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Second Order Power Corrections in the Heavy Quark Effective Theory II. Baryon Form Factors*

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The analysis of $1/m_Q^2$ corrections of the previous paper is extended to the semileptonic decays of heavy baryons. We focus on the simplest case, the ground state Λ_Q baryons, in which the light degrees of freedom are in a state of zero total angular momentum. The formalism, while identical in spirit, is considerably less cumbersome than for heavy mesons. The general results are applied to the semileptonic decay $\Lambda_b \to \Lambda_c \, \ell \, \nu$. An estimate of the leading power corrections to the decay rate at zero recoil, which are of order $1/m_Q^2$, is presented. It is pointed out that a measurement of certain asymmetry parameters would provide a direct measurement of $1/m_Q^2$ corrections. Finally, it is shown how the analysis could be extended to include excited heavy baryons such as the Σ_Q and the Σ_Q^*.

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I. INTRODUCTION

In the previous paper [1] (hereafter referred to as Ref. I), we have developed the formalism for including in the heavy quark effective theory (HQET) terms in the mass expansion of order $1/m_Q^2$. That paper focused on the case of the ground state pseudoscalar and vector mesons. Here we extend the analysis to the case of the heavy baryons, in particular the spin-$\frac{1}{2}$ Λ_Q. It turns out that the formalism is far less cumbersome than for the heavy mesons. The structure of the previous paper may be taken over almost in its entirety to the baryons, but with the number of invariant form factors considerably reduced. Hence to avoid redundancy we will abbreviate considerably those aspects of the presentation which are common to the two cases, and concentrate instead on features which distinguish the baryons from the mesons. In Sec. II we discuss the Lagrangian of HQET and the expansion of the baryon masses. Sec. III reviews the form of baryon matrix elements in the $m_Q \to \infty$ limit and the corrections of order $1/m_Q$. In Sec. IV we present the extension of this analysis to order $1/m_Q^2$. Some phenomenological applications of our results to semileptonic decays of the Λ_b are discussed in Sec. V, while Sec. VI contains a discussion of excited baryons. In Sec. VII we provide a brief summary.

For the sake of simplicity, we shall completely ignore radiative corrections in this paper. In particular, we omit the μ-dependence of the universal form factors of HQET, and ignore the short-distance coefficients in the expansion of the currents. All these effects would not change the structure of the heavy quark expansion, but they would complicate considerably the presentation. As discussed in detail in Ref. I, renormalization effects may be incorporated straightforwardly into our general formalism in a perturbative way.

II. THE LAGRANGIAN OF THE EFFECTIVE THEORY

The heavy quark effective theory provides an expansion of strong matrix elements in inverse powers of the mass of a heavy quark [2–9]. It is useful when one considers external states containing a single heavy quark, dressed by light degrees of freedom to make up a color singlet hadron. HQET is constructed by redefining the field operator $Q(x)$ of a heavy quark in such a way that the heavy quark part of the QCD Lagrangian can be expanded in powers of $1/m_Q$. This expansion is independent of the nature of the hadronic states one wants to describe. Hence the field redefinition and the construction of the effective Lagrangian and the effective heavy quark currents are the same as described in Ref. I.

In brief, then, there are two objects which one must expand to construct HQET. The first is the QCD Lagrangian. In the limit $m_Q \to \infty$, the heavy quark field $Q(x)$ is replaced by the velocity-dependent field

$$h(v,x) = e^{im_Q v\cdot x}\, P_+\, Q(x)\,, \tag{2.1}$$

where $P_+ = \frac{1}{2}(1+\not{v})$ is a positive energy projection operator. The effective Lagrangian for the strong interactions of a heavy quark becomes [7, 10, 11]

$$\mathcal{L}_{\rm HQET} = \bar h\, iv\cdot D\, h\,, \tag{2.2}$$

where $D^\alpha = \partial^\alpha - ig_s T_a A_a^\alpha$ is the gauge-covariant derivative. This is corrected by an infinite series of terms involving higher dimension operators, which are suppressed by inverse powers of m_Q:

$$\mathcal{L}_{\rm power} = \frac{1}{2m_Q}\mathcal{L}_1 + \frac{1}{4m_Q^2}\mathcal{L}_2 + \cdots\,. \tag{2.3}$$

The terms in $\mathcal{L}_{\rm power}$ are treated as ordinary perturbations of the Lagrangian $\mathcal{L}_{\rm HQET}$. Omitting operators which vanish by the equations of motion, the first and second order terms are [12–14]

$$\begin{aligned}\mathcal{L}_1 &= \bar h\,(iD)^2 h + Z\,\bar h\, s_{\alpha\beta} G^{\alpha\beta} h\,,\\ \mathcal{L}_2 &= Z_1\,\bar h\, v_\beta iD_\alpha G^{\alpha\beta} h + 2Z_2\,\bar h\, s_{\alpha\beta} v_\gamma iD^\alpha G^{\beta\gamma} h\,,\end{aligned} \tag{2.4}$$

where $s_{\alpha\beta} = -\frac{i}{2}\sigma_{\alpha\beta}$, and $G^{\alpha\beta} = [iD^\alpha, iD^\beta] = ig_s T_a G_a^{\alpha\beta}$ is the gluon field strength. Expressions for the renormalization factors have been given in Ref. I. It is necessary to perform a similar expansion of the heavy quark currents which mediate the weak decays of heavy hadrons. In the full theory these currents are of the form $\bar Q'\,\Gamma\, Q$. At tree level in the effective theory the expansion takes the form

$$\begin{aligned}\bar Q'\,\Gamma\, Q \to\ & \bar h'\,\Gamma\, h + \frac{1}{2m_Q}\,\bar h'\,\Gamma\, i\not{D}\, h + \frac{1}{2m_{Q'}}\,\bar h'\,(-i\overleftarrow{\not{D}})\,\Gamma\, h\\ &+ \frac{1}{4m_Q^2}\,\bar h'\,\Gamma\,\gamma_\alpha v_\beta G^{\alpha\beta} h - \frac{1}{4m_{Q'}^2}\,\bar h'\,\gamma_\alpha v'_\beta G^{\alpha\beta}\,\Gamma\, h\\ &+ \frac{1}{4m_Q m_{Q'}}\,\bar h'\,(-i\overleftarrow{\not{D}})\,\Gamma\, i\not{D}\, h + \cdots\,.\end{aligned} \tag{2.5}$$

A more complete form of the expansion, which allows for the inclusion of radiative corrections, is given in Ref. I.

