SLAC–PUB–5917
September 1992
T/E

QCD ON THE LIGHT CONE*

STANLEY J. BRODSKY

Stanford Linear Accelerator Center

Stanford University, Stanford, California 94309

Invited talk presented at the Workshop
"QCD – 20 Years Later"
RWTH, Aachen, Germany
June 9–13, 1992

* Work supported by the Department of Energy, contract DE–AC03–76SF00515.

QCD ON THE LIGHT CONE

STANLEY J. BRODSKY
Stanford Linear Accelerator Center, Stanford University
Stanford, CA 94309 USA

ABSTRACT

The quantization of gauge theory at fixed light-cone time $\tau = t - z/c$ provides new perspectives for solving non-perturbative problems in quantum chromodynamics. The light-cone Fock state expansion provides both a precise definition of the relativistic wavefunctions of hadrons as bound-states of quarks and gluons and a general calculus for predicting QCD processes at the amplitude level. Applications to exclusive processes and weak decay amplitudes are discussed. The problem of computing the hadronic spectrum and the corresponding light-cone wavefunctions of QCD in one space and one time dimension has been successfully reduced to the diagonalization of a discrete representation of the light-cone Hamiltonian. The problems confronting the solution of gauge theories in 3 + 1 dimensions in the light-cone quantization formalism, including zero modes and non-perturbative renormalization, are reviewed.

1. Introduction

A primary goal of particle physics is to understand the structure of hadrons in terms of their fundamental quark and gluon degrees of freedom. It is important to predict not only the spectrum of the hadrons, but also to derive from first principles the hadron structure functions that control inclusive reactions, the form of the hadron distribution amplitudes that control exclusive processes, and the behavior of the fragmentation functions which control the transition between quark and gluon jets and hadrons. Such questions will evidently require an understanding of confinement and other properties of non-perturbative quantum chromodynamics at the amplitude level. The first, but non-trivial, step toward this goal is to give a consistent definition of hadron wavefunctions, the amplitudes which describe a composite system consisting of an arbitrary number of confined relativistic quarks and gluons.

There are many reasons why detailed information on hadron wavefunctions in QCD is critical for future progress in particle physics. For example, in electroweak theory, the central unknown required for reliable calculations of weak decay amplitudes are the hadronic matrix elements: the computation of the B meson decay into particular hadron channels requires detailed knowledge of both the light and heavy hadron wavefunctions. The coefficient functions in the operator product expansion needed to compute leading and higher twist structure functions and other inclusive cross sections are also essentially unknown. Form factors and exclusive scattering processes depend in detail on the basic amplitude structure of the scattering hadrons in a general Lorentz frame. Even the calculation of the proton magnetic moment requires an understanding of hadron wavefunctions in a boosted frame.

In this talk I will discuss the light-cone quantization of gauge theories from two perspectives: as a language for representing hadrons as QCD bound-states of relativistic quarks and gluons, and also as a novel method for simulating quantum field theory on a computer. The light-cone Fock state expansion of wavefunctions at fixed light-cone time in fact provides a precise definition of the parton model and a general calculus for hadronic matrix elements. The Hamiltonian formulation of quantum field theory quantized at fixed light-cone time has led to new non-perturbative calculational tools for numerically solving quantum field theories.[1] In particular, the "discretized light-cone quantization," method (DLCQ)[2] has been successfully applied to several gauge theories, including QCD in one-space and one-time dimension, and quantum electrodynamics in physical space-time at large coupling strength. Other non-perturbative methods based on light-cone quantization, such as the transverse lattice[3] and the Light-Front Tamm-Dancoff method[4] are also being developed as new alternatives to conventional lattice gauge theory.

There have been relatively few calculations of the wavefunctions of hadrons from first principles in QCD. The most interesting progress has come from QCD sum rule calculations,[5] and lattice gauge theory[6,7] both of which have provided predictions for the lowest moments $\langle x_i^n \rangle$ of the proton's distribution amplitude, $\phi_p(x_i, Q)$. The distribution amplitude is the fundamental gauge invariant wavefunction which describes the fractional longitudinal momentum distributions of the valence quarks in a hadron integrated over transverse momentum up to the scale Q.[8] However, the results from the two analyses are in strong disagreement: the QCD sum rule analysis predicts a strongly asymmetric three-quark distribution (see Fig. 1), whereas the lattice results,[7] obtained in the quenched approximation, favor a symmetric distribution in the x_i. Models of the proton distribution amplitude based on a quark–di-quark structure suggest strong asymmetries and strong spin-correlations in the baryon wavefunctions.[9] Even less is known from first principles in non-perturbative QCD about the gluon and non-valence quark contributions to the proton wavefunction, although data from a number of experiments now suggest non-trivial spin correlations, a significant strangeness content, and a large x component to the charm quark distribution in the proton.[10]

It is also interesting to note that light-cone wavefunctions of the projectile hadron in large measure control the distributions of final state hadrons produced in the fragmentation region of inclusive processes $AB \to CX$. At high energies, the Fock states of large invariant mass $\mathcal{M}$ survive for times $T = 2P_{\rm lab}/\mathcal{M}^2$ and are materialized by the interactions of the slowest parton spectators in the target. Because

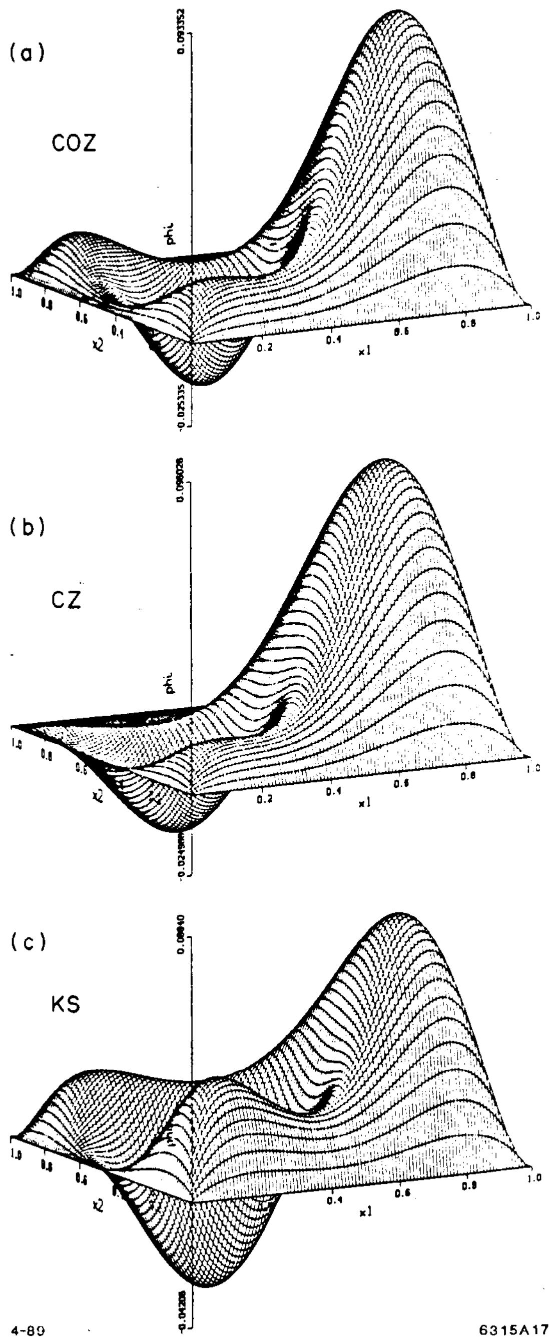

Figure 1. The proton distribution amplitude $\phi_P(x_i, \mu)$ evaluated at the scale $\mu \sim 1^G eV$ from QCD sum rules.[5] The enhancement at large x_1 correspond to a strong correlation between the a high momentum u quark with spin parallel to the proton spin.

of color screening, small color singlet configurations in the projectile Fock state can penetrate the target with minimal QCD interactions whereas large transverse size color fluctuations interact strongly in the target. These considerations can help explain many of the features of Feynman-scaling distributions, including the nuclear dependence on x_F and the size of the multiplicity fluctuations and leading charm production. Further details may be found in Ref. 11.

2. Quantization on the Light-Cone

By far the simplest and most intuitive representation of relativistic bound state wavefunctions is the light-cone Fock expansion. In 1949 Dirac[12] showed that there are remarkable advantages of quantizing relativistic field theories at fixed "light-cone time" $\tau = t+z/c$ rather than ordinary time. In the traditional equal-time Hamiltonian formulation none of the Poincare operators that generate Lorentz boosts commute with the Hamiltonian; thus computing a boosted wavefunction is as complicated a dynamical problem as diagonalizing the Hamiltonian itself. On the other hand, quantization on the light-cone can be formulated without reference to the choice of a specific Lorentz frame; the eigensolutions of the light-cone Hamiltonian, the generator of translations in τ, describe bound states of arbitrary four-momentum, allowing the computation of scattering amplitudes and other dynamical quantities. Another remarkable feature of this formalism is the apparent simplicity of the light-cone vacuum. In many theories the vacuum state of the free Hamiltonian is an eigenstate of the total light-cone Hamiltonian. In principle, the Fock expansion constructed on this vacuum state provides a complete relativistic many-particle basis for diagonalizing the full theory.

There are advantages of light-cone quantization even in ordinary quantum mechanics. Consider an experiment which could specify the initial wavefunction of a multi-electron atom. Determining $\Psi(\vec{r}_i, t = 0), i = 1, \ldots n$ would require the simultaneous measurement of the positions of the n bound electrons. In principle this could be carried out by the simultaneous Compton scattering of n independent laser beams on the atom. In contrast, determining the initial wavefunction at a fixed light-cone time τ requires only the scattering of one plane-wave laser beam since the signal reaching each of the electrons is received along the light front at the same light-cone time $\tau = t_i + z_i/c$.

In the case of perturbation theory, light-cone quantization has overwhelming advantages over standard time-ordered perturbation theory. In order to calculate a Feynman amplitude of order g^n in TOPTH one must suffer the calculation of n time-ordered graphs, each of which is a non-covariant function of energy denominators which, in turn, consist of sums of complicated square roots $p_i^0 = \sqrt{\vec{p}_i^{\,2} + m_i^2}$. On the other hand, in light-cone perturbation theory (LCPTH), only a relatively few graphs give non-zero contributions, and those that are non-zero have light-cone energy denominators which are simple sums of rational forms $p^- = (\vec{p}_{\perp i}^{\,2} + m_i^2)/p_i^+$. An analog of light-cone perturbation theory has in fact been used to calculate the anomalous magnetic moment to two loops in QED.[13]

In light-cone quantization, a free particle is specified by its four momentum $k^\mu = (k^+, k^-, k_\perp)$ where $k^\pm = k^0 \pm k^3$. Since it has positive energy, its light-cone energy is also positive: $k^- = (k_\perp^2 + m^2)/k^+ > 0$. In perturbation theory, transverse momentum $\sum k_\perp$ and the plus momentum $\sum k^+$ are conserved at each vertex. The light-cone bound-state wavefunction thus describes constituents which are on their mass shell, but off the light-cone energy shell: $P^- < \sum k_i^-$.

