# Effect of Flavor-Changing Neutral Currents in the Leptonic Asymmetry in $B_{d}$ Decays 

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#### Abstract

We evaluate the charge asymmetry in equal sign dileptons arising from the decay of a $B_{d}^{0}-\overline{B^{0}}{ }_{d}$ pair, in the presence of $Z$-mediated flavor-changing neutral currents. We compare our predictions with those of the standard model and the superweak model.


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[^0]The measurement of the CP -violating leptonic charge asymmetry in the number of equal sign lepton pairs arising from the semileptonic decay of $B^{0}-\overline{B^{0}}$ pairs:

$$
\begin{equation*}
A_{S L}=\frac{N_{++}-N_{--}}{N_{++}+N_{--}} \tag{1}
\end{equation*}
$$

constitutes an important test of the Kobayashi-Maskawa (KM) mechanism of the - standard model (SM). Recently, it has been pointed out by Cocolicchio and Maiani [1] that $A_{S L}$ may actually play a crucial rôle in distinguishing the superweak model (SW) [2] from the SM. This is specially relevant since, as pointed out by Winstein [3], there is an experimentally allowed domain of the Cabibbo-Kobayashi-Maskawa (CKM) parameters where the SM gives equal values for the CP asymmetries in the decays $B^{o} \rightarrow \psi K_{s}$ and $B^{o} \rightarrow \pi^{+} \pi^{-}$. Soares and Wolfenstein [4] have pointed out that, if one takes into account both tree and penguin contributions to the decay amplitudes, the distinction between SM and SW may still be possible, provided that a third asymmetry is measured with sufficient accuracy.

In this letter we will evaluate the leptonic charge asymmetry in the presence of $Z$-mediated flavor-changing neutral currents (FCNC) and compare our predictions with those of the SM and the SW model. For definiteness, we will consider a model (VQ) where FCNC naturally arise due to the addition of a charge $-1 / 3$ vector-like quark to the standard model. In general one has [5]:

$$
\begin{equation*}
A_{S L} \simeq \operatorname{Im} \frac{\Gamma_{12}}{M_{12}} \tag{2}
\end{equation*}
$$

where $\Gamma_{12}$ and $M_{12}$ are the width and the mass transition matrix elements between $B^{\theta}$ and $\overline{B^{0}}$ states defined by $\left\langle B^{0}\right| H_{e f f}\left|\overline{B^{0}}\right\rangle=M_{12}-i \frac{1_{12}}{2}$. We use the conventions $\left\langle\overline{B^{0}}\right\rangle=|b \bar{d}\rangle$ and $C P\left\langle\overline{B^{0}}\right\rangle=-\left|B^{0}\right\rangle$ thus fixing the overall sign of $\Gamma_{12}$ and of $M_{12}$.

It has been pointed out by various authors [6] that the most likely place where physics beyond the SM could come is in new contributions to $M_{12}$. This is indeed the case in the model we are considering, where, due to the presence of FCNC, $M_{12}$ receives a new contribution from a $Z$-mediated tree diagram. The strength of this new contribution to $M_{12}$, is closely related to the size of deviations from unitarity in the (CKM) matrix [7], [8]:

$$
\begin{equation*}
V_{t b}^{*} V_{t d}+V_{c b}^{*} V_{c d}+V_{u b}^{*} V_{u d}=Z_{b d} \tag{3}
\end{equation*}
$$

where $Z_{b d}$ is defined by:

$$
\begin{equation*}
\mathcal{L}_{Z}^{b d}=-\frac{g}{2 \cos \theta_{W}} Z_{b d} \bar{b}_{L} \gamma^{\mu} d_{L} Z_{\mu} \tag{4}
\end{equation*}
$$

- It has been shown [8], [9] that one may comply to all present constraints on FCNC and at the same time have $Z_{b d}$ large enough for the tree diagram to contribute significantly to $M_{12}$. One can write [9]:

$$
\begin{equation*}
M_{12}=M_{12}^{(0)} \Delta_{d b}=M_{0} \xi_{t}^{2} \Delta_{d b} \tag{5}
\end{equation*}
$$

where $M_{12}^{(0)}$ is the box diagram contribution, $\xi_{i}=V_{i b} V_{i d}^{*}$ and:

$$
\begin{equation*}
\Delta_{d b}=1+r_{d} e^{-2 i \theta_{b d}} \tag{6}
\end{equation*}
$$

with:

