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ON THE CORRESPONDENCE LIMIT OF RELATIVISTIC QUANTUM MECHANICS*

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ABSTRACT

One of the many problems of second quantized relativistic field theory is that the "correspondence limit" either in non-relativistic quantum mechanics for atomic systems or in non-relativistic quantum mechanics for strongly interacting nuclear systems or in classical relativistic particle mechanics is not well specified. In this paper we argue that by using a fully finite and discrete approach to relativistic quantum mechanics we can arrive at a theory which does not have these defects, yet reproduces many of the same *empirical* results which are conventionally accounted for by elementary particle physics and the related physical cosmology.

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1. THE CONVENTIONAL WISDOM

Second quantization as originally devised assigned an infinite energy to each space-time point. Renormalization met this problem for QED in an *ad hoc* way. It is claimed that non-Abelian gauge theories are constructed in such a way that no infinities can appear. This statement has not been proved to the satisfaction of some mathematicians. Even if the finiteness of non-Abelian gauge theories — and in particular of the standard model of quarks and leptons — is accepted, other problems remain.

To illustrate one difficulty with the conventional approach, consider Weinberg's discussion of the low momentum limit of massless fields whose quanta carry spin j.[1] He shows that as the momentum p carried by the field quanta approaches zero, Lorentz invariance requires the interaction to vanish like p^j. This would be a disaster for the conventional theory since only scalar quanta could survive, ruling out photons and gravitons! He saves the second quantized relativistic field theory by showing that if one *also* requires gauge invariance, a well specified limit exists. The resulting theory then predicts that the force between two identical particles mediated by a field whose quanta have spin 1 is repulsive while the force between particle and anti-particle is attractive. For spin 0 and spin 2 the predicted force is always attractive. Both predictions are consistent with currently available experimental information.

This result appears to be a triumph for the theory, particularly since subsequent developments have singled out the class of non-abelian gauge theories as appropriate for describing the observations cited in support of weak-electromagnetic unification and quantum chromodynamics. But this success has a price. It requires in some sense the reification of the concept of "potential" at a fundamental level, in contrast the classical situation where potentials have no objective significance. Although the Aharanov-Bohm effect might seem to support this point of view, there is no consensus. For instance, topological explanations of the A-B effect which invoke only forces rather than the electromagnetic vector potential have been put forward; they have not achieved wide acceptance. The fundamental theory described in the next chapter meets this problem by using as sources and sinks of the "fields" only quantum particles with quantized energy and momenta as well as angular momenta.

Yet another problem encountered in using second quantized relativistic field theory is that it does not have a well defined "correspondence limit". To "derive" the ordinary non-relativistic description of atoms and molecules used in elementary chemistry requires a mixture of approximations both in powers of the fine structure constant and in powers of mass ratios which for higher terms looks more like an art than a science. Quantum chromodynamics is notoriously unsuccessful in dealing with the region of "infrared slavery" where ordinary nuclear physics has its home. And whether classical field theory, particularly gravitation, should be a macroscopic consequence of second quantized relativistic field theory is an unsolved problem. We sketch in the next chapter how our new theory meets this problem. In the following chapter we relate our approach to recent work by Tanimura based on Dyson's reconstruction of Feynman's 1948 proof of the Maxwell Equations.

2. A NEW FUNDAMENTAL THEORY

An alternative fundamental theory,[2–5] which has been discussed at the three previous conferences in this series,[6] opens up new possibilities in the discussion about the relationship between classical physics and quantum mechanics. Define *particles* as the *conceptual* carriers of conserved quantum numbers between events and *events* as regions across which quantum numbers are conserved. Take as the basic paradigm for two events the sequential firing of two counters separated by distance L and time interval T, where the clocks recording the firings are synchronized using the Einstein convention. Define the velocity of the "particle" connecting these two events as $v = \beta c = L/T$ where c is the limiting velocity for the transfer of *information*. Given a beam of particles of this velocity selected by a collimator and counter telescope incident on two slits a distance w apart we find a double slit interference pattern at a detector array a distance D behind the slits whose maxima are separated by a distance s. Define the *deBroglie wavelength* $\lambda = ws/D$ using laboratory units of length. If a different source producing particles with the same velocity incident on the same arrangement gives a fringe spacing s', define the mass ratio $m'/m = s/s'$. Introduce Planck's constant h by the definition $\lambda = h/p$ where $\beta = pc/E$, $E^2 - p^2c^2 = m^2c^4$. *Postulate* that two events mediated by a particle of mass m and velocity βc can, but need not, take place *only* when they are separated by an integer number of deBroglie wavelengths.