In principle, the problem of computing the spectrum in QCD and the corresponding light-cone wavefunctions for each hadron can be reduced to the diagonalization of the Fock state matrix representation of the QCD light-cone Hamiltonian in analogy to Heisenberg quantum mechanics. Any hadron state must be an eigenstate of the light-cone Hamiltonian. (For convenience we will work in the "standard" frame where $\underline{P}_\pi \equiv (P^+, P_\perp) = (1, 0_\perp)$ and $P_\pi^- = M_\pi^2$.) Thus the state $|\pi\rangle$ satisfies an equation

$$(M_\pi^2 - H_{LC})\,|\pi\rangle = 0.$$

Projecting this onto the various Fock states $\langle q\overline{q}|$, $\langle q\overline{q}g|\ldots$ results in an infinite number of coupled integral eigenvalue equations,[8]

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^{\,2} + m_i^2}{x_i}\right)\begin{bmatrix}\psi_{q\overline{q}/\pi}\\ \psi_{q\overline{q}g/\pi}\\ \vdots\end{bmatrix}$$

$$= \begin{bmatrix}\langle q\overline{q}|\,V\,|q\overline{q}\rangle & \langle q\overline{q}|\,V\,|q\overline{q}g\rangle & \cdots\\ \langle q\overline{q}g|\,V\,|q\overline{q}\rangle & \langle q\overline{q}g|\,V\,|q\overline{q}g\rangle & \cdots\\ \vdots & \vdots & \ddots\end{bmatrix}\begin{bmatrix}\psi_{q\overline{q}/\pi}\\ \psi_{q\overline{q}g/\pi}\\ \vdots\end{bmatrix}$$

where V is the interaction part of H_{LC}. Diagrammatically, V involves completely irreducible interactions—*i.e.* diagrams having no internal propagators—coupling Fock states. (See Fig. 2.) The explicit forms of each matrix element of V are given in Ref. 2. In principle,the solutions to these equations determine not only the hadronic spectrum of QCD but also the light-cone wavefunctions needed to compute hadronic amplitudes.

Recently a new numerical method, discretized light-cone quantization (DLCQ), has been developed to diagonalize the light-cone Hamiltonian on a covariantly regulated discrete basis.[2] By imposing periodic or anti-periodic boundary conditions of the fields in x^- and $x_\perp$, and an upper bound on the invariant mass of the particles in the Fock space

$$\mathcal{M}_0^2 = \sum_{i=1}^{n} \frac{k_\perp^2 + m^2}{x} < \Lambda^2$$

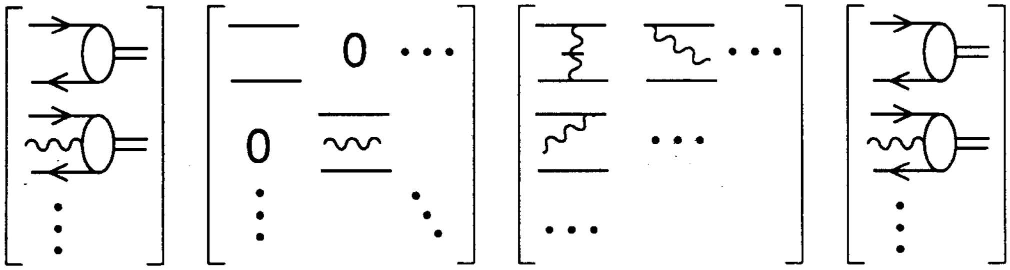

Figure 2. Coupled eigenvalue equations for the light-cone wavefunctions of a pion.

(the "global cutoff"), one obtains a discrete momentum space matrix representation of the light-cone Hamiltonian. The DLCQ method thus provides a new type of computer simulation of quantum field theories in momentum space. Since only relative coordinates appear, the formulation is completely independent of the total momentum p^+ and $\vec{p}_\perp$ of the system. By using light-cone gauge, only the minimum number of physical degrees of freedom appear in the simulation. Unlike lattice gauge theory, DLCQ has no fermion doubling problem.

The DLCQ method thus converts the problem of solving a quantum field theory to the diagonalization of the light-cone Hamiltonian on a discrete Fock-space basis

$$\langle n \,|\, H_{LC} \,|\, m \rangle \ \langle m \,|\, \psi \rangle = \mathcal{M}^2 \,\langle n \,|\, \psi \rangle \ .$$

Its most dramatic success has been the applications to quantum field theories in one-space and one-time dimensions. The DLCQ method was first used to obtain the mass spectrum and wavefunctions of Yukawa theory, $\overline{\psi}\psi\phi$, in one-space and one-time dimensions.[14] This success led to further applications including QED(1+1) for general mass fermions and the massless Schwinger model by Eller *et al.*,[15] ϕ^4 theory in 1+1 dimensions by Harindranath and Vary,[16] and QCD(1+1) for $N_C = 2,3,4$ by Hornbostel *et al.*[17] Complete numerical solutions have been obtained for the meson and baryon spectra as well as their respective light-cone Fock state wavefunctions for general values of the coupling constant, quark masses, and color. Similar results for QCD(1+1) were also obtained by Burkardt[18] by solving the coupled light-cone integral equation in the low particle number sector. Burkardt was also able to study non-additive nuclear effects in the structure functions of nuclear states in QCD(1+1). In each of these applications, the mass spectrum and wavefunctions were successfully obtained, and all results agree with previous analytical and numerical work, where they were available. More recently, Hiller[19] has used DLCQ and the Lanczos algorithm for matrix diagonalization method to compute the annihilation cross section, structure functions and form factors in 1+1 theories. Although these are just toy models, they do exhibit confinement and are excellent tests of the light-cone Fock methods.

In the case of gauge theories in one-space and one-time dimension, there are no physical gluon degrees of freedom in light-cone gauge. The computational problem is thus tractable, and it is possible to explicitly diagonalize the light-cone Hamiltonian and solve these theories numerically. In the work of Hornbostel et al,[17] complete numerical solutions for the spectrum and light-cone wavefunctions in QCD(1+1) can be obtained for any value of the coupling strength and quark masses and any number of flavor and color numbers.

A related approach, the light-front Tamm-Dancoff method (LFTD),[4] has also been proposed to solve the light-cone equation of motion. As in the traditional Tamm-Dancoff method, the light-cone Fock space is truncated to a fixed particle number, and cutoffs are imposed on the maximum transverse momentum and minimum k_i^+. Renormalization counterterms are then introduced to restore the QCD symmetries violated by the Fock space truncation.

The application of the DLCQ and LFTD methods to QCD in physical space-time is a highly challenging problem. The size of the quark and gluon Fock space and the discretization of the transverse momenta leads quickly to very large matrices. A more subtle difficulty is the necessity to include zero mode contributions enforced by the equations of motion and the imposed boundary conditions. The effective Hamiltonian must also be supplemented by terms specified by the ultraviolet renormalization procedure. Despite these challenges, the light-cone methods have been successfully been applied to QED(3+1)[20,21,22] at couplings $\alpha \sim 0.3$. For example, Kaluza and Pauli[21] have computed the structure functions of QED bound states, the lepton and photon light-cone momentum distributions of positronium. I will return to the discussion of the successes and problems of the DLCQ method in section 7.

It is thus natural to employ the light-cone Fock expansion as the basis for representing the physical states of QCD. For example, a pion with momentum $\underline{P} = (P^+, \vec{P}_\perp)$ is described by expansion over color-singlet eigenstates of the free QCD light-cone Hamiltonian:

$$|\pi : \underline{P}\rangle = \sum_{n,\lambda_i} \int \overline{\prod_i} \frac{dx_i d^2\vec{k}_{\perp i}}{\sqrt{x_i} 16\pi^3} \left| n : x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i \right\rangle \psi_{n/\pi}(x_i, \vec{k}_{\perp i}, \lambda_i)$$

where the sum is over all Fock states and helicities, and where

$$\overline{\prod_i} dx_i \equiv \prod_i dx_i\, \delta\left(1 - \sum_j x_j\right)$$

$$\overline{\prod_i} d^2\vec{k}_{\perp i} \equiv \prod_i d^2\vec{k}_{\perp i}\, 16\pi^3\, \delta^2\left(\sum_j \vec{k}_{\perp j}\right).$$

The wavefunction $\psi_{n/\pi}(x_i, \vec{k}_{\perp i}, \lambda_i)$ is the amplitude for finding partons in a specific light-cone Fock state n with momenta $(x_i P^+, x_i \overrightarrow{P}_\perp + \vec{k}_{\perp i})$ in the pion. The Fock state is off the light-cone energy shell: $\sum k_i^- > P^-$. The light-cone momentum coordinates x_i, with $\sum_{i=1}^n x_i$ and $\vec{k}_{\perp i}$, with $\sum_{i=1}^n \vec{k}_{\perp i} = \overrightarrow{0}_\perp$, are actually relative coordinates; *i.e.* they are independent of the total momentum P^+ and $P_\perp$ of the bound state. The special feature that light-cone wavefunctions do not depend on the total momentum is not surprising, since x_i is the longitudinal momentum fraction carried by the $i^{\rm th}$-parton ($0 \le x_i \le 1$), and $\vec{k}_{\perp i}$ is its momentum "transverse" to the direction of the meson. Both of these are frame-independent quantities. The ability to specify wavefunctions simultaneously in any frame is a special feature of light-cone quantization.

The coefficients in the light-cone Fock state expansion thus are the parton wavefunctions $\psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)$ which describe the decomposition of each hadron in terms of its fundamental quark and gluon degrees of freedom. The light-cone variable $0 < x_i < 1$ is often identified with the constituent's longitudinal momentum fraction $x_i = k_i^z/P_z$, in a frame where the total momentum $P^z \to \inf$. However, in light-cone Hamiltonian formulation of QCD, x_i is the boost-invariant light-cone fraction,

$$x_i \equiv \frac{k_i^+}{P^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z},$$

independent of the choice of Lorentz frame.

Given the light-cone wavefunctions, $\psi_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)$, one can compute virtually any hadronic quantity by convolution with the appropriate quark and gluon matrix elements. For example, the leading-twist structure functions measured in deep inelastic lepton scattering are immediately related to the light-cone probability distributions:

$$2M\, F_1(x, Q) = \frac{F_2(x, Q)}{x} \approx \sum_a e_a^2\, G_{a/p}(x, Q)$$

where

$$G_{a/p}(x,Q)=\sum_{n,\lambda_i}\int\overline{\prod_i}\frac{dx_i d^2\vec{k}_{\perp i}}{16\pi^3}|\psi_n^{(Q)}(x_i,\vec{k}_{\perp i},\lambda_i)|^2\sum_{b=a}\delta(x_b-x)$$

is the number density of partons of type a with longitudinal momentum fraction x in the proton. This follows from the observation that deep inelastic lepton scattering in the Bjorken-scaling limit occurs if x_{bj} matches the light-cone fraction of the struck quark. (The $\sum_b$ is over all partons of type a in state n.) However, the light-cone wavefunctions contain much more information for the final state of deep inelastic scattering, such as the multi-parton distributions, spin and flavor correlations, and the spectator jet composition.

The spacelike form factor is the sum of overlap integrals analogous to the corresponding nonrelativistic formula:[23]

$$F(Q^2)=\sum_{n,\lambda_i}\sum_a e_a\int\overline{\prod_i}\frac{dx_i\,d^2\vec{k}_{\perp i}}{16\pi^3}\,\psi_n^{(\Lambda)*}(x_i,\vec{\ell}_{\perp i},\lambda_i)\,\psi_n^{(\Lambda)}(x_i,\vec{k}_{\perp i},\lambda_i).$$

Here e_a is the charge of the struck quark, $\Lambda^2\gg\vec{q}_\perp^{\,2}$, and

$$\vec{\ell}_{\perp i}\equiv\begin{cases}\vec{k}_{\perp i}-x_i\vec{q}_\perp+\vec{q}_\perp & \text{for the struck quark}\\ \vec{k}_{\perp i}-x_i\vec{q}_\perp & \text{for all other partons.}\end{cases}$$

The general rule for calculating an amplitude involving wavefunction $\psi_n^{(\Lambda)}$, describing Fock state n in a hadron with $\underline{P}=(P^+,\vec{P}_\perp)$, has the form[8] (see Fig. 3):

$$\sum_{\lambda_i}\int\overline{\prod_i}\frac{dx_i d^2\vec{k}_{\perp i}}{\sqrt{x_i}16\pi^3}\,\psi_n^{(\Lambda)}(x_i,\vec{k}_{\perp i},\lambda_i)\;T_n^{(\Lambda)}(x_iP^+,x_i\vec{P}_\perp+\vec{k}_{\perp i},\lambda_i)$$

where $T_n^{(\Lambda)}$ is the irreducible scattering amplitude in LCPTh with the hadron replaced by Fock state n. The light-cone Fock expansion thus allows a definition of the parton model and wavefunctions. By using the light-cone gauge, $A^+=0$, only physical non-ghost degrees of freedom appear in the Fock expansion even for non-Abelian theories. Furthermore in this gauge, the numerator couplings of soft gluons inserted into hard scattering expansions remain finite in the high momentum transfer limit. Thus this

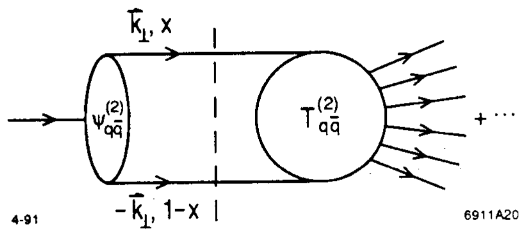

Figure 3. Calculation of hadronic amplitudes in the light-cone Fock formalism.

formalism is ideal for proving factorization theorems, i.e. the isolation of hard and soft contributions at high momentum transfer.

