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ON THE PROFILES OF JETS INITIATED BY LIGHT AND HEAVY QUARKS*

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ABSTRACT

We review some recent results in applications of the analytical perturbative technique to the description of particle distributions in QCD jets. Gluon emission in top production is briefly discussed.

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1. Introduction

The perturbative (PT) approach to QCD jet physics is based on the Modified Leading Logarithmic Approximation (MLLA).[1] In addition, the hypothesis of Local Parton-Hadron Duality (LPHD),[1] which is supported by experiment, suggests a close correspondence between the inclusive characteristics of hadron spectra and those calculated by means of PT QCD. Thus, when combined with LPHD, MLLA can hope to describe the gross features of hadronic systems.

Until now the main phenomenological successes of the MLLA–LPHD approach were connected with the description of the jets in e^+e^- annihilation, without distinction between the light and heavy primary quarks (see Ref. 2). We present here a few results, including the distributions of massive hadrons.

Prompted by these successes, and the recent availability of data on heavy quarks one would like to compare the PT predictions with the data on heavy quark events. Some PT results on the distributions in the events containing heavy particles are briefly discussed here (see for details Refs. 3 and 4). Finally we consider some effects of gluon emission in $e^+e^- \to t\bar{t}$.

2. MLLA Results for the Spectra of Massive Particles

As is shown in Refs. 2 and 5 to approximate the distributions of massive hadrons $(K, p, \eta, \ldots)$ the partonic formulae truncated at the different cutoff values Q_0 $(Q_0(m_h) > \Lambda)$ can be used. One can encode the MLLA effects in terms of a few analytically calculated shape parameters by means of the distorted Gaussian $(\ell = \ell n(1/x_p), \delta = (\ell - <\ell>)/\sigma)$[5,6]:

$$\overline{D}(\ell, Y, \lambda) = \frac{N(Y,\lambda)}{\sigma\sqrt{2\pi}} \exp\left[\frac{1}{8}k - \frac{1}{2}s\delta - \frac{1}{4}(2+k)\delta^2 + \frac{1}{6}s\delta^3 + \frac{1}{24}k\delta^4\right] . \quad (1)$$

Here $Y = \ell n\,(E/Q_0)$, $\lambda = \ell n\,(Q_0/\Lambda)$, $2E = W$ the total c.m.s. energy. Relative to the leading order, the MLLA implies that the peak is shifted to lower x, narrowed, skewed towards higher x, and flattened, with tails that fall off more rapidly than a Gaussian, see Refs. 5 and 6.

Motivated by the recent successes of the MLLA, one can make predictions at the energies of the Next Linear Collider, see Ref. 7. Figure 1 illustrates the expectation for the $\pi^{\pm}$ and $K^{\pm}$ mesons at $E = 1\ TeV$ together with the Z^0 results.

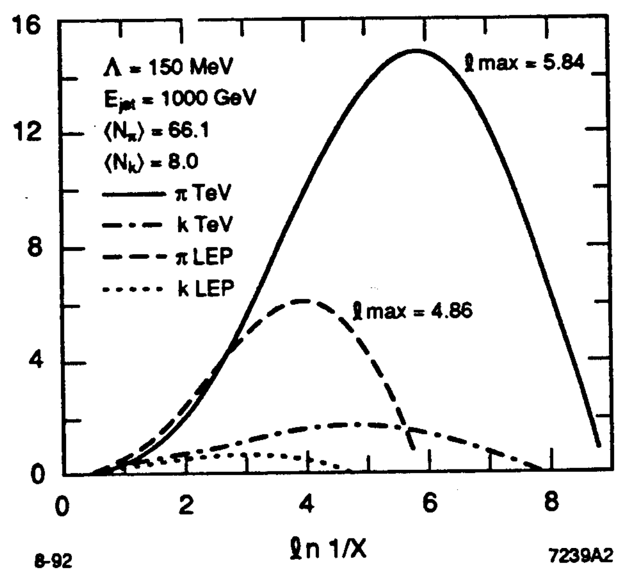

Figure 1. Distribution of the $\pi^{\pm}$ and $k^{\pm}$ mesons at $E = 45.6\ GeV$ and $E = 1\ TeV$. The MLLA curves correspond $\Lambda = 150\ MeV$, $Q_0(m_K) = 250\ MeV$.

In Ref. 5 the analytic procedure for calculations of the four moments $\ell_k \equiv \left\langle \ell^k \right\rangle$ $(k = 1, \ldots 4)$ of the truncated spectrum is described. In particular it is shown that stiffening of the distribution originating from truncated cascades is W-independent. The main effect of a finite λ on the quantity $\ell_{\max}$ is the asymptotically constant shift of the peak, see Fig. 2. This fact can be used to measure effective Q_0 values by comparative study of the energy evolution of the peak position for massive hadrons. The study of the energy dependence of ℓ_{max} shows that in a wide range of W the slopes of the curves corresponding to different Q_0 values are the same.

