5911
September 1992
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Large Penguin effects in the CP Asymmetry of $B_d^0 \to \pi^+\pi^-$ ⋆

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ABSTRACT

Penguin effects in the CP asymmetries of $B_d^0 \to \pi^+\pi^-$, $B_d^0 \to \rho^\pm\pi^\mp$ and $B_d^0 \to a_1^\pm\pi^\mp$ are studied as function of the CKM unitarity triangle α. Despite a fairly small penguin amplitude, it leads to quite sizable uncertainties in the determination of $\sin(2\alpha)$ from all but very large asymmetries. This effect is maximal for vanishing final state interaction phases, for which it can cause, for instance, an asymmetry of 40% if $\alpha = \pi/2$.

Submitted to *Physics Letters B*

⋆ Work supported by the Department of Energy, contract DE–AC03–76SF00515.
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There are two good reasons for which B mesons provide a unique opportunity for testing the Cabibbo-Kobayashi-Maskawa (CKM) mechanism of CP violation [1]. The CP asymmetries in certain decays, most notably decays to CP- eigenstates, are expected to be both large and theoretically clean. For instance, in $B_d^0 \to \psi K_S$ the time-dependent asymmetry is predicted to oscillate with an amplitude given directly by $\sin(2\beta)$ [2], where β is one of the angles of the CKM unitarity triangle. This relation between a measured asymmetry and a pure CKM phase parameter follows from having essentially a single weak phase which contributes to the decay. In the case of $B_d^0 \to \psi K_S$, where it is known that $\sin(2\beta) \geq 0.08$ [1], this relation is expected to hold within a 1% accuracy [3].

Another decay mode which seems to be very promising is $B_d^0 \to \pi^+\pi^-$, in which the asymmetry is related to the angle α. In this case the theoretical situation is somewhat less clean due to the contribution of "penguin" amplitudes [3,4,5], which may interfere with the dominant "tree" amplitude through their different weak phases. Here the ratio of penguin-to-tree amplitudes is roughly estimated to be at the level of $(10-20)\%$. This estimate may lead one to conclude that an uncertainty at this level applies also to the relation between the measured asymmetry and $\sin(2\alpha)$. This by itself would not have spoiled the testing power of the asymmetry measurement. The purpose of this note is to critically elaborate on this question. We will show that, even with a relatively small penguin contribution and with small final-state interaction phases, penguin effects on the asymmetry may, in fact, be quite large for $|\sin(2\alpha)| \leq 0.7$. Therefore, unless a very large asymmetry is measured, this would prohibit obtaining a useful value of α from the asymmetry measurement.

The formalism of studying CP asymmetries in neutral B decays to CP eigenstates in the presence of two interfering decay amplitudes was given in[3]. For completeness, we write down the basic equations and study them for the case of $B_d^0 \to \pi^+\pi^-$. We denote the amplitude of $B_d^0 \to \pi^+\pi^-$ by A and that of $\overline{B}_d^0 \to \pi^+\pi^-$ by $\overline{A}$. Each of these amplitudes obtains contributions from "tree" and from "penguin" amplitudes:

$$A = A_T e^{i\delta_T} e^{i\phi_T} + A_P e^{i\delta_P} e^{i\phi_P} \ ,$$

$$\overline{A} = A_T e^{i\delta_T} e^{-i\phi_T} + A_P e^{i\delta_P} e^{-i\phi_P} \ . \tag{1}$$

$A_{T,P}$ are real, $\phi_{T,P}$ are CKM phases and $\delta_{T,P}$ are strong interaction final state phases, all corresponding to the "tree" and "penguin" amplitudes, respectively. It should be mentioned at this point that $\delta_{T,P}$ stand for soft final state interaction phases. We neglect a phase due to the absorptive part of the physical $c\overline{c}$ quark pair in the penguin diagram[6]. This phase is very small at the inclusive $b \to u\overline{u}d$ level[7], and is not expected to be considerably larger for exclusive modes such as $\pi^+\pi^-$, where the absorptive part picks up contributions from a limited q^2 range[8].