3. Exclusive Processes and Light-Cone Wavefunctions

The dynamics of exclusive reactions reflect not only the behavior of quark-gluon scattering processes at the amplitude level, but also the fundamental structure of the hadron wavefunctions themselves. In a relativistic quantum field theory, a bound state cannot be described in terms of a fixed number of constituents. However, in the case of exclusive reactions at large momentum transfer, there is an enormous simplification: only the lowest valence-quark light-cone Fock state of each hadron contributes to a high momentum transfer exclusive scattering process. It is easy to show that in the light-cone gauge, $A^+ = 0$, higher Fock state contributions involving extra gluons are always suppressed by powers of the momentum transfer Q.[24] Furthermore, the absence of gluon radiation into the final state demands that the valence quarks in the hadron wavefunction must be at relative transverse separation $b^i_\perp$ of order $1/Q$; thus small color-dipole configurations of the hadron wavefunction control large momentum transfer exclusive processes.[25,24] Thus at high momentum transfer exclusive reactions provide an important testing ground for light-cone wavefunctions since in the light-cone gauge only the simplest valence wavefunction is involved.

On the other hand, many properties of large momentum transfer exclusive reactions can be calculated without explicit knowledge of the form of the non-perturbative light-cone wavefunctions. The main ingredients of this analysis are asymptotic freedom, and the power-law scaling relations and quark helicity conservation rules of perturbative QCD. For example, consider the light-cone convolution formula for the meson form factor at high momentum transfer Q^2. If the internal momentum transfer is large then one can iterate the gluon-exchange term in the effective potential for the light-cone wavefunctions. The result is the hadron form factors can be written in a factorized form as a convolution of quark "distribution amplitudes" $\phi(x_i, Q)$, one for each hadron involved in the amplitude, with a hard-scattering amplitude T_H.[8,26] The distribution amplitude is the fundamental gauge invariant wavefunction which describes the fractional longitudinal momentum distributions of the valence quarks in a hadron integrated over transverse momentum up to the scale Q.[8] The pion's electromagnetic form factor, for example, can be written as[8,26,27]

$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy\, \phi_\pi^*(y,Q)\, T_H(x,y,Q)\, \phi_\pi(x,Q) \left(1 + \mathcal{O}\left(\frac{1}{Q}\right)\right).$$

Here T_H is the scattering amplitude for the form factor but with the pions replaced by collinear $q\overline{q}$ pairs—*i.e.* the pions are replaced by their valence partons. We can also regard T_H as the free particle matrix element of the order $1/Q^2$ term in the effective Lagrangian for $\gamma^* q\overline{q} \to q\overline{q}$.[10]

The process-independent distribution amplitude[8] $\phi_\pi(x,Q)$ is the probability amplitude for finding the $q\overline{q}$ pair in the pion with $x_q = x$ and $x_{\overline{q}} = 1-x$. It is directly related to the light-cone valence wavefunction:

$$\begin{aligned}\phi_\pi(x,Q) &= \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^{(Q)}_{q\overline{q}/\pi}(x,\vec{k}_\perp) \\ &= P_\pi^+ \int \frac{dz^-}{4\pi}\, e^{ixP_\pi^+ z^-/2}\, \langle 0|\, \overline{\psi}(0)\, \frac{\gamma^+\gamma_5}{2\sqrt{2n_c}}\, \psi(z)\, |\pi\rangle^{(Q)} \Big|_{z^+ = \vec{z}_\perp = 0}.\end{aligned}$$

The $\vec{k}_\perp$ integration in the above equation is cut off by the ultraviolet cutoff $\Lambda = Q$ implicit in the wavefunction; thus only Fock states with invariant mass squared $\mathcal{M}^2 \le Q^2$ contribute.

The above result for exclusive amplitudes is in the form of a factorization theorem; all of the non-perturbative dynamics is factorized into the non-perturbative distribution amplitudes, which sums all internal momentum transfers up to the scale Q^2. On the other hand, all momentum transfers higher than Q^2 appear in T_H, which, because of asymptotic freedom, can be computed perturbatively in powers of the QCD running coupling constant $\alpha_s(Q^2)$.

Isgur and Llewellyn Smith[28] and also Radyushkin[29] have raised the concern that important contributions to exclusive processes could arise from non-factorizing end-point contributions of the hadron wavefunctions with $x \sim 1$ even at very large momentum transfer. However, recent work by Botts, Li, and Sterman[30] has now shown that such soft physics contributions are effectively eliminated due to Sudakov suppression. I will briefly review this work below. In addition, Kronfeld and Nizic[31] have shown how one can consistently integrate over on-shell singularities in the hard-scattering amplitude for Compton processes involving baryons. Thus the QCD predictions based on the factorization of long and short distance physics are reliable and should be valid for momentum transfers in the experimentally accessible domain beyond a few GeV. It is clearly important to test these predictions as precisely as possible.

Given the factorized structure of exclusive amplitudes at large momentum transfer, one can read off a number of general features of the PQCD predictions; *e.g.* the dimensional counting rules, hadron helicity conservation, color transparency, etc.[24] In addition, the scaling behavior of the exclusive amplitude is modified by the logarithmic dependence of the distribution amplitudes in Q^2 which is in turn determined by QCD evolution equations.[8]

Because of asymptotic freedom, the nominal power-law fall-off $\mathcal{M} \sim Q^{4-n}$ of an exclusive amplitude at large momentum transfer reflects the elementary scaling of the lowest-order connected quark and gluon tree graphs obtained by replacing each of the external hadrons by its respective collinear quarks. Here n is the total number of initial state and final state lepton, photon, or quark fields entering or leaving the hard scattering subprocess. The empirical success of the dimensional counting rules for the power-law fall-off of form factors and general fixed center-of-mass angle scattering amplitudes has given important evidence for scale-invariant quark and gluon interactions at short distances.[32] QCD also predicts calculable corrections to the nominal dimensional counting power-law behavior due to the running of the strong coupling constant, higher order corrections to the hard scattering amplitude, Sudakov effects, pinch singularities, as well as the evolution of the hadron distribution amplitudes, $\phi_H(x_i, Q)$, the basic factorizable non-perturbative wavefunctions needed to compute exclusive amplitudes.[24,5]

The fundamental non-perturbative quantities which control large momentum transfer exclusive reactions in quantum chromodynamics are the hadron distribution amplitudes[8]: $\phi_B(x_i, \lambda_i, Q)$, for the baryons with $x_1 + x_2 + x_3 = 1$, and $\phi_M(x_i, \lambda_i, Q)$, for the mesons with $x_1 + x_2 = 1$. The distribution amplitudes are the hadron wavefunctions which interpolate between the QCD bound state and their valence quarks. The constituents have longitudinal light-cone momentum fractions $x_i = (k^0 + k^z)_i/(p^0 + p^z)$, helicities λ_i, and transverse separation $b_\perp \simeq 1/Q$. If one can calculate the distribution amplitude at an initial scale Q_0, then one can determine $\phi(x_i, Q)$ at higher momentum scales via evolution equations in $\log Q^2$ or equivalently, the operator product expansion. Thus far the most important experimental constraints on the hadron distribution amplitudes has come from the normalization and scaling of form factors at large momentum transfer.

The data for hadron form factors is consistent with the onset of PQCD scaling at a momentum transfers of a few GeV, as expected from the parameters which determine the mass scales of QCD. Recently Stoler[33] has shown that the measurements of the transition form factors of the proton to the $N(1535)$ and $N(1680)$ resonances are consistent with the predicted PQCD Q^{-4} scaling to beyond $Q^2 = 20\ GeV^2$. The normalization is also in reasonable agreement with that predicted from QCD sum rule constraints on the nucleon distribution amplitudes, allowing for uncertainties from higher order QCD corrections. In the case of the proton to $\Delta(1232)$ transition, the form factor falls faster that Q^{-4}. This anomalous behavior is in fact predicted by the QCD sum rule analysis since unlike the proton, the Δ has a highly symmetric distribution amplitude with a small coupling to the QCD hard scattering amplitude. The observed scaling pattern of the transition form factors gives strong support to the QCD sum rule predictions and PQCD factorization.

The hadron distribution amplitudes can also be used for calculating weak decay transitions, structure functions at $x \sim 1$, fragmentation distributions at large z, and higher twist correlations.[34] For example, strong higher twist effects are observed in the angular and Q^2 dependence of Drell-Yan processes and deep inelastic scattering at $x \sim 1$.[35] In each of these applications, one can use factorization theorems to separate the perturbative quark and gluon dynamics which involves momentum transfer higher than Q from the non-perturbative long-distance physics contained in $\phi(x_i, Q)$. These analyses parallel the developments in leading-twist inclusive reactions, where one factorizes hard-scattering quark-gluon subprocess cross sections from the long-distance physics contained in the hadron structure functions. However, in the case of exclusive processes at large momentum transfer, the scale-separation and factorization are done at the amplitude level.

Exclusive reactions involving two real or virtual photons provide a particularly interesting testing ground for QCD because of the relative simplicity of the couplings of the photons to the underlying quark currents and the absence of significant initial state interactions—any remnant of vector-meson dominance contributions is suppressed at large momentum transfer. The angular distributions for the hadron pair production processes $\gamma\gamma \rightarrow H\overline{H}$ are sensitive to the shapes of the hadron wavefunctions.[36] Lowest order predictions for meson pair production in two photon collisions using this formalism are given in Refs. 36 and 5; the analysis of the $\gamma\gamma$ to meson pair process has been carried out to next to leading order in $\alpha_s(Q^2)$ by Nizic.[37]

The simplest example of two-photon exclusive reactions is the $\gamma^*(q)\gamma \rightarrow M^0$ process which is measurable in tagged $e^+e^- \rightarrow e^+e^-M^0$ reactions. The photon to neutral meson transition form factor $F_{\gamma \rightarrow M^0}(Q^2)$ is predicted to fall as $1/Q^2$—modulo calculable logarithmic corrections from the evolution of the meson distribution amplitude. The QCD prediction reflects the scale invariance of the quark propagator at high momentum transfer, the same scale-invariance which gives Bjorken scaling of the deep inelastic lepton-nucleon cross sections. The existing data from the TPC/$\gamma\gamma$ experiment are consistent with the predicted scaling and normalization of the transition form factors for the π^0, η_0, and η'. The Mark II and TPC/$\gamma\gamma$ measurements of $\gamma\gamma \rightarrow \pi^+\pi^-$ and $\gamma\gamma \rightarrow K^+K^-$ reactions are also consistent with PQCD expectations. A review of this work is given in Ref. 38.

