since we are concerned here with hadronic events in e^+e^- annihilation, we may take $Q = E_{\rm cm}$ for simplicity.

Since the hard-scattering amplitude T_H is slowly varying over momentum ranges $k \sim \mu$, where μ is a typical transverse momentum within the hadron, the contribution to the amplitude from formation of the meson in a state with nonzero angular momentum will be suppressed by powers of μ^2/Q^2, as will contributions with additional partons (gluons or $q\bar{q}$ pairs) in the hadron. In this work, we will neglect such terms; thus we are concerned only with the $L_z = 0$ state of smallest particle number, the *valence* Fock state of the hadron.

When dealing with mesons, we will always use $x = x_Q$ to denote the momentum fraction carried by the heavier quark constituent, and $\bar{x} \equiv 1 - x$ for the light-quark momentum fraction. Since the wavefunction favors equal parton velocities, we expect $\langle x \rangle \geq \langle \bar{x} \rangle$.

Finally, note that the wavefunction of a pseudoscalar $Q\bar{q}$ meson contains a helicity factor $(Q_\uparrow \bar{q}_\downarrow - Q_\downarrow \bar{q}_\uparrow)/\sqrt{2}$; we will absorb the $1/\sqrt{2}$ into the average over spins. (In contrast to exclusive processes, in which we must sum over the internal parton spins within the amplitude, semi-exclusive processes require that we treat the spins of the outgoing color-triplet partons as observables; we neglect quark masses, so that this determines the internal parton helicities as well.)

3. SEMI-EXCLUSIVE PRODUCTION

The distribution amplitude is a universal, process-independent property of the hadron; as such, it can be inserted into any parton-level calculation to extract the amplitude for a process involving both hadrons and partons[3]. In this paper, we propose to examine processes in which some, but not all, of the final-state partons combine into observed hadrons in the hard scattering; for this reason, we call such processes *semi-exclusive*, and refer to the creation of hadrons in hard scattering as *direct QCD production* [4]. The remaining quarks will hadronize outside the hard subprocess; we do not attempt to analyze this. Instead, we distinguish direct semi-exclusive production from inclusive production by specifying the kinematic features of the directly produced hadron. We now turn to the problem at hand: designing tests to probe the distribution amplitude in semi-exclusive processes of the form $e^+e^- \to HX$ for a hadron H. For definiteness, in this section we will take $H = K^-$.

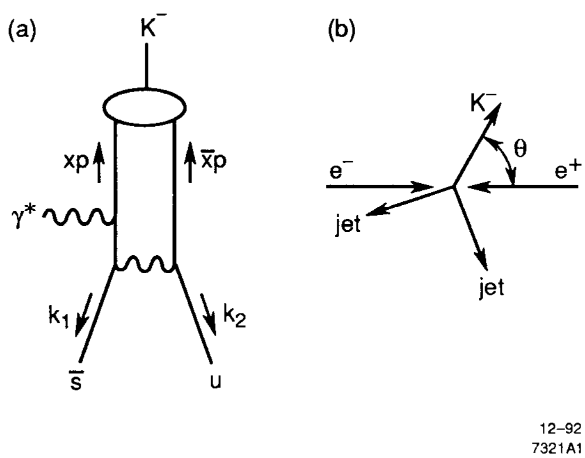

Figure 1. (a) A Feynman diagram contributing to the semi-exclusive process $e^+e^- \to K^-X$; the other diagrams are obtained from different attachments of the photon line. The remaining u and $\bar{s}$ quarks will hadronize nonperturbatively; at this level, we neglect final-state interactions and resonance physics. (b) The observed kinematics of a semi-exclusive event. Note that the plane containing the K and jets need not contain the incoming beams.

A representative Feynman diagram contributing to the direct production in which we are interested are shown in Fig. 1(a). Such amplitudes are of higher twist than the inclusive 4-jet process; hence the resulting cross-sections will be suppressed by a factor f_K^2/s where f_K is the kaon decay constant. To distinguish such a weak signal, it will be necessary to restrict ourselves to extreme kinematic regions where backgrounds are supressed. Thus the experimental signature of the events will be an extremely isolated meson recoiling against jets; since the meson is a color singlet, it will not be affected by the jet hadronization process.

The amplitudes take their simplest form in the center-of-momentum frame if we define the K momentum fraction (see Fig. 1(b))

$$z \equiv \frac{|\vec{p}_K|}{|\vec{p}_{K,\mathrm{max}}|};$$

the quark and antiquark back-momenta (light-cone momenta in the frame antiparallel to $\vec{p}_K$)

$$y_i Q \equiv |\vec{k}_i| - \frac{\vec{k}_i \cdot \vec{p}_K}{|\vec{p}_K|} \qquad (i = 1, 2),$$

with $y_1 + y_2 = 1$; θ and ϕ, where θ is the e^--to-K polar angle and ϕ is the angle between the K-q-$\bar{q}$ plane and the plane containing the beam and K [5]; and $s \equiv \sin(\theta/2)$, $c \equiv \cos(\theta/2)$. In these terms,

$$\mathrm{d}\sigma = \frac{1}{1024\pi^4}\, z\bar{z}\mathrm{d}z\ \mathrm{d}y_1\ \mathrm{dcos}\theta \mathrm{d}\phi\ \frac{1}{2}\sum_{\mathrm{spins}} |\mathcal{M}|^2;$$

the amplitude $\mathcal{M}$ for a process with three final-state particles has dimensions of mass^{-1}.