The time-dependent CP asymmetry for a neutral B meson, created at $t = 0$ as a pure B_d^0 and decaying at time t to $\pi^+\pi^-$, when compared to the corresponding decay rate of an initially $\overline{B}_d^0$, is[3]:

$$Asym(t) = \frac{1 - |\frac{\overline{A}}{A}|^2}{1 + |\frac{\overline{A}}{A}|^2} \cos(\Delta m t) - \frac{2\mathrm{Im}(\frac{\overline{A}}{A} e^{-2i\beta})}{1 + |\frac{\overline{A}}{A}|^2} \sin(\Delta m t) \ . \tag{2}$$

Δm is the mass-difference of the two neutral B mesons. The phase 2β appears in the $B_d^0 - \overline{B}_d^0$ mixing amplitude in the standard CKM phase convention. The two terms in Eq.(2) describe two different kinds of CP violating phenomena. The first $\cos(\Delta mt)$ term, which describes CP violation in the direct decay of B_d^0, vanishes when only a single CKM phase contributes to the decay process. The coefficient of the well-known $\sin(\Delta mt)$ term, which appears when the mixed B_d^0 and $\overline{B}_d^0$ decay to a comomn final state, is given by $\sin(2\alpha)$ when only one CKM phase contributes.

For $A_P/A_T \ll 1$ one finds the following expressions[3] for the two coefficients in Eq.(2):

$$\frac{1-|\frac{\overline{A}}{A}|^2}{1+|\frac{\overline{A}}{A}|^2} \approx -2\frac{A_P}{A_T}\sin(\phi_T-\phi_P)\sin(\delta_T-\delta_P) \ ,$$

$$\frac{2\mathrm{Im}(\frac{\overline{A}}{A}e^{-2i\beta})}{1+|\frac{\overline{A}}{A}|^2} \approx \sin(2\alpha)+2\frac{A_P}{A_T}\sin(\phi_T-\phi_P)\cos(2\alpha)\cos(\delta_T-\delta_P) \ . \tag{3}$$

In order to evaluate the penguin effects one must know the three quantities $(\phi_T - \phi_P)$, A_P/A_T, $(\delta_T-\delta_P)$. Only the first quantity can be studied theoretically in a reliable manner. In the standard CKM phase convention $\phi_T = phase(V_{ub}^*V_{ud}) = \gamma$. The penguin amplitude, on the other hand, obtains contributions from three diagrams in which u,c,t quarks run in a loop. Denoting the three amplitudes, from which the CKM factors are omitted, by P_u, P_c, P_t, one notes that within $\mathcal{O}(m_c^2/m_b^2)$ one has $P_c \approx P_u$. It then follows from the unitarity of the CKM matrix that $\phi_P \approx phase(V_{tb}^*V_{td}) = -\beta$, and therefore

$$\phi_T - \phi_P \approx \gamma+\beta = \pi-\alpha \ . \tag{4}$$

The ratio A_P/A_T cannot be calculated reliably at present. We will attempt to evaluate it in two different manners. To be conservative, we will try not to overestimate it. The ratio of loop-induced processes $b \to dq\overline{q}$ $(q=u,d,s,c), dg, dgg$

to the tree process $b \to u\overline{u}d$ was calculated perturbatively at the quark and gluon level for the rates of inclusive charmless-strangeless decays[9]. The penguin processes $b \to dq\overline{q}$ dominate the loop induced processes. The penguin-to-tree ratio of rates was found to decrease as a function of $|V_{ub}/V_{cb}|$ within the range $0.07 < |V_{ub}/V_{cb}| < 0.20$, from a largest possible value of 0.4 to a lowest value of 0.1, depending on m_t and on CKM parameters. The quark process $b \to du\overline{u}$ is likely to be the dominant mechanism for $B_d^0 \to \pi^+\pi^-$. Its rate is smaller by about 0.3 relative to all $b \to dq\overline{q}$ processes, which leads to a penguin-to-tree ratio of amplitudes decreasing from 0.35 to 0.15 in the above range of $|V_{ub}/V_{cb}|$. This may be a slight overestimate, since in this calculation the penguin rate was maximized over the entire acceptable CKM parameter space.