$$
\begin{align*}
r_{d} & =\frac{1}{\nu\left|\bar{E}\left(x_{t}\right)\right|}\left|\frac{Z_{b d}}{V_{t b}^{*} V_{t d}}\right|^{2}  \tag{7}\\
\theta_{b d} & =\arg \left[\frac{Z_{b d}}{V_{t b}^{*} V_{t d}}\right] \tag{8}
\end{align*}
$$

where $\nu=\frac{\alpha}{4 \pi \sin ^{2} \theta_{W}}$ and $\bar{E}\left(x_{t}=\left(\frac{m_{t}}{m_{W}}\right)^{2}\right)$ is an Inami-Lim function [10] for the top quark box diagram $\left(\nu \bar{E}\left(x_{t}\right)=-0.0046\right.$ for $\left.m_{t}=140 \mathrm{GeV}\right)$.

In the SM, $A_{S L}$ is suppressed [5], [6] due to the fact that in the limit of degenerate $m_{c}, m_{u}$ masses, $\arg \Gamma_{12}=\arg M_{12}$, and $A_{S L}$ vanishes, as it should since in that limit CP is not violated in the SM . In the VQ model, it can be easily shown that there is CP violation even in the limit of $m_{c}=m_{u}$, and as a result $A_{S L}$ can be about an order of magnitude larger than in the SM. Since the $Z$ exchange diagram contribution to the decay rate is negligible, one has [11]: :-

$$
\begin{equation*}
\Gamma_{12}=-\frac{G_{F}^{2} f_{B}^{2} B M_{B}^{3}}{8 \pi}\left[\xi_{u}^{2} X_{u u}+\xi_{c}^{2} X_{c c}+2 \xi_{u} \xi_{c} X_{u c}\right] \tag{9}
\end{equation*}
$$

where the overall sign is convention dependent as already pointed out.
Using Eq.(3) one obtains:

$$
\begin{align*}
\Gamma_{12}= & -\frac{G_{F}^{2} f_{B}^{2} B M_{B}^{3}}{8 \pi}\left[\xi_{u}^{2}\left(X_{u u}-X_{u c}\right)+\xi_{c}^{2}\left(X_{c c}-X_{u c}\right)\right.  \tag{10}\\
& \left.+\xi_{t}^{2} X_{u c}+\left(Z_{b d}^{*}\right)^{2} X_{u c}-2 Z_{b d}^{*} \xi_{t} X_{u c}\right]
\end{align*}
$$

The coefficients $X_{i j}$ that appear in $\Gamma_{12}$ were calculated by several authors by taking the absorptive part of the box diagrams and correcting the result for QCD effects [12]. There has been some discussion in the literature about the theoretical uncertainties of this calculation [13], [11] and it was pointed out that the individual terms such as $X_{u c}$ are fairly sensitive to QCD corrections whilst in the differences the effects are rather small due to cancellations. We will use the values chosen in [1], $X_{u u}=0.920, X_{c c}=0.714$ and $X_{u c}=0.820$.

Rewriting Eq.(5) in the form:

$$
\begin{equation*}
M_{12}=\left|M_{12}\right| e^{i \arg \xi_{t}^{2}} e^{i \arg \Delta_{d b}} \tag{11}
\end{equation*}
$$

and introducing this expression in Eq. (2) we obtain for $A_{S L}$ in the VQ model:

$$
\begin{align*}
A_{S L}(V Q)= & \frac{1}{\left|M_{12}\right|}\left\{\operatorname{Im}\left(\Gamma_{12} e^{-i \arg \xi_{t}^{2}}\right) \cos \left(\arg \Delta_{d b}\right)\right. \\
& \left.-\operatorname{Re}\left(\Gamma_{12} e^{-i \arg \xi_{t}^{2}}\right) \sin \left(\arg \Delta_{d b}\right)\right\} \tag{12}
\end{align*}
$$

This equation clearly shows some of the new features of our model. In fact in "the SM we only have a contribution proportional to the first term and this term is small due to the suppression mentioned before. In the VQ model the second term can give a substantial enhancement to the asymmetry. The corresponding expression for the SW model only has a contribution proportional to the second term and again the asymmetry can be much larger than in the SM.