4. Compton Scattering in Perturbative QCD

Compton scattering $\gamma p \rightarrow \gamma p$ at large momentum transfer and its s-channel crossed reactions $\gamma\gamma \to \overline{p}p$ and $\overline{p}p \to \gamma\gamma$ are classic tests of the perturbative QCD formalism for exclusive reactions. At leading twist, each helicity amplitude has the factorized form,[24] (see Fig. 4)

$$\mathcal{M}_{hh'}^{\lambda\lambda'}(s,t) = \sum_{d,i} \int [dx][dy]\phi_i(x_1,x_2,x_3,\widetilde{Q})T_i^{(d)}(x,h,\lambda;y,h',\lambda';s,t)\phi_i(y_1,y_2,y_3;\widetilde{Q}) \ .$$

The index i labels the three contributing valence Fock amplitudes at the renormalization scale $\widetilde{Q}$. The index d labels the 378 connected Feynman diagrams which contribute to the eight-point hard scattering amplitude $qqq\gamma \to qqq\gamma$ at the tree level; *i.e.* at order $\alpha\alpha_s^2(\widehat{Q})$. The arguments $\widehat{Q}$ of the QCD running coupling constant can be evaluated amplitude by amplitude using the method of Ref. 39. The evaluation of the hard scattering amplitudes $T_i^{(d)}(x,h,\lambda;y,h',\lambda';s,t)$ has now been done by several groups.[40,41,31,42]

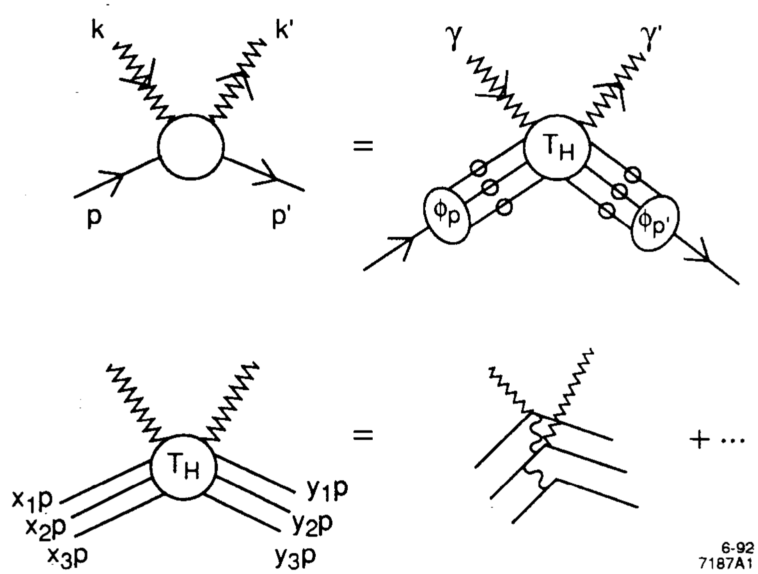

Figure 4. Factorization of the Compton amplitude in QCD.

An important simplification of Compton scattering in PQCD is the fact that pinch singularities are readily integrable and do not change the nominal power-law behavior of the basic amplitudes.[31] Physically, the pinch singularities correspond to the existence of potentially on-shell intermediate states in the hard scattering amplitudes, leading to a non-trivial phase structure of the Compton amplitudes. Such phases can in principle be measured by interfering the virtual Compton process in

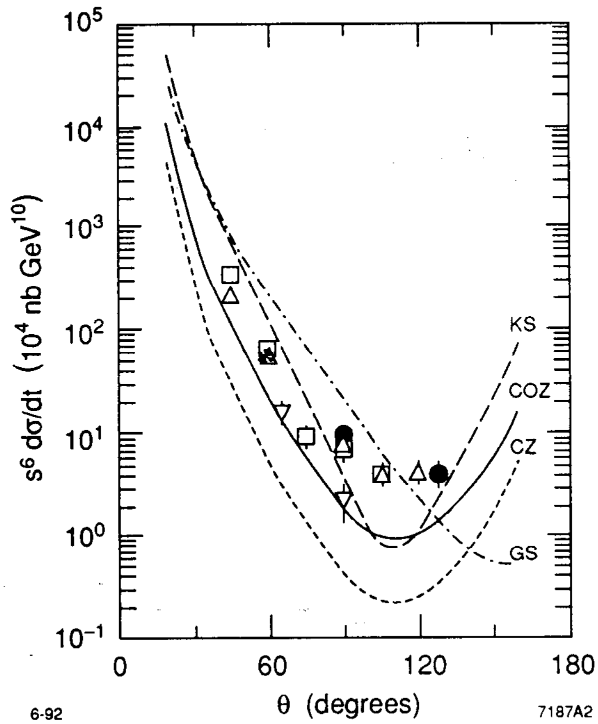

Figure 5. Comparison of the QCD prediction for the scaled unpolarized proton Compton scattering differential cross section $s^6 d\sigma/dt(\gamma p \to \gamma p)$ with experiment. The experimental data[44] are at $s = 4.63\ GeV$ (circles) $s = 6.51\ GeV$ (triangles), $s = 8.38\ GeV$ (squares) and $s = 10.26\ GeV$ (asterisk). The QCD prediction is from the calculation of Kronfeld and Nizic.[31] The QCD sum rule distribution amplitudes are listed in Ref. 5.

$e^{\pm}p \to e^{\pm}p\gamma$ with the purely real Bethe-Heitler bremmstrahlung amplitude.[43] A careful analytic treatment of the integration over the on-shell intermediate states is given by Kronfeld and Nizic.[31]

The most characteristic feature of the PQCD predictions is the scaling of the differential Compton cross section at fixed t/s or θ_{CM}.

$$s^6 \frac{d\sigma}{dt}(\gamma p \to \gamma p) = F(t/s).$$

The power s^6 reflects the fact that 8 elementary fields enter or leave the hard scattering subprocess.[32] The scaling of the existing data[44] as shown in Fig. 5 is remarkably consistent with the PQCD power-law prediction, but measurements at higher energies and momentum transfer are needed to test the predicted logarithmic corrections to this scaling behavior and determine the angular distribution of the scaled cross section over as large a range as possible.

The predictions for the normalization of the Compton cross section and the shape of its angular distribution are sensitive to the shape of the proton distribution amplitude $\phi_p(x_i, Q)$. The forms predicted for the proton distribution amplitude by QCD sum-rules by Chernyak, Oglobin, and Zhitnitskii, and also King and Sachrajda, shown in Fig. 1, appear to give a reasonable representation of the existing data. These distributions, which predict that 65% of the proton's momentum is carried by the u quark with helicity parallel to the proton's helicity also provide reasonable predictions for the normalization of the proton's form factor and the $J/\psi \to p\overline{p}$ decay rate. Kronfeld and Nizic have also given detailed predictions for the helicity and phase structure of the PQCD predictions for both proton and neutrons. The crossing behavior from the Compton scattering to the annihilation channels will also provide important tests and constraints on the PQCD formalism and the shape of the proton distribution amplitudes. Predictions for the timelike processes have been made by Farrar *et al.*,[40] Millers and Gunion[41], and Hyer.[42]

It should be emphasized that the theoretical uncertainties from finite nucleon mass corrections, the magnitude of the QCD running coupling constant, and the normalization of the proton distribution amplitude largely cancel out in the ratio of differential cross sections

$$R_{\gamma\gamma/e^+e^-}(s, \theta_{cm}) = \frac{d\sigma(\overline{p}p \to \gamma\gamma)/dt}{d\sigma(\overline{p}p \to e^+e^-)/dt},$$

which is predicted by QCD to be essentially independent of s at large momentum transfer. (See Fig. 6.) If this scaling is confirmed, then the center-of-mass angular dependence of $R_{\gamma\gamma/e^+e^-}(s, \theta_{cm})$ will be one of the best ways to determine the shape of $\phi_p(x_i, Q)$. The measurement of this ratio appears to well-suited to the Fermilab antiproton accumulator experiment $E760$ and SuperLear.

Another important characteristic of the leading-twist QCD predictions for exclusive processes is hadron-helicity conservation.[45] Because of chiral invariance, the hard-scattering amplitude is non-zero only for amplitudes that conserve quark helicity. Since the distribution amplitude projects only $L_z = 0$, this implies that the proton helicity is conserved in $\gamma p \to \gamma p$. Similarly, the baryon and x anti-baryon helicities must be opposite in the crossed reactions $\gamma\gamma \to \overline{BB}$ and $\overline{p}p \to \gamma\gamma$ at large momentum transfer. Detailed predictions for each of the leading power Compton scattering helicity amplitudes are also given by Kronfeld and Nizic.[31]

5. The Domain of Validity of PQCD Predictions for Exclusive Processes

The factorized predictions for the Compton amplitude are rigorous predictions of QCD at large momentum transfer. However, it is important to understand the kinematic domain where the leading twist predictions become valid. As emphasized by Isgur and Llewellyn Smith,[28] this question is non-trivial because of the possibility of

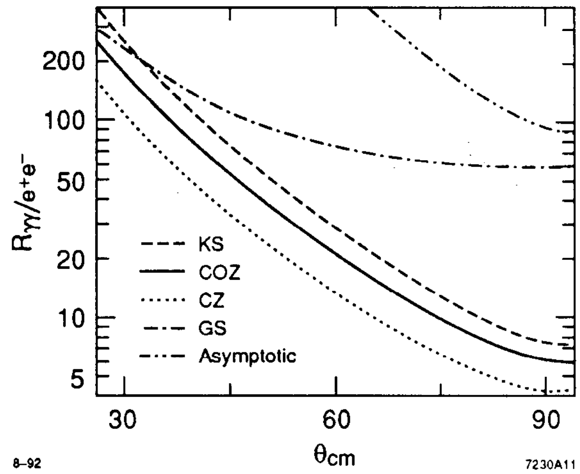

Figure 6. PQCD predictions for the ratio of the timelike Compton cross section for $\overline{p}p \to \gamma\gamma$ to the cross section for $\overline{p}p \to e^+e^-$ annihilation assuming different model forms for the proton distribution amplitude at $s = 25\ GeV^2$.[42] The predictions include the effect of Sudakov suppression in the endpoint region using the Li-Sterman formalism.

significant contributions to the scattering amplitude at the endpoint regions $x_i \to 1$ where the PQCD factorization could break down. Because of the denominator structure of the hard scattering amplitudes, *e.g.*, $T_H \propto \alpha_s/[(1-x)(1-y)Q^2]$ for the meson form factor, the endpoint integration region at $x \sim 1$ and $y \sim 1$ will be enhanced. Of more concern is the fact that such endpoint regions are even further emphasized when one assumes the strongly asymmetric forms for the nucleon distribution amplitude derived from QCD sum rules.

It is important to note that the end-point regime corresponds to scattering processes where one quark carries nearly all of the proton's momentum and is at a fixed transverse separation $b_\perp$ from the spectator quarks. However, if a quark which is isolated in space receives a large momentum transfer x_iQ, it will normally strongly radiate gluons into the final state due to the displacement of both its initial and final self-field, contrary to the requirements of exclusive scattering.[30] For example, in QED the radiation from the initial and final state charged lines is controlled by the coherent sum $\sum_i \frac{\epsilon\cdot p_i}{k\cdot p_i}\eta_i q_i$ where q_i and p_i are the charges four-momenta of the charged lines, ϵ and k are polarization and four-momentum of the radiation, and $\eta_i = \pm 1$ for initial and final state particles, respectively. Radiation will occur for any finite momentum transfer scattering as long as the photon's wavelength is less than the size of the initial and final neutral bound states.

