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Second Order Power Corrections in the Heavy Quark Effective Theory
I. Formalism and Meson Form Factors*

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In the heavy quark effective theory, hadronic matrix elements of currents between two hadrons containing a heavy quark are expanded in inverse powers of the heavy quark masses, with coefficients that are functions of the kinematic variable $v \cdot v'$. For the ground state pseudoscalar and vector mesons, this expansion is constructed at order $1/m_Q^2$. A minimal set of universal form factors is defined in terms of matrix elements of higher dimension operators in the effective theory. The zero recoil normalization conditions following from vector current conservation are derived. Several phenomenological applications of the general results are discussed in detail. It is argued that at zero recoil the semileptonic decay rates for $B \to D\,\ell\,\nu$ and $B \to D^*\ell\,\nu$ receive only small second order corrections, which are unlikely to exceed the level of a few percent. This supports the usefulness of the heavy quark expansion for a reliable determination of V_{cb}.

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I. INTRODUCTION

Recent developments in the theory of heavy quarks have increased the prospects both for a reliable determination of some of the fundamental parameters of the standard model, and for a study of nonperturbative QCD in the weak decays of heavy mesons and baryons. The excitement is driven by the discovery of a spin-flavor symmetry for heavy quarks that QCD reveals in the limit where the quark mass $m_Q \to \infty$, in which certain properties of a hadron containing the heavy quark become independent of its mass and spin [1, 2]. These symmetries are responsible for restrictive relations among weak decay amplitudes and reduce the number of independent form factors. The description of semileptonic transitions between two ground state heavy mesons [2, 3] or baryons [4–7] becomes particularly simple. In the limit where the heavy quark masses are much larger than any other hadronic scale in the process, the large set of hadronic form factors is reduced to a single universal function of the kinematic variable $v \cdot v'$, which measures the change of velocities that the heavy hadrons undergo during the transition. It depends on the quantum numbers of the light degrees of freedom, but not on the heavy quark masses and spins. In addition, the conservation of the vector current implies that this celebrated Isgur-Wise form factor is normalized at zero recoil, allowing model-independent predictions unaffected by hadronic uncertainties.

Clearly, a careful analysis of at least the leading symmetry-breaking corrections is essential for any phenomenological application of the heavy quark symmetries. An elegant framework in which to analyze such corrections is provided by the so-called heavy quark effective theory, which allows for a systematic expansion of decay amplitudes in powers of $1/m_Q$ [8–14]. The coefficients in this expansion are given by matrix elements of operators in the effective theory and can be parameterized in terms of universal form factors, which characterize the properties of the light degrees of freedom in the background of the static color source provided by the heavy quark. At leading order one recovers the Isgur-Wise limit, in which only a single function remains. But already at order $1/m_Q$ one encounters a larger set of universal form factors, which affect all but very few of the symmetry predictions that hold in the infinite quark mass limit [15–17]. An understanding of these functions is at the heart of nonperturbative QCD, but it is ultimately necessary for any quantitative analysis based on heavy quark symmetries. In the future, one might hope to compute the universal form factors from first principles by using a formulation of the effective theory on a lattice [8, 18, 19]. In the meantime, QCD sum rules [20] offer a less ambitious approach to this problem, and have recently been employed to study the decay constants of heavy mesons, the Isgur-Wise form factor, and the universal functions that appear at order $1/m_Q$ in the heavy quark expansion [17, 21–25]. One can also gain valuable information about symmetry-breaking corrections from measurements of certain ratios of form factors [25].

In this paper we analyze current-induced transitions between ground state heavy mesons at order $1/m_Q^2$ in the heavy quark expansion. Such an analysis is particularly relevant for the important cases where the leading $1/m_Q$ corrections are known to vanish at zero recoil. An example is the semileptonic decay $B \to D^*\ell\,\nu$, which therefore seems ideal for a measurement of the weak mixing parameter V_{cb} [1, 26]. In Sec. II we discuss the parameters of the effective theory that appear at subleading order. The general method of computing power corrections is outlined in Sec. III, together with a review of the analysis of the $1/m_Q$ corrections to transitions between heavy mesons. In Sec. IV we extend this analysis to second order. We identify a minimal set of universal functions and give their relation to matrix elements of higher dimension operators in the effective theory. The zero recoil normalization conditions imposed on some of these form factors are derived. Although in principle straightforward, the analysis is tedious and involves considerable technicalities of the heavy quark effective theory. The reader not interested in these details is encouraged to proceed to Sec. V, where we summarize our results and illustrate them for some specific cases of phenomenological relevance. In particular, the corrections affecting the determination of V_{cb} from exclusive semileptonic B decays are investigated. We also study the fictitious limit of vanishing chromo-magnetic interaction, which leads to great simplifications and might serve as an estimate of the dominant corrections.

Based on the analysis for heavy mesons, the $1/m_Q^2$ corrections to matrix elements between heavy baryons can readily be derived. We discuss this subject in the following paper [27].

II. PARAMETERS OF THE EFFECTIVE THEORY

The construction of the heavy quark effective theory (HQET) is based on the observation that, in the limit $m_Q \gg \Lambda_{\rm QCD}$, the velocity of a heavy quark is conserved with respect to soft processes. It is then possible to remove the mass-dependent piece of the momentum operator by a field redefinition. To this end, one introduces a field $h_Q(v,x)$, which annihilates a heavy quark with velocity v^α, by [11]

$$h_Q(v,x) = e^{im_Q v\cdot x}\,P_+(v)\,Q(x)\,, \tag{2.1}$$

where $P_+(v) = \frac{1}{2}(1+\rlap/v)$ is an on-shell projection operator, and $Q(x)$ denotes the conventional quark field in QCD. If P^α is the total momentum of the heavy quark, the new field carries the residual momentum $k^\alpha = P^\alpha - m_Q v^\alpha$.

There is obviously some ambiguity associated with the construction of HQET, since the heavy quark mass used in the definition of the field h_Q is not uniquely defined. In fact, for HQET to be consistent it is only necessary that k^α be of order of $\Lambda_{\rm QCD}$, *i.e.*, stay finite in the limit $m_Q \to \infty$. It is intuitively clear that different choices for m_Q must lead to the same answer for any physical matrix element, and this can indeed be shown to be the case [28]. Yet it is advantageous to adopt a special choice for which the resulting effective theory becomes particularly simple, in the sense that there are no "residual mass terms" for the heavy quark and the heavy quark expansion becomes a covariant derivative expansion. This prescription provides a nonperturbative definition of the heavy quark mass, which has been adopted implicitly in most previous analyses based on HQET. It is important to realize, however, that so defined, the mass m_Q is a nontrivial parameter of the effective theory.

In the limit $m_Q \to \infty$, the effective Lagrangian for the strong interactions of the heavy quark becomes [11–13]

$$\mathcal{L}_{\rm HQET} = \bar{h}_Q \, iv\!\cdot\! D \, h_Q \, , \tag{2.2}$$

where $D^\alpha = \partial^\alpha - ig_s t_a A_a^\alpha$ is the gauge-covariant derivative. For finite m_Q, there appears in the Lagrangian an infinite series of power corrections involving higher dimension operators,

$$\mathcal{L}_{\rm power} = \frac{1}{2m_Q}\,\mathcal{L}_1 + \frac{1}{4m_Q^2}\,\mathcal{L}_2 + \cdots \, . \tag{2.3}$$

Note that it is natural to expand in powers of $1/2m_Q$ since, after the field redefinition (2.1), $2m_Q$ is the mass associated with the heavy antiquark field which is integrated out [13]. Omitting an operator whose matrix elements vanish by the equation of motion, the leading term in (2.3) is given by [14]

$$\mathcal{L}_1 = \bar{h}_Q \, (iD)^2 h_Q + Z(m_Q/\mu) \, \bar{h}_Q \, s_{\alpha\beta} G^{\alpha\beta} h_Q \, , \tag{2.4}$$

where $s_{\alpha\beta} = -\frac{i}{2}\sigma_{\alpha\beta}$, and $G^{\alpha\beta} = [iD^\alpha, iD^\beta] = ig_s t_a G_a^{\alpha\beta}$ is the gluon field strength. In leading logarithmic approximation, the renormalization factor for the chromo-magnetic moment operator is

$$Z(m_Q/\mu) = \left[\frac{\alpha_s(m_Q)}{\alpha_s(\mu)}\right]^{9/\beta} ; \; \beta = 33 - 2n_f \, , \tag{2.5}$$

where n_f is the number of light quark flavors with mass below m_Q. The kinetic term in (2.4) is not renormalized.

The purpose of the heavy quark expansion is to make the m_Q-dependence of some hadronic quantity A explicit by writing

$$A(m_Q) = C_0(m_Q/\mu)\, A_0(\mu) + \frac{1}{2m_Q}\, C_1(m_Q/\mu)\, A_1(\mu) + \cdots \, , \tag{2.6}$$

in such a way that the coefficients $A_i(\mu)$ are universal, m_Q-independent parameters, and $C_i(m_Q/\mu)$ are purely perturbative coefficients, which dependent on m_Q only via the running of the strong coupling $\alpha_s(m_Q)$. The aim is to relate A_i to matrix elements of operators in HQET evaluated between the eigenstates of the lowest order Lagrangian $\mathcal{L}_{\rm HQET}$. This paper focuses on the ground state pseudoscalar and vector mesons, which form a degenerate doublet under the heavy quark spin symmetry. These mesons have the same velocity as the heavy quark which they contain. Their common mass M, however, differs from the mass of the heavy quarks by a finite amount $\bar{\Lambda} = M - m_Q$, which measures the "mass" carried by the light degrees of freedom. Because of the field redefinition (2.1), it is this mass which governs the x-dependence of states in the effective theory:

$$|M(x)\rangle_{\rm HQET} = e^{-i\bar\Lambda v\cdot x}|M(0)\rangle_{\rm HQET}\,. \tag{2.7}$$

$\bar\Lambda$ is a universal parameter which can be defined in terms of a matrix element of a higher dimension operator in HQET. Using the equation of motion $iv\cdot D\, h_Q = 0$, which follows from the effective Lagrangian $\mathcal{L}_{\rm HQET}$, it is easy to see that [28]

$$\bar\Lambda = \frac{\langle\, 0\,|\, \bar q\, iv\cdot \overleftarrow{D}\, \Gamma\, h_Q\, |M(v)\rangle}{\langle\, 0\,|\, \bar q\, \Gamma\, h_Q\, |M(v)\rangle}\,. \tag{2.8}$$

Here Γ is an appropriate Dirac matrix such that the currents interpolate the heavy meson M. This relation shows that $\bar\Lambda$ is in fact a parameter describing the properties of the light degrees of freedom in the background of the static color source provided by the heavy quark. It turns out that this mass scale also enters the leading power corrections to heavy meson form factors and determines the canonical size of deviations from the infinite quark mass limit [15, 16]. A recent analysis of $\bar\Lambda$ using QCD sum rules predicts [17]

$$\bar\Lambda = 0.50 \pm 0.07~\text{GeV}\,. \tag{2.9}$$

The eigenstates of $\mathcal{L}_{\rm HQET}$ differ from the states of the full theory. In particular, their mass M differs from the physical masses of pseudoscalar or vector mesons by an amount of order $1/m_Q$. These mass shifts are computable in HQET. The physical masses m_M obey a heavy quark expansion, which we write as $(m_M - m_Q) = \bar\Lambda + \Delta m_M^2/2m_Q + \cdots$. In the meson rest frame,

$$\Delta m_M^2 = \frac{\langle M(v)|\,(-\mathcal{L}_1)\,|M(v)\rangle}{\langle M(v)|\, h_Q^\dagger h_Q\, |M(v)\rangle}\,. \tag{2.10}$$

A convenient way to evaluate hadronic matrix elements in HQET is by associating spin wave functions

$$\mathcal{M}(v) = \sqrt{M}\, P_+(v) \begin{cases} -\gamma_5 & ;\ \text{pseudoscalar meson } P \\ \not\epsilon & ;\ \text{vector meson } V \end{cases} \tag{2.11}$$

with the eigenstates of $\mathcal{L}_{\rm HQET}$ [3, 29, 30]. These wave functions have the correct transformation properties under boosts and heavy quark spin rotations. Here ϵ^α denotes the polarization vector of the vector meson. For reasons of simplicity we shall often omit the argument v in both P_+ and $\mathcal{M}$. We note that $\mathcal{M} = P_+\,\mathcal{M}\,P_-$, where $P_\pm = \frac{1}{2}(1 \pm \not v)$. Lorentz invariance allows one to write any matrix element as a trace over these wave functions and appropriate Dirac matrices. For the matrix elements in (2.10) we define hadronic parameters λ_i by

$$\begin{aligned} \langle M|\, \bar h_Q\, (iD)^2 h_Q\, |M\rangle &= -\lambda_1\, \mathrm{tr}\{\, \overline{\mathcal{M}}\,\mathcal{M}\,\} = 2M\,\lambda_1\,, \\ \langle M|\, \bar h_Q\, s_{\alpha\beta} G^{\alpha\beta} h_Q\, |M\rangle &= -\lambda_2(\mu)\, \mathrm{tr}\{\, i\sigma_{\alpha\beta}\,\overline{\mathcal{M}}\, s^{\alpha\beta}\mathcal{M}\,\} = 2d_M M\,\lambda_2(\mu)\,, \end{aligned} \tag{2.12}$$

where $d_P = 3$ for a pseudoscalar meson, and $d_V = -1$ for a vector meson. The conservation of the vector current implies that, in the rest frame, the matrix element in the denominator is given by $\langle M|\, h_Q^\dagger h_Q\, |M\rangle = 2M$. We thus have

$$\Delta m_M^2 = -\lambda_1 - d_M\, Z(\mu)\, \lambda_2(\mu)\,. \tag{2.13}$$

The universal parameters λ_1 and λ_2 are the analogs of $\bar\Lambda$ at subleading order in the heavy quark expansion. They are independent of m_Q. Whereas λ_1 is not renormalized, $\lambda_2(\mu)$ depends on the renormalization scale in such a way that the product $Z(\mu)\,\lambda_2(\mu)$ is scale-independent.

