QCD Sum Rule Analysis of the Subleading Isgur-Wise
Form Factor $\chi_2(v \cdot v')$ *

Matthias Neubert

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

Zoltan Ligeti and Yosef Nir

Weizmann Institute of Science
Physics Department, Rehovot 76100, Israel

We present a QCD sum rule calculation of the spin-symmetry violating universal function $\chi_2(v \cdot v')$, which appears at order $1/m_Q$ in the heavy quark expansion of meson form factors. This function vanishes in the standard approximation, where radiative effects are neglected. For the first time, the complete set of diagrams arising at order α_s is evaluated. In particular, we find $\chi_2(1) = -(3.8 \pm 0.4)\%$ at zero recoil, indicating that $1/m_Q$ corrections induced by the chromo-magnetic moment operator are small.

(submitted to Physics Letters B)

arXiv:hep-ph/9209271   23 Sep 1992

I. INTRODUCTION

Recently, the discovery of a spin-flavor symmetry that QCD reveals for heavy quarks has increased the prospects both for a reliable determination of some of the weak mixing angles and a study of nonperturbative QCD in semileptonic decays of heavy mesons [1,2]. These symmetries are responsible for restrictive relations among weak decay amplitudes and reduce the number of independent form factors. The heavy quark effective theory provides a convenient framework in which to analyze such processes [3–6]. It allows a systematic expansion of decay amplitudes in powers of $1/m_Q$. The coefficients in this expansion are universal functions of the kinematic variable $y = v \cdot v'$, where v and v' denote the velocities of the initial and final mesons. They are independent of the masses and spins of the heavy quarks, but characterize the properties of the light degrees of freedom. At leading order, only a single function $\xi(y)$ appears. The conservation of the vector current implies that this celebrated Isgur-Wise form factor is normalized at zero recoil, allowing model-independent predictions unaffected by hadronic uncertainties.

Already at order $1/m_Q$, however, one encounters a set of four additional universal functions $\xi_3(y)$ and $\chi_i(y)$ for $i = 1, 2, 3$, as well as a parameter $\bar{\Lambda}$, which denotes the mass difference between a heavy meson and the heavy quark that it contains [7]. These functions break the spin-flavor symmetries and induce corrections to the relations valid in the infinite quark mass limit. At zero recoil, two of them can be shown to vanish, $\chi_1(1) = \chi_3(1) = 0$ [7], but $\xi_3(1)$ and $\chi_2(1)$ are nonzero and lead to observable effects. For instance, the leading correction to the ratio of the axial form factors describing $B \to D^* \ell \nu$ decays is given by [8]

$$\frac{4 m_B m_{D^*}}{(m_B + m_{D^*})^2} \frac{A_2(q^2_{\max})}{A_1(q^2_{\max})} = 1 - \frac{\bar{\Lambda}}{2 m_c} \left[(1 + 3r) \, \xi_3(1) + 4(1 - r) \, \chi_2(1) \right], \tag{1}$$

where $r = m_c/m_b$. An understanding of the universal form factors is at the heart of nonperturbative QCD, but is necessary for any phenomenological application of the heavy quark expansion. QCD sum rules are particularly suited for this purpose and have recently been employed to calculate the Isgur-Wise form factor [9–12], the mass parameter $\bar{\Lambda}$ [13,14], and the universal functions that appear at order $1/m_Q$ in the heavy quark expansion [8]. For these subleading form factors an interesting result was obtained. Within the approximations usually adopted in QCD sum rules, one finds the parameter-free predictions [8]

$$\xi_3(1) = \tfrac{1}{D-1} = \tfrac{1}{3}, \qquad \chi_2(y) = 0, \tag{2}$$

where D is the space-time dimension. Corrections to these relations could only come from radiative effects or from condensates of rather high dimension, both of which are expected to be small and are usually neglected. Provided those corrections are indeed negligible, the predictions (2) allow for an essentially model-independent description of the $1/m_Q$ corrections at zero recoil. For instance, using $r \simeq 1/3$ and $\bar{\Lambda} \simeq 0.5$ GeV, one finds $\simeq 0.9$ for the right-hand side of (1).

