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Generalized Einstein Theory on Solar and Galactic Scales *

Masakatsu Kenmoku [1]
Department of Physics
Nara Women's University, Nara 630, Japan

Yuko Okamoto [2]
Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94309, USA

Kazuyasu Shigemoto [3]
Department of Physics
Tezukayama University, Nara 631, Japan

Abstract

We study a generalized Einstein theory with the following two criteria: *i*) on the solar scale, it must be consistent with the classical tests of general relativity, *ii*) on the galactic scale, the gravitational potential is a sum of Newtonian and Yukawa potentials so that it may explain the flat rotation curves of spiral galaxies. Under these criteria, we find that such a generalized Einstein action must include at least one scalar field and one vector field as well as the quadratic term of the scalar curvature.

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[1] E-mail address: kenmoku@jpnyitp.bitnet

[2] E-mail address: yuko@slacvm ; on leave of absence from Department of Physics, Nara Women's University,Nara 630, Japan.

[3] E-mail address: shigemot@jpnyitp.bitnet

1. Introduction

Recent astrophysical observations of distant galaxy distributions [1],[2] and the cosmic microwave background [3] have revealed a quite astonishing picture of the universe. From the survey of relatively near galaxies, the void structure (the Great Wall) was discovered,[2] and from the pencil-beam survey of galaxies, the quasi-periodic distributions (of period about $130Mpc$) were inferred.[1] The data from COBE, on the other hand, have revealed extremely isotropic and homogeneous distribution of the 2.7 K cosmic microwave background with fluctuations of order 10^{-5}.[3] In general, it is very difficult to explain how these anistopic and inhomogeneous large-scale structures of the universe have developed from such an isotropic and homogeneous distribution of matter in the early stage of the universe. The standard solution to this difficulty totally relies on the existence of dark matter which accounts for more than 90% of matter in the universe. The evidence for dark matter was first claimed in order to explain the flat rotation curves of spiral galaxies. Since there is no established direct observation of dark matter, however, there are many attempts to explain the rotation curves without dark matter by modifying the Newtonian force [4] or by modifying the Newton's second force law.[5],[6] Other people have tried to derive such modified Newtonian force laws from the framework of general relativity. [7][8][9][10]

In a previous paper,[10] we attempted to explain not only the flat rotation curves of spiral galaxies but also the large-scale structure of the universe, starting from a simple model with the addition of a quadratic scalar curvature term to the Einstein action. Our generalized action could qualitatively explain the flatness of the rotation curves and the nearly periodic galaxy distributions. However, it turned out that our theory does not imply the unity of the coefficient γ of the Robertson expansion [11] on the solar scale.[10] This constraint ($\gamma = 1$) from the classical tests of general relativity such as the observation of the radar echo delay is quite stringent, and it is very difficult to realize this value in the generalized Einstein action.

In this paper, we construct the generalized Einstein action under the two criteria: *i*) it must give $\gamma = 1$ in the post Newtonian approximation, *ii*) the gravitational potential is a sum of Newtonian and Yukawa potentials. The second criterion is imposed, since it is the empirical gravitational potential of Sanders [4] that can quite successfully explain the flat rotation curves of spiral galaxies. We then show that the minimum ingredient of the theory that satisfies the above criteria is the R^2 term, a scalar field, and a vector field in addition to the Einstein action.

2. Generalized Einstein Action and Its Post Newtonian Approximation

The generalized Einstein action which contains quadratic terms of the scalar curvature, R^2, and Ricci tensor, $R_{\mu\nu}R^{\mu\nu}$, was introduced to regulate the ultraviolet divergences of the Einstein theory.[12] It was applied to cosmology to obtain the bounce universe to avoid the singularity at the creation of the universe.[13] The structure and the properties of the theory were further elaborated in subsequent works. [14]

In this section, we consider a further generalization of the theory by adding scalar and vector fields in addition to R^2 and $R_{\mu\nu}R^{\mu\nu}$ terms and study its post Newtonian approximation. We will investigate such a theory with two requirements: *i*) on the solar scale, it must conform with the classical tests of general relativity, *ii*) on the galactic scale, the gravitational potential is modified to give a sum of Newtonian and Yukawa potentials in order to explain the rotational velocity curves of spiral galaxies.