A somewhat different approach which leads to a similar estimate of A_P/A_T is based on calculating the low energy effective Hamiltonian for $b \to d\overline{u}u$ in the leading log approximation[5]. One finds that the penguin operators $\overline{d}_{Li}\gamma^\mu b_{Lj}\overline{u}_{Lj}\gamma_\mu u_{Li}$, $\overline{d}_{Li}\gamma^\mu b_{Lj}\overline{u}_{Rj}\gamma_\mu u_{Ri}$, ($i, j$ are color indices and L, R are left and right projections) appear with coefficients 0.026, 0.033, respectively, and are multiplied by the CKM factor $V_{tb}V_{td}^*$. On the other hand, the relevant tree operator has a slightly enhanced coefficient (= 1.11) and is multiplied by $V_{ub}V_{ud}^*$. Adding up the penguin coefficients and allowing the ratio $|V_{tb}V_{td}^*/V_{ub}V_{ud}^*|$ to vary between the values of 1 and 5[1], one finds at the quark level $A_P/A_T \sim 0.05 - 0.27$. Note that, since the operator coefficients are all positive, A_P/A_T (which does not include final state phases) is positive too.

A very rough and oversimplified approximation, which represents the above two results, somewhat on the low side, can be obtained by simply using the CKM factors and the QCD factor related to the single t-quark penguin diagram:

$$\frac{A_P}{A_T} \sim \frac{|V_{tb}^* V_{td}|}{|V_{ub}^* V_{ud}|} \frac{\alpha_s}{12\pi} \ln(\frac{m_t^2}{m_b^2}) \sim 0.04 - 0.20 \ . \tag{5}$$

The ratio of the CKM factors is the ratio of the lengths of two sides of the CKM unitarity triangle which form the angle α. As noted above, this ratio lies between the values of 1 and 5[1] . Recent preliminary data[10] , which seem to indicate that $|V_{ub}/V_{cb}|$ is only about 0.06 or even smaller, favor a large ratio. For the QCD factor $(\alpha_s/12\pi)\ln(m_t^2/m_b^2)$ we took the value 0.04, using $\alpha_s(m_b^2) \approx 0.2$. This value would be larger if α_s were to be taken at $(m_b/2)^2$.

A large uncertainty is involved in calculating the tree and the penguin operator matrix elements between the B_d^0 and the $\pi^+\pi^-$ states[11] . Certain hadronic models seem to indicate that penguin operator matrix elements may be enhanced due to their special chiral structure[12] . Since none of the existing methods of calculating hadronic matrix elements is very reliable for our case, we will make the most simplified assumption that the ratio of these matrix elements is one, and will thus use Eq.(5) as a crude approximation. This assumption has not yet been tested experimentally even in an indirect way, that is, by comparing tree-dominated to penguin-dominated processes. We feel that, since the simplified relation (5) somewhat underestimates the ratio calculated at the quark and gluon level, it allows a certain amount of penguin matrix element suppression, and does not overestimate the ratio of matrix elements. We note again that this ratio is likely to be on the high side of (5) if $|V_{ub}/V_{cb}|$ is near its present lower limit value of 0.06.

The soft final state interaction phase difference, $\delta_T - \delta_P$, is basically uncalculable. Denoting this phase-difference by δ, one finds from (2)-(4):

$$
\begin{aligned}
Asym(t) \approx -2\frac{A_P}{A_T}\sin\delta\sin\alpha\,\mathbf{cos(\Delta mt)} \\
-\ [\sin(2\alpha) + 2\frac{A_P}{A_T}\cos\delta\cos(2\alpha)\sin\alpha]\mathbf{sin(\Delta mt)}\ .
\end{aligned}
\tag{6}
$$

We note that the $\cos(\Delta mt)$ term and the $\sin(\Delta mt)$ term have a different and complementary δ-dependence. The first term, which describes CP violation in the direct decay, behaves like $\sin\delta$, while the correction to the mixing-induced asymmetry is proportional to $\cos\delta$. Thus, as function of δ, the smaller the direct CP violation $\cos(\Delta mt)$ term, the larger becomes the penguin correction to $\sin(2\alpha)$, and vice versa. In particular, when $\delta = 0$, the $\cos(\Delta mt)$ term vanishes, whereas the correction to the $\sin(2\alpha)$ coefficient becomes maximal.