We let q_s and q_u denote the fractional QED charges of the s and u quark respectively, and define $\bar{z} \equiv 1 - z$. If the outgoing s quark has the same helicity as the electron, the hard-scattering amplitude then becomes

$$
\begin{aligned}
T_H^{(+)} = C_F \frac{e^2 g_s^2}{Q^2} \Bigg[& \frac{y_2 \bar{x} q_u - y_1 x q_s}{z x \bar{x} \sqrt{y_1 y_2}} \left(s e^{-i\phi} - \sqrt{\frac{\bar{z} y_2}{y_1}} c \right) \left(c - \sqrt{\frac{\bar{z} y_1}{y_2}} s e^{-i\phi} \right) \\
& + \frac{1}{x} \sqrt{\frac{y_2}{y_1}} s e^{-i\phi} q_u \left(c - \sqrt{\frac{\bar{z} y_2}{y_1}} \frac{s e^{-i\phi}}{1 - z\bar{x}} \right) \\
& - \frac{1}{\bar{x}} \sqrt{\frac{y_1}{y_2}} c q_s \left(s e^{-i\phi} - \sqrt{\frac{\bar{z} y_1}{y_2}} \frac{c}{1 - z x} \right) \Bigg],
\end{aligned}
\tag{1}
$$

where the color factor $C_F = 4/3$. The amplitude $T_H^{(-)}$ for opposite s and e^- helicities is obtained by the substitution $se^{-i\phi} \to c$, $c \to -se^{i\phi}$. Defining

$$
\begin{aligned}
A_K(z) &\equiv \int_0^1 \frac{\phi_K(x)}{\bar{x}(1 - zx)} \mathrm{d}x, \qquad \bar{A}_K(z) \equiv \int_0^1 \frac{\phi_K(x)}{x(1 - z\bar{x})} \mathrm{d}x; \\
B_K \equiv A_K(0) &= \int \frac{\phi_K(x)}{\bar{x}} \mathrm{d}x, \qquad \text{and} \quad \bar{B}_K \equiv \bar{A}_K(0),
\end{aligned}
$$

we can rearrange Eq. 1 to obtain

$$
\begin{aligned}
\mathcal{M}^{(+)} = C_F \frac{16\pi^2 \alpha \alpha_s}{z Q^2} \Bigg[& 2 s c e^{-i\phi} \left[\sqrt{\frac{y_2}{y_1}} \bar{B}_K q_u - \sqrt{\frac{y_1}{y_2}} B_K q_s \right] \\
& + \sqrt{\bar{z}} s^2 e^{-2i\phi} \left(B_K q_s - \frac{y_2}{y_1} \left[\bar{B}_K + z \bar{A}_K(z) \right] q_u \right) \\
& - \sqrt{\bar{z}} c^2 \left(\bar{B}_K q_u - \frac{y_1}{y_2} \left[B_K + z A_K(z) \right] q_s \right) \Bigg].
\end{aligned}
\tag{2}
$$

In the exclusive limit ($z \to 1$), the $\sin^2\theta$ angular dependence of the cross-section required by hadron helicity conservation is confirmed [6,7]. Note that the naive expectation that the cross-section vanishes for $q_u \to q_s$ (zero hadron charge) is violated even when $\bar{A}(z) = A(z)$ and $y_1 = y_2$; the hard photon probes the partonic rather than the overall hadronic structure. From Eq. 2, one can see that the interference term is ϕ-dependent, leaving a cross-section proportional to $z\bar{z}^2(A^2 + \bar{A}^2)$; thus, in the exclusive limit, the cancellation again becomes complete.

Squaring $\mathcal{M}$ and averaging over spins yields a complicated expression which we choose not to present here. Instead, we integrate out the angular dependence, eliminating many cross terms in the squared amplitude. After considerable simplification, this leaves

$$
\begin{aligned}
\mathrm{d}\sigma = \frac{16\pi}{27}\frac{\alpha^2\alpha_s^2}{Q^4}\,\bar{z}\mathrm{d}z\,\mathrm{d}y_1 \Bigg(& \left[\frac{\bar{z}}{z} + y_1 y_2\right]\left|\frac{B_K q_s}{y_2} - \frac{\bar{B}_K q_u}{y_1}\right|^2 \\
& + z\bar{z}\left[\frac{y_1^2}{y_2^2}\left|A_K(z)\right|^2 q_s^2 + \frac{y_2^2}{y_1^2}\left|\bar{A}_K(z)\right|^2 q_u^2\right] \\
& + 2\bar{z}\,\mathrm{Re}\Big[\frac{y_1^2}{y_2^2}A_K^\dagger(z)B_K q_s^2 + \frac{y_2^2}{y_1^2}\bar{A}_K^\dagger(z)\bar{B}_K q_u^2 \\
& \quad - \Big(\frac{y_1}{y_2}A_K^\dagger(z)\bar{B}_K + \frac{y_2}{y_1}\bar{A}_K^\dagger(z)B_K\Big)q_u q_s\Big]\Bigg).
\end{aligned}
\tag{3}
$$

4. OBSERVABLE RESULTS

The form of Eq. 3 is clearly independent of the specific meson under consideration; the generalization to other light pseudoscalar mesons is immediate. We now propose several quantities whose measurement will yield insight into mesonic wavefunctions.