In this paper we investigate the corrections to the second relation in (2), which arise if one goes beyond the standard approximations by including diagrams of order α_s in the sum rule for $\chi_2(y)$. In particular, we calculate for the first time the two-loop radiative corrections to the perturbative triangle diagram.

2

II. DERIVATION OF THE SUM RULE

The heavy quark effective theory provides an expansion of hadronic matrix elements in powers of $1/m_Q$. Its construction is based on the observation that, in the limit $m_Q \gg \Lambda_{\mathrm{QCD}}$, the velocity v of a heavy quark Q is conserved with respect to soft processes [3]. In this limit it is possible to remove the m_Q-dependent piece of the heavy quark momentum P_Q by the field redefinition

$$h_Q(v,x) = e^{im_Q v \cdot x}\, P_+ \, Q(x) \tag{3}$$

where $P_+ = \frac{1}{2}(1+\not{v})$ is a positive-energy projection operator. For simplicity we will abbreviate $h = h_Q(v,x)$. The new field h carries the residual "off-shell" momentum $k = P_Q - m_Q v$, which is of order Λ_{QCD}. The effective Lagrangian for the strong interactions of the heavy quark becomes [3,4,15]

$$\mathcal{L}_{\mathrm{HQET}} = \bar{h}\, iv\cdot D\, h + \frac{1}{2m_Q}\Big[\bar{h}\,(iD)^2 h + Z\,\frac{g_s}{2}\,\bar{h}\,\sigma_{\alpha\beta}G^{\alpha\beta}h\Big] + \dots, \tag{4}$$

where Z is a renormalization factor, $D = \partial - ig_s t_a A_a$ is the gauge-covariant derivative, and G is the gluon field strength.

A similar expansion can be performed for a generic heavy quark current $\bar{Q}'\,\Gamma\,Q$, with Γ being a 4×4 Dirac matrix carrying any number of Lorentz indices ($\Gamma = \gamma_\mu(1-\gamma_5)$ for the weak flavor-changing current). At tree-level, it simply reads

$$\bar{Q}'\,\Gamma\,Q \to C\,\bar{h}'\,\Gamma\,h + \dots, \tag{5}$$

where $h' = h_{Q'}(v')$, and C is again a renormalization factor [5,16].

A convenient way to evaluate hadronic matrix elements in the effective theory is by using the covariant trace formalism introduced in Refs. [5,17]. Matrix elements of the leading current operator in (5) can be parameterized as ($y = v \cdot v'$)

$$\langle M'(v')|\,\bar{h}'\,\Gamma\,h\,|M(v)\rangle = -\xi(y)\,\mathrm{tr}\{\overline{\mathcal{M}}'(v')\,\Gamma\,\mathcal{M}(v)\}, \tag{6}$$

where $\xi(y)$ is the Isgur-Wise function, and

$$\mathcal{M}(v) = \sqrt{m_M}\,P_+ \begin{cases} -\gamma_5 & ;\ \text{pseudoscalar meson} \\ \not{\epsilon} & ;\ \text{vector meson} \end{cases} \tag{7}$$

is a spin wave function associated with the pseudoscalar or vector meson M, which has the right transformation properties under Lorentz boosts and heavy quark spin rotations. At subleading order in the $1/m_Q$ expansion, matrix elements receive contributions from higher-dimension operators in the effective Lagrangian (4) and in the effective current (5). A particular type of correction comes from an insertion of the chromo-magnetic operator contained in $\mathcal{L}_{\mathrm{HQET}}$. The corresponding matrix elements of

$$\mathcal{O}(x) = i\int dy\, T\Big\{[\,\bar{h}'\,\Gamma\,h\,]_x,\, [\,\tfrac{g_s}{2}\,\bar{h}\,\sigma_{\alpha\beta}G^{\alpha\beta}h\,]_y\Big\} \tag{8}$$

can be parameterized by a tensor form factor,

$$\langle M'(v')|\,\mathcal{O}\,|M(v)\rangle = -\bar{\Lambda}\,\mathrm{tr}\Big\{\chi^{\alpha\beta}(v,v')\,\overline{\mathcal{M}}'(v')\,\Gamma\,P_+\,i\sigma_{\alpha\beta}\mathcal{M}(v)\Big\}. \tag{9}$$