The eigenstates of $\mathcal{L}_{\rm HQET}$ differ from those of the full theory in the baryon sector in the same way as in the meson sector. The latter case was discussed in some detail in the previous paper. For the spin-$\frac{1}{2}$ Λ_Q baryon the situation is in fact simpler, because the light degrees of freedom carry no angular momentum and hence there is no spin symmetry violating mass splitting. We expand the mass of the physical Λ_Q as $m_\Lambda = m_Q + \bar\Lambda + \Delta m_\Lambda^2/2m_Q + \cdots$. The mass of the Λ_Q in the strict $m_Q \to \infty$ limit is given by $M \equiv m_Q + \bar\Lambda$; the next term in the series represents the leading correction to this quantity. Fixing, as usual, the heavy quark mass m_Q so that there is no residual mass term [15] in the Lagrangian (2.2), the parameter $\bar\Lambda$ is well defined and controls the phase of the effective heavy baryon state:

$$|\Lambda(x)\rangle_{\rm HQET} = e^{-i\bar\Lambda v\cdot x}|\Lambda(0)\rangle_{\rm HQET}\,. \tag{2.6}$$

Note that $\bar\Lambda$ as defined here is *not* the same as the analogous parameter $\bar\Lambda$ defined for the heavy mesons. In order to make clear the parallels with the analysis for mesons given in Ref. I, and in order to avoid a further proliferation of nomenclature, we will sometimes use the same (or similar) names for parameters and form factors appearing in the description of heavy mesons and baryons. However, under no circumstances should there be confusion that these form factors are at all related.

In the rest frame of the Λ_Q, the mass shift Δm_Λ^2 is given by

$$\Delta m_\Lambda^2 = \frac{\langle\Lambda(v,s)|\,(-\mathcal{L}_1)\,|\Lambda(v,s)\rangle}{\langle\Lambda(v,s)|\,h^\dagger h\,|\Lambda(v,s)\rangle}\,. \tag{2.7}$$

The matrix elements which appear in the numerator of (2.7) are restricted by Lorentz invariance to take the form

$$\begin{aligned}\langle\Lambda|\,\bar h\,(iD)^2 h\,|\Lambda\rangle &= 2m_\Lambda\lambda,\\ \langle\Lambda|\,\bar h\, s_{\alpha\beta}G^{\alpha\beta}h\,|\Lambda\rangle &= 0\,.\end{aligned} \tag{2.8}$$

Vector current conservation implies that the matrix element in the denominator equals $2m_\Lambda$. We thus find $\Delta m_\Lambda^2 = -\lambda$. At this order in the heavy quark expansion, then, $\bar\Lambda$ and λ are the fundamental mass parameters of the effective theory. They are independent of m_Q and of the renormalization scale μ. Unfortunately, these parameters cannot be measured directly. While one may naïvely estimate $\bar\Lambda\approx 700\,\text{MeV}$ from the constituent quark model, little is known about the higher order correction λ.

III. BARYON FORM FACTORS IN THE EFFECTIVE THEORY

Consider the semileptonic decay of a spin-$\frac12$ baryon Λ containing heavy quark Q of mass m_Q, to a spin-$\frac12$ baryon Λ' containing heavy quark Q' of mass $m_{Q'}$. This transition is governed by the hadronic matrix elements of the flavor changing vector and axial vector currents. They are conventionally parameterized in terms of six form factors f_i and g_i, defined by

$$\begin{aligned}\langle\Lambda'(p',s')|\,\bar Q'\gamma^\mu Q\,|\Lambda(p,s)\rangle &= \bar u_{\Lambda'}(p',s')\Big[f_1\,\gamma^\mu - if_2\,\sigma^{\mu\nu}q_\nu + f_3\,q^\mu\Big]u_\Lambda(p,s)\,,\\ \langle\Lambda'(p',s')|\,\bar Q'\gamma^\mu\gamma^5 Q\,|\Lambda(p,s)\rangle &= \bar u_{\Lambda'}(p',s')\Big[g_1\,\gamma^\mu - ig_2\,\sigma^{\mu\nu}q_\nu + g_3\,q^\mu\Big]\gamma^5\,u_\Lambda(p,s)\,,\end{aligned} \tag{3.1}$$

where $q^\mu = p^\mu - p'^\mu$ is the momentum transfer to the leptons. For heavy baryons it is convenient to replace this with a parameterization in terms of the velocities of the initial and final baryons. We thus define an equivalent set of form factors by

$$\begin{aligned}\langle\Lambda'(v',s')|\,\bar Q'\gamma^\mu Q\,|\Lambda(v,s)\rangle &= \bar u_{\Lambda'}(v',s')\Big[F_1\,\gamma^\mu + F_2\,v^\mu + F_3\,v'^\mu\Big]u_\Lambda(v,s)\,,\\ \langle\Lambda'(v',s')|\,\bar Q'\gamma^\mu\gamma^5 Q\,|\Lambda(v,s)\rangle &= \bar u_{\Lambda'}(v',s')\Big[G_1\,\gamma^\mu + G_2\,v^\mu + G_3\,v'^\mu\Big]\gamma_5\,u_\Lambda(v,s)\,.\end{aligned} \tag{3.2}$$

Here $u_\Lambda(p,s)$ and $u_\Lambda(v,s)$ are the same spinors, and are normalized to the physical mass m_Λ:

$$\bar u_\Lambda(v,s)\, u_\Lambda(v,s) = 2m_\Lambda\,. \tag{3.3}$$

While the form factors f_i and g_i are conventionally written in terms of the invariant momentum transfer q^2, it is more appropriate to consider F_i and G_i as functions of the kinematic variable $w = v\cdot v'$, which measures the change in velocity of the heavy baryons. Using the fact that the spinors are eigenstates of the velocity, $\not v\, u_\Lambda(v,s) = u_\Lambda(v,s)$, one can readily derive the relations among these sets of form factors. They are

$$\begin{aligned}
f_1 &= F_1 + (m_\Lambda + m_{\Lambda'})\bigg(\frac{F_2}{2m_\Lambda} + \frac{F_3}{2m_{\Lambda'}}\bigg)\,,\\
f_2 &= -\frac{F_2}{2m_\Lambda} - \frac{F_3}{2m_{\Lambda'}}\,,\\
f_3 &= \frac{F_2}{2m_\Lambda} - \frac{F_3}{2m_{\Lambda'}}\,,\\
\\
g_1 &= G_1 - (m_\Lambda - m_{\Lambda'})\bigg(\frac{G_2}{2m_\Lambda} + \frac{G_3}{2m_{\Lambda'}}\bigg)\,,\\
g_2 &= -\frac{G_2}{2m_\Lambda} - \frac{G_3}{2m_{\Lambda'}}\,,\\
g_3 &= \frac{G_2}{2m_\Lambda} - \frac{G_3}{2m_{\Lambda'}}\,.
\end{aligned} \tag{3.4}$$