Ideally, one could attempt to reconstruct y_1 and y_2 from measured jet momenta. However, in addition to the intrinsic difficulty of tagging the primary quark flavor [8], one would then also be confronted with the smearing of the momenta k_i in hadronization. Thus we adopt the more practicable strategy of selecting an experimental cut and integrating over the values of y_1 which allows it to be satisfied.

The simplest such condition is that the meson should carry a momentum fraction $z > 1/2$ and be isolated in its own momentum hemisphere in the center-of-momentum frame (that is, in the c.m. frame $\vec{p}_K \cdot \vec{k} < 0$ for any other final-state particle momentum $\vec{k}$). Such a drastic cut will be necessary to identify the weak signal against prevalent backgrounds [9]. This cut is frame-dependent, as is z; we choose it for its relative ease of implementation and the striking signature it presents. The cut is equivalent to the parton-level constraint

$$\min\{y_1, y_2\} > \frac{1-z}{2-z}.$$

Integrating over the values of y satisfying this constraint, we obtain (for ϕ_K real)

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_{\mathrm{sx}}}{\mathrm{d}z} = \frac{16\pi}{27}\frac{\alpha^2\alpha_s^2}{Q^4}\bar{z}\,\Bigg(& (2-z)\Big[\big(z\bar{A}_K(z)+\bar{B}_K\big)^2 q_u^2 + \big(zA_K(z)-B_K\big)^2 q_s^2\Big] \\
& + \frac{z^2\bar{z}}{2-z}\Big[A_K^2(z)q_s^2 + \bar{A}_K^2(z)q_u^2\Big] - \frac{z}{2-z}\Big[B_K q_s + \bar{B}_K q_u\Big]^2 \\
& + \frac{z\bar{z}}{2-z}\Big[A_K(z)q_s + \bar{A}_K(z)q_u\Big]\Big[B_K q_s + \bar{B}_K q_u\Big] \\
& + 2\ln\bar{z}\Big[z\bar{z}\big(A_K^2(z)q_s^2 + \bar{A}_K^2(z)q_u^2\big) - \frac{1}{2}\big(B_K^2 q_s^2 + \bar{B}_K^2 q_u^2\big) \\
& \quad + 4\frac{\bar{z}}{z}B_K\bar{B}_K q_u q_s + 2\bar{z}\big(A_K(z)B_K q_s^2 + \bar{A}_K(z)\bar{B}_K q_u^2\big) \\
& \quad + \bar{z}\big(\bar{A}_K(z)B_K + A_K(z)\bar{B}_K\big)q_u q_s\big)\Big]\Bigg).
\end{aligned} \tag{4}
$$

We find it advisable to restrict ourselves to values of z where the invariant mass $\sqrt{\bar{z}}Q$ of the hadronizing $\bar{s}u$ system is large enough that resonance physics may be neglected. We place the dividing line at $\sqrt{\bar{z}}Q = 2$ GeV; at $Q = 10$ GeV, for example, this corresponds to $z < z_{\mathrm{cut}} = 0.96$. Due to the overall factor of $\bar{z}$ in Eq. 3, little data will be lost to such a cut.

To proceed further, we must consider models of the valence-state distribution amplitude $\phi_K(x)$. Any such function can be expanded on the basis of Gegenbauer polynomials orthogonal over the measure with weight $x\bar{x}$:

$$
\begin{aligned}
\phi_K(x) = (f_K\sqrt{3})x\bar{x}\Big[& 1 + a_1\sqrt{5}(1-2x) + a_2\sqrt{14}(1-5x\bar{x}) \\
& + a_3\sqrt{30}(1-2x)(1-7x\bar{x}) + \dots\Big].
\end{aligned} \tag{5}
$$

The kaon decay constant f_K is measured in semileptonic decays to be about 170 MeV; we will use this value in our numerical predictions.

One model distribution amplitude, based on analysis of QCD sum rules, has been proposed by Zhitnitskiĭ *et al.* (ZZC) [10]; for ϕ_K, they give $a_1 = -0.24$,

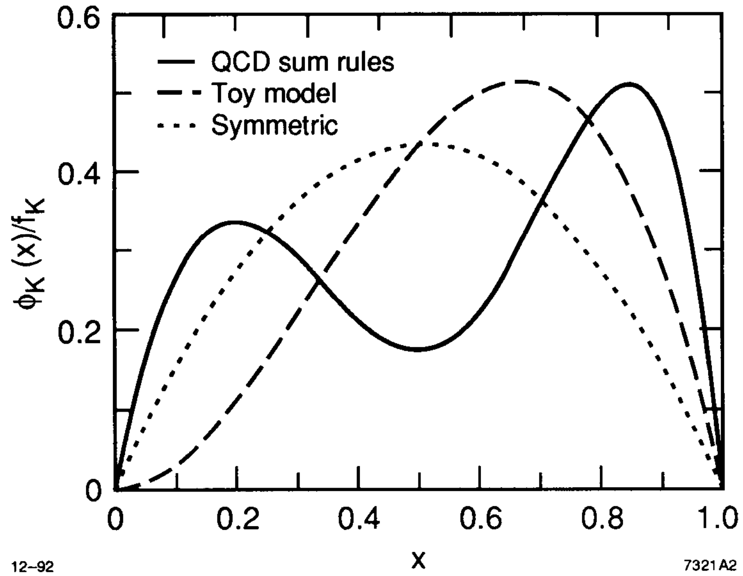

Figure 2. Three models of the distribution amplitude $\phi_K(x)$. The double-humped structure of the ZZC wavefunction [10] is a striking result of the sum-rule calculation. Note that the exact shape of the sum-rule wavefunction is determined not only by the calculated moments but by the projection onto the first four Gegenbauer polynomials.