The radiation from the colored lines in QCD have similar coherence properties:[46] because of the destructive color interference of the radiators, the momentum of the radiated gluon in a QCD hard scattering process only ranges from k of order $1/b_\perp$, where color screening occurs, up to the momentum transfer x_iQ of the scattered quarks. The one-gluon correction to the wavefunction is thus proportional to

$$\frac{C_F}{\pi}\int^{Q}\frac{d^2q_\perp}{q_\perp^2}\Big[3-\sum_{i<j}\exp\big(-i(b_i-b_j)\cdot q_\perp\big)\Big]\frac{\alpha_s(q_\perp^2)}{2\pi}\int_{q_\perp}^{Q}\frac{dq_+}{q_+}.$$

This result[30,42] and unitarity allows one to compute the probability that no radiation occurs during the hard scattering. It is given by a rapidly falling exponentiated Sudakov form factor $S = S(x_iQ, b_\perp, \Lambda_{QCD})$; thus at large Q and fixed impact separation, the Sudakov factor strongly suppresses the endpoint contribution. On the other hand, when $b_\perp = \mathcal{O}(x_iQ)^{-1}$, the Sudakov form factor is of order 1, and the radiation leads to logarithmic evolution and contributions of higher order in $\alpha_s(Q^2)$ corrections already contained in the PQCD predictions.[8,47,48] This is the starting point of the detailed analysis of the suppression of endpoint contributions to meson and baryon form factors and its quantitative effect on the PQCD predictions recently presented by Li and Sterman.[30] This analysis has now also been applied to two-photon reactions and the timelike proton form factor by Hyer.[42]

It should be emphasized that the standard PQCD contributions to large momentum transfer exclusive reactions derive from wavefunction configurations where the valence quarks are at small transverse separation $b_\perp = \mathcal{O}(1/Q)$, the regime where there is no Sudakov suppression. However, as noted by Li and Sterman, the hard scattering amplitude loses its singular end-point structure if one retains the valence quark transverse momenta in the denominators. For example, in the case of the pion form factor, the hard scattering amplitude is effectively modified to the form

$$T_H \propto \frac{\alpha_s}{(1-x)(1-y)Q^2 + (\mathbf{k}_1^\perp + \mathbf{k}_2^\perp)^2}.$$

Li and Sterman thus find that the pion form factor becomes relatively insensitive to soft gluon exchange at momentum transfers beyond 20 Λ_{QCD}. In the case of the proton Dirac form factor, the corresponding analysis by Li[30] is in good agreement with experiment at momentum transfers greater than 3 GeV.

The Botts, Li, and Sterman analysis of the Sudakov suppression of endpoint contributions makes it understandable why PQCD factorization and its predictions for exclusive processes are already applicable at momentum transfers of a few GeV, thus accounting for the empirical success of quark counting rules in exclusive process phenomenology. The Sudakov effect suppression also enhances "color transparency" phenomena, since only small color singlet wavefunction configurations can scatter at large momentum transfer.[25] Color transparency in Compton scattering can be tested by checking for the absence of final state absorption in quasi-elastic $\gamma p \to \gamma p$

scattering in heavy nuclei. Similarly, QCD color transparency implies that there will be diminished initial state absorption of the antiproton for large-angle quasi-elastic $\overline{p}p \to \gamma\gamma$ annihilation in heavy nuclear targets.

In the case of large angle proton-proton scattering, the perturbative predictions for color transparency and the spin-spin correlation A_{NN} appear to fail at $E_{CM} \sim 5\ GeV$; this effect has been attributed to the effect of the threshold for charm production in intermediate states.[49] A similar breakdown of the perturbative predictions may also occur at the corresponding energy threshold in $\overline{p}p \to \gamma\gamma$ at large angles due to charmed hadron intermediate states.

Recently Luke, Manohar, and Savage[50] have shown that the QCD trace anomaly leads to a strong, attractive, scalar potential which dominates the interaction of heavy quarkonium states with ordinary matter at low relative velocity. The scalar attraction is sufficiently strong to produce nuclear-bound quarkonium.[51] Thus it will be interesting to look for strong threshold enhancements for charm production near threshold in two-photon reactions, particularly in exclusive channels such as $\rho^0 J/\psi$ as well as $D\overline{D}$. Predictions for the threshold production of charmed mesons has also been given in Ref. 52. Evidence for excess inclusive production of charmed mesons in photon-photon collisions has been reported by the JADE collaboration.[53]

Exclusive processes, particularly two-photon reactions, thus provide one of the most important, but least explored frontiers in particle physics. The recent analyses by Botts, Li, and Sterman and by Kronfeld and Nizic have shown that the predictions based on QCD factorization theorems are applicable to measurements at present-day accelerators. It is clearly crucial for a fundamental understanding of both the perturbative and non-perturbative aspects of QCD that the predictions for exclusive amplitudes be tested as carefully as possible.

6. Exclusive Weak Decays of Heavy Hadrons

An important application of PQCD factorization is to the exclusive decays of heavy hadrons to light hadrons, such as $B^0 \to \pi^+\pi^-,\ K^+, K^-$.[54] To a good approximation, the decay amplitude $\mathcal{M} = \langle B|H_{Wk}|\pi^+\pi^-\rangle$ is caused by the transition $\overline{b} \to W^+\overline{u}$; thus $\mathcal{M} = f_\pi p_\pi^\mu \frac{G_F}{\sqrt{2}} \times \langle \pi^-|J_\mu|B^0\rangle$ where J_μ is the $\overline{b} \to \overline{u}$ weak current. The problem is then to recouple the spectator d quark and the other gluon and possible quark pairs in each B^0 Fock state to the corresponding Fock state of the final state π^-. (See Fig. 7.) The kinematic constraint that $(p_B - p_\pi)^2 = m_\pi^2$ demands that at least one quark line is far off shell: $p_{\overline{u}}^2 = (yp_B - p_\pi)^2 \sim -\mu m_B \sim -1.5\ GeV^2$, where we have noted that the light quark takes only a fraction $(1-y) \sim \sqrt{(k_\perp^2 + m_d^2)}/m_B$ of the heavy meson's momentum since all of the valence quarks must have nearly equal velocity in a bound state. In view of the successful applications[55] of PQCD factorization to form factors at momentum transfers in the few GeV^2 range, it is reasonable to assume that $\langle |p_{\overline{u}}^2| \rangle$ is sufficiently large that we can begin to apply

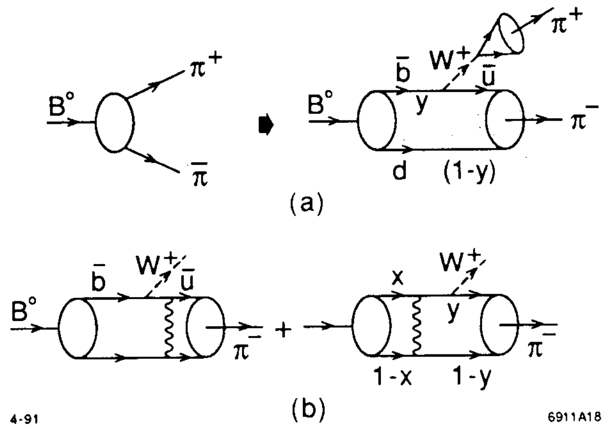

Figure 7. Calculation of the weak decay $B \to \pi\pi$ in the PQCD formalism of Ref. 54. The gluon exchange kernel of the hadron wavefunction is exposed wherever a hard momentum transfer is required.

perturbative QCD methods.

The analysis of the exclusive weak decay amplitude can be carried out in parallel to the PQCD analysis of electroweak form factors[56] at large Q^2. The first step is to iterate the wavefunction equations of motion so that the large momentum transfer through the gluon exchange potential is exposed. The heavy quark decay amplitude can then be written as a convolution of the hard scattering amplitude for $Q\overline{q} \to W^+ q\overline{q}$ convoluted with the B and π distribution amplitudes. The minimum number valence Fock state of each hadron gives the leading power law contribution. Thus T_H contains all perturbative virtual loop corrections of order $\alpha_s(\Lambda^2)$. The result is the factorized form:

$$\mathcal{M}(B \to \pi\pi) = \int_0^1 dx \int_0^1 dy \phi_B(y, \Lambda) T_H \phi_\pi(x, \Lambda)$$

which is expected to be correct up to terms of order $1/\Lambda^4$. All of the non-perturbative corrections with momenta $|k^2| < \Lambda^2$ are summed in the distribution amplitudes.

An interesting example of this analysis is "atomic alchemy",[57] i.e., the exclusive decays of muonic atoms to electronic atoms plus neutrinos. In this case the calculation requires the very high momentum tail of the atomic wavefunctions, which in turn can be obtained via the iteration of the relativistic atomic bound-state equations. Again one obtains a factorization theorem for exclusive atomic transitions where the atomic wavefunction at the origin plays the role of the distribution amplitude.

7. Discretized Light-Cone Quantization: Applications to QCD

QCD dynamics takes a rather simple form when quantized at equal light-cone "time" $\tau = t + z/c$. In light-cone gauge $A^+ = 0$, the QCD light-cone Hamiltonian

$$H_{\rm QCD} = H_0 + gH_1 + g^2 H_2$$

contains the usual 3-point and 4-point interactions plus induced terms from instantaneous gluon exchange and instantaneous quark exchange diagrams. The perturbative vacuum serves as the lowest state in constructing a complete basis set of color-singlet Fock states of H_0 in momentum space. Solving QCD is then equivalent to solving the eigenvalue problem:

$$H_{\rm QCD}|\Psi >= M^2|\Psi >$$

as a matrix equation on the free Fock basis. The set of eigenvalues $\{M^2\}$ represents the spectrum of the color-singlet states in QCD. The Fock projections of the eigenfunction corresponding to each hadron eigenvalue gives the quark and gluon Fock state wavefunctions $\psi_n(x_i, k_{\perp i}, \lambda_i)$ required to compute structure functions, distribution amplitudes, decay amplitudes, etc. For example, the e^+e^- annihilation cross section into a given $J = 1$ hadronic channel can be computed directly from its $\psi_{q\overline{q}}$ Fock state wavefunction.

The key step in obtaining a discrete representation of the light-cone Hamiltonian in a form amenable to numerical diagonalization, is the construction of a complete, countable, Fock state basis,

$$\sum_n |n\rangle \langle n| = I \ .$$

This can be explicitly done in QCD by constructing a complete set of color-singlet eigenstates of the free Hamiltonian as products of representations of free quark and gluon fields. The states are chosen as eigenstates of the constants of the motion, P^+, $\overrightarrow{P}_\perp$, J_z, and the conserved charges. In addition, one can pre-diagonalize the Fock representation by classifying the states according to their discrete symmetries, as described in the previous section. This step alone reduces the size of the matrix representations by as much as a factor of 16.

The light-cone Fock representation can be made discrete by choosing periodic (or, in the case of fermions, anti-periodic) boundary conditions on the fields: $\psi(z^-) = \pm\psi(z^- - L)$, and $\psi(x_\perp) = \psi(x_\perp - L_\perp)$. Thus in each Fock state, $P^+ = \frac{2\pi}{L}K$, and each constituent $k_i^+ = \frac{2\pi}{L}n_i$, where the positive integers n_i satisfy $\sum_i n_i = K$. Similarly $\vec{k}_{\perp i} = \frac{\pi}{L_\perp}\vec{n}_{\perp i}$, where the vector integers sum to $\overrightarrow{0}_\perp$ in the standard frame.

The positive integer K is called the "harmonic resolution." For a given choice of K, there are only a finite number of partitions of the plus momenta; thus only a finite set of rational values of $x_i = k_i^+/P^+ = n_i/K$ appear: $x_i = \frac{1}{K}, \quad \frac{2}{K}, \ldots \frac{K-1}{K}$. Thus eigensolutions obtained by diagonalizing H_{LC} on this basis determine the deep inelastic structure functions $F_I(x)$ only at the set of rational discrete points x_i. The continuum limit thus requires extrapolation to $K \to \infty$. Note that the value of L is irrelevant, since it can always be scaled away by a Lorentz boost. Since H_{LC}, P^+, $\overrightarrow{P}_\perp$, and the conserved charges all commute, H_{LC} is block diagonal.

The DLCQ program becomes especially simple for gauge theory in one-space one-time dimensions not only because of the absence of transverse momenta, but also because there are no gluon degrees of freedom. In addition, for a given value of the harmonic resolution K the Fock basis becomes restricted to finite dimensional representations. The dimension of the representation corresponds to the number of partitions of the integer K as a sum of positive integers n. The eigenvalue problem thus reduces to the diagonalization of a finite Hermitian matrix. The continuum limit is clearly $K \to \infty$.

Since continuum scattering states as well as single hadron color-singlet hadronic wavefunctions are obtained by the diagonalization of H_{LC}, one can also calculate scattering amplitudes as well as decay rates from overlap matrix elements of the interaction Hamiltonian for the weak or electromagnetic interactions. In principle, all higher Fock amplitudes, including spectator gluons, can be kept in the light-cone quantization approach; such contributions cannot generally be neglected in decay amplitudes involving light quarks.