Consider a particle bound to a center a distance r away which receives an impulsive force toward the center each time it has moved a deBroglie wavelength. Assume that the area swept out per unit time by the radial distance to the particle is constant for each step (Kepler's Second Law) and that the polygon closes after j steps. If we take $2\pi r = j\lambda$, and compute the square of the quantized angular momentum consistent with this correspondence limit we find it equal to $(j^2 - \frac{1}{4})\hbar^2 = \ell(\ell+1)\hbar^2$ where we have defined $\ell = j - \frac{1}{2}$. Assuming that the probability of the impulsive force occurring after one *Compton* wavelength is $1/137(\ell+1)$ we obtain[4] Bohr's relativistic formula $(\frac{m-\epsilon_\ell}{m})^2[1 + (\frac{1}{137(\ell+1)})^2] = 1$ for the levels of the hydrogen atom[7] in the approximation $e^2/\hbar c \approx 1/137$, and hence *his* correspondence limit. Adding a second degree of freedom gives us the Sommerfeld formula and an improvement of four significant figures[5] in our value for $e^2/\hbar c$. After deriving the commutation relations, we can invoke[8] Feynman's proof of the Maxwell Equations[9] to show that we also have the correct classical fields in the appropriate correspondence limit. For gravitational orbits about a center containing N particles of mass m, orbital velocity reaches c when $\ell = 0$ and $N = M_{Planck}/m$ where $M_{Planck} = (\frac{\hbar c}{G})^{\frac{1}{2}}$ is the Planck mass. Consequently the shortest distance (between *two* events!) in the theory is the Planck length $h/M_{Planck}c$. Thanks to the fact that our Lorentz-invariant (for finite and discrete boosts and rotations!) theory predicts both the (quantized) Newtonian interaction and spin 2 gravitons, it meets the three classical tests of general relativity. Quantitative results:

Table I. **Coupling constants and mass ratios** predicted by the finite and discrete unification of quantum mechanics and relativity. Empirical Input: $c, \hbar$ and m_p as understood in the "Review of Particle Properties", Particle Data Group, *Physics Letters*, **B 239**, 12 April 1990.

COUPLING CONSTANTS

Coupling Constant	Calculated	Observed
$G^{-1}\frac{\hbar c}{m_p^2}$	$[2^{127}+136]\times[1-\frac{1}{3\cdot 7\cdot 10}] = 1.693\ 31\ldots\times 10^{38}$	$[1.69358(21)\times 10^{38}]$
$G_F m_p^2/\hbar c$	$[256^2\sqrt{2}]^{-1}\times[1-\frac{1}{3\cdot 7}] = 1.02\ 758\ldots\times 10^{-5}$	$[1.02\ 682(2)\times 10^{-5}]$
$sin^2\theta_{Weak}$	$0.25[1-\frac{1}{3\cdot 7}]^2 = 0.2267\ldots$	$[0.2259(46)]$
$\alpha^{-1}(m_e)$	$137\times[1-\frac{1}{30\times 127}]^{-1} = 137.0359\ 674\ldots$	$[137.0359\ 895(61)]$
$G^2_{\pi N\bar{N}}$	$[(\frac{2M_N}{m_\pi})^2-1]^{\frac{1}{2}} = [195]^{\frac{1}{2}} = 13.96..$	$[13, 3(3), > 13.9?]$