Let us now review the analysis of the baryon form factors in HQET [16–19]. This will allow us to outline the procedure and to set up our conventions in such a way that the extension to the next order becomes straightforward. At each order in the heavy quark expansion, one writes the contributions to F_i and G_i in terms of universal, m_Q-independent form factors, which are defined by matrix elements in the effective theory. At leading order, one needs the matrix elements of the first operator on the right-hand side of (2.5) between baryon states in the effective theory. They have the structure [7, 18]

$$\langle\Lambda'(v',s')|\,\bar h'\,\Gamma\, h\,|\Lambda(v,s)\rangle = \zeta(w)\, \overline{\mathcal{U}'}(v',s')\,\Gamma\,\mathcal{U}(v,s)\,, \tag{3.5}$$

where $\zeta(w)$ is the Isgur-Wise function for Λ baryon transitions, and $\mathcal{U}(v,s)$ denotes the spinor for a heavy baryon in the effective theory. It is normalized to the effective mass $M = m_Q + \bar\Lambda$ of the state in HQET,

$$\overline{\mathcal{U}}(v,s)\,\mathcal{U}(v,s) = 2M\,, \tag{3.6}$$

and is thus related to the spinor of the physical state by

$$\mathcal{U}(v,s) = Z_M^{-1/2}\, u(v,s)\,, \quad Z_M = \frac{m_\Lambda}{M} = 1 - \frac{\lambda}{2m_Q^2} + \cdots\,. \tag{3.7}$$

At order $1/m_Q^2$ in the heavy quark expansion we will have to include this factor.

From (3.5) one can immediately derive expressions for the baryon form factors in the infinite quark mass limit. One finds $F_1 = G_1 = \zeta(w)$ and $F_2 = F_3 = G_2 = G_3 = 0$. One can then use the conservation of the flavor-conserving vector current to derive the normalization of the Isgur-Wise form factor at zero recoil [3]. From

$$\langle \Lambda(v,s)|\, \bar Q\, \gamma^0\, Q\, |\Lambda(v,s)\rangle = 2m_\Lambda v^0 \tag{3.8}$$

it follows that

$$\sum_{i=1,2,3} F_i(1) = 1\,, \qquad (m_\Lambda = m_{\Lambda'}) \tag{3.9}$$

which implies the normalization condition $\zeta(1) = 1$. From here on we will omit the velocity and spin labels on the states and spinors. It is to be understood that unprimed objects refer to Λ and depend on v and s, while primed objects refer to Λ' and depend on v' and s'.

As shown by Georgi, Grinstein and Wise [19], the leading power corrections to the infinite quark mass limit involve contributions of two types. The first come from terms in the expansion of the current (2.5) which involve operators containing a covariant derivative. Their matrix elements can be parameterized as

$$\langle \Lambda'|\, \bar h'\, \Gamma^\alpha\, iD_\alpha\, h\, |\Lambda\rangle = \zeta_\alpha(v,v')\, \overline{\mathcal{U}}'\, \Gamma^\alpha\, \mathcal{U}\,. \tag{3.10}$$

As in Ref. I, we do not have to specify the nature of the matrix Γ^α in the definition of the universal functions. At tree level, however, $\Gamma^\alpha = \Gamma\,\gamma^\alpha$. Matrix elements of operators containing a derivative acting on h' are, as usual, obtained from this by complex conjugation and interchange of the velocity and spin labels. The most general decomposition of ζ_α involves two scalar functions defined by [19]

$$\zeta_\alpha(v,v') = \zeta_+(w)\,(v+v')_\alpha + \zeta_-(w)\,(v-v')_\alpha\,. \tag{3.11}$$

As in the case of the mesons, one can use the equation of motion $iv\cdot Dh = 0$ and the known spatial dependence (2.6) of the states in the effective theory to put constraints on these form factors. One finds [19]

$$\begin{aligned} \zeta_+(w) &= \frac{\bar\Lambda}{2}\, \frac{w-1}{w+1}\, \zeta(w)\,, \\ \zeta_-(w) &= \frac{\bar\Lambda}{2}\, \zeta(w)\,. \end{aligned} \tag{3.12}$$

From these relations it follows that the matrix element in (3.10) vanishes at zero recoil.

The form factors also receive corrections from insertions of higher order terms in the effective Lagrangian (2.3) into matrix elements of the lowest order current $J = \bar{h}'\Gamma h$. In fact, the contribution of the chromo-magnetic operator vanishes by Lorentz invariance, and the entire effect takes the form of a correction to the Isgur-Wise function $\zeta(w)$:

$$\langle\Lambda'|\, i\int \mathrm{d}x\, T\{\, J(0), \mathcal{L}_1(x)\,\}\,|\Lambda\rangle = A(w)\, \overline{\mathcal{U}}'\,\Gamma\,\mathcal{U}\,. \tag{3.13}$$

It is now straightforward to compute the form factors F_i and G_i at subleading order in HQET in terms of $\bar\Lambda$ and the universal form factors $\zeta(w)$ and $A(w)$. Introducing the functions

$$\begin{aligned} \mathcal{B}_1(w) &= \bar\Lambda\, \frac{w-1}{w+1}\, \zeta(w) + A(w)\,, \\ \mathcal{B}_2(w) &= -\frac{2\bar\Lambda}{w+1}\, \zeta(w)\,, \end{aligned} \tag{3.14}$$

the result becomes [19]

$$\begin{aligned} F_1(w) &= \zeta(w) + \left(\frac{1}{2m_Q} + \frac{1}{2m_{Q'}}\right)\Big[\mathcal{B}_1(w) - \mathcal{B}_2(w)\Big]\,, \\ G_1(w) &= \zeta(w) + \left(\frac{1}{2m_Q} + \frac{1}{2m_{Q'}}\right)\mathcal{B}_1(w)\,, \\ F_2(w) &= \quad G_2(w) = \frac{1}{2m_{Q'}}\, \mathcal{B}_2(w)\,, \\ F_3(w) &= -G_3(w) = \frac{1}{2m_Q}\, \mathcal{B}_2(w)\,. \end{aligned} \tag{3.15}$$

For the subleading form factors, vector current conservation [cf. (3.9)] implies

$$\mathcal{B}_1(1) = 0 \quad \Leftrightarrow \quad A(1) = 0\,. \tag{3.16}$$

Thus, at zero recoil all leading power corrections are determined in terms of $\mathcal{B}_2(1) = -\bar\Lambda$, and in particular one finds that $G_1(1) = 1$ is not renormalized at this order [19].

IV. SECOND ORDER POWER CORRECTIONS

We are now in a position to extend this analysis to include corrections of order $1/m^2$ (from now on m will designate a generic heavy quark mass). As in the case of the mesons, we must discuss separately three classes of contributions: corrections to the current, corrections to the effective Lagrangian, and mixed corrections. We shall take them each in turn.