Because $v^\alpha P_+\sigma_{\alpha\beta}\mathcal{M}(v) = 0$, the most general decomposition of $\chi^{\alpha\beta}$ consistent with Lorentz invariance and the heavy quark symmetries involves two real, scalar functions χ_2 and χ_3 defined by [7]

$$\chi^{\alpha\beta}(v,v') = (v'^\alpha\gamma^\beta - v'^\beta\gamma^\alpha)\,\chi_2(y) - 2i\sigma^{\alpha\beta}\,\chi_3(y). \tag{10}$$

Since we have factored out $\bar{\Lambda}$ in (9), the functions $\chi_i(y)$ are dimensionless.

The QCD sum rule analysis for these subleading form factors proceeds along the same lines as that for the Isgur-Wise function. The procedure is outlined in detail in Refs. [8,11,14], and we shall adopt the same notations here. We are interested in the analytic properties of the three-current correlator

$$\begin{aligned}
\Xi &= \int \mathrm{d}x\,\mathrm{d}y\, e^{i(k'\cdot x - k\cdot y)}\,\langle 0\,|\,T\big\{[\,\bar{q}\,\overline{\Gamma}_{M'}h'\,]_x, \mathcal{O}(0), [\,\bar{h}\,\Gamma_M\,q\,]_y\big\}|\,0\,\rangle \\
&= \Xi_2(\omega,\omega',y)\,\mathrm{tr}\Big\{(v'^\alpha\gamma^\beta - v'^\beta\gamma^\alpha)\,\overline{\Gamma}_{M'}P'_+\Gamma\,P_+\,i\sigma_{\alpha\beta}P_+\Gamma_M\Big\} \\
&\quad + \Xi_3(\omega,\omega',y)\,\mathrm{tr}\Big\{2\sigma^{\alpha\beta}\,\overline{\Gamma}_{M'}P'_+\Gamma\,P_+\sigma_{\alpha\beta}P_+\Gamma_M\Big\},
\end{aligned} \tag{11}$$

where $P'_+ = \frac{1}{2}(1+\slashed{v}')$. Depending on the choice $\Gamma_M = -\gamma_5$ or $\Gamma_M = \gamma_\mu - v_\mu$, the heavy-light currents interpolate pseudoscalar or vector mesons, respectively. In the effective theory, the coefficients Ξ_i are analytic functions in the "off-shell energies" $\omega = 2v\cdot k$ and $\omega' = 2v'\cdot k'$, with discontinuities for positive values of these variables. They receive a double-pole contribution from the ground-state mesons M and M' associated with the heavy-light currents. Using the fact that the external momenta are $P = m_Q v + k$ and $P' = m_{Q'}v' + k'$, one finds that the pole positions $P^2 = m_M^2$ and $P'^2 = m_{M'}^2$ correspond to $\omega = \omega' = 2\bar{\Lambda}$, where $\bar{\Lambda} = m_M - m_Q = m_{M'} - m_{Q'}$ [7]. The residues are proportional to the universal functions $\chi_2(y)$ and $\chi_3(y)$, respectively. It follows that [8]

$$\Xi_i^{\mathrm{pole}}(\omega,\omega',y) = \frac{\chi_i(y)\,F^2\,\bar{\Lambda}}{(\omega - 2\bar{\Lambda} + i\epsilon)\,(\omega' - 2\bar{\Lambda} + i\epsilon)} \;;\quad i = 2,3, \tag{12}$$

where F is defined by $\langle 0\,|\,\bar{q}\,\Gamma\,h\,|M\rangle = \frac{i}{2}F\,\mathrm{tr}\{\Gamma\,\mathcal{M}\}$. It is the analog of the meson decay constant in the effective theory [11].