An estimate of the value of λ_2 can be obtained from the measured mass splitting between the B^* and B mesons, assuming that higher order corrections in the B system are small. One finds

$$m_{B^*}^2 - m_B^2 \approx \Delta m_{B^*}^2 - \Delta m_B^2 = 4\lambda_2(m_b) \approx 0.48~\mathrm{GeV}^2\,, \tag{2.14}$$

where the experimental value has been taken from Refs. [31]. Using (2.5) for the evolution of this parameter down to the low energy scale $2\bar\Lambda \approx 1$ GeV, we obtain

$$\lambda_2(2\bar\Lambda) \approx 0.15~\mathrm{GeV}^2\,. \tag{2.15}$$

Unfortunately, it is not possible directly to relate the spin-symmetry conserving parameter λ_1 to an observable quantity. Recently, QCD sum rules have been used to compute both λ_1 and λ_2 [17]. The spin-symmetry breaking correction was found in excellent agreement with experiment, $\lambda_2^{\mathrm{s.r.}} = 0.12 \pm 0.02~\mathrm{GeV}^2$, and a rather large value for the spin-symmetry conserving correction was obtained, $\lambda_1^{\mathrm{s.r.}} \approx 1~\mathrm{GeV}^2$. However, the sum rule analysis suggests that it might be more appropriate to use an effective value of λ_1 in the b and c system which could be substantially smaller, even compatible with zero. A measurement of λ_1 on a lattice could help to clarify this issue.

III. MESON FORM FACTORS IN THE EFFECTIVE THEORY

Let us now review the analysis of current-induced transitions between two heavy mesons to subleading order in HQET, as performed by Luke [15]. This will help to outline the general procedure and set up the conventions we will need in Sec. IV. The aim is to construct the heavy quark expansion (2.6) for matrix elements of the type $\langle M'(v')|\,\bar Q'\,\Gamma\, Q\,|M(v)\rangle$, where Γ is an arbitrary Dirac matrix. In this case the universal parameters are functions of the kinematic variable $w = v\cdot v'$, and the perturbative coefficients, subsequently denoted by C_j, c_j and c_j', depend on w and both heavy quark masses. The current $\bar Q'\,\Gamma\, Q$ has a short distance expansion in terms of operators of the effective theory. It reads

$$\begin{aligned}\bar Q'\,\Gamma\, Q \to {} & \sum_j C_j\, \bar h'\,\Gamma_j\, h \\ & + \frac{1}{2m_Q} \sum_j c_j\, \bar h'\,\Gamma_j^\alpha\, iD_\alpha\, h + \frac{1}{2m_{Q'}} \sum_j c_j'\, \bar h'\,(-i\overleftarrow{D}_\alpha)\,\Gamma_j'^\alpha\, h + \cdots\,,\end{aligned} \tag{3.1}$$

where we have abbreviated $h = h_Q(v)$ and $h' = h_{Q'}(v')$. The matrices Γ_j are in general different from Γ and can depend on v and v'. At tree level, however, one has

$$\sum_j C_j\,\Gamma_j \to \Gamma\,, \qquad \sum_j c_j\,\Gamma_j^\alpha \to \Gamma\,\gamma^\alpha\,, \qquad \sum_j c_j'\,\Gamma_j'^\alpha \to \gamma^\alpha\Gamma\,. \tag{3.2}$$

Using the trace formalism described in Sec. II, matrix elements of the leading term in (3.1) can be parameterized as

$$\langle M'|\,\bar h'\,\Gamma\,h\,|M\rangle = -\xi(w,\mu)\,\mathrm{tr}\{\,\overline{\mathcal{M}}'\,\Gamma\,\mathcal{M}\,\}\,, \tag{3.3}$$

where we omit the velocity argument in the states and the wave functions in order to simplify notation. It is to be understood that quantities without a prime refer to the initial state meson M, while primed quantities refer to the final state meson M'. Also, from now on m will designate a generic heavy quark mass. In general the form factor ξ could be some matrix-valued function of v and v', but in this case the projection operators contained in the spin wave functions restrict it to a scalar function of w. Eq. (3.3) implies that, to leading order in the heavy quark expansion, all matrix elements of currents between pseudoscalar or vector mesons are described by a single form factor, the Isgur-Wise function [2,3,29]. The kinematical information is contained in the trace over spin wave functions. By evaluating the special case of mesons with equal mass and velocity, one readily derives the zero recoil normalization condition $\xi(1,\mu) = 1$ as a consequence of the conservation of the vector current.

At subleading order in (3.1) one encounters current operators which contain a covariant derivative. Their matrix elements are represented by the diagrams shown in Fig. 1(a) and can be parameterized as

$$\begin{aligned} \langle M'|\,\bar h'\,\Gamma^\alpha\,iD_\alpha\,h\,|M\rangle &= -\mathrm{tr}\{\,\xi_\alpha(v,v',\mu)\,\overline{\mathcal{M}}'\,\Gamma^\alpha\mathcal{M}\,\}\,, \\ \langle M'|\,\bar h'\,(-i\overleftarrow{D}_\alpha)\,\Gamma'^\alpha h\,|M\rangle &= -\mathrm{tr}\{\,\bar\xi_\alpha(v',v,\mu)\,\overline{\mathcal{M}}'\,\Gamma'^\alpha\mathcal{M}\,\}\,. \end{aligned} \tag{3.4}$$

Note the interchange of the velocities in the second matrix element. The most general decomposition of the universal form factor ξ_α involves three scalar functions. Following Ref. [15], we define

$$\xi_\alpha(v,v',\mu) = \xi_+(w,\mu)\,(v+v')_\alpha + \xi_-(w,\mu)\,(v-v')_\alpha - \xi_3(w,\mu)\,\gamma_\alpha\,. \tag{3.5}$$

T-invariance of the strong interactions requires that these scalar functions be real. Using (2.7) and the fact that $i\partial_\alpha(\bar h'\,\Gamma\,h) = \bar h'\,i\overleftarrow{D}_\alpha\Gamma\,h + \bar h'\,\Gamma\,iD_\alpha\,h$, one finds that

$$\xi_-(w,\mu) = \frac{\bar\Lambda}{2}\,\xi(w,\mu)\,. \tag{3.6}$$

This is where the parameter $\bar\Lambda$ enters the analysis.

The equation of motion, $iv\!\cdot\!D\,h = 0$, yields an additional relation among the scalar form factors. Taking into account that under the trace ξ_α is sandwiched between projection operators, one obtains

$$P_-\,v^\alpha\xi_\alpha(v,v',\mu)\,P_-' = 0\,. \tag{3.7}$$

For the remainder of this paper we use the symbol "$\hat{=}$" for relations such as this, which are true when sandwiched between the projection operators provided by the meson wave functions. We thus write $v^\alpha \xi_\alpha(v, v', \mu) \hat{=} 0$. In terms of the scalar functions this is equivalent to

$$(w+1)\,\xi_+(w,\mu) - (w-1)\,\xi_-(w,\mu) + \xi_3(w,\mu) = 0\,. \tag{3.8}$$

We shall use this equation to eliminate ξ_+. In particular, it follows that at zero recoil $2\,\xi_+(1,\mu) + \xi_3(1,\mu) = 0$. This relation has an interesting consequence, since it implies that

$$\xi_\alpha(v,v,\mu) \hat{=} [2\,\xi_+(1,\mu) + \xi_3(1,\mu)]\,v_\alpha = 0\,, \tag{3.9}$$

showing that matrix elements of the higher dimension currents in (3.1) vanish at zero recoil. This is the first part of Luke's theorem [15]. In its above form it is obvious that this result is true to all orders in perturbation theory [32], since it does not rely on the structure of the perturbative coefficients in (3.1).

A second class of $1/m$ corrections comes from the presence of higher dimension operators in the effective Lagrangian. Insertions of operators of $\mathcal{L}_1$ in (2.3) into matrix elements of the leading order currents represent corrections to the wave functions, which appear since the eigenstates of $\mathcal{L}_{\rm HQET}$ are different from the eigenstates of the full theory. The corresponding diagrams are shown in Fig. 1(b). The relevant matrix elements can be written as

$$\begin{aligned}
\langle M'|\,i\!\int\!\mathrm{d}x\,T\{\,J(0),\mathcal{L}_1(x)\,\}\,|M\rangle &= -A_1(w,\mu)\,\mathrm{tr}\{\,\overline{\mathcal{M}}'\,\Gamma\,\mathcal{M}\,\} \\
&\quad - Z(m_Q/\mu)\,\mathrm{tr}\{\,A_{\alpha\beta}(v,v',\mu)\,\overline{\mathcal{M}}'\,\Gamma\,P_+\,s^{\alpha\beta}\mathcal{M}\,\}\,, \\
\langle M'|\,i\!\int\!\mathrm{d}x\,T\{\,J(0),\mathcal{L}'_1(x)\,\}\,|M\rangle &= -A_1(w,\mu)\,\mathrm{tr}\{\,\overline{\mathcal{M}}'\,\Gamma\,\mathcal{M}\,\} \\
&\quad - Z(m_{Q'}/\mu)\,\mathrm{tr}\{\,\overline{A}_{\alpha\beta}(v',v,\mu)\,\overline{\mathcal{M}}'\,s^{\alpha\beta}P'_+\,\Gamma\,\mathcal{M}\,\}\,.
\end{aligned} \tag{3.10}$$

where $J = \bar{h}'\,\Gamma\,h$ is a lowest order current. Noting that $v_\alpha P_+ s^{\alpha\beta}\mathcal{M} = 0$, we write the decomposition[1]

$$A_{\alpha\beta}(v,v',\mu) = A_2(w,\mu)\,(v'_\alpha\gamma_\beta - v'_\beta\gamma_\alpha) + A_3(w,\mu)\,i\sigma_{\alpha\beta}\,. \tag{3.11}$$

The four independent functions ξ_3 and A_i, as well as the mass parameter $\bar{\Lambda}$, suffice to describe the first order power corrections to any matrix element of a heavy quark current between ground state mesons. To get a picture of the structure of the

[1]Our functions are related to those defined in Ref. [15] by $A_1 = 2\chi_1, A_2 = -2\chi_2$ and $A_3 = 4\chi_3$.

corrections let us for simplicity neglect radiative corrections. In this case, there is a simple relation between the currents in HQET and the current in the full theory. Consider now the power corrections proportional to $1/m_Q$. They leave the wave function of the final state meson unaffected, but change the simple structure of $\mathcal{M}(v)$. The part proportional to the on-shell projection operator P_+ will be modified, and a component proportional to P_- will be induced, representing the "small component" of the full wave function. Hence

$$\mathcal{M}(v) \to P_+(v)\, L_+^M(v,v') + P_-(v)\, L_-^M(v,v')\,. \tag{3.12}$$

The general form of $L_\pm^M$ is

$$\begin{aligned}
L_+^P(v,v') &= \sqrt{M}\,(-\gamma_5)\, L_1(w)\,,\\
L_+^V(v,v') &= \sqrt{M}\,\Big[\not\epsilon\, L_2(w) + \epsilon\!\cdot\! v'\, L_3(w)\Big]\,,\\
L_-^P(v,v') &= \sqrt{M}\,(-\gamma_5)\, L_4(w)\,,\\
L_-^V(v,v') &= \sqrt{M}\,\Big[\not\epsilon\, L_5(w) + \epsilon\!\cdot\! v'\, L_6(w)\Big]\,.
\end{aligned} \tag{3.13}$$

The insertions of higher order terms from the effective Lagrangian in (3.10) obviously contribute to L_+^M only. On the other hand, in the absence of radiative corrections the matrix elements of the higher dimension currents can be written in the form

$$\langle M'|\,\bar h'\,\Gamma\, i\not{D}\, h\,|M\rangle = -\mathrm{tr}\{\,\overline{\mathcal{M}}'\,\Gamma\, P_-[\gamma_\alpha\, \mathcal{M}\, \xi^\alpha(v,v')]\,\}\,, \tag{3.14}$$

where we have used (3.7) to insert P_- between Γ and γ_α. Consequently, these corrections contribute to L_-^M only. This is expected since $i\not{D}\,h$ is proportional to the small component of the full heavy quark spinor. By evaluating the relevant traces one easily obtains

$$\begin{aligned}
L_1 &= A_1 + 2(w-1)A_2 + 3A_3\,,\\
L_2 &= A_1 - A_3\,,\\
L_3 &= -2A_2\,,\\[1em]
L_4 &= -\bar\Lambda\,\xi + 2\,\xi_3\,,\\
L_5 &= -\bar\Lambda\,\xi\,,\\
L_6 &= -\frac{2}{w+1}\,(\bar\Lambda\,\xi + \xi_3)\,,
\end{aligned} \tag{3.15}$$

and the complete matrix element becomes

$$\begin{aligned}
\langle M'|\,\bar Q'\,\Gamma\, Q\,|M\rangle &= -\xi(w)\,\mathrm{tr}\{\,\overline{\mathcal{M}}'\,\Gamma\,\mathcal{M}\,\}\\
&\quad - \frac{1}{2m_Q}\,\mathrm{tr}\Big\{\,\overline{\mathcal{M}}'\,\Gamma\,\Big[P_+\, L_+^M(v,v') + P_-\, L_-^M(v,v')\Big]\Big\}\\
&\quad - \frac{1}{2m_{Q'}}\,\mathrm{tr}\Big\{\,\Big[\overline{L}_+^{M'}(v',v)\, P_+' + \overline{L}_-^{M'}(v',v)\, P_-'\Big]\,\Gamma\,\mathcal{M}\,\Big\} + \cdots\,.
\end{aligned} \tag{3.16}$$

This way of organizing the corrections reduces to a minimum the effort required to compute the traces. If radiative corrections are taken into account, it still suffices to define these six functions L_i, as long as one stays with the leading logarithmic approximation for the perturbative coefficients. The corresponding expressions are given in Ref. [25], where the functions L_i were called ϱ_i.