From Eqs.(2),(5), and (10) one finally obtains the explicit expression for $A_{S L}$ in-the VQ model:

$$
\begin{align*}
A_{S L}(V Q)= & -C\left\{\left|\xi_{u}^{2}\right| \sin \left(2 \alpha-\arg \Delta_{d b}\right)\left(X_{u u}-X_{u c}\right)\right. \\
& +\left|\xi_{c}^{2}\right| \sin \left(-2 \beta-\arg \Delta_{d b}\right)\left(X_{c c}-X_{u c}\right) \\
& +\left|\xi_{t}\right|^{2} \sin \left(-\arg \Delta_{d b}\right) X_{u c}  \tag{13}\\
& +\left|Z_{b d}\right|^{2} \sin \left(-2 \theta_{b d}-\arg \Delta_{d b}\right) X_{u c} \\
& \left.-2\left|Z_{b d}\right|\left|\xi_{t}\right| \sin \left(-\theta_{b d}-\arg \Delta_{d b}\right) X_{u c}\right\}
\end{align*}
$$

where $C=\frac{G_{F}^{2} f_{B}^{2} B M_{B}^{3} \tau_{B}}{4 \pi x_{d}}$. We have taken into account that $x_{d}=\frac{\Delta M}{\Gamma} \simeq 2\left|M_{12}\right| \tau_{B}$ and we have introduced the rephasing invariant phases $\alpha=\arg \left(-\frac{V_{t d} V_{b}^{*}}{V_{u d} V_{u b}^{*}}\right)$, $\beta_{0}=\arg \left(-\frac{V_{c d} V_{c t}^{*}}{V_{t d} V_{t b}^{*}}\right)$. One can obtain each term of Eq.(12) in terms of the parameters of our model by expanding the sines in Eq.(13). For comparison with Eq.(13), it
is instructive to write down the corresponding expressions for the standard model $A_{S L}(S M)$ [11] and the superweak model $A_{S L}(S W)[1]:$

$$
\begin{align*}
A_{S L}(S M)= & -C\left\{\left|\xi_{u}^{2}\right| \sin (2 \alpha)\left(X_{u u}-X_{u c}\right)+\left|\xi_{c}^{2}\right| \sin (-2 \beta)\left(X_{c c}-X_{u c}\right)\right\}  \tag{14}\\
A_{S L}(S W)= & -C \sin \theta_{B}\left\{\left(\xi_{u}^{S W}\right)^{2}\left(X_{u u}-X_{u c}\right)\right. \\
& \left.+\left(\xi_{c}^{S W}\right)^{2}\left(X_{c c}-X_{u c}\right)+\left(\xi_{t}^{S W}\right)^{2} X_{u c}\right\} \tag{15}
\end{align*}
$$

where $\sin \theta_{B}=-\sin \left[\arg \left(M_{12}\right)\right]$ is the basic input parameter of the SW theory.
It is important to bear in mind that the angles $\alpha$ and $\beta$ in the VQ model can vary in a much wider range than the one allowed in the SM since they are defined as internal angles of a unitarity quadrangle given by Eq.(3) and depend on the weight of physics beyond the standard model (i.e., the ratio of $\left.\left|Z_{b d}\right| /\left|\xi_{t}\right|\right)$. Fig. 1 shows the graphical representation of the relevant unitarity relation for the VQ model.

The first two terms of Eq.(13) correspond to the contributions already present in the SM, with the typical suppression proportional to $X_{u u}-X_{u c} \simeq-\left(X_{c c}-X_{u c}\right)$. It is worth noting that both in the VQ model and the SW model, the dominant contribution to the asymmetry comes from a term proportional to $\left|\xi_{t}\right|^{2}$, where the above suppression no longer exists, since the coefficient is the "natural scale" $X_{u c}$. The last two terms in $A_{S L}(V Q)$ are suppressed by $\left|Z_{b d}\right|$. Obviously, in the limit $\triangle_{d b}=1$, one recovers the prediction of the Standard Model.

In order to investigate how the $V Q$ model can deviate from the $S M$ predictions we: did a numerical analysis, using central values for our variables. Our emphasis is in the study of correlations among the various asymmetries and in the comparison
with the corresponding correlations [1],[14] in the SM and SW models. We chose the values $m_{t}=140 \mathrm{GeV},\left|V_{u d}\right|=0.9744,\left|V_{c d}\right|=0.204,\left|V_{u b}\right| /\left|V_{c b}\right|=0.1,\left|V_{c b}\right|=0.041$ and , $\sqrt{B_{B} f_{B}^{2}}=0.2 G e V$, consistent [15] with $\eta_{Q C D}=0.55$ (this is the QCD correction factor to the box calculation). We took the central value of $\tau_{B}=1.28 \pm 0.06 p s$ (recent world average [16]) and of $x_{d}=0.67 \pm 0.10$ (from ARGUS, CLEO and LEP [17]). $\left|\xi_{u}\right|,\left|\xi_{c}\right|$ are thus fixed while $\left|\xi_{t}\right|$ is constrained by the equation:

$$
\begin{equation*}
\left|\xi_{t}\right|\left|\Delta_{b d}\right|^{1 / 2}=\left[\frac{6 \pi^{2}}{G_{F} \eta_{Q C D} M_{W}^{2} M_{B}}\right]^{1 / 2}\left[\frac{x_{d}^{1 / 2}}{\tau_{B}^{1 / 2} B_{B}^{1 / 2} f_{B}}\right] \frac{1}{\sqrt{E\left(x_{t}\right)}} \tag{16}
\end{equation*}
$$

We studied the asymmetry correlations for two fixed values of $r_{d}$ and let $\theta_{b d}$ vary from 0 to $2 \pi$. In each case the variables $\left|\Delta_{b d}\right|, \arg \Delta_{b d}$ are readily obtained from Eq.(6) and $\left|Z_{b d}\right|$ is fixed by Eq.(7). The angles $\alpha$ and $\beta$ can be computed using trigonometrical relations derived from Fig. 1 (the exact expressions were given in Ref. [9]).

Our numerical results are summarized in Figs.2-5. In Figs.2,3 we give $A_{S L}(V Q)$ versus $A_{\psi K_{S}}(V Q)$ for two different values of $r_{d}$. It is clear that $A_{S L}(V Q)$ can be substantially larger than the SM prediction just like $A_{S L}(S W)$ as shown by Cocolicchio and Maiani [1]. Furthermore, the VQ model is distinguishable from both the SM and the SW models since it gives a significantly different correlation curve. We cannot obtain an arbitrarily large semileptonic asymmetry in the VQ model by simply increasing the weight of physics beyond the standard model, given by $r_{d}$, because our dominant contribution is proportional to $\left|\xi_{t}\right|^{2}$ which has to obey the constraint given by Eq.(16); only for extremely large values of $r_{d}$, already ectuded by experiment, the terms in $\left|Z_{b d}\right|$ would become dominant. In Figs.4,5 we give $A_{\psi K_{S}}(V Q)$ versus $A_{\pi^{+} \pi^{-}}(V Q)$ and include, for comparison, the prediction
of the SM [14]. As recently emphasized by Soares and Wolfenstein [14], the SM predicts a strong correlation between $A_{\psi K_{S}}(S M)$ and $A_{\pi^{+} \pi^{-}}(S M)$ implying a very restricted allowed region for these asymmetries. It is clear from Figs. 4,5 that the VQ model can have drastically different predictions even with the central values chosen for the parameters.

In conclusion, we have shown that in the VQ model $A_{S L}$ can be signifi-- cantly larger than in the SM and that a study of the correlations $\left(A_{S L}, A_{\psi K_{S}}\right)$ $\left(A_{\psi K_{S}}, A_{\pi^{+} \pi^{-}}\right)$offers a promissing way of detecting new sources of CP violation and in particular distinguishing the VQ model from the SM and the SW models.

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## FIGURE CAPTIONS

Figure 1: The unitarity quadrangle in the $B_{d}$ sector, corresponding to Eq.(3).
Figure 2: $A_{S L}$ versus $A_{\psi K_{s}}$ for $r_{d}=0.6$. Several values of $\theta_{b d}$ are explicitly indicated. The arrows give the direction of variation with $\theta_{b d}$ in the range from 0 to $2 \pi$. For this figure we used $m_{t}=140 G e v$ and the other parameter values given in the text. The lines are broken because for some values of $\theta_{b d}$ the unitarity quadrangle does not close.
$\because$ Figure 3: The same as in Fig.2, but with $r_{d}=2.5$. In this case all values of $\theta_{b d}$ are allowed by Eq.(3).

Figure 4: $A_{\pi^{+} \pi^{-}}$versus $A_{\psi K_{S}}$ for $r_{d}=0.6$. The solid line gives the VQ model prediction with the choice of central values for the parameters given in the text and $m_{t}=140 \mathrm{Gev}$. The dashed line gives the SW prediction and the dotted line errcloses the allowed region in the SM, as obtained in Ref.[14], when the penguin contribution to $A_{\pi^{+} \pi^{-}}$is considered.

Figure 5: The same as in Fig 4, but with $r_{d}=2.5$. Again, we include the SW prediction and the allowed region in the SM.


Fig. 1


Fig. 2


Fig. 3


Fig. 4


Fig. 5


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