For large negative values of ω and ω', the correlator can be calculated in perturbation theory using the Feynman rules of the heavy quark effective theory [5]. The idea of QCD sum rules is that, at the transition from the perturbative to the nonperturbative region, nonperturbative effects can be accounted for by including the leading power corrections in the operator product expansion of the three-point function. They are proportional to vacuum expectation values of local quark-gluon operators, the so-called condensates [18]. One then writes the theoretical expression for Ξ_i in terms of a double dispersion integral,

$$\Xi_i^{\mathrm{th}}(\omega,\omega',y) = \int \mathrm{d}\nu\,\mathrm{d}\nu'\,\frac{\rho_i^{\mathrm{th}}(\nu,\nu',y)}{(\nu - \omega - i\epsilon)\,(\nu' - \omega' - i\epsilon)} + \text{subtractions}, \tag{13}$$

and performs a Borel transformation

4

$$\frac{1}{\tau}\,\widehat{B}_\tau^{(\omega)} = \lim_{\substack{n\to\infty \\ -\omega\to\infty}} \frac{\omega^n}{\Gamma(n)}\left(-\frac{\mathrm{d}}{\mathrm{d}\omega}\right)^n ; \quad \tau = \frac{-\omega}{n} \text{ fixed} \tag{14}$$

with respect to ω and ω'. This yields an exponential damping factor in the dispersion integral and eliminates possible subtraction terms. Because of the flavor symmetry it is natural to set the Borel parameters equal, $\tau = \tau' = 2T$. It is then convenient to introduce new variables $\omega_\pm = \frac{1}{2}(\nu \pm \nu')$ to obtain

$$\widehat{\Xi}_i^{\mathrm{th}} = \widehat{B}_{2T}^{(\omega)}\widehat{B}_{2T}^{(\omega')}\,\Xi_i^{\mathrm{th}} = \int \mathrm{d}\omega_+ \, e^{-\omega_+/T}\int \mathrm{d}\omega_- \, 2\rho_i^{\mathrm{th}}(\omega_+{+}\omega_-,\omega_+{-}\omega_-,y). \tag{15}$$

Following Refs. [11,12] we perform the integral over ω_- and employ quark-hadron duality to equate the integral over ω_+ up to a threshold ω_0 to the Borel transform of the pole contribution in (12). This gives the QCD sum rules

$$\chi_i(y)\,F^2\,\bar{\Lambda}\,e^{-2\bar{\Lambda}/T} = \int_0^{\omega_0}\mathrm{d}\omega_+ \, e^{-\omega_+/T}\,\widetilde{\rho}_i^{\mathrm{th}}(\omega_+,y) ; \quad i = 2,3. \tag{16}$$

The spectral densities $\widetilde{\rho}_i^{\mathrm{th}}$ arise after integration of the double spectral densities over ω_-.

In this paper we concentrate on the sum rule for $\chi_2(y)$. It has been shown in Ref. [8] that within the standard approximations, where one includes the bare quark loop as well as the leading nonperturbative corrections proportional to the quark condensate and the mixed quark-gluon condensate, there is no contribution to this function. The leading terms, then, are of order α_s and come from the two-loop radiative corrections to the quark loop, the one-loop radiative corrections to the quark condensate, and the gluon condensate. In general, the calculation of all diagrams arising at order α_s in the sum rule for a three-current correlator is rather tedious. In fact, the two-loop corrections to the triangle diagram have never been computed before. Because of the particular structure of the trace associated with Ξ_2 in (11), however, the calculation is considerably simplified in this case. Only the three diagrams shown in Fig. 1 contribute. Note that for each contribution the dependence of the spectral density $\widetilde{\rho}_2^{\mathrm{th}}(\omega_+,y)$ on ω_+ is known on dimensional grounds, i.e.

$$\begin{aligned}
\widetilde{\rho}_2^{\mathrm{pert}} &\propto \omega_+^3, \\
\widetilde{\rho}_2^{\langle\bar{q}q\rangle} &\propto \mathrm{const.}, \\
\widetilde{\rho}_2^{\langle GG\rangle} &\propto \delta(\omega_+).
\end{aligned} \tag{17}$$

Instead of the spectral densities, it thus suffices to calculate directly the Borel transform of the individual contributions to Ξ_2^{th}, corresponding to the limit $\omega_0 \to \infty$ in (16). The ω_0-dependence can then be deduced from (17).