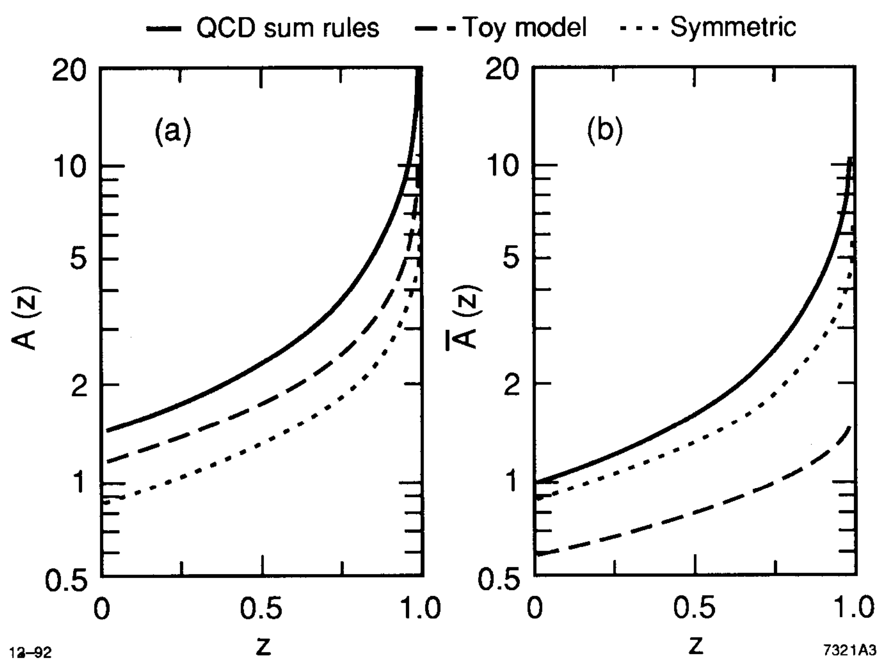

Figure 3. The transforms $A_K(z)$ and $\bar{A}_K(z)$ of the distribution amplitudes in Fig. 2. When $A \gg \bar{A}$, as in the toy model, contributions from Feynman diagrams where the photon attaches to the light quark are suppressed.

$a_2 = 0.64$, and $a_3 = -0.13$. We test the sensitivity of our conclusions to the wavefunction by also using the toy wavefunction $\phi_K(x) = 2\sqrt{3} f_K x^2 \bar{x}$ (corresponding to $a_1 = -1/\sqrt{5}$, all others zero), and the symmetric distribution $\phi_K(K) = \sqrt{3} f_K x\bar{x}$ with all $a_i = 0$. These functions are plotted in Fig. 2; the resulting transforms $A_K(z)$ and $\bar{A}_K(z)$ are shown in Fig. 3.

4.1 Ratio of charged and neutral K production.

Isospin symmetry, which [10] asserts is valid to within $\lesssim 1\%$ in mesonic wavefunctions, dictates that $\phi_{K^-}(x) = \phi_{K^0}(x)$; thus the simple substitution $q_u \to q_d$ gives the cross-section to form a hard isolated K^0. Charge conjugation symmetry gives the rates for K^+ and $\bar{K}^0$ formation. While resonance effects and final-state interactions may alter the cross-section, the ratio of charged to neutral production is a safe prediction of pQCD since the $\bar{s}u$ and $\bar{s}d$ systems (for example) have the same resonance properties.

Table I. Cross sections for semi-exclusive production of K and π mesons at CESR or B-factory energies, as described in Sec. 4. The ratio R_{K^-/K^0} is very insensitive to soft physics.

	Distribution amplitude		
meson	ZZC[8,9]	toy	symmetric
K^-	3.0 fb	1.1 fb	1.5 fb
K^0	0.7 fb	0.35 fb	0.25 fb
π^+	5.4 fb	–	1.5 fb
π^0	2.6 fb	–	0.8 fb

Table 1 shows the production cross-sections for each light meson at $Q =$ 10.58 GeV, the mass of the Υ_{4s}. Note that the ratio

$$R_{K^-/K^0} \equiv \frac{\sigma(e^+e^- \to K^-X)}{\sigma(e^+e^- \to K^0X)}$$

is very sensitive to the choice of distribution amplitude. We obtain $R_{K^-/K^0} = 4.1$ for the ZZC sum-rule distribution, 3.3 for our toy model, and 6.5 for the asymptotic limit.