A. Second Order Corrections to the Current

The effective operators appearing at second order in the expansion of the current (2.5) are all bilinear in the covariant derivative, a property which remains true even if one goes beyond tree level. It is thus sufficient to analyze the matrix element

$$\langle\Lambda'|\,\bar h'\,(-i\overleftarrow{D}_\alpha)\,\Gamma^{\alpha\beta}\,iD_\beta\,h\,|\Lambda\rangle = \psi_{\alpha\beta}(v,v')\,\overline{\mathcal{U}}'\,\Gamma^{\alpha\beta}\,\mathcal{U}\,. \tag{4.1}$$

Considering the complex conjugate of this equation leads immediately to the relation $\psi_{\alpha\beta}(v,v') = \psi^*_{\beta\alpha}(v',v)$. Decomposing the form factor into symmetric and antisymmetric parts, $\psi_{\alpha\beta} = \frac{1}{2}[\psi^S_{\alpha\beta} + \psi^A_{\alpha\beta}]$, we then write down the general decomposition

$$\begin{aligned}\psi^S_{\alpha\beta}(v,v') &= \psi^S_1(w)\,g_{\alpha\beta} + \psi^S_2(w)\,(v+v')_\alpha(v+v')_\beta + \psi^S_3(w)\,(v-v')_\alpha(v-v')_\beta\,,\\ \psi^A_{\alpha\beta}(v,v') &= \psi^A_1(w)\,(v_\alpha v'_\beta - v'_\alpha v_\beta)\,.\end{aligned} \tag{4.2}$$

The equation of motion implies $v^\beta\psi_{\alpha\beta} = 0$, yielding

$$\begin{aligned}\psi^S_1 + (w+1)\,\psi^S_2 - (w-1)\,\psi^S_3 + w\,\psi^A_1 &= 0\,,\\ (w+1)\,\psi^S_2 + (w-1)\,\psi^S_3 - \psi^A_1 &= 0\,.\end{aligned} \tag{4.3}$$

As with the mesons, it is convenient to use an integration by parts to relate (4.1) to matrix elements of operators in which two derivatives act on the same heavy quark field. We find

$$\langle\Lambda'|\,\bar h'\,\Gamma^{\alpha\beta}\,iD_\alpha iD_\beta\,h\,|\Lambda\rangle = \psi_{\alpha\beta}(v,v')\,\overline{\mathcal{U}}'\,\Gamma^{\alpha\beta}\,\mathcal{U} + \bar\Lambda\,(v-v')_\alpha\,\zeta_\beta(w)\,\overline{\mathcal{U}}'\,\Gamma^{\alpha\beta}\,\mathcal{U}\,. \tag{4.4}$$

In particular, we define form factors for the matrix elements

$$\begin{aligned}\langle\Lambda'|\,\bar h'\,\Gamma\,(iD)^2 h\,|\Lambda\rangle &= \phi_0(w)\,\overline{\mathcal{U}}'\,\Gamma\,\mathcal{U}\,,\\ \langle\Lambda'|\,\bar h'\,\Gamma^{\alpha\beta}G_{\alpha\beta}\,h\,|\Lambda\rangle &= \phi_1(w)\,(v_\alpha v'_\beta - v'_\alpha v_\beta)\,\overline{\mathcal{U}}'\,\Gamma^{\alpha\beta}\,\mathcal{U}\,.\end{aligned} \tag{4.5}$$

We may then use (4.4) and the relations given by the equation of motion to write the form factors ψ_i in terms of ϕ_i, ζ, and $\bar\Lambda$:

$$\begin{aligned}\psi^S_1 &= \phi_0 + w\,\phi_1 + \frac{w-1}{w+1}\,\bar\Lambda^2\,\zeta\,,\\ \psi^S_2 &= -\frac{1}{2(w+1)}\Big[\phi_0 + (2w-1)\,\phi_1 + \frac{(2-w)(w-1)}{w+1}\,\bar\Lambda^2\,\zeta\Big]\,,\\ \psi^S_3 &= \frac{1}{2(w-1)}\big[\phi_0 + (2w+1)\,\phi_1\big] - \frac{w}{2(w+1)}\,\bar\Lambda^2\,\zeta\,,\\ \psi^A_1 &= \phi_1 - \frac{w-1}{w+1}\,\bar\Lambda^2\,\zeta\,,\end{aligned} \tag{4.6}$$

where we omit the kinematic argument w in the form factors. It follows from (2.8) that the function $\phi_0(w)$ is normalized at zero recoil, $\phi_0(1) = \lambda$. The equation of motion then implies $\phi_1(1) = -\frac{1}{3}\lambda$. From the relations (4.6) we see that, as in the meson case, at zero recoil all matrix elements of second order currents may be written in terms of the single parameter λ, since

$$\psi_{\alpha\beta}(v,v) = \frac{\lambda}{2}\,(g_{\alpha\beta} - v_\alpha v_\beta)\,. \tag{4.7}$$

Furthermore, only the last operator in (2.5) contributes at zero recoil, yielding corrections of order $\lambda/m_Q m_{Q'}$.

B. Corrections to the Lagrangian

We now turn to $1/m^2$ corrections which come from insertions of higher dimension operators from the effective Lagrangian into matrix elements of the lowest order current $J = \bar{h}'\,\Gamma\,h$. These fall into three classes. First, there are insertions of the second order effective Lagrangian $\mathcal{L}_2$. Although there are two new operators at this order, only one of them gives a nonzero contribution. This follows simply from Lorentz invariance, for the same reason that the chromo-magnetic operator at order $1/m$ gave no contribution. We then define

$$\langle\Lambda'|\,i\int\!\mathrm{d}x\,T\{\,J(0),\mathcal{L}_2(x)\,\}\,|\Lambda\rangle = Z_1\,B(w)\,\overline{\mathcal{U}}'\,\Gamma\,\mathcal{U}\,. \tag{4.8}$$

Insertions of $\mathcal{L}_2'$ are parameterized by the same function.

Second, there are corrections which come from two insertions of the first order correction $\mathcal{L}_1$. These have the structure

$$\begin{aligned}\langle\Lambda'|\,\tfrac{i^2}{2}\int\!\mathrm{d}x\mathrm{d}y\;T\,\{\,J(0),\mathcal{L}_1(x),\mathcal{L}_1(y)\,\}\,|\Lambda\rangle&\\ = C_1(w)\,\overline{\mathcal{U}}'\,\Gamma\,\mathcal{U} + Z^2\,C_{\alpha\beta\gamma\delta}(v,v')\,\overline{\mathcal{U}}'\,\Gamma\,P_+\,s^{\alpha\beta}P_+\,s^{\gamma\delta}\,\mathcal{U}\,,&\end{aligned} \tag{4.9}$$

where we decompose

$$\begin{aligned}C_{\alpha\beta\gamma\delta}(v,v') = {}& C_2(w)\,(g_{\alpha\gamma}\,g_{\beta\delta} - g_{\alpha\delta}\,g_{\beta\gamma})\\ &+ C_3(w)\,(g_{\alpha\gamma}v'_\beta v'_\delta - g_{\beta\gamma}v'_\alpha v'_\delta - g_{\alpha\delta}v'_\beta v'_\gamma + g_{\beta\delta}v'_\alpha v'_\gamma)\,.\end{aligned} \tag{4.10}$$

The matrix elements for a double insertion of $\mathcal{L}_1'$ are given by the same formula, but with $C_{\alpha\beta\gamma\delta}(v,v')$ replaced by $C_{\gamma\delta\alpha\beta}(v',v) = C_{\alpha\beta\gamma\delta}(v',v)$.

