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Multibunch Motion with Nearest Neighbor Wakefield Coupling

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ABSTRACT

This paper discusses various aspects of multibunch motion in the presence of nearest neighbor wakefield coupling. Included are the solution to the problem for smooth focusing with equal bunch energies, an explanation for the apparent damping of the bunch amplitudes that is observed for weak coupling, and a treatment of the problem for discrete focusing based on the moments of the wakefield distributions in the structures.

1 INTRODUCTION

Many of the designs being considered for the next generation linear collider use multiple bunch operation to achieve the desired luminosity. One limitation in this approach comes from the coupling of the motion of the bunches due to the long-range transverse wakefields generated in the accelerator structures of the linac. If not controlled, these wakefields will produce a large growth in the transverse motion of the bunches that degrades the luminosity [1]. One means of reducing this growth is by detuning the dipole modes in the cells of the structures so that the sum of the wakefields generated in each structure decohere. At SLAC, for example, the X-band structures being developed for the Next Linear Collider (NLC) will have a 10% Gaussian detuning of the cells to reduce the wakefield sum by two orders of magnitude at the downstream bunch locations [2]. The frequency spread that can be accommodated for such detuning is usually limited, so the decoherence time is generally comparable to the bunch separation time. Hence the nearest downstream bunch is likely to experience the largest wakefield kick. If damping is used to suppress the dipoles modes, the neighboring bunch is also likely to receive the largest kick. In these cases it may be a good approximation when treating the problem of multibunch motion to consider only the nearest neighbor coupling, that is, the effect of the wakefield generated by each bunch on only its immediate downstream neighbor. In the following sections, we examine some of the characteristics of multibunch motion with this type of coupling.

2 MUTIBUNCH MOTION WITH SMOOTH FOCUSING

To formulate a solution to the multibunch motion problem for nearest neighbor coupling, we first need to define the linac configuration and the dynamics. To simplify the problem we assume a constant acceleration gradient linac with smooth focusing in which the beta function grows as the square root of the bunch energy. For specific applications of the results, we will use the linac parameters listed in Table I. For this set, the fractional change in the bunch energy on the distance scale of a betatron wavelength is small enough that the equations of motion can be reduced to a good approximation to those for a zero acceleration gradient linac of length

$$L \equiv \frac{2}{E_a}\left(\sqrt{E_f E_o} - E_o\right). \tag{1}$$

Using the values in the table, this yields a length of 1.9 *km* compared to the actual linac length of 4.9 *km*. In this zero acceleration gradient linac, it will be assumed that the bunch energies are constant and equal. Also, each bunch will be treated as a macro-particle of charge I_b that has a transverse offset of unity at the beginning of the linac.

Table I. NLC Linac Parameters

Quantity	Symbol	Value
Charge per bunch	I_b	$1 \cdot 10^{10}$
Initial Beta Function	β_o	4 *m*
Initial Linac Energy	E_o	16 *GeV*
Final Linac Energy	E_f	250 *GeV*
Acceleration Gradient	E_a	50 *MeV/m*

To represent the bunch-to-bunch coupling, we treat the wakefield as being independent of position along the linac and characterize its strength by the average of the dipole mode wakefields generated by a bunch trajectory with a fixed transverse offset in a structure. This representation ignores the effect of the variation of the bunch trajectory and wakefields within the structure, but is generally a good approximation when the betatron wavelength is large compared to the structure length (a formalism to include these effects is discussed in section 4). With this assumption, the bunch interaction can be expressed by

$$\frac{d\theta_i}{dz} = \frac{W_{bb} I_b}{E_o} x_{i-1} \tag{2}$$

which relates the angular kick to bunch i per unit length of the linac, $d\theta_i/dz$, to the transverse position, x_{i-1}, of the next upstream bunch. The coupling coefficient is the wakefield strength per unit length at one bunch separation, W_{bb}, and is normalized to the charge of the leading bunch, I_b.

With this definition of the problem, an exact solution to the equations of motion was derived. The transverse position of bunch n, x_n, at location z is

$$x_n(z) = \cos(z/\beta_o) + \sum_{j=1}^{n-1} A_{n,j} \frac{(fz)^j}{j!} \begin{cases} \cos(z/\beta_o)\,(-1)^{j/2} \\ \sin(z/\beta_o)\,(-1)^{(j-1)/2} \end{cases} \tag{3}$$

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where the upper expression is for j even, and the lower for j odd. Also,

$$A_{n,j} = \sum_{k=0}^{n-j-1} r^k \frac{j(j+2k-1)!}{k!\,(j+k)!}, \qquad (4)$$

$$f \equiv \frac{W_{bb}\beta_o I_b}{2E_o} \quad \text{and} \quad r \equiv \frac{W_{bb}\beta_o^2 I_b}{4E_o}. \qquad (5)$$