Let us evaluate (3.16) for the matrix elements of the vector and axial vector currents, V_μ and A_μ, between bottom and charm mesons, which can be described completely in terms of fourteen meson form factors $h_i(w)$. We define

$$\begin{aligned}
\langle D(v')|\,V_\mu\,|B(v)\rangle &= \sqrt{m_B m_D}\,\Big[h_+(w)\,(v+v')_\mu + h_-(w)\,(v-v')_\mu\Big]\,,\\
\langle D^*(v',\epsilon')|\,V_\mu\,|B(v)\rangle &= i\sqrt{m_B m_{D^*}}\;h_V(w)\,\epsilon_{\mu\nu\alpha\beta}\,\epsilon'^{*\nu}\,v'^\alpha\,v^\beta\,,\\
\langle D^*(v',\epsilon')|\,A_\mu\,|B(v)\rangle &= \sqrt{m_B m_{D^*}}\,\Big[h_{A_1}(w)\,(w+1)\,\epsilon'^*_\mu\\
&\qquad - h_{A_2}(w)\,\epsilon'^*\!\cdot v\,v_\mu - h_{A_3}(w)\,\epsilon'^*\!\cdot v\,v'_\mu\Big]\,,\\
\langle D^*(v',\epsilon')|\,V_\mu\,|B^*(v,\epsilon)\rangle &= \sqrt{m_{B^*} m_{D^*}}\,\Big\{-\epsilon\cdot\epsilon'^*\,\Big[h_1(w)\,(v+v')_\mu + h_2(w)\,(v-v')_\mu\Big]\\
&\qquad + h_3(w)\,\epsilon'^*\!\cdot v\,\epsilon_\mu + h_4(w)\,\epsilon\cdot v'\,\epsilon'^*_\mu\\
&\qquad - \epsilon\cdot v'\,\epsilon'^*\!\cdot v\,\Big[h_5(w)\,v_\mu + h_6(w)\,v'_\mu\Big]\Big\}\,,\\
\langle D^*(v',\epsilon')|\,A_\mu\,|B^*(v,\epsilon)\rangle &= i\sqrt{m_{B^*} m_{D^*}}\,\epsilon_{\mu\nu\alpha\beta}\,\epsilon^\alpha\,\epsilon'^{*\beta}\,\Big[h_7(w)\,(v+v')^\nu + h_8\,(v-v')^\nu\Big]\,.
\end{aligned} \tag{3.17}$$

At leading order in the heavy quark expansion one finds that

$$h_+ = h_V = h_{A_1} = h_{A_3} = h_1 = h_3 = h_4 = h_7 = \xi\,, \tag{3.18}$$

while the remaining six form factors vanish. The expressions arising at subleading order are given in Appendix B. Here we restrict ourselves to three important cases, namely

$$\begin{aligned}
h_+(w) &= \xi(w) + \left(\frac{1}{2m_c} + \frac{1}{2m_b}\right) L_1(w)\,,\\
h_1(w) &= \xi(w) + \left(\frac{1}{2m_c} + \frac{1}{2m_b}\right) L_2(w)\,,\\
h_{A_1}(w) &= \xi(w) + \frac{1}{2m_b}\left[L_1(w) - \frac{w-1}{w+1}\,L_4(w)\right] + \frac{1}{2m_c}\left[L_2(w) - \frac{w-1}{w+1}\,L_5(w)\right].
\end{aligned} \tag{3.19}$$

The conservation of the vector current in the limit $m_b = m_c$ implies the zero recoil normalization conditions $h_+(1) = h_1(1) = 1$, from which it follows that $L_1(1) = L_2(1) = 0$, *i.e.* [15]

$$A_1(1,\mu) = A_3(1,\mu) = 0\,. \tag{3.20}$$

This is the second part of Luke's theorem, which is again true to all orders in perturbation theory. It follows that $A_{\alpha\beta}(v,v,\mu)\hat{=}0$, so that the matrix elements in (3.10) vanish at zero recoil.

In summary, Luke's theorem implies that the *matrix elements* which describe the first order power corrections in HQET vanish at zero recoil. It is important to realize that this does not imply that the meson *form factors* are unaffected by $1/m$ corrections [33]. In fact, the theorem only applies for form factors which are not kinematically suppressed as $v' \to v$. Besides h_+ and h_1 those are h_{A_1} and h_7. The form factor h_{A_1}, which according to (3.19) is indeed seen to be unaffected by first order power corrections at zero recoil, plays an important role in the determination of V_{cb} from semileptonic decays [26]. It is one of the purposes of the next section to investigate the second order corrections to this form factor.

IV. SECOND ORDER POWER CORRECTIONS

The analysis of higher order corrections in HQET makes use of the same techniques as those developed above. The second order corrections can be divided into three classes: corrections to the current, corrections to the effective Lagrangian, and mixed corrections. We shall discuss each of them separately below. In order to keep the presentation as simple as possible we will often ignore radiative corrections; however, we will always make clear how they could be incorporated into our analysis.

A. Second Order Corrections to the Current

At tree level, the expansion of the heavy quark current reads [cf. (3.1)]

$$\begin{aligned}\bar{Q}'\,\Gamma\,Q \to \bar{h}'\,\Gamma\,h &+ \frac{1}{2m_Q}\,\bar{h}'\,\Gamma\,i\not{D}\,h + \frac{1}{2m_{Q'}}\,\bar{h}'\,(-i\overleftarrow{\not{D}}\,)\,\Gamma\,h\\ &+\frac{1}{4m_Q^2}\,\bar{h}'\,\Gamma\,\gamma_\alpha v_\beta G^{\alpha\beta} h - \frac{1}{4m_{Q'}^2}\,\bar{h}'\,\gamma_\alpha v'_\beta G^{\alpha\beta}\Gamma\,h\\ &+\frac{1}{4m_Q m_{Q'}}\,\bar{h}'\,(-i\overleftarrow{\not{D}}\,)\,\Gamma\,i\not{D}\,h + \cdots\,.\end{aligned} \tag{4.1}$$

On dimensional grounds, the operators appearing at second order are bilinear in the covariant derivative (recall that $G^{\alpha\beta} = [iD^\alpha, iD^\beta]$). This remains true in the presence of radiative corrections, although a number of additional operators are induced. It thus suffices to consider the single hadronic matrix element

$$\langle M'|\,\bar{h}'\,(-i\overleftarrow{D}_\alpha)\,\Gamma^{\alpha\beta}\,iD_\beta\,h\,|M\rangle = -\mathrm{tr}\{\,\psi_{\alpha\beta}(v,v',\mu)\,\overline{\mathcal{M}}'\,\Gamma^{\alpha\beta}\mathcal{M}\,\}\,, \tag{4.2}$$

represented by the third diagram in Fig. 2(a). From now on we shall always omit the μ-dependence of the universal form factors except in the equations which define them. Considering the complex conjugate of the above matrix element, one finds that the form factor must obey the symmetry relation

$$\overline{\psi}_{\beta\alpha}(v',v) = \psi_{\alpha\beta}(v,v')\,, \tag{4.3}$$

which reduces the number of invariant functions to seven. It is convenient to perform a decomposition into symmetric and antisymmetric parts, $\psi_{\alpha\beta} = \frac{1}{2}[\psi^S_{\alpha\beta} + \psi^A_{\alpha\beta}]$, and to define

$$\begin{aligned}
\psi^S_{\alpha\beta}(v,v') &= \psi^S_1(w)\,g_{\alpha\beta} + \psi^S_2(w)\,(v+v')_\alpha(v+v')_\beta + \psi^S_3(w)\,(v-v')_\alpha(v-v')_\beta \\
&\quad + \psi^S_4(w)\,\Big[(v+v')_\alpha\gamma_\beta + (v+v')_\beta\gamma_\alpha\Big]\,, \\
\psi^A_{\alpha\beta}(v,v') &= \psi^A_1(w)\,(v_\alpha v'_\beta - v'_\alpha v_\beta) + \psi^A_2(w)\,\Big[(v-v')_\alpha\gamma_\beta - (v-v')_\beta\gamma_\alpha\Big] \\
&\quad + i\psi^A_3(w)\,\sigma_{\alpha\beta}\,.
\end{aligned} \tag{4.4}$$

As in (3.7) one can use the equation of motion to derive relations among the scalar form factors. It follows that under the trace

$$v^\beta\psi_{\alpha\beta}(v,v') \mathrel{\hat{=}} 0, \qquad v'^\alpha\psi_{\alpha\beta}(v,v') \mathrel{\hat{=}} 0\,. \tag{4.5}$$

These conditions are equivalent because of (4.3) and lead to the three relations

$$\begin{aligned}
\psi^S_1 + (w+1)\,\psi^S_2 - (w-1)\,\psi^S_3 - \psi^S_4 + w\,\psi^A_1 - \psi^A_2 - \psi^A_3 &= 0\,, \\
(w+1)\,\psi^S_2 + (w-1)\,\psi^S_3 - \psi^S_4 - \psi^A_1 + \psi^A_2 &= 0\,, \\
(w+1)\,\psi^S_4 + (w-1)\,\psi^A_2 - \psi^A_3 &= 0\,,
\end{aligned} \tag{4.6}$$

which reduce the number of independent functions to four.

One can use an integration by parts to relate (4.2) to matrix elements of operators containing two derivatives acting on the same heavy quark field, which are represented by the first two diagrams in Fig. 2(a). It follows that

$$\begin{aligned}
\langle M'|\,\bar h'\,\Gamma^{\alpha\beta}\,iD_\alpha iD_\beta\,h\,|M\rangle &= -\mathrm{tr}\{\,\psi_{\alpha\beta}(v,v')\,\overline{\mathcal{M}}'\,\Gamma^{\alpha\beta}\mathcal{M}\,\} \\
&\quad -\bar\Lambda\,(v-v')_\alpha\,\mathrm{tr}\{\,\xi_\beta(v,v')\,\overline{\mathcal{M}}'\,\Gamma^{\alpha\beta}\mathcal{M}\,\}
\end{aligned} \tag{4.7}$$

with ξ_β as defined in (3.4). Matrix elements of operators with both derivatives acting to the left can be obtained in a similar way. In particular, we may derive from (4.7) the matrix elements

$$\begin{aligned}
\langle M'|\,\bar h'\,\Gamma\,(iD)^2 h\,|M\rangle &= -\phi_0(w)\,\mathrm{tr}\{\,\overline{\mathcal{M}}'\,\Gamma\,\mathcal{M}\,\}\,, \\
\langle M'|\,\bar h'\,\Gamma^{\alpha\beta}G_{\alpha\beta}\,h\,|M\rangle &= -\mathrm{tr}\{\,\phi_{\alpha\beta}(v,v')\,\overline{\mathcal{M}}'\,\Gamma^{\alpha\beta}\mathcal{M}\,\}\,,
\end{aligned} \tag{4.8}$$

the second of which is needed in (4.1). Choosing the same decomposition for $\phi_{\alpha\beta}$ as for $\psi^A_{\alpha\beta}$, we find

$$
\begin{aligned}
\phi_0 &= 2\psi_1^S + (w+1)\,\psi_2^S - (w-1)\,\psi_3^S - 2\psi_4^S - \bar\Lambda^2(w-1)\,\xi\,,\\
\phi_1 &= \psi_1^A + \tfrac{1}{w+1}[\bar\Lambda^2(w-1)\,\xi - 2\bar\Lambda\,\xi_3]\,,\\
\phi_2 &= \psi_2^A - \bar\Lambda\,\xi_3\,,\\
\phi_3 &= \psi_3^A\,.
\end{aligned}
\tag{4.9}
$$

We have already encountered matrix elements similar to (4.8) in the discussion of mass shifts in Sec. II, and from a comparison with (2.12) we find the zero recoil conditions

$$
\begin{aligned}
\phi_0(1) &= \lambda_1\,,\\
\phi_3(1) &= \lambda_2\,,\\
\phi_1(1) - \phi_2(1) &= -\tfrac{1}{3}\,\lambda_1 + \tfrac{1}{2}\,\lambda_2\,,
\end{aligned}
\tag{4.10}
$$

the last one being a consequence of the relations (4.6), which allow us also to express the form factors ψ_i^S in terms of the functions ϕ_i. After some algebra we find

$$
\begin{aligned}
\psi_1^S &= \phi_0 + w\,\phi_1 - \frac{w}{w+1}\,(2\phi_2 + \phi_3) + \left(\frac{w-1}{w+1}\right)\bar\Lambda^2\,\xi\,,\\
\psi_2^S &= -\frac{1}{2(w+1)}\,[\phi_0 + (2w-1)\,\phi_1]\\
&\quad + \frac{1}{2(w+1)^2}\,\Big[2\phi_2 + (2w+3)\,\phi_3 - (2-w)(w-1)\,\bar\Lambda^2\,\xi - 4(w-1)\,\bar\Lambda\,\xi_3\Big]\,,\\
\psi_3^S &= \frac{1}{2}\,(\hat\phi + \phi_1) - \frac{1}{4(w+1)}\,\Big[2\phi_2 + \phi_3 + 2w\,\bar\Lambda^2\,\xi\Big]\,,\\
\psi_4^S &= \frac{1}{w+1}\,\Big[-(w-1)\,\phi_2 + \phi_3 - (w-1)\,\bar\Lambda\,\xi_3\Big]\,.
\end{aligned}
\tag{4.11}
$$

We have introduced the function

$$
\hat\phi(w) = \frac{1}{w-1}\,\Big[\phi_0(w) + (w+2)\,\phi_1(w) - 3\,\phi_2(w) - \tfrac{3}{2}\,\phi_3(w)\Big]\,, \tag{4.12}
$$

which is nonsingular as $w \to 1$ because of (4.10).