Finally, there are corrections from an insertion of both $\mathcal{L}_1$ and $\mathcal{L}_1'$. These have the structure

$$\begin{aligned}\langle\Lambda'|\,i^2\int\!\mathrm{d}x\mathrm{d}y\;T\,\{\,J(0),\mathcal{L}_1(x),\mathcal{L}_1'(y)\,\}\,|\Lambda\rangle&\\ = D_1(w)\,\overline{\mathcal{U}}'\,\Gamma\,\mathcal{U} + ZZ'D_{\alpha\beta\gamma\delta}(v,v')\,\overline{\mathcal{U}}'\,s^{\alpha\beta}P'_+\,\Gamma\,P_+\,s^{\gamma\delta}\,\mathcal{U}\,.&\end{aligned} \tag{4.11}$$

We decompose $D_{\alpha\beta\gamma\delta}$ analogously to (4.10):

$$\begin{aligned}D_{\alpha\beta\gamma\delta}(v,v') = & D_2(w)\,(g_{\alpha\gamma}\,g_{\beta\delta} - g_{\alpha\delta}\,g_{\beta\gamma}) \\ & + D_3(w)\,(g_{\alpha\gamma}v_\beta v'_\delta - g_{\beta\gamma}v_\alpha v'_\delta - g_{\alpha\delta}v_\beta v'_\gamma + g_{\beta\delta}v_\alpha v'_\gamma)\end{aligned} \tag{4.12}$$

Note that $D_{\alpha\beta\gamma\delta}$ obeys the symmetry constraint $D_{\alpha\beta\gamma\delta}(v,v') = D_{\gamma\delta\alpha\beta}(v',v)$.

C. Mixed Corrections to the Current and the Lagrangian

Finally, we turn to second order corrections arising from insertions of $\mathcal{L}_1$ into matrix elements of first order corrections to the current. The structures of interest are

$$\begin{aligned}&\langle\Lambda'|\,i\int\mathrm{d}x\;T\,\{\,\bar h'\,\Gamma^\gamma\,iD_\gamma\,h,\mathcal{L}_1(x)\,\}\,|\Lambda\rangle \\ &\qquad = E_\gamma(v,v')\,\overline{\mathcal{U}}'\,\Gamma^\gamma\,\mathcal{U} + Z E_{\gamma\alpha\beta}(v,v')\,\overline{\mathcal{U}}'\,\Gamma^\gamma P_+\,s^{\alpha\beta}\,\mathcal{U}\,, \\ &\langle\Lambda'|\,i\int\mathrm{d}x\;T\,\{\,\bar h'\,(-i\overleftarrow{D}_\gamma)\,\Gamma^\gamma\,h,\mathcal{L}_1(x)\,\}\,|\Lambda\rangle \\ &\qquad = E'_\gamma(v,v')\,\overline{\mathcal{U}}'\,\Gamma^\gamma\,\mathcal{U} + Z E'_{\gamma\alpha\beta}(v,v')\overline{\mathcal{U}}'\,\Gamma^\gamma P_+\,s^{\alpha\beta}\,\mathcal{U}\,.\end{aligned} \tag{4.13}$$

Again, insertions of $\mathcal{L}'_1$ give rise to the conjugate matrix elements, with primed quantities interchanges with unprimed. We parameterize

$$\begin{aligned}E_\gamma(v,v') &= E_1(w)\,v_\gamma + E_2(w)\,v'_\gamma\,, \\ E'_\gamma(v,v') &= E'_1(w)\,v_\gamma + E'_2(w)\,v'_\gamma\,, \\ E_{\gamma\alpha\beta}(v,v') &= E_3(w)\,(g_{\gamma\alpha}v'_\beta - g_{\gamma\beta}v'_\alpha)\,, \\ E'_{\gamma\alpha\beta}(v,v') &= E'_3(w)\,(g_{\gamma\alpha}v'_\beta - g_{\gamma\beta}v'_\alpha)\,.\end{aligned} \tag{4.14}$$

The equation of motion implies $v^\gamma E_\gamma = v'^\gamma E'_\gamma = 0$, yielding $E_1 = -w\,E_2$ and $E'_2 = -w\,E'_1$. There are no conditions on E_3 and E'_3.

As discussed in detail in Appendix C of Ref. I, the two matrix elements in (4.13) may be related to each other by an integration by parts. Because there are fewer possible Lorentz structures for the heavy baryons than for the mesons, here these relations take a particularly simple form, namely

$$\begin{aligned}E_\gamma - E'_\gamma &= \bar\Lambda\,(v-v')_\gamma\,A + v_\gamma[\phi_0 - \lambda\,\zeta]\,, \\ E_3 - E'_3 &= 0\,.\end{aligned} \tag{4.15}$$

Hence we are left with only one new independent form factor, E_3. The others may be written

$$E_1 = -wE_2 = \frac{w}{w+1}\Big[w\,\tilde\phi + \bar\Lambda\,A\Big]\,,$$
$$E_2' = -wE_1' = \frac{w}{w+1}\Big[-\tilde\phi + \bar\Lambda\,A\Big]\,, \tag{4.16}$$

where

$$\tilde\phi(w) = \frac{\phi_0(w) - \lambda\,\zeta(w)}{w-1} \tag{4.17}$$

is a nonsingular function as $w \to 1$, since $\phi_0(1) = \lambda$.

Finally, we note that the equations of motion imply that the form factor E_γ takes the form $E_\gamma = E_1\,(v_\gamma - w\,v'_\gamma)$, which vanishes as $v \to v'$. The expression for E'_γ has a similar structure, while the kinematic structures multiplying E_3 and E'_3 vanish at zero recoil. Hence, as with the mesons, the mixed corrections give no contribution at zero recoil to form factors which are not kinematically suppressed.

D. Form Factors and Normalization Conditions

We have introduced a set of ten new universal functions which describe the $1/m^2$ corrections to heavy Λ baryon form factors in the heavy quark expansion. Two of these, ϕ_0 and ϕ_1, parameterize the corrections to the current, seven more, B, C_i and D_i, for $i = 1, 2, 3$, parameterize the effects of higher order terms in the effective Lagrangian, and one, E_3, is needed in order to include mixed corrections to the current and the Lagrangian. It is now straightforward to express the vector and axial vector form factors F_i and G_i up to order $1/m^2$ in terms of these universal functions. To this end it is useful, as in the meson case, to collect certain combinations of universal form factors by introducing the functions