Equation 3 shows that the growth of the bunch amplitudes is characterized by a power series in fz. Thus to keep the increase in the amplitudes small at the end of the linac, one wants

$$fL < 1 \quad \text{or} \quad W_{bb} < 4.2\ MeV/m^2/10^{10}e \qquad (6)$$

for our example. In this case,

$$r < 1.1 \cdot 10^{-3} \qquad (7)$$

so the k=0 term dominates in equation 4, and hence

$$A_{n,j} \approx 1. \qquad (8)$$

Using this approximation, the results for the first three bunches are

$$\begin{aligned} x_1(z) &= \cos(z/\beta_0) \\ x_2(z) &= \cos(z/\beta_0) + fz\sin(z/\beta_0) \\ x_3(z) &= \cos(z/\beta_0) + fz\sin(z/\beta_0) - \tfrac{1}{2}(fz)^2\cos(z/\beta_0). \end{aligned} \qquad (9)$$

In studies of the NLC linac performance, the relevant quantities are the betatron amplitudes of the bunches at the end of the linac. Figure 1 shows these amplitudes for the first 6 bunches for four values of fL ranging from .25 to 1.0. For bunches beyond the sixth, the amplitudes are essentially unity. Therefore to limit the amplitude growth to below 10%, fL should be less than about 0.5.

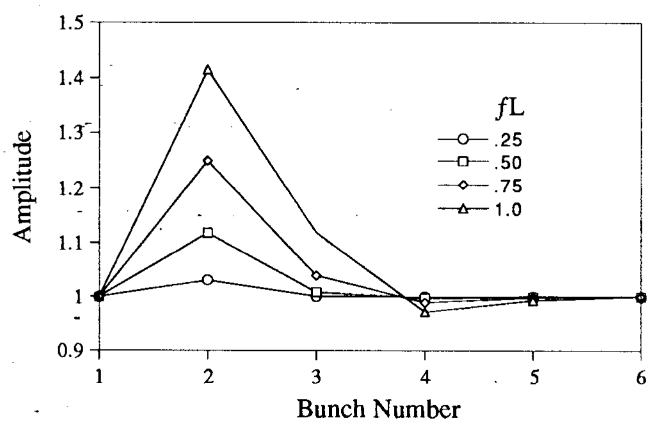

Figure 1. Amplitude versus bunch number at the end of the linac for four values of fL.

3 SUPPRESSION OF RESONANT GROWTH

The results in figure 1 are somewhat surprising in that one would naively guess that the effect of bunch 1 on bunch 2 would be the same as any other bunch pair, so the amplitudes would increase monotonically with bunch number. What is observed, however, is an apparent damping of the amplitudes after the second bunch. To see why this occurs, we first consider the motion of bunch 2 which we rewrite in the form

$$x_2 = \sqrt{1 + f^2z^2}\,\cos[kz - \tan^{-1}(fz)] \qquad (10)$$

where $k=1/\beta_0$. Since $fz \approx \tan^{-1}(fz)$ in the regime that we are considering,

$$x_2 \approx \sqrt{1 + f^2z^2}\,\cos[(k-f)z] \qquad (11)$$

so the wakefield coupling effectively lowers the wave number of bunch 2 by f. As a consequence, bunch 3 is driven off resonance by an amount that essentially cancels the "beating" term that arises in such cases. That is, to order f, the equation of motion for bunch 3,

$$\frac{d^2x_3}{dz^2} + k^2 x_3 = 2kf\cos[(k-f)z] \qquad (12)$$

has the solution

$$x_3 = \cos[(k-f)z] - 2\left\{1 - \frac{2kf}{k^2-(k-f)^2}\right\}\sin[(k-f/2)z]\sin[(f/2)z] \qquad (13)$$

which reduces to

$$x_3 = \cos[(k-f)z]. \qquad (14)$$

Likewise, bunch n, for n > 3, has a similar solution. Thus all bunches but the first effectively have an increased energy of

$$\Delta E/E = 2f/k = 4.2 \cdot 10^{-3} \text{ for } fL = 1. \qquad (15)$$

If the bunch energies vary randomly by amounts of this order or larger, one would expect that the apparent damping of the amplitudes would not occur. To investigate this, the equations for motion for unequal bunch energies were solved recursively and the solutions evaluated for different sets of Gaussianly distributed energy differences. For rms energy spreads in the range of 10^{-3} to 10^{-2}, the computed bunch amplitudes appear to vary randomly about unity as a function of bunch number. The size of this variation scales with fL, and is of the order of the increase in amplitude of bunch 2 in figure 1. For the NLC, where the energy spread tolerance on the bunches is a few parts in a thousand, the variation of the amplitudes will likely be somewhere between the "damped" and the "random" case since some of the energy variation will be correlated.

4 TWO BUNCH MOTION IN A FODO ARRAY

In defining the coupled motion problem in section 2, we ignored the effect of the variation of the wakefields and the bunch trajectories in the structures. A formalism to account for these effects readily follows from the treatment of the multibunch motion problem in linac FODO arrays where the trajectories are essentially straight lines through the structures. Such trajectories simplify the integration of the wakefield kicks along the structure and lead to a characterization of the wakefields in terms of the moments of their distribution. Assuming exact linear trajectories for the purpose of computing the driving terms in the equations of motion is generally a good approximation since the trajectory deviations due to intra-bunch transverse wakefields, acceleration and beam loading are usually small relative to the unperturbed betatron motion.