We consider the following generalized action with the scalar curvature R, the Ricci tensor $R_{\mu\nu}$, a scalar field σ, and a vector field A_μ:

$$\begin{aligned} I &= \int d^4x\sqrt{-g}\Big\{-\frac{1}{16\pi G}(R + c_1R^2 + c_2R^{\mu\nu}R_{\mu\nu}) \\ &\quad -\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{\mu^2}{2}\sigma^2 + g_1\sigma R \\ &\quad -\frac{1}{2}D_\mu A_\nu D^\mu A^\nu - \frac{m^2}{2}A^\mu A_\mu + g_2 D_\mu A^\mu R + L^{matter}\Big\} , \end{aligned} \tag{1}$$

where G is the gravitational constant, μ and m are the masses of the scalar and vector particles, D_μ is the covariant derivative, and L^{matter} is the matter Lagrangian. The coefficients c_1, c_2, and G have the dimension of $(mass)^{-2}$, while the coefficient g_1 has the dimension of mass and g_2 is dimensionless.

In order to calculate the coefficient γ in the Robertson expansion,[11] we introduce the weak fields ϕ and ψ defined by

$$g_{00} = -1 - 2\phi, \quad g_{ij} = \delta_{ij}(1 + 2\psi) \ . \tag{2}$$

In the post Newtonian approximation, we must take into account up to the quadratic term of the weak fields and source in the action, and the necessary formulae to the first order in the weak fields are

$$\sqrt{-g} \approx 1 + \phi + 3\psi \ , \quad R_{00} \approx -\Delta\phi \ ,$$

$$R_{ij} \approx \partial_i\partial_j(\phi + \psi) + \delta_{ij}\Delta\psi \ , \quad R \approx 2\Delta(\phi + 2\psi) \ , \tag{3}$$

and the necessary formulae up to the second order in the weak fields and source in the action are

$$\sqrt{-g}R \approx 2\Delta(\phi + 2\psi) + 4\phi\Delta\psi + 2\psi\Delta\psi \ ,$$

$$\sqrt{-g}R^{\mu\nu}R_{\mu\nu} \approx 2(\Delta\phi)^2 + 4\Delta\phi\Delta\psi + 6(\Delta\psi)^2 \ , \tag{4}$$

$$\sqrt{-g}R^2 \approx 4\{\Delta(\phi + 2\psi)\}^2 \ , \quad \sqrt{-g}L^{matter} \approx -\rho\phi \ ,$$

where we have suppressed the total derivative terms.[15] We substitute the weak field expressions Eqs. (3) and (4) into Eq. (1), and retain up to the quadratic terms of the weak fields $\phi, \psi, \sigma, A_\mu$ and source ρ to obtain

$$\begin{aligned} I \approx \int d^4x \Big[& -\frac{1}{16\pi G}\{2\Delta(\phi + 2\psi) + 4\phi\Delta\psi + 2\psi\Delta\psi + 4c_1(\Delta(\phi + 2\psi))^2 \\ & +4c_2(\frac{1}{2}(\Delta\phi)^2 + \Delta\phi\Delta\psi + \frac{3}{2}(\Delta\psi)^2\} \\ & +\frac{1}{2}\sigma\Delta\sigma - \frac{\mu^2}{2}\sigma^2 + \frac{1}{2}A_i\Delta A_i - \frac{m^2}{2}A_i^2 - \frac{1}{2}A_0\Delta A_0 + \frac{m^2}{2}A_0^2 \\ & +2g_1\sigma\Delta(\phi + 2\psi) + 2g_2\partial_i A_i\Delta(\phi + 2\psi) - \rho\phi \Big] \ . \end{aligned} \tag{5}$$

Since the field A_0 decouples from the other fields and source, we do not consider this field hereafter. It is convenient to introduce the following variables $\bar{\phi}$ and $\bar{\psi}$:

$$\bar{\phi} = \phi + 2\psi \ , \quad \bar{\psi} = \psi - \phi \ , \tag{6}$$

which in turn gives

$$\phi = \frac{\bar{\phi} - 2\bar{\psi}}{3} \ , \quad \psi = \frac{\bar{\phi} + \bar{\psi}}{3} \ . \tag{7}$$

