NUCLEON FORM FACTORS

IN A RELATIVISTIC QUARK MODEL*

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Abstract

We demonstrate that a relativistic constituent-quark model can give nucleon form factors in agreement with data for both low- and high-momentum transfer. The relativistic features of the model and the specific form of the wave function are essential for the result.

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The purpose of this report is to show that a simple constituent-quark model can yield the elastic nucleon form factors in agreement with all available data up to more than 30 GeV^2. Very high momentum transfer (Q^2) behavior of elastic form factors can be obtained from perturbative QCD [1], while low energy quantities are calculated within various models. But there is still an open question of the energy scale at which the perturbative contributions are important. A model analysis of the pion [2] concluded that the nonperturbative contributions are much larger at 2 GeV^2. An analysis based on general features of models [3] found also for the nucleon that perturbative terms are unimportant in the region of present experiments. It is therefore important to have a model that is valid for all values of Q^2.

The hadronic matrix element of the radiative transition of the nucleon $N \rightarrow N'\gamma$ is represented in terms of the form factors as

$$\langle N', p' \left| J^\mu \right| N, p \rangle = \bar{u}(p') \left[F_1(Q^2)\gamma^\mu + \frac{F_2(Q^2)}{2M_N} i\sigma^{\mu\nu} Q_\nu \right] u(p) , \qquad (1)$$

with momentum transfer $Q = p' - p$, nucleon mass M_N, and quark current $J^\mu = \bar{q}\gamma^\mu q$. The Sachs form factor for the magnetic transition is given by $G_M = F_1 + F_2$. The matrix elements can be calculated within a relativistic constituent quark model on the light cone [4–6]. This approach has been extended to asymmetric wave functions [7], which provide an excellent and consistent picture of electroweak transitions of the baryon octet. In this report we focus on the high-energy behavior of the wave function.

Usually harmonic-oscillator-type wave functions are used [4–6]

$$\phi(M) = N \exp\left(-M^2/2\alpha^2\right) , \qquad (2)$$

with α being the confinement scale of the bound state and N being the normalization. The operator M is the free-mass operator of the noninteracting three-body system, and it is a function of the internal momentum variables $\vec{q}_i$ of the quarks and the quark mass m:

$$M = \sum_i \sqrt{\vec{q}_i^{\,2} + m^2} \, . \tag{3}$$

With this special form of the wave function the form factors fall off exponentially for high Q^2. This is why the form factors calculated with Eq. (2) are only valid up to 4–6 GeV^2, an energy scale well below the perturbative region [3].

The orbital wave function we use is

$$\phi(M) = \frac{N}{(M^2 + \alpha^2)^{3.5}} \, , \tag{4}$$

with a scale α and normalization factor N different from Eq. (2). The two parameters of the model, the confinement scale α and the quark mass m, have to be determined by comparison with experimental data. We find a quark mass $m = 263$ MeV and a scale $\alpha = 607$ MeV by fitting the magnetic moment of the proton $\mu(p)$ and the neutron $\mu(n)$. In addition, these parameters give also excellent results for the magnetic moments and the semileptonic decays of the baryon octet [7].

For reference, we calculate the form factors with Eq. (2) using parameters $\alpha = 560$ MeV and $m = 267$ MeV as well. The results are shown in Figs. 1–3. Note that the low-energy behavior of both wave functions is almost identical, while only the wave function in Eq. (4) fits the data. It is therefore significant to choose an appropriate Ansatz for the orbital wave function. The relativistic features of the

model are also important. In the nonrelativistic limit, $\alpha/m \to 0$, the form factor G_M for the proton is (for small Q^2)

$$\frac{G_M}{\mu(p)} = \begin{cases} \exp\left(-Q^2/\alpha^2\right) & \text{[Eq. (2)]} , \\ \left(1 + \dfrac{2Q^2}{\alpha^2 + 9m^2}\right)^{-3.5} & \text{[Eq. (4)]} , \end{cases} \tag{5}$$

which is too small for any reasonable value of α and m (compare Figs. 1 and 2). This limit shows that the relativistic treatment of the problem increases the form factors significantly, even for low values of the momentum transfer. The same effect has also been found for the pion [8]. While the asymptotic falloff for the wave function in Eq. (4) is still larger than Q^{-4}, it shows up only at very high Q^2 of over 1000 GeV^2.

