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BEAM LOADING COMPENSATION WITH VARIABLE GROUP VELOCITY*

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Abstract

Consider a section with linearly variable group velocity and a beam pulse shorter than the section fill time. Choose the current amplitude so that the gradient of the last bunch equals the gradient of the first bunch. For beam pulses less than about 15% of fill time, the voltage deviation during the beam pulse is small, but as the pulse width increases the voltage deviation also increases. We show that by decreasing the output to input group velocity ratio, we can reduce the first order voltage deviation, and that we can remove the remaining second-order voltage deviation by linearly decreasing the section input power by a small amount starting at beam injection time. This way we can increase the beam pulse width to more than half the fill time, and thereby increase the RF to beam energy transfer efficiency and the luminosity without increasing the voltage deviation.

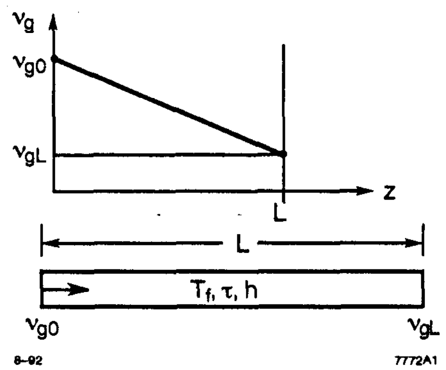

Figure 1. Variable group velocity section.

Introduction

Consider the section shown in Fig. 1, of length L, in which the group velocity varies linearly with distance z according to

$$v_g(z) = v_{go}(1 + gz/L) = v_{go}(1 + mz) = v_{go} + v_g' z \ , \quad (1)$$

having an output-to-input velocity ratio

$$h \equiv v_{gL}/v_{g0} \equiv 1 + g \equiv 1 + mL \ . \quad (2)$$

The propagation time to position z is given by [1]

$$t = \int_0^z \frac{dz}{v_g(z)} = \frac{L}{v_{g0}} \, \frac{\ln(1 + gz/L)}{g} \ . \quad (3)$$

If $z = L$, then $t = T_f$ and

$$v_{g0} = (L/T_f)\ [\ln(1+g)/g] \ .$$

Substitute for v_{g0} and obtain

$$t = T_f \, \frac{\ln(1 + gz/L)}{\ln(1+g)} \ , \qquad z = L\, \frac{h^{t/T_f} - 1}{g} \ . \quad (4)$$

We will derive the expressions for RF and beam induced gradients.

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RF Induced Section Gradient

The accelerating field as a function z is [2]

$$E(z) = \sqrt{\frac{P_o e^{-2t/T_a} s}{v_{go}(1 + mz)}} = \sqrt{\frac{P_o e^{-2\tau t/T_f} s}{v_{go} h^{t/T_f}}} \ . \quad (5)$$

Here s is the elastance/m and T_a is the TW time constant. The RF induced voltage as a function of time starting at the beginning of the input RF pulse, V_r, is obtained [2] by integrating $E(z)$ from zero to z and then substituting t for z. It can be expressed as the product of, the steady state voltage of a lossless constant impedance section, V_{rm}, and a normalized voltage, V_{rn}, that depends only on h, T_a, t, and T_f.

$$V_r = V_{rm}\ V_{rn} = \sqrt{sP_oT_fL}\ \left[(h^{x_1 t/T_f} - 1)/(x_1\sqrt{g \ln h}\right] \quad (6)$$

$$\tau = T_f/T_a \ , \qquad x_1 = 0.5 - (\tau/\ln h) \ . \quad (7)$$

Define the section efficiency [2], $\eta_s \equiv V_{rn}^2(T_f)$. Thus the steady state voltage

$$V_{rf} = \sqrt{\eta_s s P_o T_f L} \ . \quad (8)$$

For a CZ section, V_{rn} reduces to the expression given by Wilson [3] and for a CG section V_{rn} it reduces to the expression given by Wang [4].

To obtain the section gradient, E_r divide V_r by the section length L.

$$E_r(t) \equiv V_r(t)/L = \sqrt{sP_oT_f/L} \times V_{rn}(t) = E_{rm} \times V_{rn}(t) \ . \quad (9)$$

Beam Induced Section Gradient

For a given DC current amplitude I_o, the time dependent beam induced voltage, V_b, can also be expressed as the product of the steady state voltage of a lossless constant impedance section, V_{bm}, and a normalized time dependent beam induced voltage, V_{bn}, that depends only on h, T_a, t, and T_f.