A heuristic argument for factorization of tree amplitudes in certain two body B decays[13] implies that δ_T is negligible and that perhaps also δ_P is small. *If* δ *were small, then the* $\cos(\Delta mt)$ *term may be too small to be observed and the time-dependent asymmetry measurement would not provide evidence for a penguin contribution. Still, in this case the penguin amplitude effect on the coefficient of the* $\sin(\Delta mt)$ *asymmetry becomes maximal and may be large. This is the danger of penguin amplitudes.*

The crucial point is that, whereas one would naively expect that the penguin amplitude modifies $\sin(2\alpha)$ in a multiplicative manner, the correction is in fact an additive one and involves a factor of two from the interference with the tree amplitude. This means that with e.g. $A_P/A_T = 0.2$, the correction to $\sin(2\alpha)$ can be as large as ± 0.4 and is not merely a relative 20% correction. To be quantitative, let us assume that δ is negligibly small, and study the consequences of (6) on the determination of $\sin(2\alpha)$ from an asymmetry measurement. Fig. 1 shows the coefficient of the $-\sin(\Delta m t)$ term as function of the actual value of $\sin(2\alpha)$ for $45^0 \leq \alpha \leq 135^0$. The range bounded by the two solid lines describes this coefficient for $\delta = 0$ and for A_P/A_T in the range (5). The straight dashed line gives the corresponding relation in the absence of a penguin contribution. The maximal deviation from the straight line is given by $2A_P/A_T$. We note that points with largest deviations from this line correspond to the largest value of A_P/A_T and thus to the smallest values of $|V_{ub}/V_{cb}|$. The danger of the penguin amplitude is best demonstrated for $\alpha = \pi/2$, where its effect on the asymmetry is maximal. For this case, an asymmetry as large as 0.4 can possibly be measured, although $\sin(2\alpha) = 0$. Such a substantial CP asymmetry measurement would be an important observation by itself, however it could not be related to the angle α. The deviation of the asymmetry from $\sin(2\alpha)$ decreases gradually, as one moves away from $\alpha = \pi/2$. It is still at a level of 30% at $\alpha = 65^0,\ 115^0$ ($\sin(2\alpha) = +0.77,\ -0.77$), where one expects large asymmetries. The effects become much smaller outside the range $45^0 \leq \alpha \leq 135^0$ plotted in Fig. 1. Unfortunately, the asymmetry measurement, which is related to the value of $\sin(2\alpha)$, cannot distinguish between angles which lie outside and inside this range. Note that as far as the CP asymmetry measurement is concerned, the corrections are potentially large for all but very large asymmetries. The range $65^0 < \alpha < 115^0$, where the corrections are larger than 30%[14], is presently allowed[1]. It is possible that future theoretical and experimental progress in quantities such as f_B (the B decay constant), $|V_{ub}/V_{cb}|$ and m_t will rule out this range[15].

If one takes the very conservative viewpoint that the sign of the penguin amplitude relative to the tree amplitude is unknown, then the uncertainty in determining

α from the asymmetry becomes twice as large. Reversing the sign of A_P/A_T corresponds to flipping the allowed range for the asymmetry coefficient to the other side of the straight dashed line. It would therefore be useful to at least theoretically determine the sign corresponding to $\delta \leq \pi/2$. Since the penguin correction to $\sin(2\alpha)$ is proportional to $\cos\delta$, it becomes substantially smaller than in Fig. 1 only for large values of the final state phase difference. In this case one expects to observe also the $\cos(\Delta m t)$ term in the time-dependent asymmetry. This depends, of course, on the value of α and on the sensitivity of the experiment. With no observation of such a term, and without a theory of final state interaction phases, one would have to assume the worst of all cases, namely $\delta \sim 0$ (or even $\delta \sim \pi$, if the sign of the penguin amplitude is undetermined).

Penguin contributions appear also in $B_d^0 \to \rho^\pm \pi^\mp$ and in $B_d^0 \to a_1^\pm \pi^\mp$. The effects on a determination of α from the asymmetry of the related time-dependent rates is expected to be as large as in $B_d^0 \to \pi^+\pi^-$. We wish to briefly demonstate this effect in the decays to $\rho\pi$. The general formalism dealing with this kind of final states (which, although being non-CP-eigenstates, are common to B_d^0 and $\overline{B}_d^0$ decays), was described in[16] . The essential difference with respect to $B_d^0 \to \pi^+\pi^-$ is that here the tree and penguin amplitudes for $B_d^0 \to \rho^+\pi^-$ are not the same as those for $B_d^0 \to \rho^-\pi^+$. In both cases the corresponding amplitudes carry the same CKM phases as in $B_d^0 \to \pi^+\pi^-$, $\phi_T = \gamma$ and $\phi_P = -\beta$, respectively:

$$A_f \equiv A(B_d^0 \to \rho^+\pi^-) = A_T e^{i\delta_T} e^{i\phi_T} + A_P e^{i\delta_P} e^{i\phi_P} \ ,$$

$$A_{\overline{f}} \equiv A(B_d^0 \to \rho^-\pi^+) = \overline{A}_T e^{i\overline{\delta}_T} e^{i\phi_T} + \overline{A}_P e^{i\overline{\delta}_P} e^{i\phi_P} \ . \quad (7)$$

The corresponding amplitudes for the charge-conjugated processes, $\overline{A}_{\overline{f}} \equiv A(\overline{B}_d^0 \to$

$\rho^-\pi^+$), $\overline{A}_f \equiv A(\overline{B}_d^0 \to \rho^+\pi^-)$, are obtained simply by changing the sign of the weak phases.

One can measure four different time-dependent decay rates, for cases in which initially $B_d^0(\overline{B}_d^0)$ decay to $\rho^\pm\pi^\mp$. If A_P could be neglected then these four rates would be, in principle, sufficient for a determination of α[17]. As in decays to CP eigenstates, this method involves a measurement of the coefficient of the $\sin(\Delta mt)$ term in the time-dependent rate. For an initially B_d^0 decaying to $\rho^+\pi^-$ this coefficient is given by

$$\mathrm{Im}(\frac{\overline{A}_f}{A_f}e^{-2i\beta}) = \frac{\overline{A}_T}{A_T}\sin(2\alpha + \overline{\delta}_T - \delta_T)\ . \tag{8}$$

If all final state phases were negligible, this coefficient would determine $\sin(2\alpha)$. (Otherwise, the phase difference $\overline{\delta}_T - \delta_T$ can be determined separately from the four rates). In the presence of the penguin amplitudes, this coefficient becomes, to lowest order in A_P/A_T and $\overline{A}_P/\overline{A}_T$, and for negligible final state phases,

$$\mathrm{Im}(\frac{\overline{A}_f}{A_f}e^{-2i\beta}) = \frac{\overline{A}_T}{A_T}[\sin(2\alpha) + \frac{A_P}{A_T}\sin(3\alpha) - \frac{\overline{A}_P}{\overline{A}_T}\sin\alpha]\ . \tag{9}$$

The correction to $\sin(2\alpha)$ can be estimated in a way similar to the correction in $B_d^0 \to \pi^+\pi^-$. Within our approximation, Eq.(5) applies to both A_P/A_T and $\overline{A}_P/\overline{A}_T$. In fact, if one takes $A_P/A_T = \overline{A}_P/\overline{A}_T$ (which should hold only for CP-eigentates) the correction term obtains exactly the form of the correction term of Eq.(6). In general, when these ratios are in the range (5), the effect of the penguin amplitude on determining $\sin(2\alpha)$ are expected to be as large as in $B_d^0 \to \pi^+\pi^-$.

In summary, we have shown that relatively small penguin amplitudes may prohibit a useful determination of $\sin(2\alpha)$ from the CP asymmetries of $B_d^0 \to \pi^+\pi^-$, $B_d^0 \to \rho^\pm\pi^\mp$ and $B_d^0 \to a_1^\pm\pi^\mp$. Asymmetries as large as 0.4 may be measured even when $\sin(2\alpha) = 0$. The uncertainty becomes small only for very large asymmeries. It would decrease if $|V_{ub}/V_{cb}|$ were found to be on the high side of the presently allowed range. The penguin complication may be avoided to a large degree if future studies of the CKM matrix exclude the range $65^0 \leq \alpha \leq 115^0$ in which the corrections are large. Our analysis was based primarily on the estimate (5) and on the observation that the correction to $\sin(2\alpha)$ in (6) is additive rather than multiplicative, and becomes maximal when $\delta \to 0$. We assumed that the hadronic matrix element of the penguin operator is neither dynamically suppressed nor enhanced relative to the tree amplitude. It goes without saying that this issue deserves serious studies, both theoretical and experimental. The mere determination of the sign of the penguin amplitude would be useful.