DLCQ has been used to successfully obtain the complete color-singlet spectrum of QCD in one-space and one-time dimension for $N_C = 2, 3, 4$.[17] The hadronic spectra are obtained as a function of quark mass and QCD coupling constant (see Fig. 8). Where they are available, the spectra agree with results obtained earlier; in particular, the lowest meson mass in SU(2) agrees within errors with lattice Hamiltonian results. The meson mass at $N_C = 4$ is close to the value predicted by 't Hooft in the large N_C limit. The DLCQ method also provides the first results for the baryon spectrum in a non–Abelian gauge theory. The lowest baryon mass is shown in Fig. 8 as a function of coupling constant. The ratio of meson to baryon mass as a function of N_C also agrees at strong coupling with results obtained by bosonization methods.[58] Precise values for the mass eigenvalue can be obtained by extrapolation to large K by fitting to forms with the correct functional dependence in $1/K$.

When the light-cone Hamiltonian is diagonalized at a finite resolution K, one gets a complete set of eigenvalues corresponding to the total dimension of the Fock state basis. A representative example of the spectrum is shown in Fig. 9 for baryon states ($B = 1$) as a function of the dimensionless variable $\lambda = 1/(1 + \pi m^2/g^2)$. Notice that spectrum automatically includes continuum states with $B = 1$.

The structure functions for the lowest meson and baryon states in SU(3) at two different coupling strengths $m/g = 1.6$ and $m/g = 0.1$ are shown in Figs. 10 and

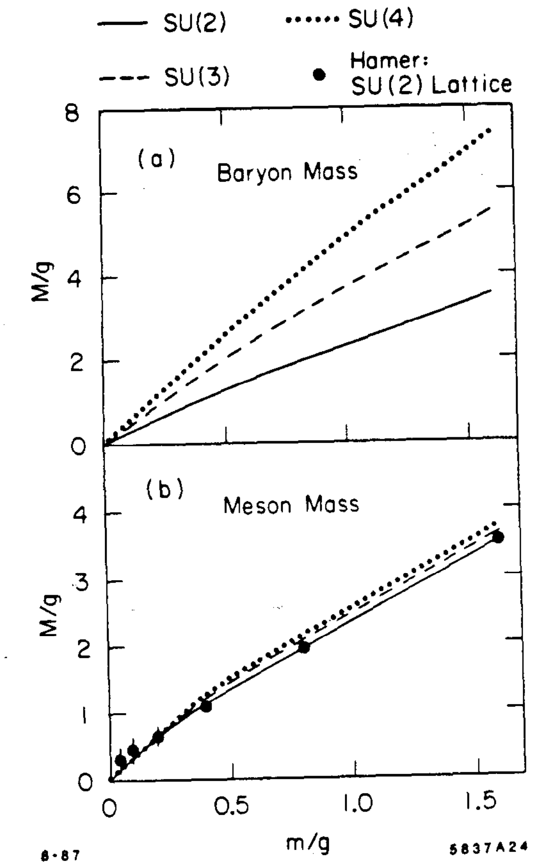

Figure 8. The baryon and meson spectrum in QCD(1+1) computed in DLCQ for $N_C = 2, 3, 4$ as a function of quark mass and coupling constant.

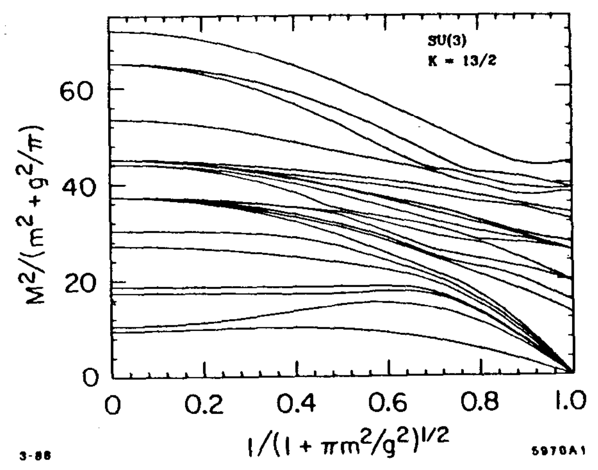

Figure 9. Representative baryon spectrum for QCD in one-space and one-time dimension.

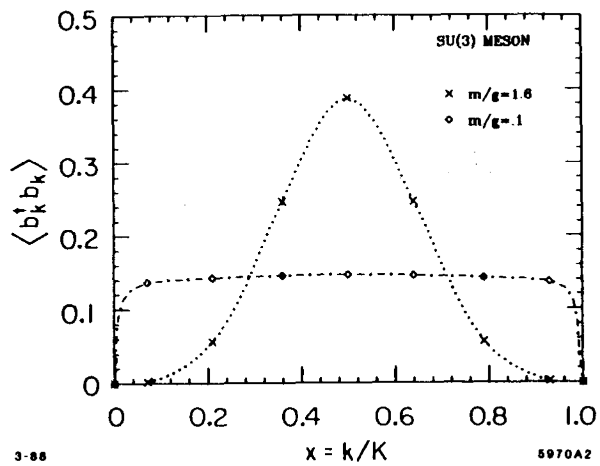

Figure 10. The meson quark momentum distribution in QCD[1+1] computed using DLCQ.

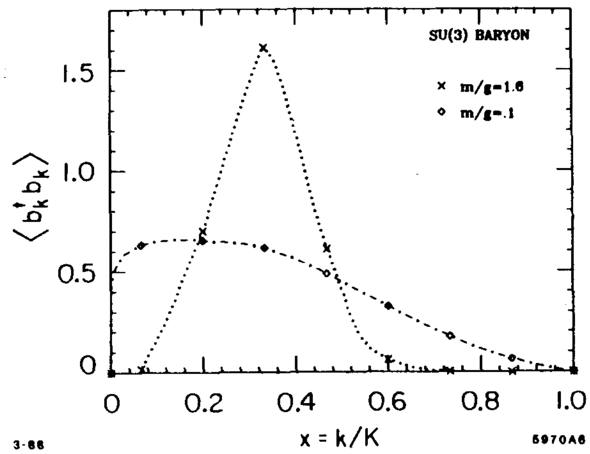

Figure 11. The baryon quark momentum distribution in QCD[1+1] computed using DLCQ.

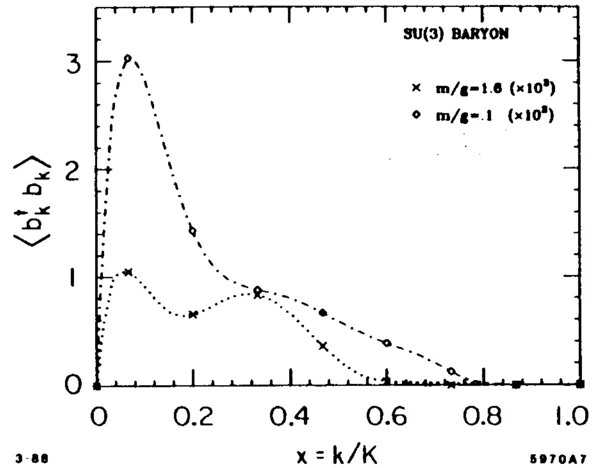

Figure 12. Contribution to the baryon quark momentum distribution from $qqq\overline{q}\overline{q}$ states for QCD[1+1].

11. Higher Fock states have a very small probability; representative contributions to the baryon structure functions are shown in Fig. 12. Although these results are for one-time one-space theory they do suggest that the sea quark distributions in physical hadrons may be highly structured.

8. The Heavy Quark Content of the Proton

The DLCQ results for sea quark distributions in QCD(1+1) may have implications for the heavy quark content of physical hadrons. One of the most intriguing unknowns in nucleon structure is the strange and charm quark structure of the nucleon wavefunction.[59] The EMC spin crisis measurements indicate a significant $s\overline{s}$ content of the proton, with the strange quark spin strongly anti-correlated with the proton spin. Just as striking, the EMC measurements[60] of the charm structure function of the Fe nucleus at large $x_{bj} \sim 0.4$ appear to be considerably larger than that predicted by the conventional photon-gluon fusion model, indicating an anomalous charm content of the nucleon at large values of x. The probability of intrinsic charm has been estimated[60] to be 0.3%.

Figure 13 shows recent results obtained by Hornbostel[61] for the structure functions of the lowest mass meson in QCD(1+1) wavefunctions for $N_C = 3$ and two quark flavors. As seen in the figure, the heavy quark distribution arising from the $q\overline{q}Q\overline{Q}$ Fock component has a two-hump character. The second maximum is expected since the constituents in a bound state tend to have equal velocities. The result is insensitive to the value of the Q^2 of the deep inelastic probe. Thus intrinsic charm is a feature of exact solutions to QCD(1+1). Note that the integrated probability for the Fock states containing heavy quarks falls nominally as g^2/m_Q^4 in this super-

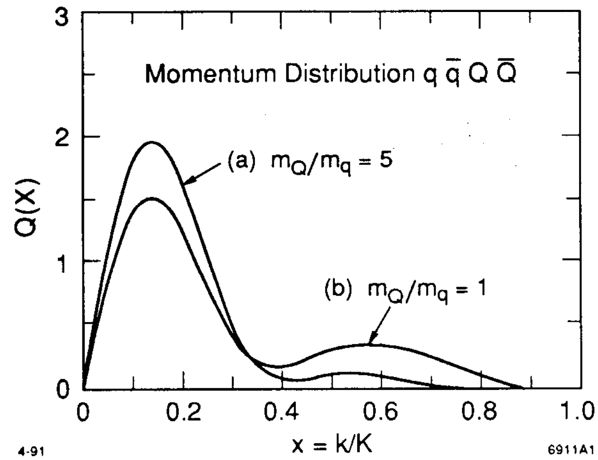

Figure 13. The heavy quark structure function $Q(x) = G_{Q/M}(x)$ of the lightest meson in QCD(1+1) with $N_c = 3$ and $g/m_q = 10$. Two flavors are assumed with (a) $m_Q/m_q = 1.001$ and (b) $m_Q/m_q = 5$. The curves are normalized to unit area. The probability of the $q\overline{q}Q\overline{Q}$ state is 0.56×10^{-2} and 0.11×10^{-4}, respectively. The DLCQ method for diagonalizing the light-cone Hamiltonian is used with anti-periodic boundary conditions. The harmonic resolution is taken at $K = 10/2$. (From Ref. 17.)

renormalizable theory, compared to g^2/m_Q^2 dependence expected in renormalizable theories.

In the case of QCD(3+1), we also expect a two-component structure for heavy-quark structure functions of the light hadrons. The low x_F enhancement reflects the fact that the gluon-splitting matrix elements of heavy quark production favor low x. On the other hand, the $Q\overline{Q}q\overline{q}$ wavefunction also favors equal velocity of the constituents in order to minimize the off-shell light-cone energy and the invariant mass of the Fock state constituents. In addition, the non-Abelian effective Lagrangian analysis discussed above allows a heavy quark fluctuation in the bound state wavefunction to draw momentum from all of the hadron's valence quarks at order $1/m_Q^2$. This implies a significant contribution to heavy quark structure functions at medium to large momentum fraction x. The EMC measurements of the charm structure function of the nucleon appear to support this picture.[60]

It is thus useful to distinguish *extrinsic and intrinsic* contributions to structure functions. The extrinsic contributions are associated with the substructure of a single quark and gluon of the hadron. Such contributions lead to the logarithmic evolution of the structure functions and depend on the momentum transfer scale of the probe. The intrinsic contributions involve at least two constituents and are associated with the bound state dynamics independent of the probe. The intrinsic gluon distributions[62] are closely related to the retarded mass-dependent part of the bound-state potential of the valence quarks.[63] In addition, because of asymptotic freedom, the hadron wavefunction has only an inverse power $\mathcal{M}^{-2}$ suppression for high mass fluctuations, whether one is considering heavy quark pairs or light quark pairs at high invariant mass $\mathcal{M}$. This "intrinsic hardness" of QCD wavefunctions leads to a number of interesting phenomena, including a possible explanation for "cumulative production," high momentum components of the nuclear fragments in nuclear collisions. This is discussed in detail in Ref. 64.