Substituting Eq. (7) into Eq. (5), we can rewrite the action as

$$I \approx \int d^4x \Big[-\frac{1}{16\pi G}\{2\triangle\bar{\phi} + \frac{2}{3}\bar{\phi}\triangle\bar{\phi} - \frac{2}{3}\bar{\psi}\triangle\bar{\psi} + 4c_1(\triangle\bar{\phi})^2 + \frac{4c_2}{3}((\triangle\bar{\phi})^2 + \frac{1}{2}(\triangle\bar{\psi})^2)\}$$
$$+\frac{1}{2}\sigma\triangle\sigma - \frac{\mu^2}{2}\sigma^2 + \frac{1}{2}A_i\triangle A_i - \frac{m^2}{2}A_i^2 + 2g_1\sigma\triangle\bar{\phi} + 2g_2\partial_i A_i\triangle\bar{\phi} - \frac{\rho\bar{\phi}}{3} + \frac{2\rho\bar{\psi}}{3}\Big] . \quad (8)$$

Taking the variation with respect to the weak fields $\bar{\phi}, \bar{\psi}, A_i$ and σ, we obtain the equation of motion of the weak fields in the post Newtonian approximation:

$$\triangle\{1 + (6c_1 + 2c_2)\triangle\}\,\bar{\phi} - 24\pi G(g_1\triangle\sigma + g_2\triangle\partial_i A_i) = -4\pi G\rho \ , \quad (9)$$

$$\triangle(1 - c_2\triangle)\bar{\psi} = -8\pi G\rho \ , \quad (10)$$

$$(\triangle - m^2)A_i - 2g_2\triangle\partial_i\bar{\phi} = 0 \ , \quad (11)$$

$$(\triangle - \mu^2)\sigma + 2g_1\triangle\bar{\phi} = 0 \ . \quad (12)$$

From Eqs. (11) and (12), we have

$$\partial_i A_i = \frac{2g_2\triangle^2\bar{\phi}}{\triangle - m^2} \ , \quad \sigma = -\frac{2g_1\triangle\bar{\phi}}{\triangle - \mu^2} \ . \quad (13)$$

Substituting this expression into Eqs.(9) and (10), we then obtain the following:

$$\triangle\left\{1 + (6c_1 + 2c_2)\triangle - 48\pi G(-\frac{g_1^2\triangle}{\triangle - \mu^2} + \frac{g_2^2\triangle^2}{\triangle - m^2})\right\}\bar{\phi} = -4\pi G\rho \ , \quad (14)$$

$$\triangle(1 - c_2\triangle)\bar{\psi} = -8\pi G\rho \ . \quad (15)$$

In the next section, we will consider the possibility that the coefficient of the Robertson expansion γ becomes 1. In order to obtain this result, it is necessary that both $\bar{\phi}$ and $\bar{\psi}$ behave $\sim 1/r$ in the limit $r \to 0$. (We remark from Eq. (7): if $\bar{\phi}$ and $\bar{\psi}$ behave like $\bar{\phi} \sim$ const. and $\bar{\psi} \sim 1/r$ in the limit $r \to 0$, we obtain $\gamma = 1/2$, while if $\bar{\phi}$ and $\bar{\psi}$ behave like $\bar{\phi} \sim 1/r$

and $\bar{\psi} \sim$ const. in the limit $r \to 0$, we obtain $\gamma = -1$.) In the region $r \to 0$, the mass term is negligible, and the necessary condition to have $\gamma = 1$ is that the highest derivative terms ($\propto \Delta^2$) on the left-hand sides of Eqs. (14) and (15) must vanish (for $\mu^2 = m^2 = 0$). This condition reads

$$6c_1 + 2c_2 - 48\pi G g_2^2 = 0 \ , \qquad c_2 = 0 \ . \tag{16}$$

Near the origin we then have

$$(1 - 6c_1 m^2 + 48\pi G g_1^2)\Delta\bar{\phi} \ \approx -4\pi G\rho \ , \tag{17}$$

$$\Delta\bar{\psi} \ \approx -8\pi G\rho \ . \tag{18}$$

We assume that the density takes the point-like distribution of the form

$$\rho(\vec{r}) = M\delta(\vec{r}) \ .$$

Using the formula $4\pi G\rho/\Delta = -GM/r$, we then obtain

$$\bar{\phi} \approx -\frac{4\pi G\rho}{k\Delta} = \frac{GM}{kr} \ , \tag{19}$$

$$\bar{\psi} \approx -\frac{8\pi G\rho}{\Delta} = \frac{2GM}{r} \ , \tag{20}$$

where

$$k = 1 - 6c_1 m^2 + 48\pi G g_1^2 \ . \tag{21}$$

Therefore we obtain the gravitational potential of the form

$$\phi \approx \frac{\bar{\phi} - 2\bar{\psi}}{3} = -\frac{GM}{r}(\frac{4k-1}{3k}) \equiv -\frac{\bar{G}M}{r} \ , \tag{22}$$