One way to overcome this potential difficulty is to measure in addition to the asymmetry in $B_d^0 \to \pi^+\pi^-$ also the rates of $B_d^0 \to \pi^0\pi^0$, $B^+ \to \pi^+\pi^0$. This isospin-based method[18] can provide a way to eliminate the penguin contribution altogether and to experimentally determine its magnitude, provided that the integrated rate into two neutral pions is measurable. A similar isospin analysis for the $\rho\pi$ modes is unlikely to work in practice due to the too many amplitudes involved and to certain ambiguities which appear in the analysis[19] .

I wish to thank J. Bjorken and H. Quinn for useful discussions.

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14. In general, there is a certain phenomenological correlation between α and the allowed values of the ratio of the CKM factors in (5)[1] . We use (5), although it seems that this ratio must be somewhat smaller than 5 for $65^0 \leq \alpha \leq 115^0$ if $|V_{ub}/V_{cb}| > 0.06$. The ratio of CKM factors may become larger with smaller values of $|V_{ub}/V_{cb}|$[10] .
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FIGURE CAPTION

FIG.1. Coefficient of $-\sin(\Delta m t)$ in the asymmetry of $B_d^0 \to \pi^+\pi^-$ as function of $\sin(2\alpha)$ for $45^0 \leq \alpha \leq 135^0$. We take $\delta = 0$. Area between solid lines corresponds to $A_P/A_T = 0.04 - 0.20$; dashed line corresponds to the absence of a penguin amplitude.

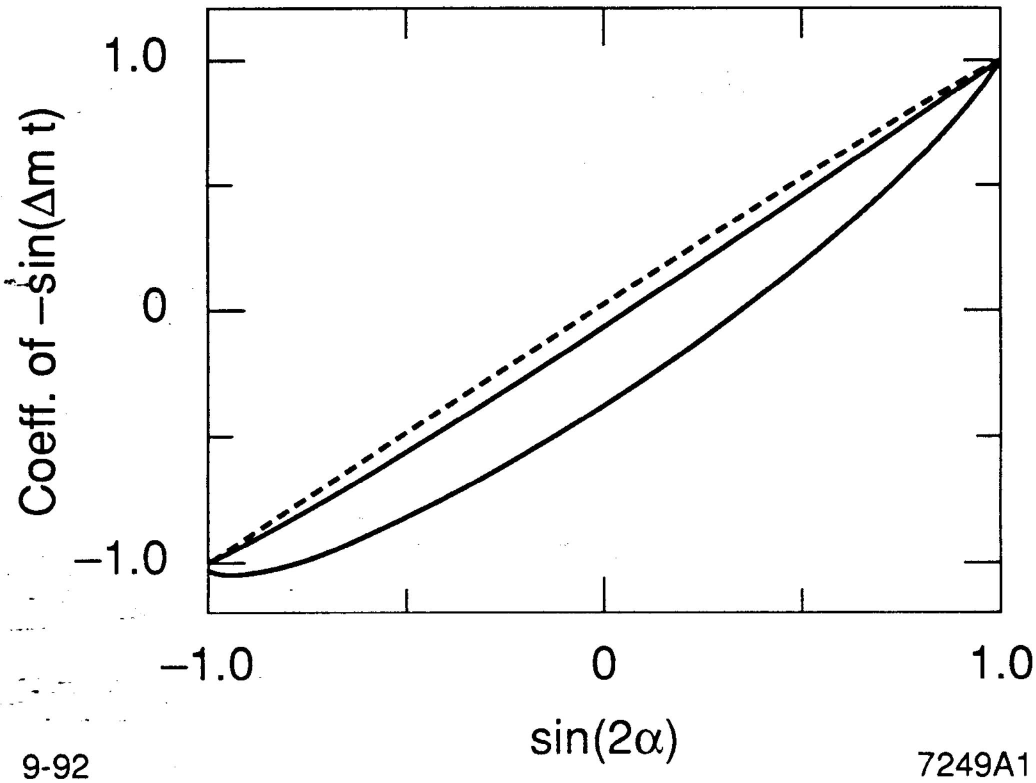

Fig. 1