9. Renormalization and Ultra-Violet Regulation of Light-Cone-Quantized Gauge Theory

An important element in the light-cone Hamiltonian formulation of quantum field theories is the regulation of the ultraviolet region. In order to define a renormalizable theory, a covariant and gauge invariant procedure is required to eliminate states of high virtuality. The physics beyond the scale Λ is contained in the normalization of the mass $m(\Lambda)$ and coupling constant $g(\Lambda)$ parameters of the theory, modulo negligible corrections of order $1/\Lambda^n$ from the effective Lagrangian. The logarithmic dependence of these input parameters is determined by the renormalization group equations. In Lagrangian field theories the ultraviolet cut-off is usually introduced via a spectrum of Pauli-Villars particles or dimensional regulation.

In the case of QCD (3+1), the renormalization of the light-cone Hamiltonian in light-cone gauge is not yet completely understood, but a number of methods are now under consideration. In Ref. 8 Lepage and I showed that by using invariant cutoffs for both the interactions in the light-cone Hamiltonian and the Fock space, one could verify the renormalization group behavior of the gauge-invariant distribution amplitude. The result is consistent with results obtained from the Bethe-Salpeter equation or the operator product expansion. Thus one has a reason to believe that a properly regulated and truncated light-cone Hamiltonian can be constructed consistent with the known renormalization group structure of QCD.

In DLCQ, one needs to provide *a priori* some type of truncation of the Fock state basis. Since wavefunctions and Green's functions decrease with virtuality, one expects that states very far off the light-cone energy shell will have no physical effect on a system, except for renormalization of the coupling constant and mass parameters. Thus it is natural to introduce a "global" cut-off such that a Fock state $|n\rangle$ is retained only if

$$\sum_{i\epsilon n} \frac{\vec{k}_{\perp}^{2}{}_i + m_i^2}{x_i} - M^2 < \Lambda^2 .$$

Here M is the mass of the system in the case of the bound state problem, or the total invariant mass $\sqrt{s}$ of the initial state in scattering theory. One can also regulate the ultraviolet region by introducing a "local" cutoff on each matrix element $\langle n|H_{LC}|m\rangle$

by requiring that the change in invariant mass squared

$$\left| \sum_{i\epsilon n} \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i} - \sum_{i\epsilon m} \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i} \right| < \Lambda^2 .$$

This avoids spectator-dependent renormalization counterterms.[20] Similarly, one can use a lower cutoff on the invariant mass difference to regulate the infrared region.[65] Global and local cutoff methods were used in Ref. 8 to derive factorization theorems for exclusive and inclusive processes at large momentum transfer in QCD. In particular, the global cut-off defines the Fock-state wavefunctions $\psi^\Lambda(x, \vec{k}_\perp, \lambda)$ and distribution amplitude $\phi(x, \Lambda)$, the non-perturbative input for computing hadronic scattering amplitudes. The renormalization group properties of the light-cone wavefunctions and the resulting evolution equations for the structure functions and distribution amplitudes are also discussed in Ref. 8. The calculated anomalous dimensions γ_n for the moments of these quantities agree with results obtained using the operator product expansion.[66]

In general, light-cone quantization using the global or local cutoff can lead to terms in H_{LC}^Λ of the form $\delta m \overline{\psi} \frac{\gamma^+}{i\partial^+} \psi$. Such terms arise in order g^2 as a result of normal-ordering of the four-point interaction terms. Although such a term is invariant under the large class of light-cone Lorentz transformations, it is not totally invariant. Burkardt and Langnau[67] have suggested that the extra counterterms can be fixed by *a posteriori* imposing rotational symmetry on the bound state solutions, so that all Lorentz symmetries are restored.

10. The Zero-Mode Problem in Light-Cone-Quantized Gauge Theory

The role of zero modes in the light-cone quantization of 1+1 gauge theories has now been greatly clarified by the work of Heinzl, Kruschke, and Werner,[68] McCartor and Robertson,[69] Griffin[70] and Hornbostel.[71] In general, zero mode (field excitations with $k^+ = 0$) must be retained consistent with the constraints imposed by the field equations of motion and the imposed boundary conditions. In the case of massless QED (1+1) (the Schwinger model), one needs to retain the zero mode at the A^+ field, since this degree of freedom leads to the labeling of the degenerate θ-vacua of the theory and the corresponding fermion condensates. In the case of theories such as $\phi^4(1+1)$, the zero mode of the ϕ field provides the degree of freedom usually associated with the spontaneous breaking of the vacuum. It is also clear that zero modes play an important role in implementing the correct degrees of freedom in the effective light-cone Hamiltonian for quantum field theories in 3+1 dimensions. Again, one must allow for quantum excitations with $k^+ = 0$ and any value of $\vec{k}_\perp$ so that the equations of motion and the boundary conditions are fulfilled. In the case of DLCQ, the assumed anti-periodic boundary conditions automatically exclude zero modes for the fermion fields, but zero modes are generally needed to describe the boson fields. Hiller and Wivoda[72] have shown that in the $\lambda\phi\psi\overline{\psi}$ theory, the convergence of the DLCQ solutions to the known Wick-Cutkowsky solutions is greatly increased by the inclusion of the $\phi(k^+ = 0)$ modes.

Zero modes are also required for the implementation of the light-cone gauge $(A^+ = 0)$ in gauge theories in 1+1 dimensions. One of the most serious complications of the light-cone gauge quantization of QED (3+1) is the appearance of an apparently unregulated $1/k^+$ singularity in the expression for electron-electron scattering due to the $1/(k \cdot \eta)$ terms in the photon propagator. Although this singularity vanishes for on-shell scattering, it confounds the proper interpretation of the effective potential for positronium in the effective light-cone potential. However, Soper[73] has now shown that the Leibbrandt-Mandelstam prescription for the light-cone propagator with

$$\frac{1}{k^+} \Rightarrow \frac{k^-}{k^+k^- + i\epsilon}$$

automatically generates a subtraction term in the QED effective Hamiltonian which eliminates the gauge singularity at $k^+ = 0$. This solution corresponds to a ghost zero mode, first identified by Bassetto[74] to be necessary for the consistent implementation of the light-cone gauge with periodic boundary conditions. A similar subtraction at $k^+ = 0$ also occurs in the definition of the evolution kernel for the distribution amplitude.[8]

11. Advantages of Light-Cone Quantization

As I have discussed in this talk, the method of discretized light-cone quantization provides a relativistic, frame-independent discrete representation of quantum field theory amenable to computer simulation. In principle, the method reduces the light-cone Hamiltonian to diagonal form and has the remarkable feature of generating the complete spectrum of the theory: bound states and continuum states alike. DLCQ is also useful for studying relativistic many-body problems in relativistic nuclear and atomic physics. In the nonrelativistic limit the theory is equivalent to the many-body Schrödinger theory. DLCQ has been successfully applied to a number of field theories in one-space and one-time dimension, providing not only the bound-state spectrum of these theories, but also the light-cone wavefunctions needed to compute structure functions, intrinsic sea-quark distributions, and the e^+e^- annihilation cross section.

Although the primary goal has been to apply light-cone methods to non-perturbative problems in QCD in physical space-time, it is important to validate these techniques for the much simpler Abelian theory of QED. The discretized quantization of quantum electrodynamics on the light-cone in principle allows practical numerical solutions for obtaining its spectrum and wavefunctions at arbitrary coupling strength α. We also have discussed a frame-independent and approximately gauge-invariant particle number truncation of the Fock basis which is useful both for computational purposes and physical approximations. In this method[20] ultraviolet and infrared regularizations are kept independent of the discretization procedure, and are identical to that of the continuum theory. One thus obtains a finite discrete representation of the gauge theory which is faithful to the continuum theory and is completely independent of the choice of Lorentz frame.

Light-cone quantization appears to have the potential for solving important non-perturbative problems in gauge theories. It has a number of intrinsic advantages:

- The formalism is independent of the Lorentz frame—only relative momentum coordinates appear. The computer does not know the Lorentz frame!
- Fermions and derivatives are treated exactly; there is no fermion-doubling problem.
- The ultraviolet and infrared regulators can be introduced as frame independent momentum space cut-offs of the continuum theory, independent of the discretization.
- The field theoretic and renormalization properties of the discretized theory are faithful to the continuum theory. No non-linear terms are introduced by the discretization.
- One can use the exact global symmetries of the continuum Lagrangian to prediagonalize the Fock sectors.
- The discretization is denumerable; there is no over-counting. The minimum number of physical degrees of freedom are used because of the light-cone gauge. No Gupta-Bleuler or Faddeev-Popov ghosts occur and unitarity is explicit.
- Gauge invariance is lost in a Hamiltonian theory. However, the truncation can be introduced in such a way as to minimize explicit breaking of the gauge symmetries.[20]
- The output of H_{LC} matrix diagonalization is the full color-singlet spectrum of the theory, both bound states and continuum, together with their respective light-cone wavefunctions.

There are, however, a number of difficulties that need to be resolved:

- The number of degrees of freedom in the representation of the light-cone Hamiltonian increases rapidly with the maximum number of particles in the Fock state. Although heavy quark bound states probably only involve a minimal number of gluons in flight, this is most likely not true for light hadrons.
- Some problems of ultraviolet and infrared regulation remain. Although Pauli-Villars ghost states and finite photon mass can be used to regulate Abelian theories, it is not suitable method in non-Abelian theories.[65]

- The renormalization procedure is not completely understood in the context of non-perturbative problems. However, a non-perturbative recursive representation for electron mass renormalization has been successfully tested in QED(3+1).[20]
- The Coulomb singularity in the effective gluon-exchange potential is poorly approximated in the discrete form. An analytic trick must be used to speed convergence. Such a method has been tested successfully in the case of the positronium spectrum in QED(3+1).[22]
- The vacuum in QCD is not likely to be trivial since the four-point interaction term in $g^2 G^2_{\mu\nu}$ can introduce new zero-mode color-singlet states which mix with the free vacuum state. Thus a special treatment of the QCD vacuum is required. In the case of zero mass quarks, there may be additional mixing of the perturbative vacuum with fermion zero-modes.

In addition to its potential for solving the problems of the hadronic spectrum and wavefunctions of QCD, light-cone quantization has already led to many new insights into the quantization of gauge theories. It has also brought a refocus of both theory and experiment to the novel features of QCD phenomena at the amplitude level.

12. Acknowledgements

I wish to thank Peter Zerwas and the other members of the organizing committee for their outstanding hospitality at Aachen. I also thank Tom Hyer, Peter Lepage, Gary McCartor, Hans Christian Pauli, and Steve Pinsky for helpful discussions. This work was supported by the U.S. Department of Energy under Contract No. DE-AC03-76SF00515.