The above relations allow us to prove a theorem which is the analog of the first part of Luke's theorem:

> *Theorem 1:* At zero recoil, matrix elements of second order currents in the heavy quark expansion can be expressed in terms of λ_1 and λ_2.

For the proof we note that, to all orders in perturbation theory, the relevant operators contain two covariant derivatives. Because of (4.2) and (4.7) the corresponding matrix elements at zero recoil only involve $\psi_{\alpha\beta}(v,v)$ sandwiched between projection operators. Using (4.10) we find that

$$\psi_{\alpha\beta}(v,v) \mathrel{\hat{=}} [g_{\alpha\beta} - v_\alpha v_\beta]\,\frac{\lambda_1}{3} + i\sigma_{\alpha\beta}\,\frac{\lambda_2}{2}\,, \tag{4.13}$$

which proves the theorem. Furthermore, we note that at tree level only the last term in (4.1) contributes at zero recoil, since $v^\beta \phi_{\alpha\beta}(v,v) \mathrel{\hat{=}} 0$. The corresponding corrections are of order $\lambda_i / m_Q m_{Q'}$.

B. Second Order Corrections to the Lagrangian

Apart from operators whose matrix elements vanish by the equation of motion, the most general form of the coefficient $\mathcal{L}_2$ appearing at second order in the expansion of the effective Lagrangian in (2.3) contains two terms,

$$\mathcal{L}_2 = Z_1(m_Q/\mu)\,\bar{h}\,v_\beta\,iD_\alpha G^{\alpha\beta}h + 2Z_2(m_Q/\mu)\,\bar{h}\,s_{\alpha\beta}\,v_\gamma\,iD^\alpha G^{\beta\gamma}h\,. \tag{4.14}$$

In leading logarithmic approximation the renormalization factors are given by [34]

$$\begin{aligned} Z_1(m_Q/\mu) &= \frac{55}{9} - \frac{46}{9}\left[\frac{\alpha_s(m_Q)}{\alpha_s(\mu)}\right]^{6/\beta}, \\ Z_2(m_Q/\mu) &= \frac{19}{9} - \frac{10}{9}\left[\frac{\alpha_s(m_Q)}{\alpha_s(\mu)}\right]^{9/\beta}. \end{aligned} \tag{4.15}$$

These operators have the same Dirac structure as the operators in $\mathcal{L}_1$ in (2.4), and consequently their matrix elements are of the same form as those of $\mathcal{L}_1$. In analogy to (3.10) we thus define

$$\begin{aligned} \langle M'|\, i\!\int\! \mathrm{d}x\, T\{\, J(0), \mathcal{L}_2(x)\,\}\, |M\rangle &= -Z_1\, B_1(w,\mu)\,\mathrm{tr}\{\,\overline{\mathcal{M}}'\,\Gamma\,\mathcal{M}\,\} \\ &\quad - Z_2\,\mathrm{tr}\{\, B_{\alpha\beta}(v,v',\mu)\,\overline{\mathcal{M}}'\,\Gamma\,P_+\,s^{\alpha\beta}\mathcal{M}\,\}\,, \end{aligned} \tag{4.16}$$

and similarly for an insertion of $\mathcal{L}_2'$. As before, $J = \bar{h}'\,\Gamma\,h$ denotes a lowest order current. The corresponding diagrams are the first two shown in Fig. 2(b). The decomposition of $B_{\alpha\beta}$ is of the same form as that for $A_{\alpha\beta}$ in (3.11). It involves two functions B_2 and B_3.

Another type of $1/m^2$ corrections comes from a double insertion of the first order correction $\mathcal{L}_1$, as shown in the third and fourth diagrams in Fig. 2(b). The corresponding matrix elements have a more complicated structure. We define

$$\begin{aligned} &\langle M'|\, \tfrac{i^2}{2}\!\int\! \mathrm{d}x\mathrm{d}y\; T\,\{\, J(0), \mathcal{L}_1(x), \mathcal{L}_1(y)\,\}\, |M\rangle \\ &\qquad = -C_1(w,\mu)\,\mathrm{tr}\{\,\overline{\mathcal{M}}'\,\Gamma\,\mathcal{M}\,\} - Z\,\mathrm{tr}\{\, C_{\alpha\beta}(v,v',\mu)\,\overline{\mathcal{M}}'\,\Gamma\,P_+\,s^{\alpha\beta}\mathcal{M}\,\} \\ &\qquad\quad - Z^2\,\mathrm{tr}\{\, C_{\alpha\beta\gamma\delta}(v,v',\mu)\,\overline{\mathcal{M}}'\,\Gamma\,P_+\,s^{\alpha\beta}P_+\,s^{\gamma\delta}\mathcal{M}\,\}\,. \end{aligned} \tag{4.17}$$

Again, the corresponding matrix elements with two insertions of $\mathcal{L}_1'$ can be obtained by conjugating the matrix elements as in (3.10). The decomposition of $C_{\alpha\beta}$ is the same as that for $A_{\alpha\beta}$, involving two form factors C_2 and C_3. The most general decomposition of the four-index object $C_{\alpha\beta\gamma\delta}$ involves nine invariant functions, C_4 to C_{12}. They can be defined by

$$\begin{aligned}
C_{\alpha\beta\gamma\delta}(v,v') = {} & C_4(w)\,(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}) + C_5(w)\,\sigma_{\gamma\delta}\sigma_{\alpha\beta} \\
& + C_6(w)\,(g_{\alpha\gamma}i\sigma_{\beta\delta} - g_{\beta\gamma}i\sigma_{\alpha\delta} - g_{\alpha\delta}i\sigma_{\beta\gamma} + g_{\beta\delta}i\sigma_{\alpha\gamma}) \\
& + C_7(w)\,(v'_\gamma\gamma_\delta - v'_\delta\gamma_\gamma)(v'_\alpha\gamma_\beta - v'_\beta\gamma_\alpha) \\
& + C_8(w)\,(g_{\alpha\gamma}v'_\beta v'_\delta - g_{\beta\gamma}v'_\alpha v'_\delta - g_{\alpha\delta}v'_\beta v'_\gamma + g_{\beta\delta}v'_\alpha v'_\gamma) \\
& + C_9(w)\,(g_{\alpha\gamma}v'_\beta\gamma_\delta - g_{\beta\gamma}v'_\alpha\gamma_\delta - g_{\alpha\delta}v'_\beta\gamma_\gamma + g_{\beta\delta}v'_\alpha\gamma_\gamma) \\
& + C_{10}(w)\,(g_{\alpha\gamma}\gamma_\beta v'_\delta - g_{\beta\gamma}\gamma_\alpha v'_\delta - g_{\alpha\delta}\gamma_\beta v'_\gamma + g_{\beta\delta}\gamma_\alpha v'_\gamma) \\
& + C_{11}(w)\,(i\sigma_{\alpha\gamma}v'_\beta\gamma_\delta - i\sigma_{\beta\gamma}v'_\alpha\gamma_\delta - i\sigma_{\alpha\delta}v'_\beta\gamma_\gamma + i\sigma_{\beta\delta}v'_\alpha\gamma_\gamma) \\
& + C_{12}(w)\,(i\sigma_{\alpha\gamma}\gamma_\beta v'_\delta - i\sigma_{\beta\gamma}\gamma_\alpha v'_\delta - i\sigma_{\alpha\delta}\gamma_\beta v'_\gamma + i\sigma_{\beta\delta}\gamma_\alpha v'_\gamma)\,.
\end{aligned} \tag{4.18}$$

Finally, there are corrections resulting from insertions of both $\mathcal{L}_1$ and $\mathcal{L}'_1$, as shown in the last diagram in Fig. 2(b). They have the form

$$\begin{aligned}
\langle M'|\, i^2 \!\int & \mathrm{d}x\mathrm{d}y\ T\,\{\,J(0), \mathcal{L}_1(x), \mathcal{L}'_1(y)\,\}\,|M\rangle \\
& = -D_1(w,\mu)\,\mathrm{tr}\{\,\overline{\mathcal{M}}'\,\Gamma\,\mathcal{M}\,\} \\
& \quad -\frac{Z}{2}\,\mathrm{tr}\{\,D_{\alpha\beta}(v,v',\mu)\,\overline{\mathcal{M}}'\,\Gamma\,P_+\,s^{\alpha\beta}\mathcal{M}\,\} \\
& \quad -\frac{Z'}{2}\,\mathrm{tr}\{\,\overline{D}_{\alpha\beta}(v',v,\mu)\,\overline{\mathcal{M}}'\,s^{\alpha\beta}P'_+\,\Gamma\,\mathcal{M}\,\} \\
& \quad -ZZ'\,\mathrm{tr}\{\,D_{\alpha\beta\gamma\delta}(v,v',\mu)\,\overline{\mathcal{M}}'\,s^{\alpha\beta}P'_+\,\Gamma\,P_+\,s^{\gamma\delta}\mathcal{M}\,\}\,.
\end{aligned} \tag{4.19}$$

The form factor $D_{\alpha\beta}$ is again of the same form as $A_{\alpha\beta}$ and involves two functions, D_2 and D_3. The most general decomposition of the four-index object $D_{\alpha\beta\gamma\delta}$ is similar to that of $C_{\alpha\beta\gamma\delta}$. However, because of the symmetry of the matrix element (4.19) this quantity has to obey the constraint

$$D_{\alpha\beta\gamma\delta}(v,v') = \overline{D}_{\gamma\delta\alpha\beta}(v',v), \tag{4.20}$$

which allows only seven independent functions, D_4 to D_{10}. We choose the decomposition

$$\begin{aligned}
D_{\alpha\beta\gamma\delta}(v,v') = {} & D_4(w)\,(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}) + D_5(w)\,\sigma_{\gamma\delta}\sigma_{\alpha\beta} \\
& + D_6(w)\,(g_{\alpha\gamma}i\sigma_{\beta\delta} - g_{\beta\gamma}i\sigma_{\alpha\delta} - g_{\alpha\delta}i\sigma_{\beta\gamma} + g_{\beta\delta}i\sigma_{\alpha\gamma}) \\
& + D_7(w)\,(v'_\gamma\gamma_\delta - v'_\delta\gamma_\gamma)(v_\alpha\gamma_\beta - v_\beta\gamma_\alpha) \\
& + D_8(w)\,(g_{\alpha\gamma}v_\beta v'_\delta - g_{\beta\gamma}v_\alpha v'_\delta - g_{\alpha\delta}v_\beta v'_\gamma + g_{\beta\delta}v_\alpha v'_\gamma) \\
& + D_9(w)\,\Big[g_{\alpha\gamma}v_\beta\gamma_\delta - g_{\beta\gamma}v_\alpha\gamma_\delta - g_{\alpha\delta}v_\beta\gamma_\gamma + g_{\beta\delta}v_\alpha\gamma_\gamma \\
& \qquad + g_{\alpha\gamma}\gamma_\beta v'_\delta - g_{\beta\gamma}\gamma_\alpha v'_\delta - g_{\alpha\delta}\gamma_\beta v'_\gamma + g_{\beta\delta}\gamma_\alpha v'_\gamma\Big]
\end{aligned}$$

$$+ D_{10}(w) \Big[v_\beta \gamma_\delta i\sigma_{\alpha\gamma} - v_\alpha \gamma_\delta i\sigma_{\beta\gamma} - v_\beta \gamma_\gamma i\sigma_{\alpha\delta} + v_\alpha \gamma_\gamma i\sigma_{\beta\delta} + i\sigma_{\alpha\gamma}\gamma_\beta v'_\delta - i\sigma_{\beta\gamma}\gamma_\alpha v'_\delta - i\sigma_{\alpha\delta}\gamma_\beta v'_\gamma + i\sigma_{\beta\delta}\gamma_\alpha v'_\gamma \Big] . \quad (4.21)$$

In total, twenty-five universal functions B_i, C_i and D_i are necessary to parameterize the effects of second order corrections to the effective Lagrangian of HQET. Unlike the corrections to the current, there are no relations imposed on these form factors by the equation of motion.

Before proceeding, we have to discuss an additional source of second order corrections, which is related to the ones encountered above. As discussed in Sec. II, the mass M in the wave functions that we associate with the eigenstates of $\mathcal{L}_{\rm HQET}$ is different from the physical mass m_M. It is the physical mass, however, that appears in the normalization of matrix elements of the vector current, which one uses to derive zero recoil conditions for some of the universal form factors. At second order in the heavy quark expansion one has to take into account this difference and perform a mass renormalization of the wave function,

$$\mathcal{M}(v) \to Z_M^{1/2}\, \mathcal{M}(v), \qquad Z_M^{1/2} = \sqrt{\frac{m_M}{M}} \qquad (4.22)$$

in the first term in (3.16). This is compensated by a counterterm

$$-\Big[1 - Z_M^{1/2} Z_{M'}^{1/2}\Big]\, \xi(w)\, {\rm tr}\{\, \overline{\mathcal{M}}'\, \Gamma\, \mathcal{M}\,\} = \left(\frac{\Delta m_M^2}{4m_Q^2} + \frac{\Delta m_{M'}^2}{4m_{Q'}^2} \right) \xi(w)\, {\rm tr}\{\, \overline{\mathcal{M}}'\, \Gamma\, \mathcal{M}\,\} , \qquad (4.23)$$

which, according to (2.13), effectively adds $\lambda_1\,\xi$ to C_1 and $\lambda_2\,\xi$ to C_3.