MASS RATIOS

Mass ratio	Calculated	Observed
m_p/m_e	$\frac{137\pi}{\frac{3}{14}\left(1+\frac{2}{7}+\frac{4}{49}\right)\frac{4}{5}} = 1836.15\ 1497\ldots$	$[1836.15\ 2701(37)]$
$m_\pi^{\pm}/m_e$	$275[1-\frac{2}{2\cdot 3\cdot 7\cdot 7}] = 273.12\ 92\ldots$	$[273.12\ 67(4)]$
m_{π^0}/m_e	$274[1-\frac{3}{2\cdot 3\cdot 7\cdot 2}] = 264.2\ 143\ldots$	$[264.1\ 373(6)]$
m_μ/m_e	$3\cdot 7\cdot 10[1-\frac{3}{3\cdot 7\cdot 10}] = 207$	$[206.768\ 26(13)]$

COSMOLOGICAL PARAMETERS

Parameter	Calculated	Observed
N_B/N_γ	$\frac{1}{256^4} = 2.328....\times 10^{-10}$	$\approx 2\times 10^{-10}$
M_{dark}/M_{vis}	≈ 12.7	$M_{dark} > 10 M_{vis}$
$N_B - N_{\bar{B}}$	$(2^{127}+136)^2 = 2.89...\times 10^{78}$	*compatible*
ρ/ρ_{crit}	$\approx \frac{4\times 10^{79} m_p}{M_{crit}}$	$.05 < \rho/\rho_{crit} < 4$

3. THE FEYNMAN-DYSON-TANIMURA PROOF OF CLASSICAL FIELD EQUATIONS

3.1 TANIMURA's ANALYSIS OF THE PROOF

An alternative route to obtaining the correspondence limit of *relativistic* quantum mechanics in classical relativistic field equations has been opened up by Tanimura. In contrast to our finite and discrete approach, he uses continuum ideas and mathematics. Starting from Feynman's proof[9] of the Maxwell Equations, he has recently shown[10] that "...the only possible fields that can consistently act on a quantum mechanical particle are scalar, gauge and gravitational fields." Feynman's proof[9] starts from the non-relativistic position-velocity commutation relations relation $m[x_i, \dot{x}_j] = i\hbar\delta_{ij}$ and Newton's Second Law $\mathcal{F}_k = m\ddot{x}_k$.

In his relativistic generalization, Tanimura says that the proof does not require the existence of non-commuting *operators* but only that the bracket expression [,] satisfies bilinearity,

$$[\lambda A + \mu B, C] = \lambda[A, C] + \mu[B, C],$$

$$[A, \lambda B + \mu C] = \lambda[A, B] + \mu[A, C];$$

anti-symmetry,

$$[A, B] = -[B, A];$$

the Jacobi identity,

$$[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0;$$

Leibniz rule I,

$$[A, BC]] = [A, B]C + B[A, C]];$$

and Leibniz rule II,

$$\frac{d}{dt}[A,B] = [\frac{dA}{dt}, B] + [B, \frac{dA}{dt}].$$

He comments: "It is one of the virtues of Feynman's proof that there is no need of a prior existence of Hamiltonian, Lagrangian, canonical equation, or Heisenberg equation."

3.2 BASIS FOR A SCALE INVARIANT PROOF

If one examines the starting point of the proof from the point of view of *dimensional analysis*, by factoring out the mass of the particle which appears in Newton's Second Law and the bracket expression, one sees that the postulates depends only on length and time units, and hence are *scale invariant.* Then the three postulates are that

$$[x_i, x_j] = 0; \ [x_i, v_j] = i\kappa\delta_{ij}; \ f_k(x, v; t) = \dot{v}_k \tag{3.1}$$

where $f_k \equiv \mathcal{F}_k/m$ is the force per unit mass, and $\kappa = \hbar/m$ is some unit of angular momentum per unit mass. We have chosen the symbol "κ" for the unit which appears in the bracket expression and call it "Kepler's Constant" because it has dimensions of area per unit time and is the quantity which is conserved in motion past a center.[11] As usually stated, Kepler's Second Law is: *The radius vector from sun to planet sweeps out equal areas in equal times.* As extended by Newton, this includes free particle and hyperbolic motions as well as closed orbits. Thus Feynman's second postulate is equivalent to a scale invariant quantization of Kepler's Second Law.