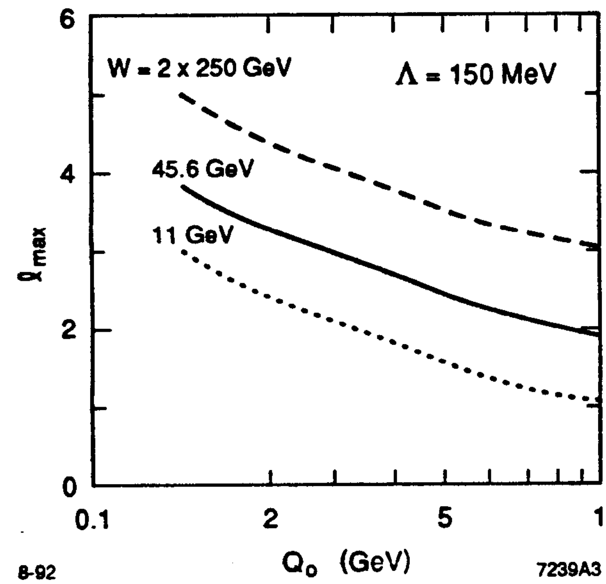

Figure 2. Q_0 dependence of the maximum of MLLA truncated distribution.[5]

As it was pointed out in Ref. 7 the prospective way of the experimental studies of the gluon jet could arise from the process $\gamma\gamma \to gg$. This reaction provides a unique environment in which two gluon jets are produced in a colour singlet state (the only counterpart to the celebrated $e^+e^- \to q\bar{q}$). At large angles its cross section is lower, approximately by an order of magnitude, than the cross section of $\gamma\gamma \to q\bar{q}$. This background could be substantially reduced by using the polarized $\gamma\gamma$ facility to ensure that the colliding photons have the same helicities.

3. QCD BREMMSTRAHLUNG IN HEAVY Q PRODUCTION

The difference in many properties of hadronic jets produced by heavy quarks (excluding the products of their decays), from that of light quarks, originates from the restriction of the phase space available to gluon radiation associated with the effects of the quark mass M_Q, see Refs. 3 and 4. One of the consequences of the suppression of forward radiation is that the multiplicity of light hadrons accompanying the heavy quark is less than the particle yield in a light quark jet at the same W.

Quantitatively, to MLLA accuracy,

$$N(e^+e^- \to Q\overline{Q}; W) = \\ N(e^+e^- \to q\overline{q}; W) - N(e^+e^- \to q\overline{q}; \sqrt{e} \cdot M_Q) + \mathcal{O}(\alpha_s(M_Q^2)N(M_Q)) \tag{2}$$

where $N(e^+e^- \to q\overline{q}(Q\overline{Q});\ W)$ is the mean charged multiplicity in light (heavy) quark events.

The difference $\delta_{b\ell} \equiv \overline{n}_b - \overline{n}_\ell$ between the measured bottom and light quark multiplicities can be written as

$$\delta_{b\ell} = \overline{n}_b^{dk} - \overline{n}_\ell(\sqrt{e}\, M_b) \ . \tag{3}$$

The results for the existing measurements of $\delta_{b\ell}$ are displayed in Fig. 3 (see Ref. 8 for details). To the available accuracy, the results are seen to be independent of W, in marked contrast to the steeply rising total multiplicity, and are thus consistent with the MLLA.

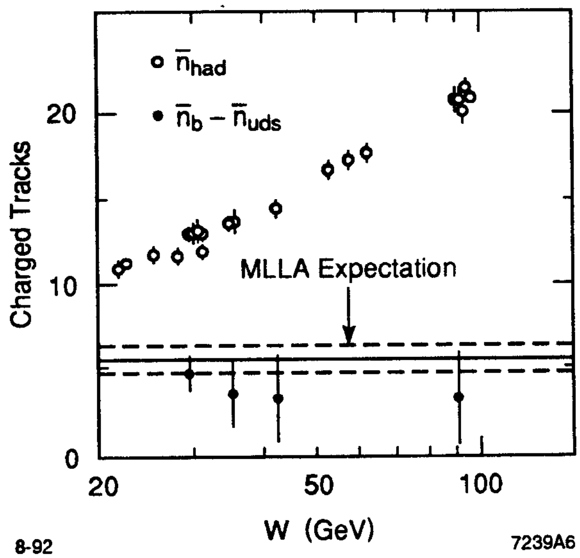

Figure 3. Energy dependence of total multiplicity (open points) and the multiplicity difference $\delta_{b\ell}$ between b and light quark production (filled point).[8]