We conclude that quark models with reasonable parameters can give agreement with data for all Q^2. The nonrelativistic limit is not adequate, and the specific form of the wave function is essential.

Acknowledgments

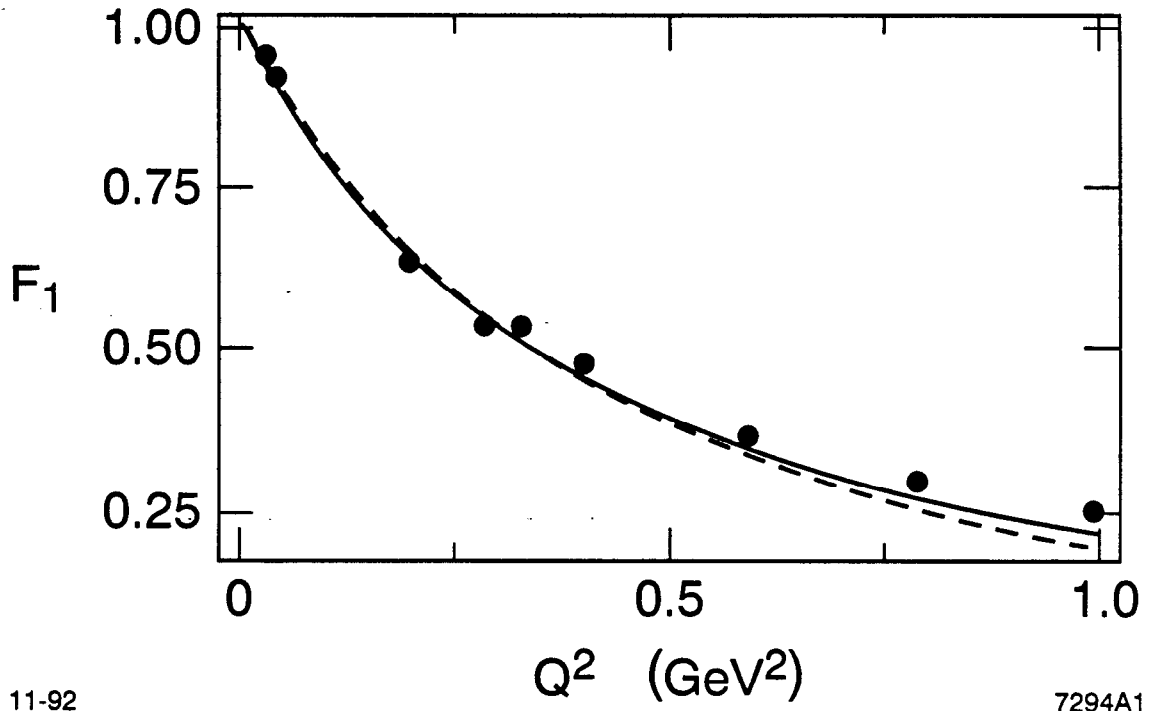I would like to thank W. Jaus for helpful discussions.