$$\begin{aligned}
b_1 &= \lambda\,\zeta + B + C_1 - 3C_2 + 2(w^2-1)C_3 \\
&\quad + (w-1)\,(\phi_1 - 2E_3) + \frac{w-1}{w+1}\,(w\tilde\phi + \bar\Lambda A)\,, \\
b_2 &= -2\,(\phi_1 - 2E_3) - \frac{2}{w+1}\,(w\tilde\phi + \bar\Lambda A)\,, \\
b_3 &= D_1 + D_2 - \phi_1 + \frac{w-1}{w+1}\Big[\bar\Lambda^2\zeta - 2\,(\tilde\phi - \bar\Lambda A)\Big]\,, \\
b_4 &= \frac{4}{w+1}\,(\tilde\phi - \bar\Lambda A)\,, \\
b_5 &= -2D_2 - 2(w-1)D_3 - 3\frac{w-1}{(w+1)^2}\,\bar\Lambda^2\zeta \\
&\quad + \frac{1}{w+1}\Big[-\phi_0 + (2-w)\,\phi_1 + 2\,(\tilde\phi - \bar\Lambda A)\Big]\,, \\
b_6 &= 2D_2 + 2(w+1)D_3 - \frac{\bar\Lambda^2}{w+1}\,\zeta
\end{aligned}$$

$$+\frac{1}{w-1}\left[\phi_0+(2+w)\,\phi_1\right]+\frac{2}{w+1}\,(\widetilde{\phi}-\bar{\Lambda}A)\,. \tag{4.18}$$

Note that the term $\lambda\zeta$ in b_1 arises from substituting the relation (3.7) between the physical baryon spinors $u(v,s)$, which appear in the definition of the form factors F_i and G_i, and the effective spinors $\mathcal{U}(v,s)$ of HQET, into the leading order matrix elements (3.5). Let us furthermore specialize to transitions of the type $\Lambda_b\to\Lambda_c$, and abbreviate $\varepsilon_b=1/2m_b$ and $\varepsilon_c=1/2m_c$. We then find

$$\begin{aligned}
F_1 &= \zeta+(\varepsilon_c+\varepsilon_b)\,[\mathcal{B}_1-\mathcal{B}_2]+(\varepsilon_c^2+\varepsilon_b^2)\,[b_1-b_2]+\varepsilon_c\varepsilon_b\,[b_3-b_4]\,,\\
F_2 &= \varepsilon_c\,\mathcal{B}_2+\varepsilon_c^2\,b_2+\varepsilon_c\varepsilon_b\,b_5\,,\\
F_3 &= \varepsilon_b\,\mathcal{B}_2+\varepsilon_b^2\,b_2+\varepsilon_c\varepsilon_b\,b_5\,,\\
\\
G_1 &= \zeta+(\varepsilon_c+\varepsilon_b)\,\mathcal{B}_1+(\varepsilon_c^2+\varepsilon_b^2)\,b_1+\varepsilon_c\varepsilon_b\,b_3\,,\\
G_2 &= \quad\varepsilon_c\,\mathcal{B}_2+\varepsilon_c^2\,b_2+\varepsilon_c\varepsilon_b\,b_6\,,\\
G_3 &= -\varepsilon_b\,\mathcal{B}_2-\varepsilon_b^2\,b_2-\varepsilon_c\varepsilon_b\,b_6\,.
\end{aligned} \tag{4.19}$$

Recall that $\widetilde{\phi}$ was defined in terms of other universal functions in (4.17).

Order by order in the heavy quark expansion, the normalization condition (3.9) imposes a constraint on the universal functions of HQET. Hence, in addition to $\zeta(1)=1$ and $A(1)=0$, there is a relation at zero recoil between the form factors which arise at order $1/m^2$. Evaluating the sum of F_i for equal masses, we obtain

$$2b_1(1)+b_3(1)-b_4(1)+2b_5(1)=0\,, \tag{4.20}$$

which is equivalent to

$$2B(1)+2C_1(1)+D_1(1)-6C_2(1)-3D_2(1)=-\lambda\,. \tag{4.21}$$

V. APPLICATIONS TO SEMILEPTONIC Λ_b DECAYS

In this section we apply our results to semileptonic Λ_b decays and give some estimates of the size of the second order corrections. For simplicity, and in order to focus on what is new in our analysis, we shall continue to ignore radiative corrections.

A. $\Lambda_b\to\Lambda_c\,\ell\,\nu$ Decays Near Zero Recoil

The semileptonic decay $\Lambda_b\to\Lambda_c\,\ell\,\nu$ is particularly simple to analyze near the zero recoil point $w=1$, where the invariant mass q^2 of the lepton pair takes on its maximum value $q^2_{\rm max}=(m_{\Lambda_b}-m_{\Lambda_c})^2$. In the limit of vanishing lepton mass, angular momentum conservation requires that the weak matrix element $\langle\Lambda_c(v,s')\,|\,(V^\mu-A^\mu)\,|\,\Lambda_b(v,s)\rangle$ depend only on the function $G_1(1)$. The differential decay rate near this point is given by

$$\lim_{w\to 1} \frac{1}{\sqrt{w^2-1}} \frac{\mathrm{d}\Gamma(\Lambda_b \to \Lambda_c\,\ell\,\nu)}{\mathrm{d}w} = \frac{G_F^2\,|\,V_{cb}|^2}{4\pi^3} m_{\Lambda_c}^3\,(m_{\Lambda_b} - m_{\Lambda_c})^2\,|G_1(1)|^2\,. \quad (5.1)$$

The form factor $G_1(1)$ is protected against corrections at order $1/m$ [19], but it receives contributions from order $1/m^2$ effects. Incorporating the normalization condition (4.21), we find

$$G_1(1) = 1 + (\varepsilon_c - \varepsilon_b)^2\, b_1(1) + \varepsilon_c\varepsilon_b[b_4(1) - 2b_5(1)]\,. \quad (5.2)$$

We may estimate the size of the corrections to $G_1(1)$ by considering the form of the corresponding vector current matrix element at zero recoil, given by

$$\langle\Lambda_c(v,s')\,|\,V^\mu\,|\,\Lambda_b(v,s)\rangle = 2\sqrt{m_{\Lambda_c} m_{\Lambda_b}}\,F(1)\,v^\mu\,, \quad (5.3)$$

where

$$F(1) \equiv \sum_{i=1,2,3} F_i(1) = 1 + (\varepsilon_c - \varepsilon_b)^2\, b_1(1)\,. \quad (5.4)$$

The function $F(1)$ measures the overlap of the wavefunctions of the light degrees of freedom between a Λ_b and a Λ_c baryon. While the light quarks and gluons were insensitive to the mass of the heavy quark in the strict $m \to \infty$ limit and in precisely the same configuration in a Λ_b and a Λ_c, at order $1/m^2$ the wavefunctions differ from each other and the overlap is incomplete ($F(1) < 1$). We may estimate the size of this difference in a nonrelativistic model in which a Λ_Q baryon is composed of a constituent diquark of mass $m_{qq} \approx \bar\Lambda \approx 700\,\mathrm{MeV}$, orbiting about the heavy quark. In this case the m_Q-dependence of the overlap integral comes from the m_Q-dependence of the reduced mass $m_{qq}^{\mathrm{red}} = m_{qq}m_Q/(m_{qq} + m_Q)$ of the diquark. We then obtain the estimate

$$b_1(1) \approx -3\bar\Lambda^2 \approx -1.5\,\mathrm{GeV}^2\,. \quad (5.5)$$

This combination is the same as appears in the first term which corrects $G_1(1)$.