$$V_b = V_{bm} \times V_{bn} = (sI_oT_fL/4) \times V_{bn} \ . \quad (10)$$

V_{bn} was derived by Wilson [5] and is given by

$$\eta \equiv -\ln h/2\tau \ , \qquad g = h - 1 = e^{-2\eta\tau} - 1 \ .$$

$$V_{bn} = \frac{2}{g\tau}\left[\frac{(1 - e^{-t(1+\eta)/T_a}}{1+\eta} - \frac{e^{-2\eta\tau}(1 - e^{-t(1-\eta)/T_a})}{1-\eta}\right] \ . \quad (11)$$

Here t starts when the beam is injected. Making the appropriate substitutions, this reduces to the expressions for V_{bn}, obtained by Wilson [3] and Wang [4]. The beam induced section gradient E_b, is V_b divided by the section length L.

$$E_b(t) = (sI_oT_f/4) \times V_{bn}(t) = E_{bm} \times V_{bn}(t) \ . \quad (12)$$

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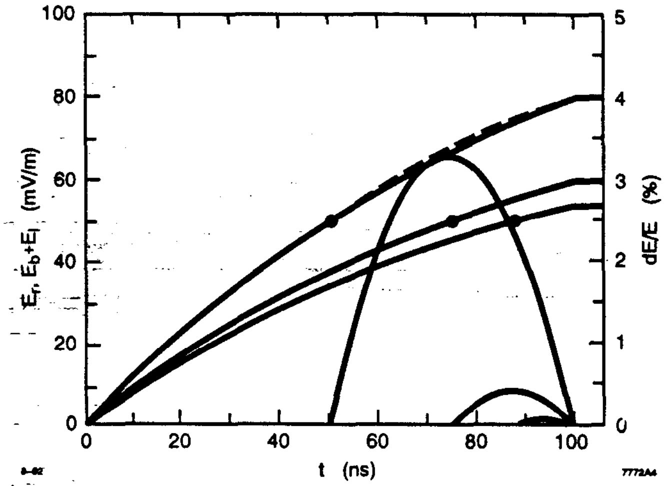

Figure 2. RF and beam induced (dash) gradients and voltage deviation versus time.

Charge and Voltage Deviation

For a given beam pulse width and a given input power or loaded gradient, we choose a current that makes the gradient at the end of the beam pulse equal to the gradient at the beginning of the beam pulse. In between the beginning and end of the beam pulse, the beam induced gradient (wakefield) is nearly neutralized by the increase in the RF induced gradient. Inject the beam at $T_i = T_f - T_b$. The change in RF induced voltage during T_b is

$$\Delta E = E_r(T_f) - E_r(T_i) = \sqrt{\frac{sP_oT_f}{L}}\left[\sqrt{\eta_s} - V_{rn}(T_i)\right] \ . \quad (13)$$

Choose P_o so that for any beam pulse width, the gradient at injection time, $E_r(T_i)$, equals a fixed value, E_i. Hence,

$$P_o = \frac{E_iL}{sT_fV_{rn}^2(T_i)} \ , \text{ and } \Delta E = \frac{E_i[\sqrt{\eta_s} - V_{rn}(T_i)]}{V_{rn}(T_i)} \ . \quad (14)$$

The loaded gradient and the voltage deviation during the pulse, are, respectively,

$$E_l(t) = E_r(t) - E_b(t - T_i) \ , \quad dE/E = (E_l(t) - E_i)/E_i \ . \quad (15)$$

To make the gradient of the last bunch equal to that of the first bunch, equate ΔE to $E_b(T_b)$ and solve for the equalizing current,

$$I_o = \frac{4\Delta E}{sT_fV_{bn}(T_b)} = \frac{4E_i[\sqrt{\eta_s} - V_{rn}(T_i)]}{V_{rn}(T_i)sT_fV_{bn}(T_b)} \ . \quad (16)$$

Use this I_o for calclating the beam induced gradient as a function of time. The charge per pulse, the charge per bunch and the RF to beam energy conversion efficiency, are, respectively

$$q_p = I_oT_b \ , \quad q_b = I_o\Delta t \ , \quad \eta_{rb} = E_iLq_p/(P_oT_f) \ . \quad (17)$$

The beam energy is

$$B_e = E_iLIT_b = E_iL\,(q_b/\Delta t)\,T_b \ .$$

Let E_{rf}=Steady state unloaded gradient, P_k=RF input power, and T_k=RF input power pulse width. Then, the RF pulse energy and the RF to beam transfer efficiency, are, respectively,

$$R_{fe} = P_kT_k \ , \quad E_{rb} = B_e/R_{fe} \ . \quad (18)$$

The luminosity varies as q_b^2, T_b and as the pulse repetition rate f_r. Thus

$$L_{um} \propto q_b^2\,T_b\,f_r \propto \left(q_b^2T_b\,P_a\right)/P_kT_k \ . \quad (19)$$