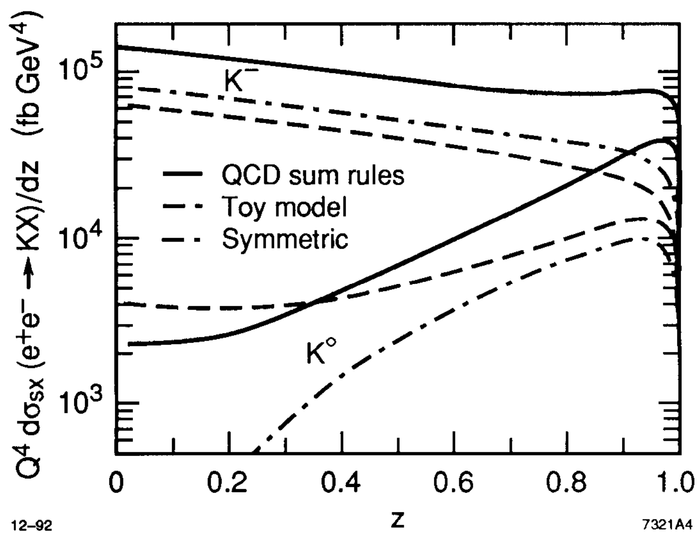

Figure 4. The differential cross-section for semi-exclusive K production at Q near 10 GeV, from Eq. 4; here the y-dependence has been integrated out.

Note that the discussion above is predicated on a scarcity of experimental statistics. Operation of a B-factory would obviate this concern; it would then be possible to measure $d\sigma/dz$ over a range of z and Q^2 and, using Eq. 4, to verify the sealing properties and extract the distribution amplitude directly! Figure 4 shows the behavior of $d\sigma/dz$ for each of the distribution amplitudes under consideration. This promises to be a stringent test of any proposed $\phi_K(x)$.

For comparison, the inclusive cross-section for K^- production at $\sqrt{s} = 10.58$ GeV is predicted by the LUND Monte Carlo as $\mathrm{d}\sigma/\mathrm{d}z = 50(1-z)$ pb for z near 1; thus we would expect $\sim 10^4$ inclusive events at a given z for each semi-exclusive event.

4.2 Semi-Exclusive pion production.

The pion distribution amplitude $\phi_\pi(x)$ is symmetric under $x \leftrightarrow \bar{x}$; thus $a_{2n+1} = 0$ in its polynomial expansion and we simply write

$$\phi_\pi(x) = (f_\pi\sqrt{3})(x\bar{x})\big(1 + a_2\sqrt{14}(1 - 5x\bar{x}) + \ldots\big),$$

where the pion decay constant $f_\pi = 133$ MeV is measured in π^+ decay. The π^0 distribution amplitude is identical; we need not consider interference between the $d\bar{d}$ and $u\bar{u}$ states of the π^0, as our neglect of resonance effects is tantamount to treating the flavors of the hadronizing quarks as observables.

The result of QCD sum rule analysis [11] is $a_2 = 1.07$ for the π. This distribution amplitude and its transform are shown in Fig. 5; note that $A_\pi(z) \equiv \bar{A}_\pi(z)$. Cross-sections for semi-exclusive π production are also given in Table 1.

To date, CLEO at CESR has over 1 fb^{-1} of integrated luminosity; thus some events of this type have probably already been recorded. This is especially true if the QCD sum-rule analysis, which predicts comparatively large cross-sections, is valid.

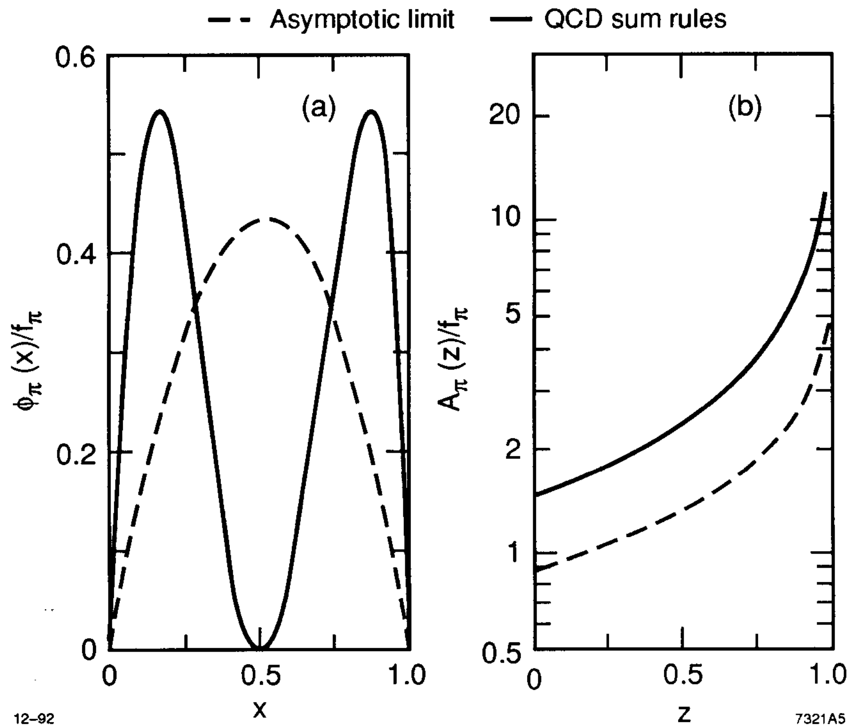

Figure 5.(a) The distribution amplitude of [11] from QCD sum rules; and the asymptotic limit of $\phi_\pi(x;Q^2)$ as $q^2 \to \infty$, the symmetric distribution [1]. (b) The resulting transforms $A_\pi(z) = \bar{A}_\pi(z)$. Because $\phi(x)$ is more concentrated near the endpoints, $A_\pi(z)$ is much larger for the sum-rule distribution than for the asymptotic.