C. Combined Corrections to the Current and the Lagrangian

The third and last type of $1/m^2$ corrections arises from the combination of first order corrections both to the current and to the Lagrangian, as shown in Fig. 2(c). The relevant structures are

$$\begin{aligned} \langle M'|\, i\!\int\! {\rm d}x\, T\{\, \bar h'\, \Gamma^\gamma\, iD_\gamma\, h, \mathcal{L}_1(x)\,\}\, |M\rangle &= -{\rm tr}\{\, E_\gamma(v,v',\mu)\, \overline{\mathcal{M}}'\, \Gamma^\gamma \mathcal{M}\,\} \\ &\quad - Z\, {\rm tr}\{\, E_{\gamma\alpha\beta}(v,v',\mu)\, \overline{\mathcal{M}}'\, \Gamma^\gamma P_+\, s^{\alpha\beta} \mathcal{M}\,\} , \end{aligned} \qquad (4.24)$$

$$\begin{aligned} \langle M'|\, i\!\int\! {\rm d}x\, T\{\, \bar h'\, (-i\overleftarrow{D}_\gamma)\, \Gamma\, h, \mathcal{L}_1(x)\,\}\, |M\rangle &= -{\rm tr}\{\, E'_\gamma(v,v',\mu)\, \overline{\mathcal{M}}'\, \Gamma^\gamma \mathcal{M}\,\} \\ &\quad - Z\, {\rm tr}\{\, E'_{\gamma\alpha\beta}(v,v',\mu)\, \overline{\mathcal{M}}'\, \Gamma^\gamma P_+\, s^{\alpha\beta} \mathcal{M}\,\} . \end{aligned}$$

As previously, insertions of $\mathcal{L}_1'$ give rise to the conjugate matrix elements. The form factor E_γ may be parameterized as

$$E_\gamma(v,v') = E_1(w)\,v_\gamma + E_2(w)\,v'_\gamma + E_3(w)\,\gamma_\gamma\,. \tag{4.25}$$

The most general decomposition of $E_{\gamma\alpha\beta}$ involves eight functions, which we define by

$$\begin{aligned} E_{\gamma\alpha\beta}(v,v') = & (v'_\alpha\gamma_\beta - v'_\beta\gamma_\alpha)\Big[E_4(w)\,v_\gamma + E_5(w)\,v'_\gamma + E_6(w)\,\gamma_\gamma\Big] \\ & + i\sigma_{\alpha\beta}\Big[E_7(w)\,v_\gamma + E_8(w)\,v'_\gamma + E_9(w)\,\gamma_\gamma\Big] \\ & + \Big\{g_{\alpha\gamma}\Big[E_{10}(w)\,v'_\beta + E_{11}(w)\,\gamma_\beta\Big] - (\alpha\leftrightarrow\beta)\Big\}\,. \end{aligned} \tag{4.26}$$

The equation of motion implies $v^\gamma E_\gamma \hat{=} 0$ and $v^\gamma E_{\gamma\alpha\beta} \hat{=} 0$, which is equivalent to

$$\begin{aligned} E_1 + w\,E_2 - E_3 &= 0\,, \\ E_4 + w\,E_5 + E_6 &= 0\,, \\ E_7 + w\,E_8 - E_9 &= 0\,. \end{aligned} \tag{4.27}$$

Here we have used the fact that $v_\alpha P_+ s^{\alpha\beta}\mathcal{M} = 0$.

We define form factors $E'_i(w)$ by identical decompositions. In this case, the equation of motion leads to the relations

$$\begin{aligned} w\,E'_1 + E'_2 - E'_3 &= 0\,, \\ w\,E'_4 + E'_5 - E'_6 + E'_{11} &= 0\,, \\ w\,E'_7 + E'_8 - E'_9 &= 0\,. \end{aligned} \tag{4.28}$$

These functions are not independent of E_i, however, the reason being that the matrix elements in (4.24) are related to each other by an integration by parts. This relation has its subtleties, since insertions of $\mathcal{L}_1$ renormalize the masses of the states in the effective theory and, therefore, modify the x-dependence of the "bare" states in (2.7). In addition, there is a contact term arising from the action of the derivative on the θ-functions in the time-ordered product. We discuss these issues in Appendix C. One finds that the differences $(E_i - E'_i)$ are in fact computable in terms of form factors introduced earlier. The relations are

$$\begin{aligned} E_\gamma - E'_\gamma &= \bar\Lambda\,(v-v')_\gamma\,A_1 + v_\gamma\,(\phi_0 - \lambda_1\,\xi)\,, \\ E_{\gamma\alpha\beta} - E'_{\gamma\alpha\beta} &= \bar\Lambda\,(v-v')_\gamma\,A_{\alpha\beta} - v_\gamma(v'_\alpha\gamma_\beta - v'_\beta\gamma_\alpha)\,\phi_2 \\ &\quad + i v_\gamma\sigma_{\alpha\beta}\,(\phi_3 - \lambda_2\,\xi)\,. \end{aligned} \tag{4.29}$$

In particular, it follows that $E'_i = E_i$ for $i = 3,6,9,10,11$, and we will choose these five functions as a basis. Then a convenient way of writing the solution of the constraints imposed by the equation of motion is

$$\begin{aligned} E_1 + w\,E_2 &= w\,E'_1 + E'_2 = E_3\,, \\ E_1 - E_2 &= \bar\Lambda A_1 + w\,\tilde\phi_0\,, \end{aligned}$$

$$E'_1 - E'_2 = -\bar\Lambda A_1 + \tilde\phi_0 \,,$$

$$\begin{aligned}
E_4 + w\,E_5 &= -E_6 \,, \\
w\,E'_4 + E'_5 &= E_6 - E_{11} \,, \\
E_4 + E_5 &= -\frac{1}{w+1} \Big[E_{11} + w\,\phi_2 - \bar\Lambda\,(w-1) A_2 \Big] \,, \\
E'_4 + E'_5 &= -\frac{1}{w+1} \Big[E_{11} - \phi_2 - \bar\Lambda\,(w-1) A_2 \Big] \,,
\end{aligned}$$

$$\begin{aligned}
E_7 + w\,E_8 &= w\,E'_7 + E'_8 = E_9 \,, \\
E_7 - E_8 &= \bar\Lambda A_3 + w\,\tilde\phi_3 \,, \\
E'_7 - E'_8 &= -\bar\Lambda A_3 + \tilde\phi_3 \,,
\end{aligned} \tag{4.30}$$

where we have introduced the nonsingular functions

$$\tilde\phi_0(w) = \frac{\phi_0(w) - \lambda_1\,\xi(w)}{w-1} \,, \qquad \tilde\phi_3(w) = \frac{\phi_3(w) - \lambda_2\,\xi(w)}{w-1} \,. \tag{4.31}$$

Consistency of the equations determining $E_{4,5}$ and $E'_{4,5}$ furthermore requires that, at zero recoil,

$$2E_6(1) - E_{11}(1) = \phi_2(1) \,. \tag{4.32}$$

The constraints imposed by the equation of motion allow us to prove a second theorem:

> *Theorem 2:* Matrix elements describing the mixed first order corrections to the current and to the Lagrangian vanish at zero recoil.

It follows from the fact that, under the traces,

$$\begin{aligned}
E_\gamma(v,v) &\hat= v_\gamma \Big[E_1(1) + E_2(1) - E_3(1) \Big] = 0 \,, \\
E_{\gamma\alpha\beta}(v,v) &\hat= i\sigma_{\alpha\beta}\, v_\gamma \Big[E_7(1) + E_8(1) - E_9(1) \Big] = 0 \,,
\end{aligned} \tag{4.33}$$

with identical relations for E'_i. Thus, at zero recoil only genuine second order corrections to the current or to the Lagrangian contribute to hadronic form factors that are not kinematically suppressed. The conservation of the vector current in the limit of equal masses then leads to relations between the universal functions which describe the corrections to the Lagrangian, and the parameters λ_1 and λ_2 which, according to Theorem 1, describe the corrections to the current. These normalization conditions are the subject of the following subsection.

D. Modified Wave Functions and Normalization Conditions at Zero Recoil

In this section we have shown that at second order in the heavy quark expansion a total of $4+25+5=34$ universal functions is necessary to parameterize, respectively, the effects of corrections to the current, of corrections to the effective Lagrangian, and of the combined corrections to both. The richness of the structures that arise might seem both impressive and frustrating, and the effort required to compute the various traces is quite considerable. However, only certain combinations of form factors appear in the final expression for any hadronic matrix element, and it is time to organize our results in a more transparent and convenient way by employing the concept of modified wave functions introduced in Sec. III. The corrections proportional to $1/m_Q^2$ change the wave function for the initial state meson, but leave the final state unaffected (and vice versa for the terms proportional to $1/m_{Q'}^2$). Their effects can therefore be accounted for as in (3.12). On the other hand, the corrections proportional to $1/m_Q m_{Q'}$ affect both mesons and can only be accounted for by a combined wave function. We can thus extend (3.16) to second order by writing

$$\begin{aligned}\langle M'|\,\bar Q'\,\Gamma\,Q\,|M\rangle = &-Z_M^{1/2} Z_{M'}^{1/2}\,\xi(w)\,\mathrm{tr}\{\,\overline{\mathcal{M}}'\,\Gamma\,\mathcal{M}\,\} \\ &-\frac{1}{2m_Q}\,\mathrm{tr}\Big\{\,\overline{\mathcal{M}}'\,\Gamma\,\Big[P_+\Big(L_+^M+\frac{1}{2m_Q}\,\ell_+^M\Big)+P_-\Big(L_-^M+\frac{1}{2m_Q}\,\ell_-^M\Big)\Big]\Big\} \\ &-\frac{1}{2m_{Q'}}\,\mathrm{tr}\Big\{\,\Big[\Big(\overline{L}_+^{M'}+\frac{1}{2m_{Q'}}\,\overline{\ell}_+^{M'}\Big)P_+'+\Big(\overline{L}_-^{M'}+\frac{1}{2m_{Q'}}\,\overline{\ell}_-^{M'}\Big)P_-'\Big]\,\Gamma\,\mathcal{M}\,\Big\} \\ &-\frac{1}{4m_Q m_{Q'}}\,\mathrm{tr}\Big\{\,\Gamma\,\Big[P_+\,m_{++}^{MM'}P_+' + P_-\,m_{--}^{MM'}P_-' \\ &\qquad + P_+\,m_{+-}^{MM'}P_-' + P_-\,m_{-+}^{MM'}P_+'\Big]\Big\} + \cdots\,,\end{aligned} \tag{4.34}$$

where we have performed the mass renormalization for the leading term. Here a "bar" denotes Dirac conjugation combined with an exchange of velocities, polarizations, and masses. The virtue of (4.34) is that it allows an interpretation in terms of large and small components, reducing to a minimum the effort required to perform the traces. The structure of ℓ^M is the same as that of $L_\pm^M$ in (3.13) and involves six functions $\ell_i(w)$. The structure of $m^{MM'}$ is more complicated and requires the introduction of twenty-four functions $m_i(w)$. They are defined in Appendix A.

Let us now discuss how the various second order corrections fit into this pattern. We start with the corrections to the Lagrangian, which according to (4.16) and (4.17) preserve the P_+ projectors for the initial and final state. Hence, the fifteen universal functions B_i and C_i contribute to ℓ_+^M only and appear in the three combinations ℓ_1,ℓ_2 and ℓ_3. Similarly, the functions D_i contribute to $m_{++}^{MM'}$ and enter in the combinations m_1 to m_7. For the discussion of the corrections to the current we restrict ourselves to the operators in (4.1), which are obtained from tree level matching of QCD and HQET. As explained in Sec. III, one can identify $i\not{D}\,h$ with the small component of the full heavy quark spinor, and those terms lead to P_- projectors in the modified wave functions. The last operator in (4.1) contains two such terms and consequently contributes to $m_{--}^{MM'}$ only. In fact, using the equation of motion its matrix elements can be written as

$$\langle M'|\,\bar h'\,(-i\overleftarrow{\not D}\,)\,\Gamma\, i\not D\, h\,|M\rangle = -\mathrm{tr}\{\,\Gamma\, P_-[\gamma^\beta \mathcal{M}\, \psi^{\alpha\beta}\, \overline{\mathcal{M}}'\,\gamma^\alpha]\, P'_-\,\}\,. \tag{4.35}$$

By evaluating the bracket one readily computes the functions m_8 to m_{14}, which appear in the parameterization of $m_{--}^{MM'}$. Because $\bar h'\,\Gamma\,\gamma_\alpha v_\beta G^{\alpha\beta} h = -\bar h'\,\Gamma\, iv{\cdot}D\, i\not D\, h$, the other second order currents in (4.1) contain one small component and thus contribute to ℓ_-^M. To see this, we employ the equation of motion to write

$$\langle M'|\,\bar h'\,\Gamma\,\gamma_\alpha v_\beta G^{\alpha\beta} h\,|M\rangle = -\mathrm{tr}\{\,\overline{\mathcal{M}}'\,\Gamma\, P_-[\gamma^\alpha \mathcal{M}\, v^\beta \phi^{\alpha\beta}]\,\}\,. \tag{4.36}$$

The mixed corrections to the current and the Lagrangian have the same structure, since

$$\langle M'|\, i\!\int\! \mathrm{d}x\, T\{\,\bar h'\,\Gamma\, i\not D\, h, \mathcal{L}_1(x)\,\}\,|M\rangle = -\mathrm{tr}\{\,\overline{\mathcal{M}}'\,\Gamma\, P_-[\gamma^\gamma \mathcal{M}\, E_\gamma + \gamma^\gamma P_+\, s^{\alpha\beta} \mathcal{M}\, E_{\gamma\alpha\beta}]\,\}\,. \tag{4.37}$$

Thus, both ϕ_i and E_i enter in the functions ℓ_4, ℓ_5 and ℓ_6. Finally, the second matrix element in (4.24) determines the functions m_{15} to m_{24}, which appear in the decompositions of $m_{+-}^{MM'}$ and $m_{-+}^{MM'}$.