In Ref. 4 the inclusive energy distribution of a quark with mass M_Q produced in $e^+e^- \to Q\overline{Q}$ has been derived

$$\overline{D}_Q^W(x_E; W, M_Q) = \int \frac{dj}{2\pi i}\, x_E^{-j} \exp\left\{ v^2 \cdot \int\limits_{2M_Q/W}^{1} dz\, \left[z^{j-1} - 1\right] \cdot \mathcal{R}(z) \right\} , \quad (4a)$$

$$\begin{aligned} \mathcal{R}(z) = \Phi_F^F(z) \Bigg\{ &\xi\left(\frac{z(1-z)^2 W^2}{1-z+\gamma} \cdot \chi_W(z)\right) \\ &- \xi\left(\frac{(1-z)^2 M_Q^2}{z} \cdot \chi_M(z)\right) \Bigg\} \end{aligned} \quad (4b)$$

with

$$\chi_M(z) = \exp\left\{\frac{2z}{1+z^2}\right\} , \quad (4c)$$

$$\chi_W(z) = \exp\left\{\frac{2}{1+z^2} \left[\frac{z(1-z)}{2(1-z+\gamma)}\right]^2\right\} \cdot \chi_M^{-1}(z) , \quad (4d)$$

and Φ_F^F the standard $q \to q$ splitting function. Here $\gamma \equiv M_Q^2/W^2 \ll 1$ and $v^2 \equiv 1 - 4\gamma$. The characteristic function ξ in Eq. (4b) is

$$\xi(Q^2) = \int^{Q^2} \frac{d\kappa^2}{\kappa^2} \, \frac{\alpha_s(\kappa^2)}{4\pi} \approx \frac{1}{b} \, \ell n \, \ell n \, \frac{Q^2}{\Lambda^2} + \text{const.} \tag{5}$$

Equations (4) incorporate the standard LLA result at finite $(1 - x_E) \sim 1$ and take into full account effects of the Sudakov suppression at $\alpha_s \ell n^2(1 - x_E) \sim 1$. The radiation factor (4b) embodies the exact first order result and, moreover, accounts for running coupling effect in $\mathcal{O}(\alpha_s)$ terms.

It is important to notice that the PT distribution (4a) exhibits by itself the basic feature which one would expect to be due to hadronization. The very dependence of $\Delta\xi$ on z in (4b) results in a peak in $\overline{D}_Q^Q(x_E)$ which appears at $(1 - x) > \Lambda/M_Q$. The PT-controlled peak emerges provided the confinement scale μ is set not too high to affect $\Delta\xi$. The corresponding condition reads

$$\{k_\perp^2\}_{\text{min}} = (1 - x_{\text{peak}})^2 M_Q^2 > \mu^2 \ , \qquad \Lambda \leq \mu < \Lambda \cdot \left(\frac{W^2}{M\Lambda}\right)^\kappa \ , \tag{6}$$

with

$$\kappa = \left[2 \exp\left(\frac{b}{4C_F}\right) - 1\right]^{-1} \approx 0.10 \text{ for } n_f = 3 \ .$$

The $(1 - x_{\text{peak}})$ value increases with W so that the PT-peak would sooner or later reveal itself and move away from dangerous non-PT region. This observation makes it worth trying to model fragmentation effects by choosing $\mu = \Lambda$.

In the heavy quark case the LPHD could pretend to describe the x_E distributions averaged over heavy-flavoured hadrons. Such a mixture naturally appears, e.g., when studying inclusive hard leptons. Though the pure PT treatment might look rather naive, it at least is free from the problem of "double counting."

By modifying the coupling $\alpha_s(Q^2)$ to make it free from the formal singularity at finite Q^2, one can make the PT approach applicable down to small momenta.[4] We have discussed in Ref. 4 the various generalizations of the evolution parameter (5) at $k^2 \lesssim \Lambda^2$. For example, one can regularized α_s by taking

$$\xi(Q^2) = \frac{1}{b} \ell n \, \ell n \left(\frac{Q^{2p}}{\Lambda^{2p}} + C \right) \, , C \geq 1 \tag{7}$$

which well preserves the large-Q^2 asymptotics. We have checked that the W-dependence of $\langle x_E \rangle$ as well as the scaling violations in the position of the peak turn out to be infrared stable.