Let us now present the results of our calculation. The contributions from condensates are obtained by evaluating the diagrams shown in Fig. 1(b) and (c), using the one-loop tensor integrals given in Ref. [8]. We find

$$\widehat{\Xi}_2^{\mathrm{cond}}(T,y) = \frac{\alpha_s\langle\bar{q}q\rangle\,T}{6\pi}\left[\frac{1-r(y)}{y-1} + \frac{1}{y+1}\right] - \frac{\langle\alpha_s GG\rangle}{96\pi}\left(\frac{2}{y+1}\right), \tag{18}$$

5

where the function

$$r(y) = \frac{1}{\sqrt{y^2-1}} \ln\left(y + \sqrt{y^2-1}\right) \tag{19}$$

satisfies $r(1) = 1$ and $r'(1) = -\frac{1}{3}$, such that there is no singularity in (18) as $y \to 1$. The calculation of the two-loop diagram depicted in Fig. 1(a) is more tedious. In an intermediate step, one encounters the two-loop tensor integral

$$\int \frac{\mathrm{d}^D s}{(2\pi)^D} \frac{\mathrm{d}^D t}{(2\pi)^D} \frac{(s-t)_\alpha\, s_\beta\, t_\gamma}{(\omega + 2v\cdot s)(\omega + 2v\cdot t)(\omega' + 2v'\cdot t)\, s^2\, t^2\, (s-t)^2}. \tag{20}$$

For its evaluation it is convenient to represent the light quark propagators by Fourier transforms in a D-dimensional Euclidean space (see [19]) and use an exponential integral representation of the heavy quark propagators. After performing the integrations over the loop momenta, one is left with five parameter integrals, two of which become trivial after Borel transformation. The remaining three integrals stay finite for $D = 4$ and can be calculated in closed form. Our result is

$$\widehat{\Xi}_2^{\mathrm{pert}}(T, y) = -\frac{\alpha_s T^4}{8\pi^3} \left(\frac{2}{y+1}\right)^2 \left[\frac{1 - r(y)}{y - 1} + 2\right]. \tag{21}$$

Based on (16) and (17), we can now introduce back the dependence of the various contributions on the continuum threshold ω_0 to obtain the final sum rule

$$\begin{aligned}
\chi_2(y)\, F^2\, \bar{\Lambda}\, e^{-2\bar{\Lambda}/T} = {}& -\frac{\alpha_s T^4}{8\pi^3} \left(\frac{2}{y+1}\right)^2 \left[\frac{1 - r(y)}{y - 1} + 2\right] \delta_3\!\left(\frac{\omega_0}{T}\right) \\
& + \frac{\alpha_s \langle \bar{q}q \rangle\, T}{6\pi} \left[\frac{1 - r(y)}{y - 1} + \frac{1}{y + 1}\right] \delta_0\!\left(\frac{\omega_0}{T}\right) \\
& - \frac{\langle \alpha_s GG \rangle}{96\pi} \left(\frac{2}{y+1}\right),
\end{aligned} \tag{22}$$

where

$$\delta_n(x) = \frac{1}{\Gamma(n+1)} \int_0^x \mathrm{d}z\, z^n\, e^{-z}. \tag{23}$$

III. EVALUATION OF THE SUM RULE

For the QCD parameters in (22) we take the standard values

$$\begin{aligned}
\alpha_s/\pi &= 0.1, \\
\langle \bar{q}q \rangle &= -(0.23\,\mathrm{GeV})^3, \\
\langle \alpha_s GG \rangle &= 0.038\,\mathrm{GeV}^4.
\end{aligned} \tag{24}$$

The value of the strong coupling corresponds to the scale $\mu = 2\bar{\Lambda} \simeq 1$ GeV, which is appropriate for evaluating radiative corrections in the effective theory [11]. In addition, the

sum rule depends on the continuum threshold ω_0 and on the Borel parameter T. These quantities have recently been determined from the analysis of a QCD sum rule for the correlator of two heavy-light currents in the effective theory [14]. One finds good stability for $\omega_0 = 2.0 \pm 0.3$ GeV, and the consistency of the theoretical calculation requires that the Borel parameter be in the range $0.6 < T < 1.0$ GeV.