The complete set of expressions for ℓ_i and m_i is given in Appendix A, and in Appendix B we compute the meson form factors h_i in terms of these functions. Let us now use these results to derive the normalization conditions which follow from the conservation of the vector current in the limit of equal heavy quark masses, $m_{Q'} = m_Q$. It implies that at zero recoil

$$\langle M(v)|\,\bar Q\,\gamma_\mu\, Q\,|M(v)\rangle = 2m_M\, v_\mu \tag{4.38}$$

for both pseudoscalar and vector mesons, which in terms of the meson form factors is equivalent to $h_+(1) = h_1(1) = 1$. It is now important that we have performed a mass renormalization in the first term in (4.34), since m_M in (4.38) is the physical meson mass. Using the normalization of the Isgur-Wise function and Luke's theorem, we find from Appendix B in the equal mass limit

$$\begin{aligned} h_+(1) &= 1 + \frac{1}{4m_Q^2}\,\Big[2\ell_1(1) + m_1(1) - m_8(1)\Big] + \cdots\,, \\ h_1(1) &= 1 + \frac{1}{4m_Q^2}\,\Big[2\ell_2(1) + m_4(1) + m_5(1) - m_{11}(1) - m_{12}(1)\Big] + \cdots\,. \end{aligned}$$

Setting the coefficients of the second order terms to zero we obtain two conditions, which at tree level may be written as

$$2B_1(1) + 2C_1(1) + D_1(1) - 3\Big[2C_4(1) + D_4(1) + 2C_5(1) + D_5(1)\Big] = -\lambda_1\,,$$
$$2B_3(1) + 2C_3(1) + D_3(1) - 2\Big[2C_5(1) + D_5(1) + 2C_6(1) + D_6(1)\Big] = -\lambda_2\,. \qquad (4.39)$$

More restrictive relations could be derived by including renormalization effects and requiring that the logarithmic dependence on m_Q be the same on both sides of (4.39). The results of such an analysis will be presented elsewhere.

V. APPLICATIONS AND SUMMARY

Let us summarize the main results of our analysis. In total, thirty-four universal functions appear in second order of the heavy quark expansion of meson form factors. We have proved two theorems stating that, at zero recoil, the leading meson form factors do not receive contributions from mixed corrections to the current and the Lagrangian, and that the corrections to the current can be expressed in terms of λ_1 and λ_2. The number of universal functions is strongly reduced if one ignores radiative corrections and only considers the phenomenologically interesting cases of $P \to P$ and $P \to V$ transitions induced by a vector or axial current. Then all matrix elements can be parameterized in terms of ℓ_1 to ℓ_6 and the five combinations $(m_1 - m_8)$, $(m_2 + m_9)$, $(m_3 - m_{10})$, $(m_{16} + m_{18})$ and $(m_{17} - m_{19})$. This can be seen from the relations given in Appendix B.

In the following paragraphs we apply our results to semileptonic B decays and give estimates for some of the second order corrections. We also discuss the corrections to Luke's theorem, which arise at second order. For simplicity, we shall ignore radiative corrections.

A. Elastic Form Factors and $B \to D\,\ell\,\nu$ Decays

As pointed out in the introduction, the universal form factors of HQET describe the properties of the light degrees of freedom in the background of the color field of the heavy quark. From this point of view, the Isgur-Wise function is the elastic form factor that describes the overlap of the wave functions of the light degrees of freedom in the initial and final mesons moving at velocities v and v'. The normalization of $\xi(w)$ at zero recoil reflects the complete overlap of the configurations of the light constituents in two infinitely heavy mesons with the same velocity. If finite-mass corrections are taken into account, the overlap decreases. In HQET the corresponding corrections are described by the functions L_i and ℓ_i, which represent the corrections to the wave function of a pseudoscalar ($i = 1$) or a vector meson ($i = 2$). At zero recoil, the first order corrections vanish, and using the expression for h_+ and h_1 from Appendix B we obtain at second order

$$\langle D(v)|\, V_\mu\, |B(v)\rangle = 2\sqrt{m_B m_D}\, v_\mu \Big\{ 1 + (\varepsilon_c - \varepsilon_b)^2 \ell_1(1) + \cdots \Big\}\,, \qquad (5.1)$$

$$\langle D^*(v)|\, V_\mu\, |B^*(v)\rangle = -2\sqrt{m_{B^*} m_{D^*}}\, \epsilon\cdot\epsilon'^*\, v_\mu \Big\{ 1 + (\varepsilon_c - \varepsilon_b)^2 \ell_2(1) + \cdots \Big\} ,$$

where $\varepsilon_Q = 1/2m_Q$.

In the nonrelativistic constituent quark model, the m_Q-dependence of the overlap integral comes from the m_Q-dependence of the reduced mass of the light constituent quark, $m_q^{\rm red} = m_q m_Q/(m_Q + m_q)$. For an estimate of $\ell_i(1)$ we use the wave functions of the ISGW model [35] to obtain

$$\ell_1(1) = \ell_2(1) = -3 m_q^2 \approx -0.75 \text{ GeV}^2 . \tag{5.2}$$

For the numerical estimate we have identified the constituent mass of the light quark with $\bar\Lambda$, since $m_M = m_Q + m_q$ in the ISGW model.

The matrix element of the vector current between a B and a D meson enters the theoretical description of the decay rate for the semileptonic process $B \to D\,\ell\,\nu$. After contraction with the leptonic current a combination of the form factors h_+ and h_- appears [33],

$$\begin{aligned} \frac{\mathrm{d}\Gamma(B \to D\,\ell\,\nu)}{\mathrm{d}w} &= \frac{G_F^2\, |\, V_{cb}|^2}{48\pi^3}\, m_D^3\, (m_B + m_D)^2\, (w^2 - 1)^{3/2} \\ &\quad \times \Big| h_+(w) - \sqrt{S}\, h_-(w) \Big|^2 , \end{aligned} \tag{5.3}$$

where $S = (\frac{m_B - m_D}{m_B + m_D})^2 \approx 0.23$ is the Voloshin-Shifman factor [1]. At leading order in the heavy quark expansion the form factor is normalized at zero recoil, offering the possibility of a reliable determination of V_{cb} for $w \gtrsim 1$, provided that the corrections to the infinite quark mass limit are small. The first order power corrections are indeed suppressed by the Voloshin-Shifman factor and have been estimated to be $\approx +2\%$ [25]. Including the second order corrections, we find from Appendix B

$$\begin{aligned} h_+(1) - \sqrt{S}\, h_-(1) &= 1 - (\varepsilon_c + \varepsilon_b)\, S\, L_4(1) + (\varepsilon_c - \varepsilon_b)^2 [\ell_1(1) - \ell_4(1)] + 4\varepsilon_c \varepsilon_b\, S\, \bar\Lambda \\ &\approx 1 - 0.7\% - 1.3\% \times \left[\frac{\ell_4(1)}{\bar\Lambda^2} \right] , \end{aligned} \tag{5.4}$$

where we have used the heavy quark masses $m_c = 1.5$ GeV and $m_b = 4.8$ GeV, the constituent quark model estimate (5.2), and the QCD sum rule results $\bar\Lambda \approx 0.5$ GeV and $L_4(1) \approx -\bar\Lambda/3$ [25]. For simplicity, the radiative corrections to h_+ and h_- have been neglected. We conclude that, unless the coefficient $\ell_4(1)$ were unusually large, both the first and second order power corrections are small. Although not protected by Luke's theorem, the decay $B \to D\,\ell\,\nu$ thus allows for a reliable measurement of V_{cb}.

B. Determination of V_{cb} from $B \to D^* \ell\, \nu$ Decays

It has been observed in Refs. [1, 26] that semileptonic B decays into D^* vector mesons offer an almost model-independent measurement of V_{cb}, since the $1/m_Q$ corrections to the decay rate vanish at zero recoil. In terms of the meson form factors one finds

$$\lim_{w\to 1} \frac{1}{\sqrt{w^2-1}}\, \frac{\mathrm{d}\Gamma(B\to D^*\ell\,\nu)}{\mathrm{d}w} = \frac{G_F^2\,|\,V_{cb}|^2}{4\pi^3}\, m_{D^*}^3\,(m_B - m_{D^*})^2\,|h_{A_1}(1)|^2\,, \tag{5.5}$$

and $h_{A_1}(1)$ is protected by Luke's theorem [15]. Thus the determination of V_{cb} from an extrapolation of the spectrum to $w \gtrsim 1$ is model-independent up to terms of order $1/m^2$. From Appendix B we obtain at second order

$$h_{A_1}(1) = 1 + (\varepsilon_c - \varepsilon_b)\,[\varepsilon_c\,\ell_2(1) - \varepsilon_b\,\ell_1(1)] + \varepsilon_c\varepsilon_b\,\Delta\,, \tag{5.6}$$

where

$$\begin{aligned} \Delta &= \ell_1(1) + \ell_2(1) + m_2(1) + m_9(1) \\ &= \tfrac{4}{3}\,\lambda_1 + 2\,\lambda_2 + 4\Big[D_4(1) + 2D_5(1) + D_6(1)\Big]\,. \end{aligned} \tag{5.7}$$

Using (5.2), the first correction in (5.6) is estimated to be $-3\bar\Lambda^2(\varepsilon_c - \varepsilon_b)^2 \approx -3.9\%$. Concerning the second term we observe that, except for λ_1 and λ_2, the coefficient Δ depends only on form factors which arise from a double insertion of the chromo-magnetic moment operator of $\mathcal{L}_1$ in (2.4). We shall argue below that these terms are expected to be very small. Neglecting them, and using (2.15) as well as the sum rule estimate $\lambda_1 \approx 1~\mathrm{GeV}^2$ [17], we obtain $\varepsilon_c\varepsilon_b\,\Delta \approx 5.7\%$, and thus

$$h_{A_1}(1) - 1 \approx 2\%\,. \tag{5.8}$$

The main uncertainty in this estimate arises from the uncertainty in λ_1, as discussed at the end of Sec. II. In the extreme case $\lambda_1 = 0$ we would obtain $h_{A_1}(1) - 1 \approx -3\%$ instead of (5.8). However, in any case the second order correction is small because of a partial cancellation of the two terms in (5.6), suggesting that the theoretical uncertainty in this method of extracting V_{cb} is less than a few percent.

It has been claimed in Ref. [36] that QCD sum rules would predict a second order correction to $h_{A_1}(1)$ of as much as -10%.[2] In view of our estimate (5.8) this assertion seems unacceptable. Even for $\lambda_1 = 0$ it would imply that $\ell_1(1)$ and $\ell_2(1)$ would have to exceed the quark model prediction (5.2) by a factor of three.

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[2] It has been pointed out in Ref. [25] that the argument given in Ref. [36] has no theoretical foundation.

C. Second Order Corrections to Luke's Theorem

At the end of Sec. III, we discussed the fact that Luke's theorem protects the meson form factors h_+, h_{A_1}, h_1, and h_7 from first order power corrections at zero recoil. Although our results show that there is no such nonrenormalization theorem at second order, the structure of the $1/m^2$ corrections to these four form factors is particularly simple and allows for a semi-quantitative estimate. At zero recoil, the expression for h_7 is

$$h_7(1) = 1 + (\varepsilon_c - \varepsilon_b)^2 \ell_2(1) + \varepsilon_c \varepsilon_b \, \Delta' \tag{5.9}$$

with

$$\Delta' = \tfrac{4}{3}\,\lambda_1 - 2\,\lambda_2 + 4\,[D_4(1) - D_6(1)]\,. \tag{5.10}$$

The other three form factors have been given in (5.1) and (5.6). We observe that there is always a correction involving $\ell_1(1)$ or $\ell_2(1)$, depending on whether one deals with a pseudoscalar or a vector meson, respectively. Using the quark model estimate (5.2), this term becomes approximately -4%. It smallness naturally results from the squared difference $(\varepsilon_c - \varepsilon_b)^2$. In addition, for h_{A_1} and h_7 there is a term proportional to $\varepsilon_c \varepsilon_b$, which depends on the mass parameters λ_1 and λ_2 as well as on form factors arising from a double insertion of the chromo-magnetic moment operator. Neglecting these latter terms, this correction can be estimated based on a model calculation of λ_1, since λ_2 is known from the experimentally observed mass splitting between vector and pseudoscalar mesons. QCD sum rules predict that λ_1 is positive, and the corresponding correction tends to cancel the terms proportional to ℓ_i, which are negative. As a result, the form factors h_{A_1} and h_7 can only receive small $1/m^2$ corrections at zero recoil.

D. Limit of Vanishing Chromo-Magnetic Interaction

Detailed QCD sum rule analyses of the universal functions that appear at order $1/m$ in the heavy quark expansion show that the form factors A_2 and A_3, which arise from the insertion of the chromo-magnetic moment operator in $\mathcal{L}_1$, are much smaller than the other two functions, A_1 and ξ_3 [25, 37]. The coarse pattern of the $1/m$ corrections can be well described by setting A_2 and A_3 to zero, corresponding to the fictitious limit of vanishing field strength, $G^{\alpha\beta} \to 0$. Let us see what kind of simplifications the same approximation implies at order $1/m^2$.

We start with the corrections to the current. In the limit $G^{\alpha\beta} \to 0$ the functions ϕ_1, ϕ_2 and ϕ_3 vanish, and according to (4.10) this implies the vanishing of λ_1 and λ_2. It then follows that $\phi_0 = (w-1)\,\hat{\phi}$ vanishes at zero recoil, and all corrections to the current can be described by the single function $\hat{\phi}$. Similar simplifications occur for the corrections to the Lagrangian. Here all universal functions except B_1, C_1, and D_1 vanish in the limit $G^{\alpha\beta} \to 0$. At tree level, one obtains from (4.39) the zero recoil condition

$$B_1(1) + C_1(1) + \tfrac{1}{2}\, D_1(1) = 0\,. \qquad (G^{\alpha\beta} \to 0) \tag{5.11}$$

Finally, the combined corrections to the current and to the Lagrangian are entirely parameterized by the form factor E_3, since E_6, E_9, E_{10}, and E_{11} vanish in the limit of vanishing field strength.