To formulate this approach, we first consider the simple case of only two bunches traversing a single unpowered structure of half-length L_h. In the approximation that the first bunch follows a straight line trajectory, its net effect on the trajectory of the second bunch can be written

$$\begin{bmatrix} \Delta x \\ \Delta\theta \end{bmatrix} = \frac{I_b}{E_o}\begin{bmatrix} -M_1 L_h & -M_2 L_h^2 \\ M_0 & M_1 L_h \end{bmatrix}\begin{bmatrix} x \\ \theta \end{bmatrix} \quad (16)$$

where Δx and $\Delta\theta$ are the effective change in the position and angle of the second bunch at the center of the structure, and x and θ are the position and angle of the first bunch at this same location. The M_i's are the moments of the wakefield distribution and are defined for an uncoupled oscillator model of a structure as

$$M_i \equiv \frac{W_o}{N}\sum_{m=1}^{N}\left(\frac{s_m}{L_h}\right)^i \sin(\omega_m l_s/c) \quad (17)$$

where s_m is the longitudinal position of cell m relative to the center of the structure, ω_m is the dipole mode frequency of cell m, and l_s is the bunch separation. In this model, the N cells of a structure are treated as independent single mode oscillators whose excitation strength is proportional to the transverse offset of the bunch trajectory at the center of the cell [2]. For simplicity, we ignore the effect of the Q's of the modes and assume that the wakefield strength, W_o, which is normalized to the length of the structure, is the same for all cells. With these definitions, the moment M_0 is the function that one usually tries to minimize by detuning the cells. It is related to W_{bb} in the smooth focusing example by

$$M_0 = 2\,L_h W_{bb}. \quad (18)$$

The additional moments, M_1 and M_2, result from the distribution of the wakefields over a finite length. This distribution can lead to a change in the trajectory of the second bunch even if the first bunch has an average offset of zero through the structure.

For a given linac configuration, the equations of motion can be solved using the representation of the bunch coupling in equation 16. For the two bunch example, we are interested in the size of the betatron amplitude induced in the second bunch from its coupling to betatron motion of the first bunch. For the simple linac configuration of a zero acceleration gradient linac with a single structure between each quadrupole, the calculation yields an expression for the induced amplitude that is independent of M_1 if the bunches travel an integral number of FODO cells in which the net betatron phase advance is an integral multiple of π. The cancellation of the contribution from M_1 is related to the symmetry of the FODO cells and also occurs to a large degree if the bunches are accelerated. Thus for linacs with many FODO cells, this moment of the wakefields can generally be ignored. In contrast, the M_2 contribution from groups of cells having π phase advance is additive in a constant energy linac, and so it increases linearly with distance as does the contribution from M_0.

As a specific example of the induced motion, we consider the linac configuration in which the bunch energy increases linearly with distance, and the beta function and quadrupole spacing increase as the square root of the bunch energy. We assume that the same type of structure is used throughout the linac, and that the structures fill the entire the space between quadrupoles starting initially with a single structure. This requires an approximation of fractional structures since the quadrupole spacing is generally not an integral number of structure lengths. For this configuration, the induced amplitude, A, of the second bunch at the end of the linac, per unit amplitude of the first bunch, is to a good approximation,

$$A = \frac{M_0 N_q \beta_c I_b}{2E_o}\left[1 + \frac{g}{3} + g\left(\frac{M_2}{M_0} - \frac{1}{3}\right)\sqrt{E_o/E_f}\right] \quad (19)$$

where

$$g \equiv \frac{\sin^2(\phi/2)}{1+\cos^2(\phi/2)}\,. \quad (20)$$

Here ϕ is the phase advance per FODO cell, β_c is the beta function at the midpoint between the first two quadrupoles, and N_q is number of quadrupoles which equals $L/(2L_h)$.

The expression for A shows that with acceleration, the relative contribution of M_2 scales as $\sqrt{E_o/E_f}$. This is actually due to the increase of the betatron wavelength as the energy increases. The relative size of M_2 and M_0 depends on the detuning parameters and the bunch separation. For the parameter ranges being considered for the NLC, the M_2/M_0 amplitude ratio computed in the uncoupled oscillator model is generally of order unity at distances of many bunch spacings, but can be as large as 20 at the nearest bunch location. For this worst case with $\phi = 90°$ and the energy values listed in table I, A is 2.5 times larger than in the short structure limit ($L_h \rightarrow 0$ with constant quadrupole spacing) where $M_2/M_0 = 1/3$. Thus it is important to also consider the size of M_2 when choosing the detuning parameters. As a final comment, we note that the induced amplitude in bunch 2 in the short structure limit is equal to the result from the smooth focusing example in section 2 when

$$\beta_o = \beta_c\,(1 + g/3). \quad (21)$$

Acknowledgments

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References

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