REFERENCES

1. For a recent review of the light-cone quantization of gauge theories see S. J. Brodsky, G. McCartor, H. C. Pauli, S. S. Pinsky, SLAC-PUB-5811, (1992).
2. For further discussions on many of the topics of this talk and further references, see S. J. Brodsky and H. C. Pauli in *Recent Aspects of Quantum Fields*, H. Mitter and H. Gausterer, Eds.; Lecture Notes in Physics, Vol. 396, Springer-Verlag, Berlin, Heidelberg, (1991).
3. W. A. Bardeen, R. B. Pearson, and E. Rabinovici, *Phys. Rev.* **D21** (1980) 1037; P. A. Griffin, *Nucl. Phys.* **B372** (1992) 270.
4. D. Mustaki, S. Pinsky, J. Shigemitsu, K. Wilson *Phys. Rev.* **D43** (1991) 3411; S. Glazek, A. Harindranath, S. Pinsky, J. Shigemitsu, K. Wilson, Ohio State University preprint OHSTPY-HEP-T-92-004, and references therein.
5. V. L. Chernyak and A. R. Zhitnitskii, *Phys. Rept.* **112** (1984) 173; V. L. Chernyak, A. A. Oglobin, and I. R. Zhitnitskii, *Sov. J. Nucl. Phys.* **48** (1988) 536; I. D. King and C. T. Sachrajda, *Nucl. Phys.* **B297** (1987) 785; M. Gari and N. G. Stefanis, *Phys. Rev.* **D35** (1987) 1074; and references therein.
6. A. S. Kronfeld and D. M. Photiadis, *Phys. Rev.* **D31** (1985) 2939.
7. G. Martinelli and C. T. Sachrajda, *Phys. Lett.* **B217** (1989) 319. Equal-time Fock state Coulomb gauge position space wavefunctions have been computed in quenched approximation lattice QCD in M. W. Hecht and T. DeGrand, *Phys. Rev.* **D46** 2155 (1992).
8. G. P. Lepage and S. J. Brodsky, *Phys. Rev.* **D22**, 2157 (1980); *Phys. Lett.* **87B** (1979) 359; *Phys. Rev. Lett.* **43** (1979) 545, 1625E.
9. See P. Kroll, Wuppertal University preprint WU-B-90-17 (1990), and references therein.
10. For a review, see S. J. Brodsky, SLAC-PUB 5529 (1991), published in the proceedings of the Lake Louise Winter Institute (1991).
11. G. Bertsch, S. J. Brodsky, A. S. Goldhaber, J.F. Gunion, *Phys. Rev. Lett.* **47** (1981) 297; S. J. Brodsky, P. Hoyer, *Phys. Rev. Lett.* **63** (1989) 1566; S. J. Brodsky, P. Hoyer, A. H. Mueller, W. - K. Tang, *Nucl. Phys.* **B369** (1992) 519. H. Heiselberg, G. Baym, B. Blaettel, L. L. Frankfurt, M. Strikman *Phys. Rev. Lett.* **67** (1991) 2946.
12. P.A.M. Dirac, *Rev. Mod. Phys.* **21** (1949) 392.
13. S. J. Brodsky, R. Roskies, and R. Suaya, *Phys. Rev.* **D8** (1973) 4574. See also S. J. Brodsky and A. Langnau, SLAC-PUB-5667 (1991), and references therein.
14. H. C. Pauli and S. J. Brodsky, *Phys. Rev.* **D32** (1985) 1993; *Phys. Rev.* **D32**1 (1985) 2001.
15. T. Eller, H. C. Pauli, S. J. Brodsky, *Phys. Rev.* **D35** (1987) 1493.
16. A. Harindranath and J. P. Vary, *Phys. Rev.* **D36** (1987) 1141.

17. K. Hornbostel, S. J. Brodsky, H. C. Pauli, *Phys. Rev.* **D41** (1990) 3814. S. J. Brodsky and K. J. Hornbostel, to be published.
18. M. Burkardt, *Nucl. Phys.* **A504** (1989) 762.
19. J. R. Hiller, University of Minnesota preprints (1990), and *Phys. Rev.* **D43** (1991) 2418.
20. A. C. Tang, S. J. Brodsky, and H. C. Pauli, *Phys. Rev.* **D44** (1991) 1842.
21. M. Kaluza and H.C. Pauli, *Phys. Rev.* **D45** (1992) 2968.
22. M. Krautgartner, H.C. Pauli, F. Wolz *Phys. Rev.* **D45** (1992) 3755.
23. S. D. Drell and T. M. Yan, *Phys. Rev. Lett.* **24** (1970) 181.
24. For reviews of the theory of exclusive processes in QCD and further references see S. J. Brodsky and G. P. Lepage in *Perturbative Quantum Chromodynamics*, edited by A. Mueller (World Scientific, Singapore, 1989). A comprehensive comparison of baryon form factors with PQCD is given in P. Stoler, Rensselaer Polytechnic Institute preprint (1992), to be published in Physics Reports.
25. S. J. Brodsky and A. H. Mueller, *Phys. Lett.* **206B** (1988) 685.
26. General QCD analyses of exclusive processes are given in Ref. 8, S. J. Brodsky and G. P. Lepage, SLAC-PUB-2294, presented at the Workshop on Current Topics in High Energy Physics, Cal Tech (Feb. 1979), S. J. Brodsky, in the Proc. of the La Jolla Inst. Summer Workshop on QCD, La Jolla (1978), A. V. Efremov and A. V. Radyushkin, *Phys. Lett.* **B94** (1980) 245, V. L. Chernyak, V. G. Serbo, and A. R. Zhitnitskii, *Yad. Fiz.* **31,** (1980) 1069, S. J. Brodsky, Y. Frishman, G. P. Lepage, and C. Sachrajda, *Phys. Lett.* **91B** (1980) 239, and A. Duncan and A. H. Mueller, *Phys. Rev.* **D21** (1980) 1636.
27. QCD predictions for the pion form factor at asymptotic Q^2 were first obtained by V. L. Chernyak, A. R. Zhitnitskii, and V. G. Serbo, *JETP Lett.* **26** (1977) 594, D. R. Jackson, Ph.D. Thesis, Cal Tech (1977), and G. Farrar and D. Jackson, *Phys. Rev. Lett.* **43** (1979) 246. See also A. M. Polyakov, *Proc. of the Int. Symp. on Lepton and Photon Interactions at High Energies*, Stanford (1975), and G. Parisi, *Phys. Lett.* **84B** (1979) 225. See also S. J. Brodsky and G. P. Lepage, in *High Energy Physics–1980*, proceedings of the XXth International Conference, Madison, Wisconsin, edited by L. Durand and L. G. Pondrom (AIP, New York, 1981); p. 568. A. V. Efremov and A. V. Radyushkin, Rev. Nuovo Cimento **3**, 1 (1980); *Phys. Lett.* **94B** (1980) 245. V. L. Chernyak and A. R. Zhitnitskii, *JETP Lett.* **25** (1977) 11; M. K. Chase, *Nucl. Phys.* **B167** (1980) 125.
28. N. Isgur and C. H. Llewellyn Smith, *Phys. Rev. Lett.* **52** (1984) 1080; *Phys.Lett* **B217** (1989) 535.
29. A. V. Radyushkin, *Nucl. Phys.* **A532** (1991) 141.
30. J. Botts and G. Sterman, *Nucl. Phys.* **B325** (1989) 62; *Phys. Lett.* **B224** (1989) 201; J. Botts, J.-W. Qiu, and G. Sterman, *Nucl. Phys.* **A527** (1991) 577. H. N. Li and G. Sterman, Stony Brook preprint ITP-SB-92-10 (1991); H. N. Li, Stony Brook preprint ITP-SB-92-25 (1991).

31. A. N. Kronfeld and B. Nizic, *Phys. Rev.* **D44** (1991) 3445. B. Nizic *Phys. Rev.* **D35**(1987) 80.
32. S. J. Brodsky and G. R. Farrar, *Phys. Rev.* **D11** (1975) 1309.
33. P. Stoler, Phys. Rev. **D44** (1991) 73, *Phys. Rev. Lett.* **66** (1991) 1003.
34. Higher twist amplitudes in the factorized PQCD formalism are computed in S. J. Brodsky, E. L. Berger, and, G. Peter Lepage, published in proceedings of the Drell Yan Workshop, Fermilab (1982); E. L. Berger and S. J. Brodsky, *Phys. Rev. Lett.* **42** (1979) 940. For a recent analysis and additional references see S. S. Agaev, Baku State University preprint BSU HEP-0003 (1992).
35. See, e.g., J. S. Conway et al., *Phys. Rev.* **D39** (1989) 92.
36. S. J. Brodsky and G. P. Lepage, *Phys. Rev.* **D24** (1981) 1808.
37. B. Nizic, *Fizika* **18** (1986) 113.
38. For a recent review of exclusive two-photon processes, see S. J. Brodsky, SLAC-PUB-5088 in the Proceedings of the *Tau-Charm Workshop*, Stanford, CA (1989).
39. S. J. Brodsky, G. P. Lepage, and P. B. Mackenzie, *Phys. Rev.* **D28** (1983) 228.
40. G. R. Farrar, *et al. Nucl. Phys.* **B311** (1989) 585.
41. D. Millers and J. F. Gunion, *Phys. Rev.* **D34** (1986) 2657.
42. T. Hyer, SLAC-PUB 5889 (1992).
43. S. J. Brodsky, F. E. Close, J. F. Gunion, *Phys. Rev.* **D6** (1972) 177.
44. M. A. Shupe, *et al.*, *Phys. Rev.* **D19** (1979) 1921.
45. G. P. Lepage and S. J. Brodsky, *Phys. Rev.* **D24** (1981) 2848.
46. See e.g., S. J. Brodsky and J. F. Gunion, *Phys. Rev. Lett.* **37** (1976) 402.
47. A. Duncan, and A. H. Mueller, *Phys. Lett.* **90B** (1980) 159.
48. A. Szczepaniak and L. Mankiewicz, *Phys. Lett.* **B266** (1991) 153.
49. S. J. Brodsky and G. F. de Teramond, *Phys. Rev. Lett.* **60** (1988) 1924.
50. M. Luke, A. V. Manohar, M. J. Savage, preprint UCSD-PTH-92-12, (1992).
51. S. J. Brodsky, and G. F. de Teramond, and I. A. Schmidt, *Phys. Rev. Lett.* **64** (1990) 1011.
52. S. J. Brodsky, G. Kopp, and P. Zerwas, *Phys. Rev. Lett.* **58** (1987) 443.
53. W. Bartel, *et al.*, *Phys. Lett.* **184B** (1987) 288.
54. A. Szczepaniak, E. M. Henley, S. J. Brodsky, *Phys. Lett.* **B243** (1990) 287.
55. P. Stoler, *Phys. Rev. Lett.* **66** (1991) 1003; and to be published in *Phys. Rev. D*.
56. S. J. Brodsky, G. P. Lepage, and S. A. A. Zaidi, *Phys. Rev.* **D23** (1981) 1152.
57. S. J. Brodsky, C. Greub, C. Munger, and D. Wyler, to be published.
58. G. D. Date, Y. Frishman, J. Sonnenschein, *Nucl. Phys.* B283 365, (1987); Y. Frishman, J. Sonnenschein WIS-92-54-PH, (1992).

59. For a recent discussion and further references, see C. S. Kim, *Nucl. Phys.* **B353** (1991) 87.

60. J. J. Aubert, *et al.*, *Nucl. Phys.* **B213** (1983) 31. See also E. Hoffmann and R. Moore, *Z. Phys.* **C20** (1983) 71.

61. K. Hornbostel, private communication; S. J. Brodsky and K. Hornbostel, to be published.

62. S. J. Brodsky and I. A. Schmidt, *Phys. Lett.* **B234** (1990) 144; *Phys. Rev.* **D43** (1991) 179.

63. G. P. Lepage, S. J. Brodsky, T. Huang, P. B. Mackenzie, published in the *Proceedings of the Banff Summer Institute*, 1981.

64. S. J. Brodsky and P. Hoyer, SLAC-PUB-5422, (1991).

65. It also should be noted that in Gribov's approach to quark confinement, Pauli-Villars or dimensional regulation cannot even be used in principle for strong coupling problems in QCD because of the way it eliminates the negative energy sea.

66. S. J. Brodsky, Y. Frishman, G. P. Lepage and C. Sachrajda, *Phys. Lett.* **91B** (1980) 239. M. E. Peskin, *Phys. Lett.* **88B** (1979) 128.

67. M. Burkardt, A. Langnau, SLAC-PUB-5394, (1990), and to be published.

68. T. Heinzl, S. Kruschke, E. Werner, *Phys. Lett.* **272B** (1991) 54.

69. D. G. Robertson, SMUHEP/92-03 (1992); G. McCartor, *Z. Phys.* **C52** (1992) 611; G. McCartor and D. G. Robertson, **Z. Phys. C53** (1992) 679.

70. P. A. Griffin, University of Florida preprint UFIFT-HEP-92-17.

71. K. Hornbostel, *Phys. Rev.* **D45** (1992) 3781.

72. J. J. Wivoda and J. R. Hiller, University of Minnesota Preprint (1992).

73. D. Soper, to be published.

74. A. Bassetto, Padova preprint PDFPD/91/TH/19 (1991).