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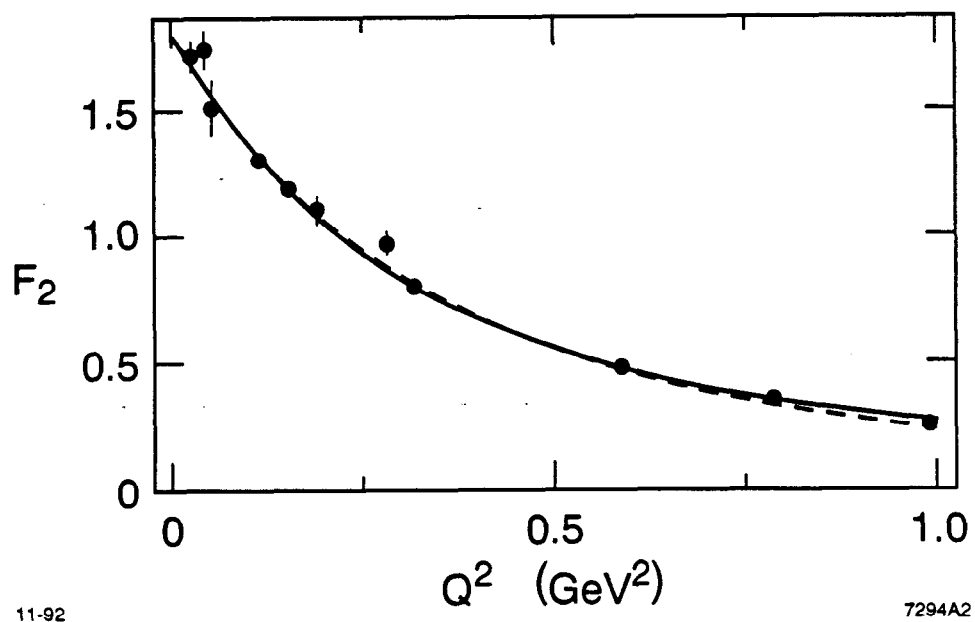
Figure Captions

1. Proton form factor $F_1(Q^2)$: continuous line, Eq. (4); broken line, Eq. (2). Experimental points are from Ref. [9].

2. Proton form factor $F_2(Q^2)$: continuous line, Eq. (4); broken line, Eq. (2). Experimental points are from Ref. [9].

3. Proton form factor $G_M(Q^2)$: continuous line, Eq. (4); broken line, Eq. (2). Experimental points are from Ref. [10].
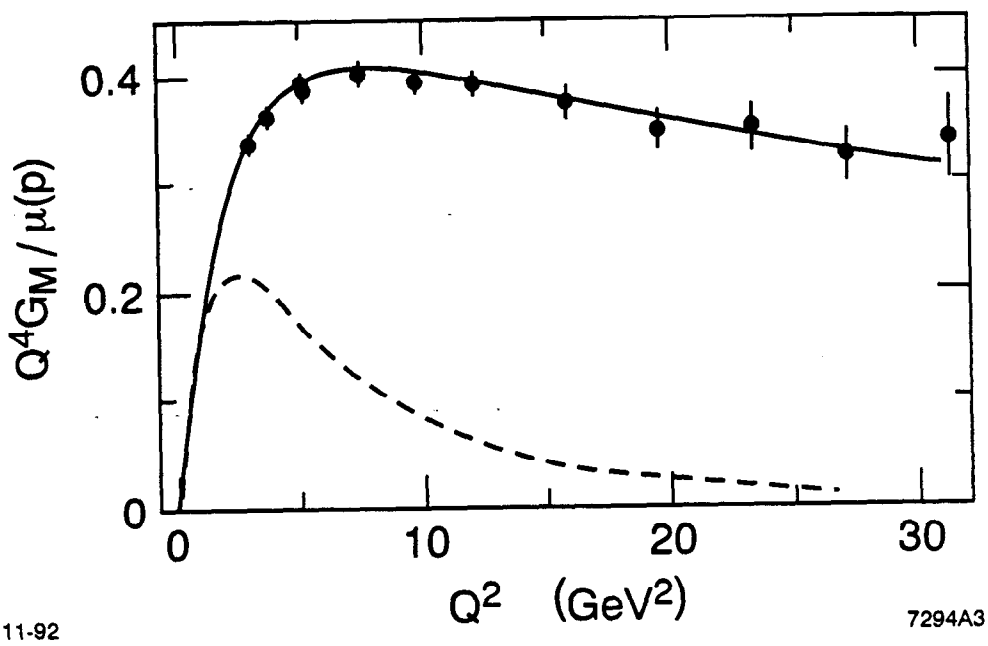
Fig. 1

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Fig. 2

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Fig. 3