The second term, $b_4(1) - 2b_5(1) = \frac{4}{3}\lambda + 4D_2(1)$, is harder to estimate. However, we note that the function D_2 arises from the double insertion of the chromo-magnetic operator in $\mathcal{L}_1$, and there are indications from QCD sum rules that it is likely to be quite small [20]. Furthermore, for heavy mesons sum rules predict a value for the analog of λ which is positive and about 1 GeV [21]. Let us for the sake of argument assume such a value here. Using $m_c = 1.5$ GeV and $m_b = 4.8$ GeV, we then we obtain

$$G_1(1) \approx 1 - 7.7\% + 4.6\%\,. \quad (5.6)$$

While this estimate is of course quite rough, it is reasonable to expect at least that the signs of the two terms are as we claim, such that there is a partial cancellation of the two contributions. Then if the magnitudes are even approximately correct, one may argue that $1/m^2$ corrections to $G_1(1)$ at the level of ten percent would be surprising. Consequently, we expect the semileptonic decay $\Lambda_b \to \Lambda_c\,\ell\,\nu$ to be well described by HQET.

B. Asymmetry Parameters in $\Lambda_b \to \Lambda_c \,\ell\, \nu$ Decays

The angular distributions in the cascade $\Lambda_b \to \Lambda_c \,\ell\, \nu\nu \to \Lambda\, X\, \ell\, \nu$ provide an efficient analysis of polarization effects in semileptonic Λ_b decay. This is particularly true at the kinematic endpoint $q^2 = 0$, where only the helicity amplitudes in which a longitudinal virtual W boson is emitted contribute. Such effects are discussed at length in Ref. [22], to which we refer the interested reader for details. Here we shall merely cite the final expressions.

There are several asymmetry parameters which are particularly interesting at $q^2 = 0$ within the heavy quark expansion. The simplest comes from the distribution in the angle θ_Λ between the Λ and Λ_c directions. The differential decay width in this variable is given by

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_\Lambda} \propto 1 + \alpha\,\alpha_{\Lambda_c} \cos\theta_\Lambda\,, \tag{5.7}$$

where α is the asymmetry parameter of the Λ_b decay, and α_{Λ_c} is the measured asymmetry parameter in the decay $\Lambda_c \to \Lambda\, X$. For the nonleptonic decay $\Lambda_c^+ \to \Lambda\,\pi^+$, a particularly useful mode, there are recent measurements $\alpha_{\Lambda_c} = -1.0^{+0.4}_{-0.0}$ [23] and $\alpha_{\Lambda_c} = -0.96 \pm 0.42$ [24].

Two additional asymmetry parameters which have interesting HQET expansions may be defined for the decay of polarized Λ_b baryons. Let P be the degree of polarization of the Λ_b, and θ_P the angle between the Λ_b polarization and the direction of the Λ_c. Then the parameter α_P is defined by the form of the differential distribution,

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_P} \propto 1 - \alpha_P P \cos\theta_P\,. \tag{5.8}$$

Further, let χ_P be the angle between the plane of the Λ_c decay and the plane formed by the Λ_b polarization and the Λ_c direction. Then the angular distribution in χ_P is given by

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2\,\mathrm{d}\chi_P} \propto 1 - \gamma_P\,\frac{\pi^2}{16}\,P\alpha_{\Lambda_c} \cos\chi_P\,, \tag{5.9}$$

where γ_P yet is another asymmetry parameter.

At $q^2 = 0$, the expressions for α, α_P and γ_P take simple forms,

$$\begin{aligned} \alpha = -\alpha_P &= -\frac{1 - |\,\epsilon\,|^2}{1 + |\,\epsilon\,|^2}\,, \\ \gamma_P &= \frac{2\,\mathrm{Re}(\epsilon)}{1 + |\,\epsilon\,|^2}\,, \end{aligned} \tag{5.10}$$

where

$$\epsilon = \frac{f_1(0) - g_1(0)}{f_1(0) + g_1(0)}\,. \tag{5.11}$$

At leading order in HQET, this ratio vanishes since $f_1 = g_1 = \zeta$. In this limit the asymmetries are predicted to be $\alpha = -\alpha_P = -1$ and $\gamma_P = 0$ [22]. Using (3.4) and (4.19), we find that there are no $1/m$ corrections to these predictions. The leading power correction comes at order $1/m^2$:

$$\epsilon_{1/m^2} = \frac{1}{4\zeta(w)}\Big\{(\varepsilon_c - \varepsilon_b)^2\Big[b_5(w) - b_6(w)\Big] + 2\varepsilon_c\varepsilon_b\Big[2b_5(w) - b_4(w)\Big]\Big\}, \quad (5.12)$$

where $w = (m_{\Lambda_b}^2 + m_{\Lambda_c}^2)/2m_{\Lambda_b}m_{\Lambda_c}$ corresponding to $q^2 = 0$. Based on our previous estimates we expect ϵ_{1/m^2} to be of the order of a few percent. A contribution of similar magnitude comes from perturbative corrections to the heavy quark currents at leading order in HQET. It is given by [25, 26]

$$\epsilon_{\rm QCD} = -\frac{2\alpha_s}{3\pi}\,\frac{m_b m_c}{m_b^2 - m_c^2}\,\ln\frac{m_b}{m_c} \approx -2.4\%\,, \quad (5.13)$$

where we have used $\alpha_s/\pi = 0.09$.

In view of its smallness, it will be virtually impossible to determine $|\,\epsilon\,|$ from a measurement of α or α_P, since these parameters depend only on $|\,\epsilon\,|^2$ and should, therefore, be very close to the asymptotic values given above. A measurement of a nonzero asymmetry γ_P, on the other hand, would provide a direct determination of $\mathrm{Re}(\epsilon)$ and could yield valuable information about the size of $1/m^2$ corrections.

VI. MATRIX ELEMENTS OF EXCITED BARYONS

The entire analysis presented here could be extended to matrix elements involving excited baryons, in particular to baryons of higher spin. To order $1/m$, this was done by Mannel, Roberts and Ryzak [27]. The Λ_Q baryons which we have been considering are extremely simple, because the light degrees of freedom are in a state of zero total angular momentum, and hence the polarization of the baryon is the same as the polarization of the heavy quark. There is, however, an excited state in which the spins of the light quarks are aligned so that the light degrees of freedom have angular momentum $s_\ell = 1$. When combined with the heavy quark, this state becomes a degenerate doublet of an excited spin-$\frac{1}{2}$ baryon, the Σ_Q, and a spin-$\frac{3}{2}$ baryon, the Σ_Q^*. The analysis of the semileptonic decays of and into these states is analogous to that for the mesons and Λ_Q baryons, except that the states have to be represented differently, and the counting of form factors is modified accordingly. Rather than elaborate the entire analysis yet again, we shall simply indicate how it differs from the cases already presented.