With this set of parameters one finds $F = 0.30 \pm 0.05$ GeV$^{3/2}$ and $\bar{\Lambda} = 0.50 \pm 0.07$ GeV for the hadronic parameters appearing on the left-hand side of (22). For reasons of consistency, however, one should rather work directly with the QCD sum rule for these quantities. This procedure minimizes the T-dependence of the final result. We thus use [14]

$$F^2 \, \bar{\Lambda} \, e^{-2\bar{\Lambda}/T} = \frac{9T^4}{8\pi^2} \, \delta_3\Big(\frac{\omega_0}{T}\Big) - \frac{m_0^2 \langle \bar{q}q \rangle}{4T}, \tag{25}$$

where $m_0^2 \simeq 0.8$ GeV^2 is the ratio of the mixed quark-gluon condensate and the quark condensate. Combining (22) and (25), we express $\chi_2(y)$ as a function of ω_0, T, and the QCD parameters. The numerical analysis of the resulting sum rule is shown in Fig. 2. We observe excellent stability against variation of the Borel parameter T, and it supports the self-consistency of the approach that the stability region coincides with that determined in Ref. [14]. Also the dependence on the threshold parameter is weak. Varying ω_0 between 1.7 and 2.3 GeV changes $\chi_2(y)$ by no more than $\pm 0.4\%$. For instance, we obtain from Fig. 2b the zero recoil prediction $\chi_2(1) = -(3.8 \pm 0.4)\%$.

Finally, we note that a rough estimate of the form factor can be obtained by neglecting the nonperturbative terms in (22) and (25). Independently of ω_0 and T, this gives the simple function

$$\chi_2^{\text{pert}}(y) = -\frac{\alpha_s}{9\pi} \left(\frac{2}{y+1}\right)^2 \Big[\frac{1 - r(y)}{y - 1} + 2\Big], \tag{26}$$

which is smaller than the complete sum rule result by roughly a factor $\frac{2}{3}$. At zero recoil, for instance, one finds $\chi_2^{\text{pert}}(1) = -\frac{7}{27}(\alpha_s/\pi) \simeq -2.6\%$.

In summary, we have presented the complete order-α_s QCD sum rule analysis of the subleading Isgur-Wise function $\chi_2(y)$. We find that this form factor is small, typically of the order of -2% to -4%. When combined with the analysis of Ref. [8], which predicts similarly small values for $\chi_3(y)$, but much larger values for the remaining two subleading form factors $\chi_1(y)$ and $\xi_3(y)$, these results strongly indicate that power corrections in the heavy quark expansion which are related to the chromo-magnetic moment operator are small.

ACKNOWLEDGMENTS

REFERENCES

[1] M.B. Voloshin and M.A. Shifman, Yad. Fiz. **45**, 463 (1987) [Sov. J. Nucl. Phys. **45**, 292 (1987)]; **47**, 801 (1988) [**47**, 511 (1988)].

[2] N. Isgur and M.B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).

[3] H. Georgi, Phys. Lett. B **240**, 447 (1990).

[4] T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. **B368**, 204 (1992).

[5] A.F. Falk, H. Georgi, B. Grinstein, and M.B. Wise, Nucl. Phys. **B343**, 1 (1990).

[6] A.F. Falk, B. Grinstein, and M.E. Luke, Nucl. Phys. **B357**, 185 (1991).

[7] M.E. Luke, Phys. Lett. B **252**, 447 (1990).

[8] M. Neubert, SLAC preprint SLAC–PUB–5826, 1992 (to appear in Phys. Rev. D **46**, Vol. 9).

[9] M. Neubert, V. Rieckert, B. Stech, and Q.P. Xu, in *Heavy Flavours*, edited by A.J. Buras and M. Lindner, Advanced Series on Directions in High Energy Physics (World Scientific, Singapore, 1992).

[10] A.V. Radyushkin, Phys. Lett. B **271**, 218 (1991).

[11] M. Neubert, Phys. Rev. D **45**, 2451 (1992).

[12] B. Blok and M. Shifman, Santa Barbara preprint NSF–ITP–92–100, 1992.

[13] E. Bagan, P. Ball, V.M. Braun, and H.G. Dosch, Phys. Lett. B **278**, 457 (1992).

[14] M. Neubert, Phys. Rev. D **46**, 1076 (1992).