4.3 Production at lower energies.

At smaller Q^2, for example $Q \simeq 4$ GeV at a τ-charm factory, the assumption that the light mesons are massless begins to break down. Nonetheless, we present predictions for this case as well.

At this energy, the semi-exclusive production cross-section is concentrated in the region $\sqrt{\bar{z}}Q = 1 - 2$ GeV, where resonance effects may not be negligible. However, the scaling behavior in Q of $d\sigma/dz$ can be tested; if it shows the expected power-law behavior, our faith in the assumption of duality will be justified. [12]

Table 2. Semi-exclusive cross sections for K and π production at $Q = 4$ GeV. The normalizations are somewhat unreliable, as we cannot entirely avoid the resonance region; however, the ratio R_{K^-/K^0} is again a valid prediction of the perturbative calculation.

	Distribution amplitude		
meson	ZZC[8,9]	toy	symmetric
K^-	370 fb	130 fb	180 fb
K^0	110 fb	50 fb	35 fb
π^+	700 fb	–	180 fb
π^0	410 fb	–	90 fb

Table 2 shows our results for semi-exclusive light meson production cross sections at $Q = 4$ GeV, with the cutoff $\sqrt{\bar{z}}Q > 1$ GeV. As before, the ratio R_{K^-/K^0}, which is nearly independent of resonance physics, is a sensitive test of the K distribution amplitude; at this energy, $R_{K^-/K^0} = 3.3$ for the ZZC model, 2.6 for our toy model, and 5.2 for the asymptotic distribution.

4.4 Semi-exclusive D production.

Finally, we consider D meson production at Υ energies. Here the neglect of quark masses is open to question, but provides a reasonable first approximation; the errors so introduced will be on the order of $m_c^2/Q^2 \simeq 5\%$.

The D distribution amplitude is strongly peaked at $\bar{x} \ll 1$, so that $\bar{A}_D(z) \ll A_D(z)$; thus the ratio of charged to neutral production is no longer a sensitive test. Instead, we hope to measure the absolute cross-section, which we will show is a measure of the extent to which the wavefunction is concentrated near

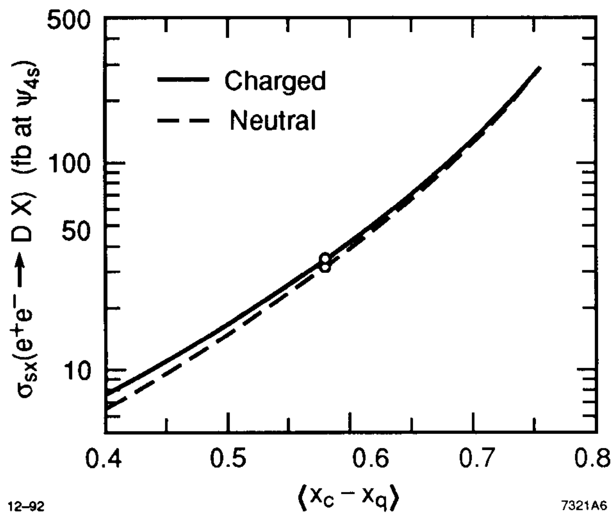

Figure 6. The semi-exclusive cross-section $\sigma_{\rm sx}(e^+e^- \rightarrow DX)$ at B-factory energies. Here we have used the toy wavefunction, Eq. 5. The points marked correspond to the prediction of [10] for $\langle x \rangle$; [10] does not itself advance a model wavefunction.

$x = 1$. For definiteness, we use the toy wavefunction

$$\phi_D(x) = (f_D\sqrt{3})\ \frac{(1-x)(x-x_0)}{(1-x_0)^3}\ \theta(x-x_0). \tag{6}$$

We expect [10] that $f_D \simeq f_K$; in this work we will simply assume $f_B = f_D = f_K$.

To avoid resonance effects, which will decrease the reliability of our normalization, we restrict ourselves to the region where $\sqrt{\bar{z}}Q > 4$ GeV, so that $z_{\rm cut} = 0.86$. The predicted cross-sections for D^+ and D^0 production as a function of $x_0 = \langle x - \bar{x} \rangle = 2\langle x \rangle - 1$ are shown in Fig. 6. These predictions do not include corrections from the charm quark mass; a careful treatment of such terms will be given elsewhere [9]. The first-order results are encouraging; if f_D can be measured independently, then $\langle x_c \rangle$ can be measured to within 5% at a B factory.

5. AT THE Z^0 PEAK

The same methods can be used to calculate the semi-exclusive decay rate $\Gamma_{sx}(Z^0 \to HX)$. Due to the much higher energy, the signal for this type of higher-twist process will be weaker; thus we must find a less stringent cut than the condition of isolation in a hemisphere.