From the point of view of dimensional analysis, the special theory of relativity is scale invariant because it only requires a special value for a single speed as a ratio of dimensional units. In SI units this universal constant c is fixed by convention to be the *integer*

$$c \equiv 299\ 792\ 458\ m\ sec^{-1} \tag{3.2}$$

This makes it sensible to restrict the velocities we consider below to the space of rational fractions.

If we set our system of units by using measurement accuracy to assign a minimum length Δx and a minimum time Δt within which it makes no operational sense to talk about length and time, we can define both c and κ in an obviously scale invariant way by

$$c \equiv \frac{\Delta x}{\Delta t}; \ \kappa \equiv \frac{\Delta x^2}{2\pi\Delta t} = \frac{c\Delta x}{2\pi} \tag{3.3}$$

Thus the minimum motion in our theory is a *Zitterbewegung* with steps of lengths Δx executed at the speed of light. The minimum angular momentum per unit mass is that of a "point" which moves at radius Δx around a center with velocity c. Thus, although Feynman starts with the Galilean invariant Second Law of Newton, and the non-relativistic commutation relation stated in terms of mass times velocity rather than momentum, dimensional analysis allows us to recast his postulate in a form that applies to the scale invariant theories of special relativistic particle mechanics and classical relativistic fields. The non-scale-invariant version of the proof given by Tanimura is obviously recovered if we have some means of measuring the *universal constant* $\hbar$ and take $\Delta x = 2\pi\hbar/mc$.

We now construct an obviously Lorentz boost invariant finite and discrete relativistic kinematics for free particles which is restricted to the space of integers. Let x be an integer in the range $-T \leq x \leq +T$ and $x\Delta x$, $t\Delta t$ the corresponding dimensional distance and time in some coordinate system and any system of length-time (LT) units. Then $v_x \equiv x/T \equiv \beta_x c$ is a rational fraction. If T is fixed, then we cannot measure the rational fraction β to an accuracy greater than $1/T$.

We now state our relativistic kinematics in both dimensionless and integer form. We pick a finite *integer* U and require that this integer have a Lorentz invariant significance. We call any integer u in the range $-U \leq u \leq +U$, the 4-velocity. We relate the 4-velocity u_x to the velocity v_x by requiring that

$$\beta_x^2 = \frac{u_x^2}{u_x^2+1} = \frac{x^2}{T^2} \tag{3.4}$$

and also require r_i, t_i to be integers. The advantage of taking 4-velocity rather than 4-momentum as basic is that then the invariant mass parameter m *factors* in the definitions

$$P \equiv mu; \; E = \beta P c \tag{3.5}$$

Our definitions now guarantee the invariance of the relation $E^2 - P^2c^2 = m^2c^4$ in any system of MLT units. Then, until we have some means of setting an ***absolute*** mass scale, scale invariance of our integer LT physics is guaranteed if there is no way to define a universal length; the generalization to MLT physics is immediate for rational mass ratios. Lorentz boosts specified by[12]

$$\beta_{ik} = \frac{\beta_{ij} + \beta_{jk}}{1 + \beta_{ij}\beta_{jk}}; \; \beta_{ij} = \frac{n_i - n_j}{n_i + n_j} \tag{3.6}$$

do not take us out of the space of rational fraction velocities provided n_i, n_j, n_k are themselves integers.

If we now consider a 4-velocity component at right angles to position, we can define another integer quantum number $2k_{xy} \equiv xu_y$ for integers and half-integers which lie in the range $\frac{1}{2} \leq k_{xy} \leq +\frac{1}{2}UT$, and a related quantum number k_z in the range $-k_{xy} \leq k_z \leq +k_{xy}$. Geometrically, these correspond to angular momenta per unit mass and can be expressed in units of κ. We now have all the ingredients in hand for a derivation of rotationally invariant, boost invariant *and scale invariant* integer and half-integer quantum numbers and bracket expressions possessing the first four of Tanimura's five properties.