As for the pseudoscalar and vector mesons, it is convenient to assemble the degenerate doublet $(\Sigma_Q,\, \Sigma_Q^*)$ into a single object. This allows us to implement the spin symmetries in a compact formalism. Let us represent the spin-$\frac{1}{2}$ Σ_Q by the spinor ψ and the spin-$\frac{3}{2}$ Σ_Q^* by the Rarita-Schwinger vector-spinor ψ^μ. In the heavy quark limit, these objects satisfy $\rlap/v\,\psi = \psi$, $\rlap/v\,\psi^\mu = \psi^\mu$, $v_\mu\psi^\mu = \gamma_\mu\psi^\mu = 0$. Then the doublet is represented by [18, 28]

$$\Psi^\mu = \psi^\mu + \tfrac{1}{\sqrt{3}}(\gamma^\mu + v^\mu)\,\gamma^5\,\psi\,, \tag{6.1}$$

which satisfies the constraints $v_\mu\Psi^\mu = 0$ and $\not v\,\Psi^\mu = \Psi^\mu$. It is straightforward to construct the analogs of Ψ^μ for baryons of arbitrary spin [28].

From here on the heavy quark expansion proceeds almost exactly as before. For example, for semileptonic transitions of the form $\Lambda_Q \to \Sigma_{Q'}$ or $\Lambda_Q \to \Sigma^*_{Q'}$, one repeats the analysis of Secs. III and IV, but with an additional index μ on all universal form factors. There is, however, a subtlety which must be considered. The spin-parity s_ℓ^P of the light degrees of freedom may be either in the series $0^+, 1^-, 2^+, \ldots$, in which case it is "natural", or in the series $0^-, 1^+, 2^-, \ldots$, in which case it is "unnatural". As noted in Ref. [29], there are additional restrictions on the universal functions which describe the transitions between "natural" and "unnatural" baryons [18, 28]. These restrictions may be imposed [27] by constructing form factors which are pseudotensors, rather than tensors.

In particular, the Λ_Q is a "natural" baryon, while the Σ_Q and Σ^*_Q are "unnatural". Hence at leading order, $\Sigma \to \Sigma$ transitions are governed by a tensor form factor of the form

$$\begin{aligned} \langle\Sigma'|\,\bar h'\,\Gamma\,h\,|\Sigma\rangle &= K_{\mu\nu}(v,v')\,\overline{\Psi}'^\mu\,\Gamma\,\Psi^\nu \\ &= \Big[K_1(w)\,g_{\mu\nu} + K_2(w)\,v_\mu v'_\nu\Big]\,\overline{\Psi}'^\mu\,\Gamma\,\Psi^\nu\,, \end{aligned} \tag{6.2}$$

while the leading $\Lambda \to \Sigma$ transitions would require a pseudovector form factor. However,

$$\langle\Sigma'|\,\bar h'\,\Gamma\,h\,|\Lambda\rangle = K_\mu(v,v')\,\overline{\Psi}'^\mu\,\Gamma\,u = 0\,, \tag{6.3}$$

since no such object K_μ can be built from the available vectors v and v'.

Once this subtlety has been taken into account, the construction of the heavy quark expansion proceeds just as before. Order by order, one identifies the (pseudo) tensor-valued functions which describe a given type of correction, performs a general decomposition in terms of velocities to obtain the complete list of universal functions, and then writes the physical matrix elements in terms of them. The restrictions imposed by the heavy quark spin symmetries are built into the formalism from the start. For example, one might consider the corrections to $\Sigma \to \Sigma'$ transitions which arise from insertions of the first order corrections to the effective Lagrangian. One finds five form factors L_i, defined by

$$\begin{aligned} &\langle\Sigma'|\,i\!\int\!\mathrm{d}x\;T\,\{\,J(0), [h\,(iD)^2 h]_x\,\}\,|\Sigma\rangle \\ &\qquad = \Big[L_1(w)\,g_{\mu\nu} + L_2(w)\,v_\mu v'_\nu\Big]\,\overline{\Psi}'^\mu\,\Gamma\,\Psi^\nu\,, \\ &\langle\Sigma'|\,i\!\int\!\mathrm{d}x\;T\,\{\,J(0), [\bar h\,s^{\alpha\beta}G_{\alpha\beta}h]_x\,\}\,|\Sigma\rangle \end{aligned} \tag{6.4}$$

$$= \Big[L_3(w)\,(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu}) + L_4(w)\,(g_{\alpha\nu}v'_\beta v_\mu - g_{\beta\nu}v'_\alpha v_\mu)$$
$$+ L_5(w)\,(g_{\alpha\mu}v'_\beta v'_\nu - g_{\beta\mu}v'_\alpha v'_\nu)\Big]\,\overline{\Psi}'^\mu\,\Gamma\,P_+\,s^{\alpha\beta}\Psi^\nu\,.$$

This procedure clearly becomes more tedious as the spin of the baryons increases and with higher order in the $1/m$ expansion; however, the enumeration of form factors is straightforward, systematic and complete.

Finally, we note that in the case of $b \to c$ weak decays, it is only the transitions of the form $\Lambda_b \to \Lambda_c$, Σ_c, $\Sigma_c^*, \ldots$ which are likely to be of experimental interest. This is because the excited bottom baryons will decay strongly (if the mass splitting is sufficient to allow pion emission) or electromagnetically to the ground state Λ_b, and thus their weak decays will not be observable. On the other hand, the decays $\Lambda_b \to \Sigma_c, \Sigma_c^*$ will be particularly interesting, since they arise solely due to effects of order $1/m_c$ and higher.

VII. SUMMARY

We have extended the analysis of $1/m^2$ corrections in the heavy quark effective theory to the heavy baryons. We have focused in detail on the simplest case, the weak matrix elements relevant to the decay of a heavy Λ_Q to a heavy $\Lambda_{Q'}$. Due to the trivial Lorentz structure of the light degrees of freedom in this system, the description of the power corrections is considerably simpler than for the heavy mesons. At order $1/m^2$, one needs a set of ten new m_Q-independent Isgur-Wise functions of the kinematic variable $v\cdot v'$, and a single new dimensionful parameter λ. Vector current conservation forces a certain combination of form factors to vanish at zero recoil.

We have given a rough estimate of the size of the second order corrections for the semileptonic decay $\Lambda_b \to \Lambda_c\,\ell\,\nu$. We find a partial cancellation of $1/m^2$ corrections at zero recoil, with the conclusion that large deviations from the infinite quark mass limit are unlikely, and the heavy quark expansion is well under control. Investigating briefly the asymmetry parameters which may be defined in this decay, we have suggested a particular measurement which would probe the $1/m^2$ corrections directly. Finally, we have sketched the extension of the formalism to excited heavy baryons of arbitrary spin.

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