Bjorken *et al.* [13-15] have shown that the 'rapidity gap' is a natural and effective tool for identification of subprocesses producing color singlets. That is, we require that the K be isolated in rapidity (or pseudorapidity) with respect to its own jet axis by some gap ΔY. In [13], it is estimated that the process of hadronization smears Y by about 0.7 in each jet; thus we use $\Delta Y \equiv \Delta Y_{\text{eff}} + 0.7$ at the parton level, where ΔY_{eff} is the experimentally observed rapidity gap after fragmentation. For a meson of mass M, the equivalent parton-level condition is

$$\min\{y_1, y_2\} > \frac{\bar{z}}{\bar{z} + \zeta z^2} \qquad \text{where} \quad \zeta \equiv \left(\frac{m_Z}{2M} e^{-\Delta Y}\right)^2 .$$

The quantity $\frac{1}{2} \ln \zeta$ can be considered as the "rapidity cushion" by which the meson rapidity can exceed the rapidity gap.

Again, we integrate over all y_1 satisfying this condition. For simplicity, we use the notation of [14], where

$$\mathbf{Q}_f \equiv \begin{pmatrix} q_{f_R} \\ q_{f_L} \end{pmatrix}$$

contains both the left- and right-handed weak charge. We then obtain

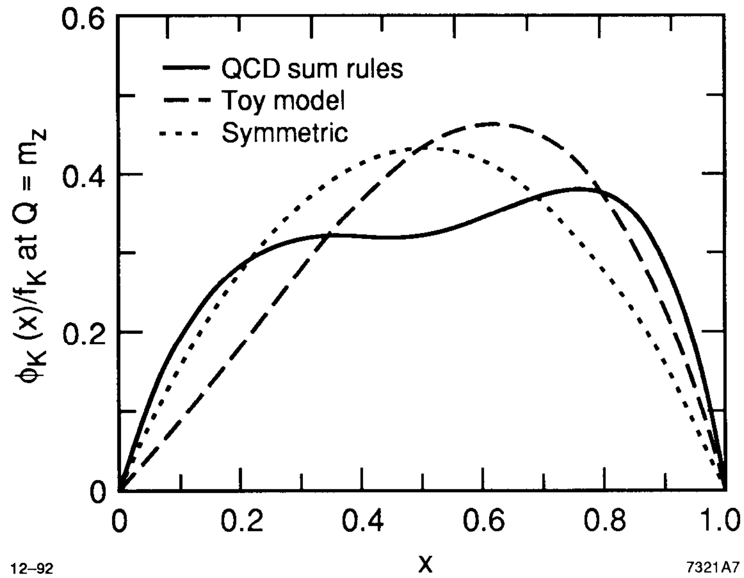

Figure 7. The distribution amplitudes $\phi_K(x;Q=m_Z)$. We have used the evolution equation for the pion distribution amplitude [1].

(for $\theta(x)$ real)

$$
\begin{aligned}
\frac{d\sigma_{\mathrm{sx}}}{dz}(\Delta Y) &= \frac{16\pi}{27}\frac{g_W^2\alpha_s^2}{2m_Z^2\Gamma_Z^2}\times \\
&\times\Bigg(\Big(\zeta z+\frac{\bar z^2}{\zeta z^3}\Big)\Big[\big(z\bar A_K(z)+\bar B_K\big)^2\mathbf{Q}_u^2+\big(zA_K(z)-B_K\big)^2\mathbf{Q}_s^2\Big] \\
&\quad+\frac{\zeta z^2-\bar z}{\zeta z^2+\bar z}\Big[z\bar z\big(A_K^2(z)\mathbf{Q}_s^2+\bar A_K^2(z)\mathbf{Q}_u^2\big)-\big(B_K\mathbf{Q}_s+\bar B_K\mathbf{Q}_u\big)^2 \\
&\quad+\bar z\big(A_K(z)\mathbf{Q}_s+\bar A_K(z)\mathbf{Q}_u\big)\cdot\big(B_K\mathbf{Q}_s+\bar B_K\mathbf{Q}_u\big)\Big] \\
&\quad-2\ln\Big(\zeta\frac{z^2}{\bar z}\Big)\Big[z\bar z\big(A_K^2(z)\mathbf{Q}_s^2+\bar A_K^2(z)\mathbf{Q}_u^2\big)-\frac{1}{2}\big(B_K^2\mathbf{Q}_s^2+\bar B_K^2\mathbf{Q}_u^2\big) \\
&\qquad+2\bar z\big(A_K(z)B_K\mathbf{Q}_s^2+\bar A_K(z)\bar B_K\mathbf{Q}_u^2\big)+4\frac{\bar z}{z}B_K\bar B_K\mathbf{Q}_u\cdot\mathbf{Q}_s \\
&\qquad+\bar z\big(\bar A_K(z)B_K+A_K(z)\bar B_K\big)\mathbf{Q}_u\cdot\mathbf{Q}_s\Big]\Bigg)
\end{aligned}
\tag{6}
$$

At these energies, we cannot neglect the running of the distribution amplitude [1]; since the Gegenbauer polynomials of Eq. 5 are the eigenfunctions of the evolution equation, we can easily evolve them to $Q = m_Z$. The result is shown in Fig. 7; note the extent to which the sum-rule distribution comes to resemble the asymtotic limit. The Q-dependence of the distribution amplitude reduces the cross-sections by 38% in the ZZC model, but by only 5% in the toy model we have used for comparison. Thus it is possible in principle (if not in practice) to observe directly the running of the distribution amplitude.