To get the fifth, we need to be able to relate bracket expressions to space and time derivatives acting on functions of $x, \dot{x}$ and t. This is easy if we simply take

$$[x_i, \dot{x}_j] \equiv \kappa[x_i \frac{\partial}{\partial x_j} - \frac{\partial}{\partial x_i} x_j] \tag{3.7}$$

Note that in a scale invariant theory whether the constant we use in this definition of the bracket expression is κ,or $i\kappa$, or any other arbitrary, fixed parameter whose partial and total derivatives are zero, the proof goes through. We will discuss the connection between measurement accuracy, this definition and scale invariance on another occasion.

4. CONCLUSIONS

We assume that "fields" are to be measured by the acceleration of a "test particle" which belongs to a class of particles whose ratios of charge to mass and gravitational to inertial mass are Lorentz invariant. We relate the measurement accuracy in space, Δx, and in time, Δt, by the scale invariant definition of two constants c, and κ: $\frac{\Delta x}{c\Delta t} \equiv 1$; $\frac{\Delta x^2}{\kappa \Delta t} \equiv 2\pi$. We relate space and time derivatives of functions of $x, \dot{x}, t$ to measurement accuracy by defining $[x_i, \dot{x}_j] \equiv \kappa[x_i \frac{\partial}{\partial x_j} - \frac{\partial}{\partial x_i} x_j]$. Then it is a *deductive* consequence that the only fields which can act on such particles are structurally indistinguishable from electromagnetic and gravitational fields in the sense that they satisfy the free space Maxwell Equations and Einstein Gravitational Equations. Such a scale invariant theory becomes the proper correspondence limit for any relativistic particle theory which breaks scale invariance by taking $m_e \kappa = \hbar$. Here we use m_e because it defines the threshold distance for position measurement, $h/2m_e c$, below which the non-classical process of electron-positron pair creation is observed, and above which that phenomenon cannot be *directly* observed.

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REFERENCES

1. S.Weinberg, *Phys. Rev.*, **4 B**, 988 (1965).
2. H.P.Noyes, *Physics Today*, pp 99-100, March, 1992.
3. D.O.McGoveran and H.P.Noyes, "Foundations for a Discrete Physics", SLAC-PUB-4526 (June, 1989).
4. H.P.Noyes and D.O.McGoveran, *Physics Essays*, **2**, 76-100 (1989).
5. D.O.McGoveran and H.P.Noyes, *Physics Essays*, **4**, 115-120 (1991).
6. H.P.Noyes, et.al., SFMP, "The Measurement Problem in Program Universe",1985; "Measurement by Phase Severance",1987; "A Finite, Rational Model for the EPR-Bohm Experiment",1990.
7. N.Bohr, *Phil Mag*, **332**, Feb. 1915.
8. H.P.Noyes, "Comment on 'Feynman's Proof...'...", SLAC-PUB-5588, Nov. 1991 (unpublished).
9. F.J.Dyson, *Am. J. Phys.*, **58**, 209 (1990).
10. S.Tanimura, *Annals of Physics* **220**, 229-247 (1992).
11. H.P.Noyes, "Quantized Conic Sections; Quantum Gravity", in *Logic, Computation and Measurement; Proc. ANPA WEST 9, 1993*, F.Young, ed., ANPA WEST, 409 Lealand Ave, Stanford, CA 94306, and SLAC-PUB-6057.
12. H.P.Noyes, "The RQM Triangle: A Paradigm for Relativistic Quantum Mechanics", in *Reconstructing the Object; Proc. ANPA WEST 8, 1992*, F.Young, ed., ANPA WEST, 409 Lealand Ave, Stanford, CA 94306, and SLAC-PUB-5752.