We assume that the averge RF power is given, hence the luminosity is proportional to a parameter

$$L_{un} \equiv \left(q_b^2T_b\right)/P_kT_s \ . \quad (20)$$

The RF induced gradient, $E_r(t)$, the beam induced gradients plus the gradient at injection time, $E_b(t - T_i) + E_i$ and the fractional voltage variation, dE/E, where E is the gradient at injection time T_i and at the end of the beam pulse, all as functions of time, are plotted in Figs. 2, for $\tau = 0.505$, for several beam pulse widths. We used the NLCTA parameters: $L = 1.8\ m$, $T_f = 100\ ns$, $T_a = 198.0\ ns$, $s = 815\ V/pc/m$ $E_i = 50\ MV/m$. The charge per bunch, the efficiency, and the luminosity increase with increased pulse width as does the voltage deviation. If the beam pulse is much shorter than the fill time, the voltage deviation is low. But it increases rapidly as the beam pulse becomes a large fraction of the section fill time. Decreasing h reduces the voltage variation.

Plots of P_k, E_{rb} and L_{un}, as a function of h, with $\tau = 0.505$, $T_{bn} = 0.25$, are shown in Fig. 3. Plots of q_p, q_b, and dE/E, also as a function of h, are shown in Fig. 4. Plots of P_k, E_{rb} and L_{un}, as a function of T_{bn}, are shown Fig. 5, for a CG section. Plots of q_p, q_b, and dE/E, also as a function of T_{bn}, are shown Figure 6. The luminosity increases as the beam pulse width increases, as does the peak power and voltage deviation. The increase in peak power is the price we pay for the higher beam loading, but we can reduce the voltage deviation to zero.

Short pulse BLC with Modulation

We can reduce the voltage deviation by linearly decreasing the section input field, that is the square root of the RF input power, E_g, by a small amount starting at beam injection time. The effect of the modulation is illustrated in Fig. 7 which shows, as a function of T_{bn} the RF and the beam induced gradients with and without modulation. It is simpler to phase modulate. The equivalent phase variation is $cos\ \phi(t) = E_g(t)$. The cost is a small increase in RF power resulting in a small decrease in luminosity.

Beam loading compensation with modulation for a long pulse equal to or greater than the fill time, is reported on at this conference [6]. The accelerator section is pre-filled with a linearly increasing field prior to beam injection. The fractional initial input field, i.e. P_m, determines the ratio of the steady state gradient to the injection time gradient. We choose the current, hence the charge per bunch, so that, unlike for the short pulse, the steady state loaded gradient equals the gradient at injection time. The input field profile, the RF and beam induced gradients and the voltage deviation, all as a function of T_{bn}, are shown in Fig. 8. Also shown are the same plots for a beam loading compensated short pulse whoose duration was chosen to yield the same luminosity as that of the long pulse. Table 1 compares the parameters of the short pulse with that of the long pulse. The RF energy per pulse and the beam energy per pulse for the short beam pulse case are slightly less than half the corresponding values for the long pulse case. This is an advantage because there are limits on the klystron output energy and on the beam energy. Also the shorter pulse is less suceptible to beam break up. The increase in f_r for the shorter pulse does not effect beam breakup.

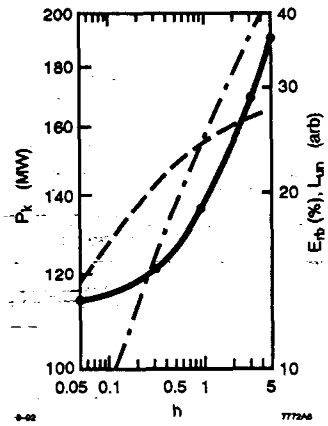

Figure 3. Peak power, P_k, RF to beam energy transfer efficiency, E_{rb}, Luminosity, L_{un}.

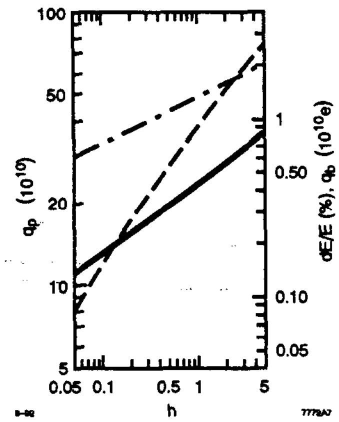

Figure 4. Charge per pulse, charge per bunch (dotdash) and voltage deviation (dash).