[15] A.F. Falk, M. Neubert, and M.E. Luke, SLAC preprint SLAC–PUB–5771, 1992 (to appear in Nucl. Phys. **B**).

[16] M. Neubert, Phys. Rev. D **46**, 2212 (1992); Nucl. Phys. **B371**, 149 (1992).

[17] J.D. Bjorken, *Proceedings of the 18th SLAC Summer Institute on Particle Physics*, pp. 167, Stanford, California, July 1990, edited by J.F. Hawthorne (SLAC, Stanford, 1991).

[18] M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979).

[19] P. Pascual and R. Tarrach, *QCD: Renormalization for the Practitioner*, Lecture Notes in Physics No. 194, edited by H. Araki *et al.* (Springer, Berlin–Heidelberg–New York–Tokyo, 1984).

FIG. 1.   Diagrams contributing at order α_s to the sum rule for the universal form factor $\chi_2(v \cdot v')$: (a) perturbative contribution, (b) quark condensate contribution, (c) gluon condensate contribution. Heavy quark propagators are drawn as double lines, and a square represents the chromo-magnetic moment operator.

FIG. 2.   Numerical evaluation of the sum rules (22) and (25): (a) form factor $\chi_2(v \cdot v')$ evaluated for $\omega_0 = 2\,\mathrm{GeV}$ and $0.6 < T < 1.0\,\mathrm{GeV}$; (b) dependence of $\chi_2(1)$ on the Borel parameter T for $\omega_0 = 2.3,\, 2.0,\, 1.7\ \mathrm{GeV}$ (top to bottom).
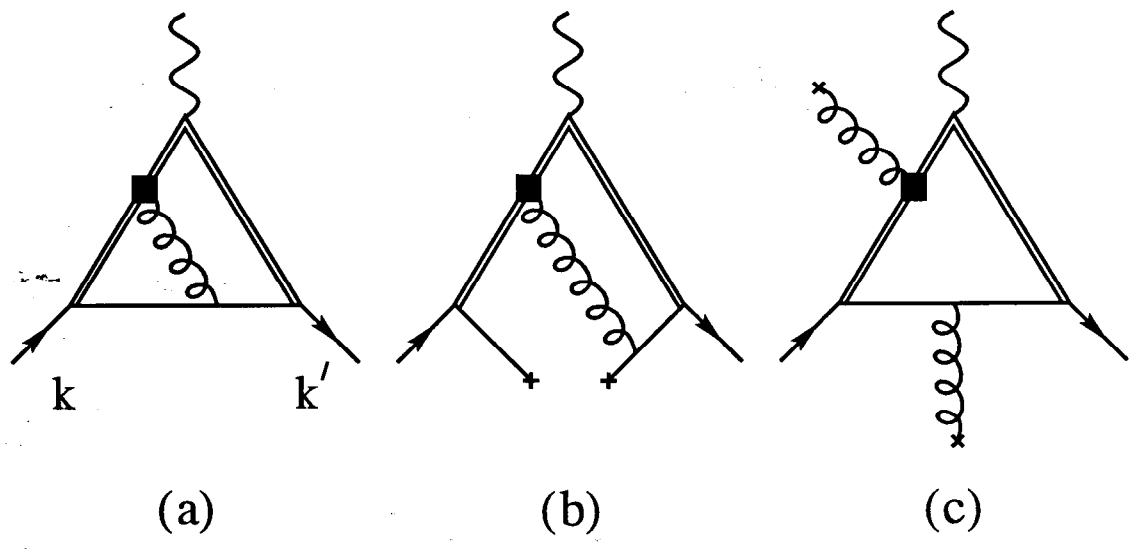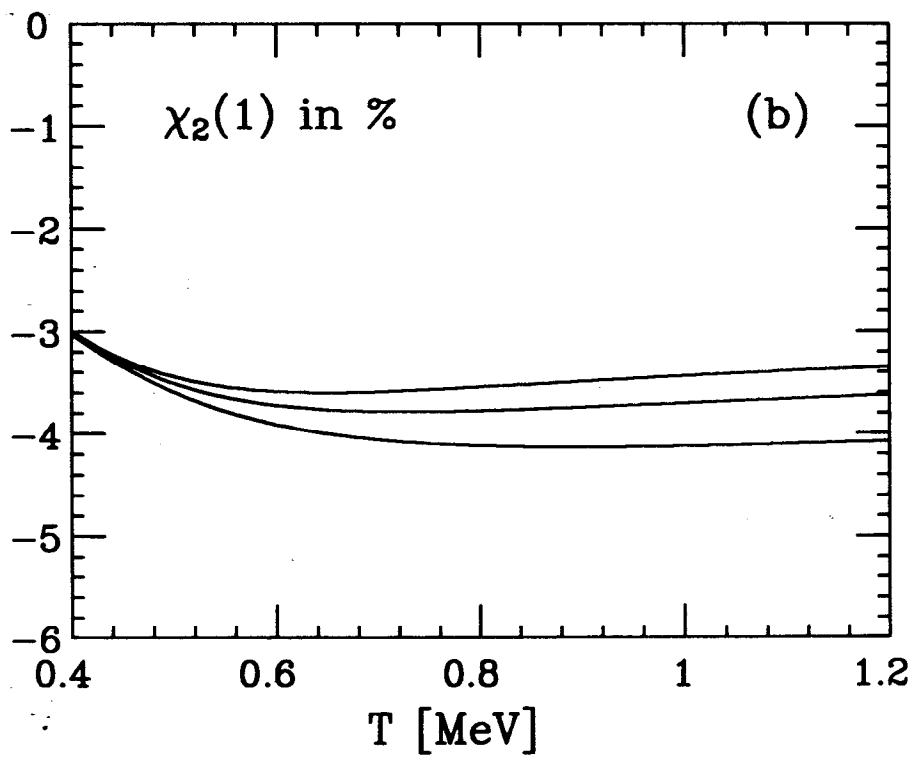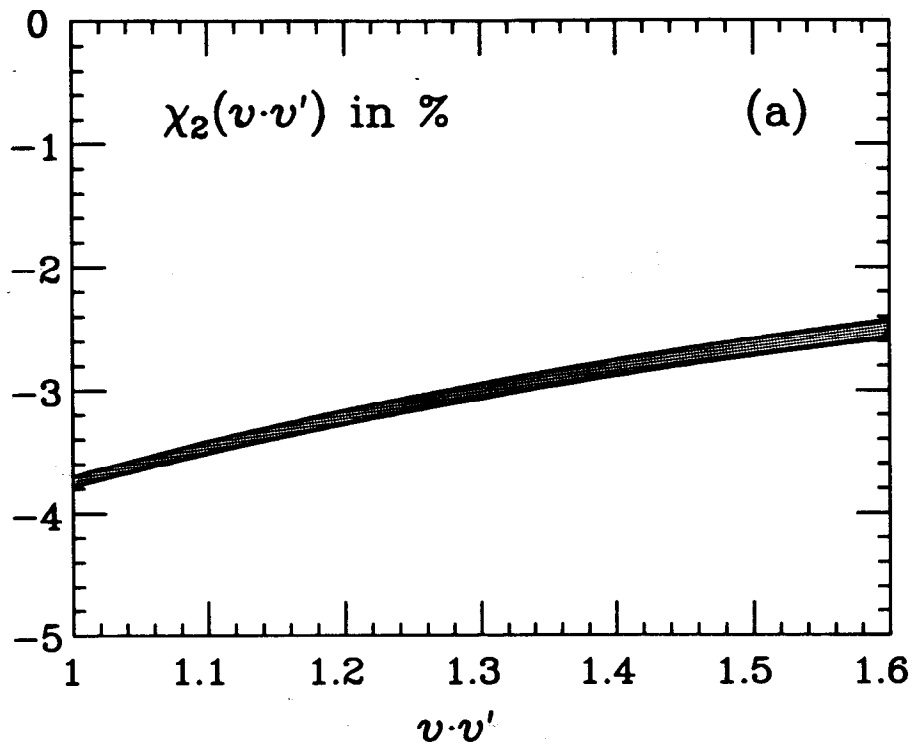
(a)   (b)   (c)

Fig.1

Fig.2