$$\psi \approx \frac{\bar{\phi} + \bar{\psi}}{3} = \frac{GM}{r}(\frac{2k+1}{3k}) \equiv \frac{\bar{G}M}{r}\gamma \ , \tag{23}$$

where

$$\bar{G} = \frac{4k-1}{3k}G \ . \tag{24}$$

This leads to the following formula for γ :

$$\gamma = \frac{2k+1}{4k-1} . \tag{25}$$

Here, by taking Morikawa model [16] as an example, we demonstrate how it is difficult to satisfy the stringent condition $\gamma = 1$ in a modified Einstein theory in general.

Morikawa model is a Brans-Dicke type theory of the form [16]

$$\begin{aligned} I^{Morikawa} &= \int d^4x\sqrt{-g}\big\{-\frac{1}{16\pi G}(R-2\Lambda) \\ &-\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\mu^2}{2}\varphi^2 + \lambda\varphi + \frac{\xi\varphi^2}{2}R + L^{matter}\big\} , \end{aligned} \tag{26}$$

where we have added the cosmological term Λ and tadpole term $\lambda\varphi$ to the original Morikawa action, since we consider the case that the scalar field φ takes an expectation value v. We write $\varphi = v + \sigma$ and consider this σ field as the weak field. We then obtain the weak field approximation of the form

$$\begin{aligned} I^{Morikawa} &\approx \int d^4x\sqrt{-g}\big\{-\frac{R}{16\pi G}(1-8\pi G\xi v^2) \\ &-\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{\mu^2}{2}\sigma^2 + \xi v\sigma R + L^{matter}\big\} . \end{aligned} \tag{27}$$

Here, we have tuned Λ and λ in such a way that

$$(\frac{\Lambda}{8\pi G} - \frac{\mu^2 v^2}{2} + \lambda v) = 0 , \quad \lambda - \mu^2 v = 0 ,$$

for a given v. If we denote $1/\tilde{G} = (1-8\pi G\xi v^2)/G$, then according to our formula Eq. (21) (with the replacements $m \to 0, G \to \tilde{G}$, and $g_1 \to \xi v$), we have the expression $k = 1 + 48\pi\tilde{G}\xi^2 v^2$, which gives

$$\gamma = \frac{1-8\pi G(\xi - 4\xi^2)v^2}{1-8\pi G(\xi-8\xi^2)v^2} . \tag{28}$$

In Morikawa's case, [16] the scale factor of the universe oscillate with time, where the period is converted to the scale of $130Mpc$ through the velocity of light, and it is unclear whether φ

here takes an expectation value on the solar scale. We obtain $\gamma = 0.88$ by using Morikawa's unit $1/G = 4\pi/3$ and his values $\xi = 10$ and $v = 0.008$, where the value of v is assumed to be of the order of the initial value of φ . (Of course, if we set $v = 0$ in Morikawa model, we have $\gamma = 1$.)

Through this example, we understand that the construction of the generalized Einstein action with $\gamma = 1$ is quite non-trivial.

3. Generalized Einstein Action on Solar and Galactic Scales

In this section, we show that the generalized Einstein theory, in which the gravitational potential is a sum of Newtonian and Yukawa potentials, must include at least scalar and vector fields in addition to the quadratic term of the scalar curvature in order that the condition $\gamma = 1$ is satisfied.