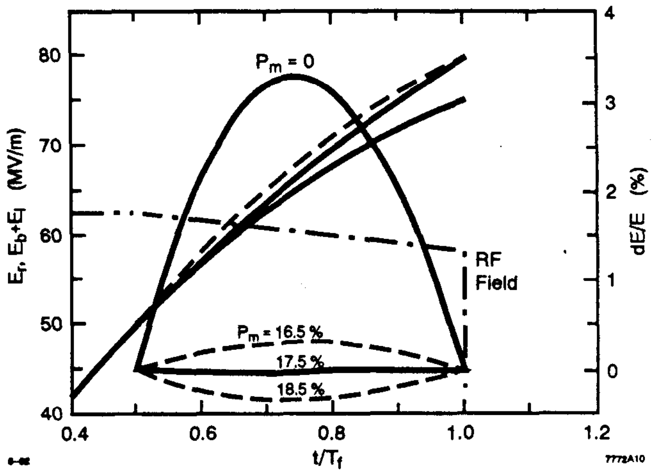

Figure 7. RF and beam induced (dash) gradients and voltage deviation, with and without modulation, versus time divided by fill time.

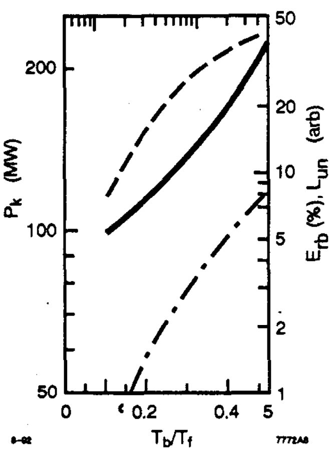

Figure 5. Peak power, P_k, RF to beam energy transfer efficiency, E_{rb}, Luminosity, L_{un} versus beam pulse width divided by fill time.

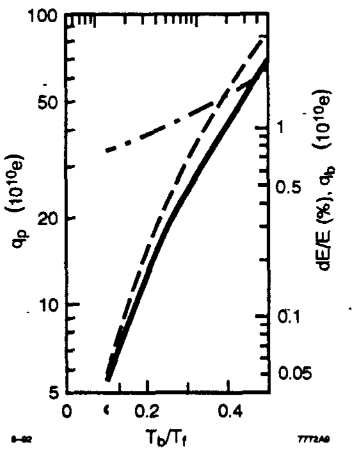

Figure 6. Charge per pulse, charge per bunch (dotdash) and voltage deviation (dash).

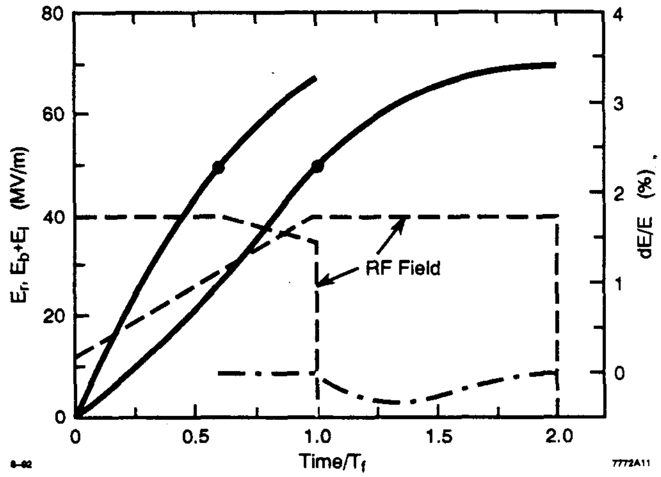

Figure 8. RF and beam induced (dash) gradients and voltage deviation (dotdash) versus time divided by fill time, for a long and a short pulse with equal luminosities.

Table 1. Comparison of short and long pulse parameters. Equal Luminosities.

T_b/T_f	P_k MW	E_{rf} MV/m	R_{fe} Ws	B_e Ws	E_{rb} %	q_b 10^{10}	L_{un}	P_m %
1.00	169	69	33.9	10.3	30.5	1.01	2.99	70.5
0.34	149	65	15.0	4.00	26.6	1.14	2.96	10.0
2.00	169	69	50.9	20.7	40.6	1.01	3.98	70.5
0.40	171	70	17.1	5.35	31.1	1.30	3.93	12.5

Conclusion

We derived the Beam Loading parameters for a linearly variable group velocity section with arbitrary output to input group velocity ratio, for a beam pulse shorter than the fill time. We showed that, for any beam pulse width, as the output to input velocity ratio decreases the voltage deviaton also decreases. Moreover, the voltage deviation can be reduced to zero with modest modulation (decrease) of the input field during the beam pulse. The smaller h and T_{bn}, the smaller the percentage modulation. We compared short pulse with long pulse modulation and showed that the RF energy per pulse and the beam energy per pulse for the short beam pulse case are slightly less than half of the corresponding values for the long beam pulse case and therfore the short pulse is disireable for high gradient accelerators.

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