Existing experimental information on $\langle x_E \rangle$ for D^* mesons and $\langle x_E \rangle$ extracted from semi-leptonic c and b decays, when compared to Eqs. (4), results in

$$\Lambda^{(3)} = 440 \pm \begin{matrix} 180 \\ 120 \end{matrix} \; MeV \, , \tag{8}$$

insensitive to the particular form of the coupling. Assuming different α_s^{eff} shapes ($p = 1, \ldots, 4$ in Eq. (7)) we found that each of them can be tuned to describe $\langle x_E \rangle$ of c and b quarks. For example, for the simplest model in which α_s is kept frozen at small momenta, one gets

$$\left\{\frac{\alpha_s}{\pi}\right\}^{\max} = 0.22 \pm \begin{matrix} 0.08 \\ 0.05 \end{matrix} > \left\{\frac{\alpha_s}{\pi}\right\}^{\text{crit}} = C_F^{-1} \left[1 - \sqrt{\frac{2}{3}} \right] \approx 0.14 \tag{9}$$

above the *critical value* one needs to switch on the *light quark confinement mechanism.* Empirically one observes that the characteristic integral

$$\int_0^{1\,GeV} d\kappa \, \frac{\alpha_s^{\text{eff}}(\kappa^2)}{\pi} \approx 0.2 \; GeV \, . \tag{10}$$

turns out to be a *fit-invariant* quantity.

The pure PT prediction for the quark mean energy losses is given by the expression

$$\langle x_E\rangle = \exp\left\{ -v^2\cdot\frac{8C_F}{3}\left(\xi(W^2)-\xi(M_Q^2)-\frac{21}{6}\cdot\frac{\alpha_s(W^2)}{4\pi}+\frac{10}{6}\cdot\frac{\alpha_s(M_Q^2)}{4\pi}\right)\right\}. \tag{11}$$

In the case of top quark production, the energy losses could be of practical importance for precise determination of the top quark parameters M_t, Γ_t in future experiments.

To predict t-spectra an account of effects of gluon bremsstrahlung off the top at the "fragmentation" stage is necessary in addition to precise knowledge of initial parton distributions and production cross sections. The non-PT effects are expected to decrease linearly with M_Q and should be negligible for the case of the top while the PT bremsstrahlung is commonly believed to be small.

To this end one may use the exact first order expression[4] since the kinematics when multiple radiation should be taken into account seems unrealistic for the foreseeable future. In Fig. 4 numerical results are shown for $M_t = 150\ GeV$ together with the improved LLA formula, Eq. (11).

As we see, relative losses start to exceed 3% already from $E_t/M_t \gtrsim 2$. This means quite sizeable *absolute* energy loss of about 10 GeV for quark energy $\sim$ 300 GeV.

As is well known, the large mass of the top quark leads to a large weak decay width Γ_t. This, in turn, leads to the possibility of interference between radiation of soft gluons in top production and decay.[4,10] In Ref. 10 the behaviour of the radiation pattern is analyzed in order to determine under what circumstances sensitivity to the top width is obtained. Figure 5 illustrates the sensitivity of the distribution to

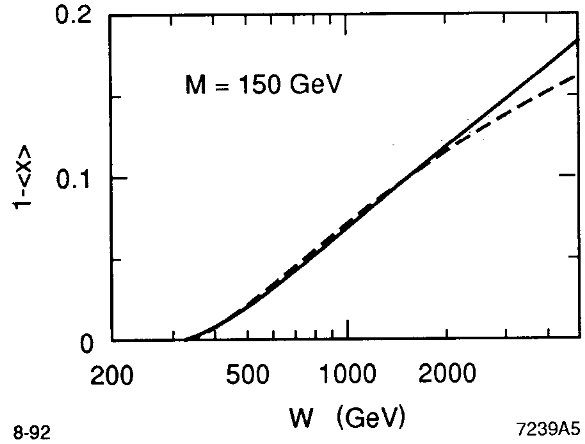

Figure 4. Top energy losses according to the exact first order formula[4] with $\alpha_s(M_t^2)$ for $M_t = 150 GeV$ (solid) and the improved LLA Eq. 11 (dashed).

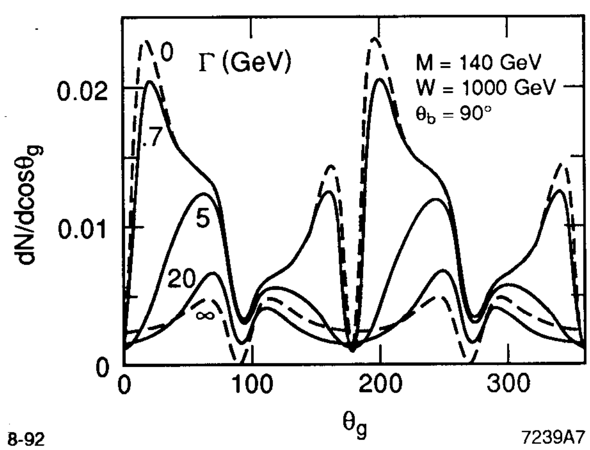

Figure 5. Soft gluon distribution in $e^+e^- \rightarrow t\bar{t}$.[10] the t and b are dt 90°. $M_t = 140\ GeV$, $W = 1\ TeV$, $\omega = 5\ GeV$ and Γ_t as marked.

Γ_t for t's and b's of right angle.

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