The condition $\gamma = 1$ implies $k = 1$ in Eq. (25), which in turn implies $c_1 m^2 = 8\pi G g_1^2$ (see Eq. (21)). Taking into account Eq. (16) also, the necessary condition to have $\gamma = 1$ is summarized as follows:

$$\begin{aligned} c_1 &= 8\pi G g_2^2, \quad c_2 = 0 \ , && (29) \\ c_1 m^2 &= 8\pi G g_1^2 \ . && (30) \end{aligned}$$

This condition is classified into the following possibilities:

$$\begin{array}{llll} I) & g_2 \neq 0 \ , & & \\ & \quad a) & g_1 = 0 \ , & (c_1 \neq 0, \ m = 0), \\ & \quad b) & g_1 \neq 0 \ , & (c_1 \neq 0, \ m \neq 0), \\ II) & g_2 = 0 \ , & & (c_1 = 0, \ g_1 = 0). \end{array}$$

We first consider case Ia), where the scalar decouples from other fields. In this case Eqs. (14) and (15) become

$$\Delta\bar{\phi} = -4\pi G\rho, \quad \Delta\bar{\psi} = -8\pi G\rho \ , \tag{31}$$

which in turn gives

$$\bar{\phi} = -\frac{4\pi G\rho}{\Delta} = \frac{GM}{r} \ , \quad \bar{\psi} = -\frac{8\pi G\rho}{\Delta} = \frac{2GM}{r} \ . \tag{32}$$

Hence, we obtain the same result as the Newtonian approximation of the ordinary Einstein theory:

$$\phi = -\frac{GM}{r} \ , \quad \psi = \frac{GM}{r} \ , \tag{33}$$

in which the desired Yukawa term is absent.

In case *II*), the vector field decouples and there is no higher derivative term such as R^2 and $R_{\mu\nu}R^{\mu\nu}$. In this case, Eq. (14) takes the same form as Eq. (31), and we obtain the result of the Newtonian approximation again.

The final case *Ib*) turns out the one that satisfies our criteria. In this case, from the relation $c_1 m^2 = 8\pi G g_1^2$, Eq. (14) becomes

$$\begin{aligned} \frac{\triangle}{(\triangle - m^2)(\triangle - \mu^2)} \Big\{ \triangle^2 - \triangle \left(m^2 + \mu^2 + 48\pi G g_1^2 (m^2 - \mu^2) \right) \\ + m^2 \mu^2 \Big\} \bar{\phi} = -4\pi G \rho \ . \end{aligned} \tag{34}$$

We then define α and β by

$$\begin{aligned} \alpha + \beta &= m^2 + \mu^2 + 48\pi G g_1^2 (m^2 - \mu^2) \ , \\ \alpha\beta &= m^2 \mu^2 \ , \end{aligned} \tag{35}$$

and assume $\alpha > \beta$. We can also define constants k_1, k_2 and k_3 by

$$\begin{aligned} \bar{\phi} &= -\frac{4\pi G (\triangle - m^2)(\triangle - \mu^2)}{\triangle(\triangle - \alpha)(\triangle - \beta)} \rho \ , \\ &= -4\pi G \Big\{ \frac{k_1}{\triangle} + \frac{k_2}{\triangle - \alpha} + \frac{k_3}{\triangle - \beta} \Big\} \rho \ , \end{aligned} \tag{36}$$

which gives the relations among k_1, k_2 and k_3 as follows:

$$\begin{aligned} & k_1 + k_2 + k_3 = 1 \ , \\ & k_1(\alpha + \beta) + k_2 \beta + k_3 \alpha = m^2 + \mu^2 \ , \\ & k_1 \alpha\beta = m^2 \mu^2 \ . \end{aligned} \tag{37}$$

From Eq. (35) we obtain $k_1 = 1$, so that k_2 and k_3 are related by $k_3 = -k_2$. Using this relation and substituting Eq. (35) into Eq. (37), we obtain

$$k_2 = -k_3 = \frac{48\pi G g_1^2 (m^2 - \mu^2)}{\alpha - \beta} \ . \tag{38}$$

The modified gravitational potential in this case becomes

$$\begin{aligned}
\bar{\phi} &= -4\pi G\left(\frac{1}{\Delta}+\frac{k_2}{\Delta-\alpha}-\frac{k_2}{\Delta-\beta}\right)\rho\ , \\
&= \frac{GM}{r}\left(1+k_2 e^{-\sqrt{\alpha}r}-k_2 e^{-\sqrt{\beta}r}\right)\ , \qquad (39) \\
\bar{\psi} &= \frac{2GM}{r}\ .
\end{aligned}$$

Therefore we have the gravitational potential of the desired form:

$$\phi = \frac{\bar{\phi}-2\bar{\psi}}{3} = -\frac{GM}{r}\left(1-\frac{k_2}{3}e^{-\sqrt{\alpha}r}+\frac{k_2}{3}e^{-\sqrt{\beta}r}\right)\ . \qquad (40)$$