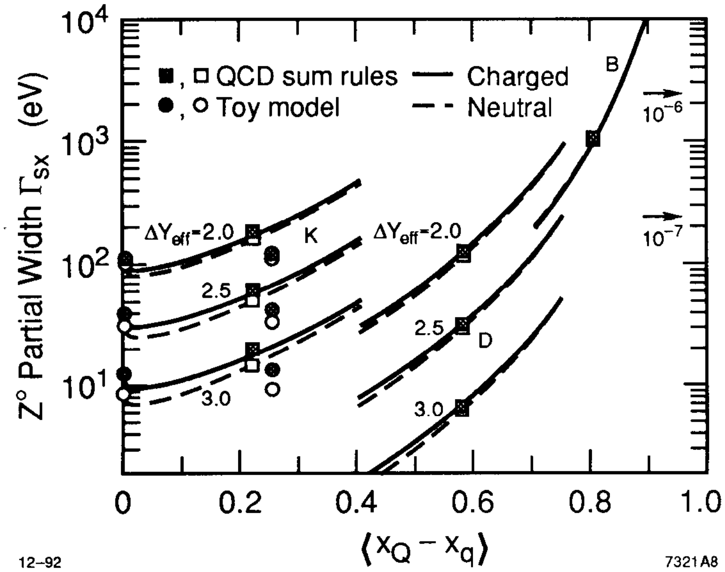

Figure 8. Partial widths per channel $d\Gamma_{\mathrm{sx}}(Z^0 \to MX)$ for mesons $M = K, D, B$ using the toy distribution amplitude, Eq. 5. The points marked on the curves for D and B mesons correspond to the values of $\langle x \rangle$ derived in [10]. We also show points for each $\phi_K(x)$ under consideration.

Unfortunately, the branching ratios for such events are less than 10^{-6} for all flavors; the loss of phase space satisfying the rapidity-gap condition offsets the increase in $A(z)$ as we move to heavier mesons. The partial widths for K, D and B production as a function of the parameter x_0 in Eq. 6 are shown in Fig. 8, along with the predictions from the wavefunction of [10] and our toy model. For B

mesons, the rapidity-gap condition is in fact far more severe than that of hemisphere isolation; thus we choose to show the results for hemisphere isolation for $\Gamma_{\mathrm{sx}}(Z^0 \to BX)$.

One prediction we obtain is that $-(\mathrm{d}/(\mathrm{d}\Delta Y)) \ln \Gamma_{\mathrm{sx}}(\Delta Y) = 2.1$ for K events, 2.6 for D and 3.6 for B (*e.g.*, decreasing ΔY by 1 increases the number of semi-exclusive K events by a factor $e^{2.1} \simeq 8$) [15]. This might be used at smaller rapidity gaps to detect the onset of the harder semi-exclusive processes within the soft background due to statistical fluctuation.

6. CONCLUSIONS

To be confident that we are calculating a meaningful quantity, we must consider the possibility that our results are unduly sensitive to the endpoint behavior of $\phi_K(x)$, especially at large z. However, the behavior $\phi_K(x) \propto x\bar{x}$ is sufficient to ensure that $\sqrt{\bar{z}} A_K(z) \to 0$ as $z \to 1$, which means that large-z divergences do not appear and the endpoints in x are not overly important. In addition, Sudakov suppression [16] ensures that endpoint contributions will not dominate the cross-section.

Some experimental issues will complicate the measurement; chief among these are π/K discrimination at high momentum and the blurring of angles induced by the hadronization process. Neither of these seems insurmountable; it is our hope that future detectors will have the necessary meson identification ability.

The cross-sections at $Q \simeq 10$ GeV are on the order of 1-10 fb; thus measurements of σ_{sx} are barely feasible with current machines, but precise extraction of $d\sigma_{sx}/dz$ should be possible with future high-luminosity colliders. With this information one can measure moments of meson distribution amplitudes; these parameters of QCD can then be compared to theoretical calculations.

On the other hand, Γ_{sx} in Z decays is extremely small under even the most favorable assumptions. Thus the discovery of more than a handful of such events in the current LEP data sample, if they were not simply due to statistical fluctuations, would require reconsideration of our results.

In sum, we have demonstrated that semi-exclusive production is a powerful probe of meson distribution amplitudes in the valence Fock state. Its advantage over traditional analysis through exclusive processes arises from two considerations. First, only a single hadron is formed in the hard QCD event: thus the total cross-section for semi-exclusive meson production scales as Q^{-4}, as opposed to the Q^{-6} scaling of exclusive meson pair production. Second, the observable z in the final state is directly related to the distribution amplitude; instead of a single measurement such as a form factor, we can measure a differential cross-section and analyze the shape of the distribution amplitude in detail. Thus we believe that these processes represent a fruitful field for experimental investigation.

7. Acknowledgements

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$$\frac{\mathrm{d}\sigma}{\mathrm{d}k_T^2\ \mathrm{d}z\ \mathrm{d}\cos\theta} \propto \frac{1}{k_T^4}\left[\bar{z}^2(1+\cos^2\theta)+\frac{4}{9}\frac{k_T^2}{Q^2}\sin^2\theta\right].$$

   Despite its appearance, this is in agreement with our result; to convert to our notation, use $k_T^2 = \bar{z}y_1y_2Q^2$. A similar direct process is analyzed in E. Berger and S.J. Brodsky, *Phys. Rev.* **D24**, 2428 (1981).

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