In the fictitious limit of vanishing chromo-magnetic interaction, the set of thirty-four universal form factors is thus reduced to only five functions, a combination of which vanishes at zero recoil. Although we are aware of the fact that such an approximation can only give us a very simplified picture, we still believe that it might be useful for an analysis of the structure of the dominant terms. The expressions arising for the functions ℓ_i and m_i in this limit can readily be obtained from the general formulas given in Appendix A.

E. Summary

Using the heavy quark effective theory, we have performed the expansion of matrix elements of heavy quark currents between pseudoscalar or vector mesons up to second order in inverse powers of the heavy quark masses. The general description of the power corrections arising at order $1/m^2$ involves a set of thirty-four Isgur-Wise form factors, which are universal, m_Q-independent functions of the kinematic variable $w = v \cdot v'$. These form factors are defined in terms of matrix elements of higher dimension operators in the effective theory.

Apart from some normalization conditions imposed by vector current conservation, the universal functions are hadronic quantities which cannot yet be predicted from first principles. Nevertheless, we have argued that in certain cases of phenomenological interest the $1/m^2$ corrections are parameterically suppressed. In particular, the corrections to the semileptonic decay rates for $B \to D\,\ell\,\nu$ and $B \to D^*\ell\,\nu$ at zero recoil are estimated to be small, not exceeding a few percent. Our results thus support the usefulness of the heavy quark symmetries for an accurate determination of the weak mixing parameter V_{cb} from these decay modes.

Although the structure of second order corrections to various decay rates is quite complex, we believe that a classification in terms of universal form factors is still a useful concept. In particular, this might provide a framework in which to analyze various models. For instance, we have shown that the second order corrections to elastic form factors arising from the m_Q-dependence of the reduced mass of the light constituent quark in a nonrelativistic quark model are accounted for by our functions ℓ_1 and ℓ_2, and an estimate of the effect gives $\ell_1(1) \approx \ell_2(1) \approx -0.75\,\mathrm{GeV}^2$. This information can then be used to predict corrections to other form factors, whose dependence on ℓ_1 and ℓ_2 is known from heavy quark symmetry. We have also suggested that, for an estimate of the dominant corrections, one might consider the limit of vanishing chromo-magnetic interaction, in which only five of the thirty-four universal form factors remain. The usefulness of such an approximation is supported by QCD sum rule calculations of the form factors appearing at order $1/m$ in the heavy quark expansion.

The analysis presented here for mesons can straightforwardly be extended to other hadrons containing a single heavy quark. The particularly interesting case of the Λ baryons is discussed in Ref. [27].

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APPENDIX A: COMPUTATION OF THE MODIFIED WAVE FUNCTIONS

According to (3.13), the general structure of the modified wave functions $\ell_\pm^M$ introduced in (4.34) is

$$\begin{aligned}
\ell_+^P &= \sqrt{M}\,(-\gamma_5)\,\ell_1\,,\\
\ell_+^V &= \sqrt{M}\,\Big[\not\epsilon\,\ell_2 + \epsilon\cdot v'\,\ell_3\Big]\,,\\
\ell_-^P &= \sqrt{M}\,(-\gamma_5)\,\ell_4\,,\\
\ell_-^V &= \sqrt{M}\,\Big[\not\epsilon\,\ell_5 + \epsilon\cdot v'\,\ell_6\Big]\,.
\end{aligned}\tag{A1}$$

The coefficients ℓ_i are functions of $w = v\cdot v'$. Similarly, we choose the following decomposition for the product wave functions $m_{++}^{MM'}$:

$$\begin{aligned}
m_{++}^{PP} &= \sqrt{MM'}\; m_1\,(-\gamma_5)\,\gamma_5 = -\sqrt{MM'}\; m_1\,,\\
m_{++}^{PV} &= \sqrt{MM'}\,\Big[m_2\,(-\gamma_5)\,\not\epsilon'^{*} + m_3\,(-\gamma_5)\,\epsilon'^{*}\cdot v\Big]\,,\\
m_{++}^{VP} &= \sqrt{MM'}\,\Big[m_2'\not\epsilon\,\gamma_5 + m_3'\,\epsilon\cdot v'\,\gamma_5\Big] = \overline{m}_{++}^{PV}\,,\\
m_{++}^{VV} &= \sqrt{MM'}\,\Big[m_4\not\epsilon\not\epsilon'^{*} + m_5\,\epsilon\cdot\epsilon'^{*} + m_6\not\epsilon\,\epsilon'^{*}\cdot v + m_6'\not\epsilon'^{*}\,\epsilon\cdot v' + m_7\,\epsilon\cdot v'\,\epsilon'^{*}\cdot v\Big]\,.
\end{aligned}\tag{A2}$$

An identical decomposition with functions m_8 to m_{14} applies for $m_{--}^{MM'}$. Finally, we define

$$\begin{aligned}
m_{+-}^{PP} &= -\sqrt{MM'}\; m_{15}\,,\\
m_{+-}^{PV} &= \sqrt{MM'}\,\Big[m_{16}\,(-\gamma_5)\,\not\epsilon'^{*} + m_{17}\,(-\gamma_5)\,\epsilon'^{*}\cdot v\Big]\,,\\
m_{+-}^{VP} &= \sqrt{MM'}\,\Big[m_{18}\not\epsilon\,\gamma_5 + m_{19}\,\epsilon\cdot v'\,\gamma_5\Big]\,,\\
m_{+-}^{VV} &= \sqrt{MM'}\,\Big[m_{20}\not\epsilon\not\epsilon'^{*} + m_{21}\,\epsilon\cdot\epsilon'^{*} + m_{22}\not\epsilon\,\epsilon'^{*}\cdot v + m_{23}\not\epsilon'^{*}\,\epsilon\cdot v' + m_{24}\,\epsilon\cdot v'\,\epsilon'^{*}\cdot v\Big]\,,
\end{aligned}\tag{A3}$$

and $m_{-+}^{MM'}$ is described by a set of related functions m_i', since $m_{-+}^{MM'} = \overline{m}_{+-}^{M'M}$. In these expressions a "bar" means Dirac conjugation combined with an exchange of velocities, polarizations, and meson masses. Also, because of radiative corrections the functions ℓ_i and m_i depend logarithmically on the heavy quark masses, and m_i' are related to m_i by an interchange of m_Q and $m_{Q'}$ in the renormalization factors. At tree level there is no such difference, and ℓ_i and m_i are universal, m_Q-independent functions.

The tree level expressions for these functions can be obtained by evaluating the various traces, as explained in Sec. IV. We find:

$$\ell_1 = (\lambda_1 + 3\lambda_2)\,\xi + B_1 + 2(w-1)B_2 + 3B_3 + C_1 + 2(w-1)C_2 + 3C_3$$

$$
\begin{aligned}
&\quad -3C_4-9C_5-6C_6+2(w^2-1)(2C_7+C_8)-4(w-1)(C_9+C_{12})\\
\ell_2 &= (\lambda_1-\lambda_2)\,\xi+B_1-B_3+C_1-C_3-3C_4-C_5+2C_6\\
&\quad +2(w-1)\Big[(w+1)C_8-C_9-C_{10}+3C_{11}+C_{12}\Big]\\
\ell_3 &= -2B_2-2C_2+4(w+1)C_7+2C_9-2C_{10}-10C_{11}-10C_{12}\\
\ell_4 &= -\bar\Lambda L_1-w(\tilde\phi_0+3\tilde\phi_3)-(w+1)\phi_1+4\phi_2+3\phi_3\\
&\quad -2E_3-4(w+1)E_6-6E_9+2(w+1)E_{10}-4E_{11}\\
\ell_5 &= -\bar\Lambda L_2-w(\tilde\phi_0-\tilde\phi_3)-(w+1)\phi_1+2\phi_2+\phi_3+2(w+1)E_{10}-2E_{11}\\
\ell_6 &= -\tfrac{2}{w+1}\Big[\bar\Lambda L_2+\tfrac{\bar\Lambda}{2}(w-1)L_3+w(\tilde\phi_0-\tilde\phi_3)+(w+1)\phi_1-\phi_2\\
&\qquad -E_3+2(w+1)E_6+E_9-2(w+1)E_{10}+E_{11}\Big]
\end{aligned}
\tag{A4}
$$

$$
\begin{aligned}
m_1 &= D_1+2(w-1)D_2+3D_3-(2w+1)D_4-9D_5-6D_6\\
&\quad -2(w-1)\Big[(w+1)(2D_7+D_8)-4D_9-8D_{10}\Big]\\
m_2 &= D_1+(w-1)D_2+D_3+D_4+3D_5+2D_6-2(w-1)(D_9+D_{10})\\
m_3 &= -2D_2+2D_4+2(w+1)(2D_7+D_8)-2D_9-10D_{10},\\
m_4 &= D_1-D_3+(2w-1)D_4-D_5-2(2w-1)D_6\\
&\quad +2(w-1)\Big[(w+1)D_8-2D_9+2D_{10}\Big]\\
m_5 &= -4wD_4+4(2w-1)D_6-4(w-1)\Big[(w+1)D_8-2D_9+2D_{10}\Big]\\
m_6 &= -2D_2-2D_4+4D_6-2(w+1)D_8+6D_9-2D_{10}\\
m_7 &= 4D_4-8D_6-4D_7+4wD_8-8D_9+8D_{10}
\end{aligned}
\tag{A5}
$$

$$
\begin{aligned}
m_8 &= \phi_0+\tfrac{6}{w+1}\phi_3+\tfrac{w-1}{w+1}\Big[(w+1)\phi_1-6\phi_2-2\bar\Lambda L_4\Big]\\
m_9 &= \tfrac{1}{w+1}\Big[-(w+1)\phi_1+2(2-w)\phi_2+3\phi_3-\bar\Lambda(w-1)L_4\Big]\\
m_{10} &= \hat\phi+\tfrac{1}{w+1}\Big[3\phi_2+\tfrac{3}{2}\phi_3+\bar\Lambda L_4\Big]\\
m_{11} &= -\phi_0-(w+1)\phi_1+2\phi_2+2\phi_3\\
m_{12} &= 2\phi_0+2w\phi_1-\tfrac{2}{w+1}\Big[2w\phi_2+(2w+1)\phi_3+\bar\Lambda(w-1)L_5\Big]\\
m_{13} &= -\hat\phi+\tfrac{1}{w+1}\Big[\phi_2+\tfrac{1}{2}\phi_3+\bar\Lambda(L_4-2L_5)\Big]\\
m_{14} &= -\tfrac{2w}{w+1}\hat\phi+\tfrac{1}{(w+1)^2}\Big[4(w+1)\phi_1-2(3w+4)\phi_2+(w+2)\phi_3\\
&\qquad +4\bar\Lambda L_4-2\bar\Lambda(4-w)L_5\Big]
\end{aligned}
\tag{A6}
$$

$$
\begin{aligned}
m_{15} &= -\bar\Lambda L_1+\tilde\phi_0+3\tilde\phi_3-2\phi_2\\
&\quad -2E_3-4(w+1)E_6-6E_9+2(w+1)E_{10}-4E_{11}\\
m_{16} &= -\bar\Lambda L_2+\tilde\phi_0-\tilde\phi_3-2E_3+2E_9+2E_{11}
\end{aligned}
$$

$$
\begin{aligned}
m_{17} &= -\tfrac{2}{w+1}\Big[\tfrac{\bar\Lambda}{2}(w-1)L_3 - \phi_2 - 2(w+1)E_6 + (w+1)E_{10} + E_{11}\Big] \\
m_{18} &= -\bar\Lambda L_1 + \tilde\phi_0 + 3\tilde\phi_3 - 2\phi_2 \\
m_{19} &= -\tfrac{2}{w+1}\Big[\bar\Lambda L_1 - \tilde\phi_0 - 3\tilde\phi_3 + 2\phi_2 - E_3 - 2(w+1)E_6 - 3E_9 + (w+1)E_{10} - 2E_{11}\Big] \\
m_{20} &= -\bar\Lambda L_2 + \tilde\phi_0 - \tilde\phi_3 - 2(w+1)E_{10} + 2E_{11} \\
m_{21} &= 4(w+1)E_{10} - 4E_{11} \\
m_{22} &= -\tfrac{1}{w+1}\Big[\bar\Lambda(w-1)L_3 - 2\phi_2 - 2(w+1)E_{10} + 2E_{11}\Big] \\
m_{23} &= \tfrac{2}{w+1}\Big[-\bar\Lambda L_2 + \tilde\phi_0 - \tilde\phi_3 + E_3 - E_9 - (w+1)E_{10}\Big] \\
m_{24} &= -\tfrac{4}{w^2-1}\Big[\tfrac{\bar\Lambda}{2}(w-1)L_3 - \phi_2 + (w+1)E_6 + (w^2-1)E_{10} - wE_{11}\Big] \qquad \text{(A7)}
\end{aligned}
$$

Eq. (4.32) ensures that there is no pole in m_{24} as $w \to 1$. Note that the first term in ℓ_1 and ℓ_2 compensates the mass renormalization performed in (4.34).