We now examine the condition for this gravitational potential to explain the flat rotation curves of spiral galaxies. We know that the gravitational potential in Sanders' form [4]

$$\phi = -\frac{GM}{r}\left(\frac{1+\alpha_S\ e^{-ar}}{1+\alpha_S}\right) \qquad (41)$$

can account for the rotation curves in a satisfactory way when $\alpha_S = -0.9$. For our potential Eq. (40) to have a similar form on the galactic scale, it is necessary to assume (for $\alpha > \beta > 0$)

$$\sqrt{\alpha} = \mathcal{O}\left(\frac{1}{r_0}\right)\ , \qquad (42)$$

$$\alpha \gg \beta > 0\ , \qquad (43)$$

where r_0 is a distance on galactic scale ($\sim$ a few $10kpc$). Note that these relations imply

$$\exp\left(-\sqrt{\beta}r_0\right) \approx 1\ . \qquad (44)$$

For the rotation curves to be flatter than the Newtonian results, we need $\alpha_S < 0$ in Eq. (41). In our potential Eq. (40) this requirement together with Eqs. (43) and (44) implies

$$k_2 > 0\ . \qquad (45)$$

By Eq. (38) this is equivalent to

$$m^2 > \mu^2\ . \qquad (46)$$

Hence, the question is whether we can satisfy the above conditions Eqs. (42), (43), and (45) (or (46)) in a consistent way. From Eqs. (35) and (46), one obvious solution is

$$\begin{aligned} \alpha &= \mathcal{O}\left(m^2\right) , \\ \beta &= \mathcal{O}\left(\mu^2\right) , \\ m^2 &= \mathcal{O}\left(\frac{1}{r_0^2}\right) \gg \mu^2 , \end{aligned} \tag{47}$$

where r_0 is the galactic scale. In this case, from Eq. (35) α can be written as

$$\alpha \approx m^2(1 + 48\pi G g_1^2) . \tag{48}$$

By defining $x = 48\pi G g_1^2 (> 0)$ and neglecting β and μ^2 in Eq. (38), we have

$$k_2 = \frac{x}{1+x} , \tag{49}$$

so that k_2 can change in the range

$$0 < k_2 < 1 . \tag{50}$$

The coefficient α_S of Eq. (41) in our theory is then written as

$$\alpha_S = -\frac{k_2}{3+k_2} = -\frac{x}{3+4x} . \tag{51}$$

Note that our α_S takes a value in the range

$$-0.25 < \alpha_S < 0 , \tag{52}$$

while Sanders takes $\alpha_S = -0.9$ to fit the rotation curves.

In this way, starting from the generalized Einstein action with additional scalar and vector fields, we can tune the parameters so that we not only have $\gamma = 1$ on the solar scale but also obtain the empirical Sanders type gravitational potential on the galactic scale.

4. Summary and Discussion

In this paper, we have examined the generalized Einstein theory which contains higher derivative terms, R^2 and $R^{\mu\nu}R_{\mu\nu}$, and satisfies the criteria: *i*) on the solar scale, it must be consistent with the classical tests of general relativity, *ii*) on the galactic scale, the gravitational potential is a sum of Newtonian and Yukawa potentials so that it may explain the flat rotation curves of spiral galaxies. We have shown that it is non-trivial to satisfy the above criteria and that at least additional scalar and vector fields are required for a consistent theory.

We have tuned the parameters of the theory so that the coefficient α_S of the Yukawa term (in Sanders' gravitational potential) is negative, which is necessary to explain the flatness of rotation curves of spiral galaxies. It will be interesting to see how well our theory can fit the rotation curves quantitatively. Numerical calculations in this direction are now in progress.

In our generalized Einstein action, even after the tuning of parameters, there are still two parameters, the vector mass m and the scalar μ, to set the scale of interest. We have chosen $1/m \sim$ galaxy scale, and it may be possible to explain the "periodic" large-scale structure of the universe by choosing $1/\mu \sim 130 Mpc$. Numerical calculations of distant galaxy distributions are also in progress.

Therefore, by taking the values of $1/m$ and $1/\mu$ as above, our generalized Einstein theory may be consistent with observations over three different distance scales: the solar, galactic and beyond galactic scales of the universe.

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