APPENDIX B: MESON FORM FACTORS

Let us set $\varepsilon_Q = 1/2m_Q$. Then to second order in the heavy quark expansion the meson form factors h_i introduced in (3.17) are given by:

$$
\begin{aligned}
h_+ &= \xi + (\varepsilon_c + \varepsilon_b)\,L_1 + (\varepsilon_c^2 + \varepsilon_b^2)\,\ell_1 + \varepsilon_c\varepsilon_b\,(m_1 - m_8) \\
h_- &= (\varepsilon_c - \varepsilon_b)\,L_4 + (\varepsilon_c^2 - \varepsilon_b^2)\,\ell_4 \qquad \text{(B1)}
\end{aligned}
$$

$$
\begin{aligned}
h_V &= \xi + \varepsilon_c\,(L_2 - L_5) + \varepsilon_b\,(L_1 - L_4) \\
&\quad + \varepsilon_c^2\,(\ell_2 - \ell_5) + \varepsilon_b^2\,(\ell_1 - \ell_4) + \varepsilon_c\varepsilon_b\,[(m_2 + m_9) - (m_{16} + m_{18})] \qquad \text{(B2)}
\end{aligned}
$$

$$
\begin{aligned}
h_{A_1} &= \xi + \varepsilon_c\Big(L_2 - \tfrac{w-1}{w+1}L_5\Big) + \varepsilon_b\Big(L_1 - \tfrac{w-1}{w+1}L_4\Big) \\
&\quad + \varepsilon_c^2\Big(\ell_2 - \tfrac{w-1}{w+1}\ell_5\Big) + \varepsilon_b^2\Big(\ell_1 - \tfrac{w-1}{w+1}\ell_4\Big) + \varepsilon_c\varepsilon_b\Big[(m_2 + m_9) - \tfrac{w-1}{w+1}(m_{16} + m_{18})\Big] \\
h_{A_2} &= \varepsilon_c\,(L_3 + L_6) + \varepsilon_c^2\,(\ell_3 + \ell_6) + \varepsilon_c\varepsilon_b\,[(m_3 - m_{10}) - (m_{17} - m_{19})] \\
h_{A_3} &= \xi + \varepsilon_c\,(L_2 - L_3 - L_5 + L_6) + \varepsilon_b\,(L_1 - L_4) \\
&\quad + \varepsilon_c^2\,(\ell_2 - \ell_3 - \ell_5 + \ell_6) + \varepsilon_b^2\,(\ell_1 - \ell_4) \\
&\quad + \varepsilon_c\varepsilon_b\,[(m_2 + m_9) - (m_3 - m_{10}) - (m_{16} + m_{18}) - (m_{17} - m_{19})] \qquad \text{(B3)}
\end{aligned}
$$

$$
\begin{aligned}
h_1 &= \xi + (\varepsilon_c + \varepsilon_b)\,L_2 + (\varepsilon_c^2 + \varepsilon_b^2)\,\ell_2 + \varepsilon_c\varepsilon_b\,[(m_4 - m_{11}) + (m_5 - m_{12})] \\
h_2 &= (\varepsilon_c - \varepsilon_b)\,L_5 + (\varepsilon_c^2 - \varepsilon_b^2)\,\ell_5 \\
h_3 &= \xi + \varepsilon_c\,[L_2 + (w-1)L_3 + L_5 - (w+1)L_6] + \varepsilon_b\,(L_2 - L_5) \\
&\quad + \varepsilon_c^2\,[\ell_2 + (w-1)\ell_3 + \ell_5 - (w+1)\ell_6] + \varepsilon_b^2\,(\ell_2 - \ell_5) \\
&\quad + \varepsilon_c\varepsilon_b\,[(m_4 - m_{11}) + (m_5 - m_{12}) - (w-1)(m_6 - m_{13}) - (w+1)(m_{22} + m_{23})]
\end{aligned}
$$

$$h_4 = h_3(\varepsilon_c \leftrightarrow \varepsilon_b)$$
$$h_5 = \varepsilon_c\,(L_3 - L_6) + \varepsilon_c^2\,(\ell_3 - \ell_6) + \varepsilon_c\varepsilon_b\,[(m_6 - m_{13}) + (m_7 - m_{14}) - (m_{22} + m_{23})]$$
$$h_6 = h_5(\varepsilon_c \leftrightarrow \varepsilon_b) \tag{B4}$$

$$h_7 = \xi + (\varepsilon_c + \varepsilon_b)\,L_2 + (\varepsilon_c^2 + \varepsilon_b^2)\,\ell_2 + \varepsilon_c\varepsilon_b\,(m_4 - m_{11})$$
$$h_8 = h_2 \tag{B5}$$

These relations are valid at tree level. The radiative corrections to the leading and subleading terms in the heavy quark expansion have been calculated in Refs. [3, 38, 39].

APPENDIX C: MODIFIED WARD IDENTITIES

Here we derive Ward identities which relate the derivative of the matrix elements in (3.10) to the matrix elements in (4.24), in which a derivative acts on the current. These identities are needed in Sec. IVC to express the universal functions E_i' in terms of E_i and other form factors. Let us consider the following matrix element:

$$\langle M'|\,J(z)\,|M + \delta M\rangle \equiv \langle M'|\,J(z)\,|M\rangle + \frac{1}{2m_Q}\,\langle M'|\,i\!\int\!\mathrm{d}x\,T\{\,J(z),\mathcal{L}_1(x)\,\}\,|M\rangle\,. \tag{C1}$$

$J(z)$ is a heavy quark current in the effective theory, $|M\rangle$ is an eigenstate of $\mathcal{L}_{\mathrm{HQET}}$, and $|M + \delta M\rangle$ denotes an eigenstate of $\mathcal{L}_{\mathrm{HQET}} + \frac{1}{2m_Q}\mathcal{L}_1$. In contrast to (2.7), we have

$$|M + \delta M\rangle_z = \exp\bigg[-i\Big(\bar\Lambda + \frac{\Delta m_M^2}{2m_Q}\Big)v\cdot z\bigg]\,|M + \delta M\rangle_0\,. \tag{C2}$$

Using this fact, we find to order $1/m_Q$

$$\begin{aligned} i\partial_\gamma^z\,\langle M'|\,J(z)\,|M + \delta M\rangle &= i\partial_\gamma^z\,\langle M'|\,J(z)\,|M\rangle + \frac{\Delta m_M^2}{2m_Q}\,v_\gamma\,\langle M'|\,J(z)\,|M\rangle \\ &\quad + \frac{\bar\Lambda}{2m_Q}\,(v - v')_\gamma\,\langle M'|\,i\!\int\!\mathrm{d}x\,T\{\,J(z),\mathcal{L}_1(x)\,\}\,|M\rangle\,. \end{aligned} \tag{C3}$$

Collecting terms of order $1/m_Q$, we thus obtain

$$\Big[i\partial_\gamma^z - \bar\Lambda\,(v - v')_\gamma\Big]\,\langle M'|\,i\!\int\!\mathrm{d}x\,T\{\,J(z),\mathcal{L}_1(x)\,\}\,|M\rangle = \Delta m_M^2\,v_\gamma\,\langle M'|\,J(z)\,|M\rangle\,. \tag{C4}$$

On the other hand, carrying out the derivative acting on the time-ordered product gives

$$\begin{aligned} i\partial_\gamma^z \,\langle M'|\, i\int\!\mathrm{d}x\, T\{\, J(z), \mathcal{L}_1(x)\,\}\,|M\rangle &= \langle M'|\, i\int\!\mathrm{d}x\, T\{\, i\partial_\gamma J(z), \mathcal{L}_1(x)\,\}\,|M\rangle \\ &\quad - v_\gamma\,\langle M'|\,\bar h'\,\Gamma\, P_+\,\Big[(iD)^2 + Z\, s_{\alpha\beta} G^{\alpha\beta}\Big]\, h\,|M\rangle\,. \end{aligned} \tag{C5}$$

Combining (C4) and (C5), we find for the universal form factors the relations given in (4.29).

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FIGURES

FIG. 1. Diagrams representing the first order power corrections to meson form factors in HQET: (a) corrections to the current, and (b) corrections to the effective Lagrangian. The squares represent operators of order $1/m_Q$ or $1/m_{Q'}$.

FIG. 2. Diagrams representing the second order power corrections to meson form factors in HQET: (a) corrections to the current, (b) corrections to the effective Lagrangian, and (c) mixed corrections to the current and the effective Lagrangian. The black squares represent operators of order $1/m_Q$ or $1/m_{Q'}$, the open ones denote operators of order $1/m_Q^2$, $1/m_{Q'}^2$, or $1/m_Q m_{Q'}$.

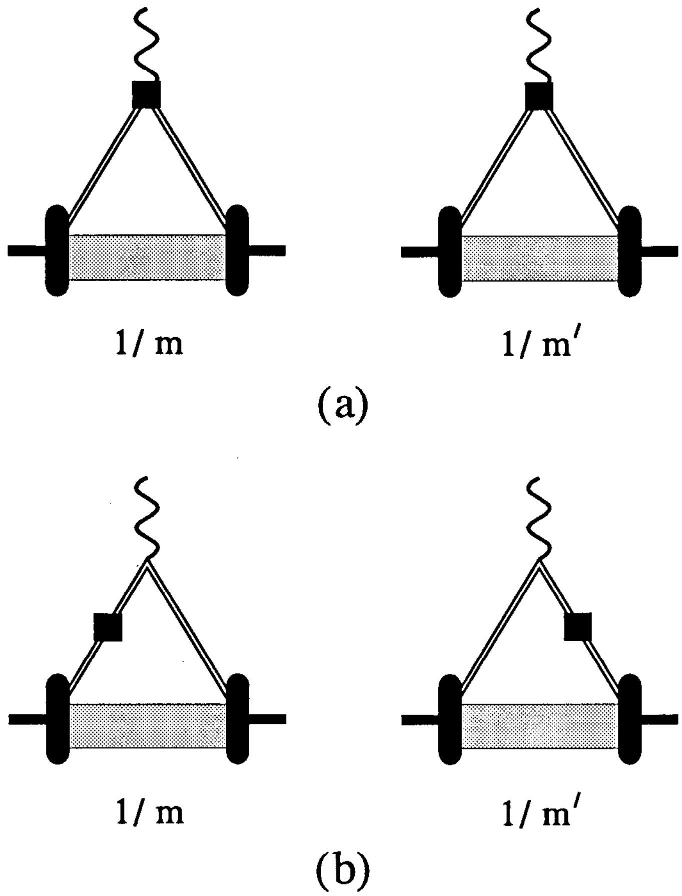

Fig.1

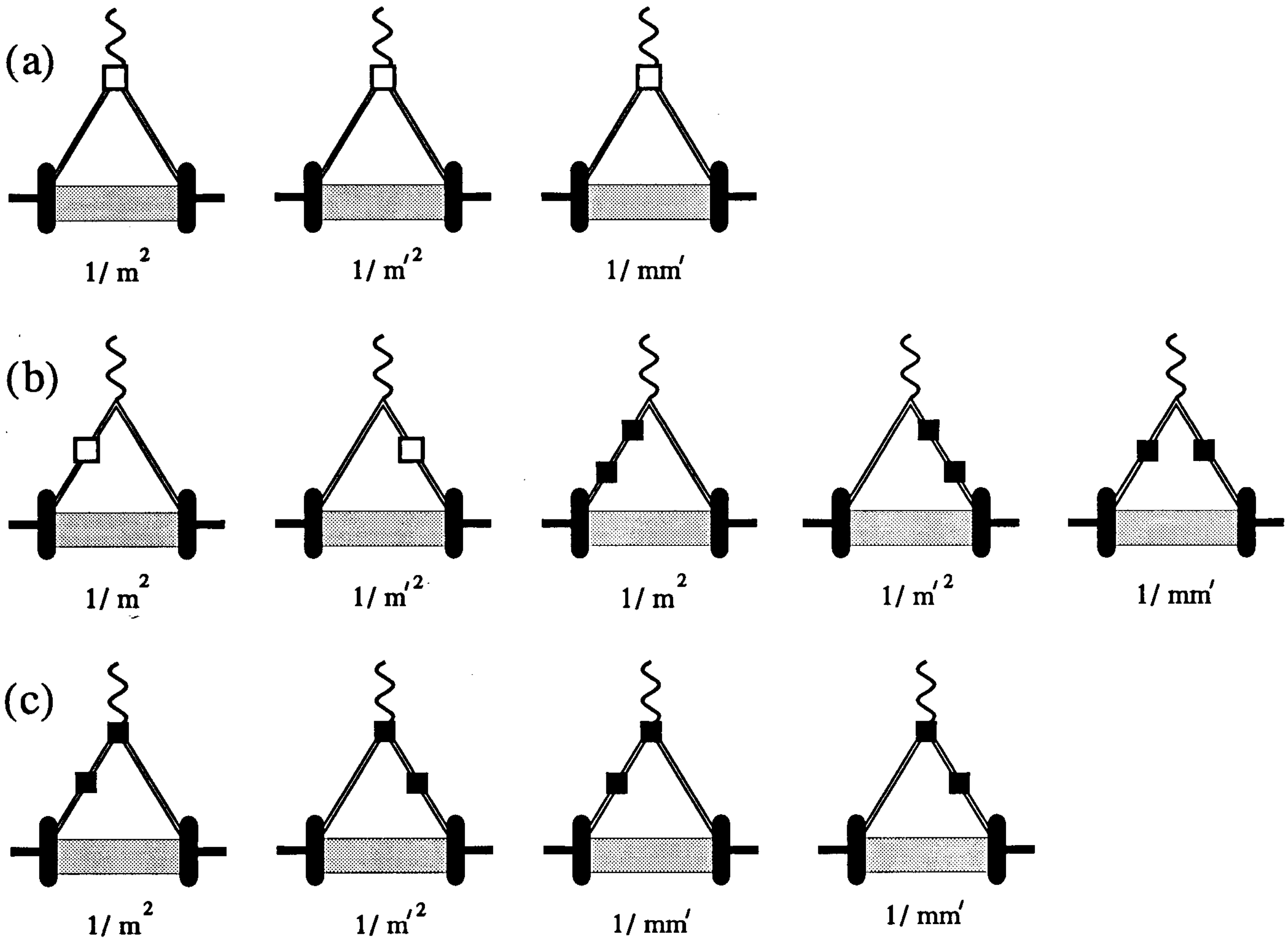

Fig.2