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A Top Quark CP-violating Asymmetry in Supersymmetric Models

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ABSTRACT

The difference in the transverse energy distribution of leptons and antileptons from $t\bar{t}$ events at hadron colliders is a potential signal of non-standard-model CP violation. We investigate contributions to the asymmetry that may arise in supersymmetric models. The effect may be as large as a few times 10^{-3}.

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1. Introduction

Despite over twenty-five years of effort, nature has yielded surprisingly little information about the origin of CP violation. Our only window into this phenomenon is through the mixing and decays of the neutral K mesons. The standard theory of Kobayashi and Maskawa[1] appears to be consistent with the measured values of ϵ and ϵ'[2], but in order to investigate all of the possibilities we must look elsewhere. Much work has been devoted to the effects of CP violation in the electron and neutron electric dipole moments and in the mixing and decays of B mesons at a future B-factory. Recently there has been interest in the possibilities for observing CP-violating effects in top quark production and decay.[3–8]

One promising signature that can occur in $t\bar{t}$ production is an asymmetry in the energies of the leptons and antileptons emitted from the decaying tops and antitops.[8] This signature is particularly well-suited to hadron supercolliders, such as the SSC and the LHC, which can produce large samples of top quarks with controllable backgrounds. The asymmetry was considered previously in the context of a model with CP violation in the Higgs sector. In this letter we show that the effect may also arise in supersymmetric models.

The asymmetry is easy to understand if we first consider the helicities of the tops and antitops in $gg \to t\bar{t}$ production. At high energy, helicity conservation implies that gluons dominantly produce left-handed top quarks (t_L) with right-handed antitops ($\bar{t}_R$) or vice versa ($t_R\bar{t}_L$). However, near threshold, there is also substantial production of $t_L\bar{t}_L$ and $t_R\bar{t}_R$. These latter states go into each other under CP, so any asymmetry in their production rates is a signal of CP violation.

Next, we note that when a top quark decays leptonically, the charged lepton momentum is highly correlated with the top spin. A t_R will decay to an l^+ in the forward direction which is then boosted to a higher energy, while a t_L will decay to a less energetic l^+ in the backward direction. For antitops we have the same effect but with L and R interchanged. This implies that $t_L\bar{t}_L$ events produce relatively slow l^+'s and fast l^-'s, while $t_R\bar{t}_R$ events produce slow l^-'s and fast l^+'s. Thus, a difference in the production of $t_L\bar{t}_L$ and $t_R\bar{t}_R$ leads to a charge asymmetry in the lepton energy distributions which does not vanish when averaged over angle.

2. The supersymmetric model

We will examine this effect in supersymmetric models, which have several new possibilities for CP violation in the top quark sector. The relevant phases can occur in the supersymmetric Higgs mass, $\mu H_2 H_1$, and in the soft supersymmetry-breaking gluino mass, $m_\lambda \lambda^a \lambda^a/2$, and trilinear couplings, $m_{3/2} A_{ij} g_{ij} \tilde{u}^*_{iL} h_2 \tilde{q}_{jR}$, where g_{ij} are the up-quark Yukawa couplings and $m_{3/2}$ is the gravitino mass. Ignoring flavor mixing, these phases can be subsumed into a single phase ϕ occuring in the top quark-squark-gluino vertex:

$$\delta\mathcal{L} \;=\; i\sqrt{2}g\left[e^{i\phi}\,\tilde{t}^*_L \mathrm{T}^a(\overline{\lambda}^a t_L) \;+\; e^{-i\phi}\,\tilde{t}^*_R \mathrm{T}^a(\overline{\lambda}^a t_R)\right] \;+\; h.c.\,, \tag{1}$$

where the squarks can be written in terms of the mass eigenstates, $\tilde{t}_1$ and $\tilde{t}_2$, by the transformation

$$\begin{aligned} \tilde{t}_L &= \tilde{t}_1 \cos\alpha + \tilde{t}_2 \sin\alpha \\ \tilde{t}_R &= -\tilde{t}_1 \sin\alpha + \tilde{t}_2 \cos\alpha\,. \end{aligned} \tag{2}$$

The top squark mixing angle is given by

$$\sin 2\alpha \;=\; \frac{2m_t\,|A_{33}m_{3/2} \;+\; \mu\cot\beta|}{(m_2^2 - m_1^2)}\,, \tag{3}$$

with $\tan\beta = v_2/v_1$, the ratio of the Higgs vacuum expectation values.

The CP violating asymmetry is proportional to $(\sin 2\alpha \sin 2\phi)$ and vanishes for equal mass squarks. This combination of mixing angles also arises in the neutron electric dipole moment, so we must discuss the limits on this quantity arising from the experimental bound[9], $d_n < 1.2 \times 10^{-25}$. If we only consider the bound specifically on the top quark-squark-gluino CP phase, the largest contribution comes from Weinberg's three gluon operator.[10] Using the naive dimensional analysis and the low energy operator renormalization suggested by Weinberg, and sending the heavier squark mass to infinity, we obtain the bound,[11]

$$|\sin 2\alpha \sin 2\phi| \ < \ 1/4 \ , \tag{4}$$

for $m_t = m_\lambda = 150$ GeV and $m_1 = 100$ GeV. Alternatively, we can set this quantity to its maximum value, $|\sin 2\alpha \sin 2\phi| = 1$, and obtain a limit on the squark mass difference. For the same values of m_t, m_λ, and m_1, we find that

$$m_2 \ < \ 500\text{GeV} \tag{5}$$

will satisfy the experimental limit on the neutron electric dipole moment. It should be noted that these bounds are fairly loose due to uncertainties in the low energy operator renormalization scale and the use of naive dimensional analysis.[12]

On the other hand, a much stronger limit on the CP violating angles can be derived from the assumptions of minimal supergravity models, which we have not applied up to this point. In minimal supergravity,

$$A_{ij} g_{ij} \ = \ A g_{ij} \ , \tag{6}$$

which implies that all up quarks couple to the same CP-violating phase. With this assumption the largest contributions to the neutron electric dipole moment are from the light quark electric and chromo-electric dipole moments[13]. These can be translated into a limit in the top quark sector of

$$\frac{(m_2^2 - m_1^2)}{(150 \text{ GeV})^2} |\sin 2\alpha \sin 2\phi| \ < \ 10^{-1} \tag{7}$$

for $m_t = m_\lambda = 150$ GeV, $m_{\tilde{u}} = 100$ GeV. The assumption of minimal supergravity would impose severe constraints on any high energy CP-violating signatures involving the top quark.

3. $t\bar{t}$ asymmetry

We now begin by computing the following CP asymmetry:

$$\Delta N_{LR} = \left(\#(t_L\bar{t}_L) - \#(t_R\bar{t}_R) \right) / \left(\text{all } t\bar{t}\right) \tag{8}$$

for the parton subprocess $gg \to t\bar{t}$ at fixed center of mass energy $\sqrt{s}$. It is well known that the gluon-gluon fusion process dominates top quark production at multi-TeV energies, so we shall neglect the $q\bar{q}$ contribution throughout. The CP violation arises in the diagrams shown in Fig. 1. Since we are producing helicity eigenstates, we require that the amplitudes be real in order to interfere with the tree level result. Thus, we need both a CP-violating phase and an absorptive phase in each diagram.

The diagrams of Fig. 1(a) contribute if the center-of-mass energy is large enough to produce on-shell gluino pairs. The asymmetry can be formulated in terms of eight linearly independent form factors. For unpolarized gluons we obtain the contribution

$$\begin{aligned} \frac{d\sigma(t_L\bar{t}_L)}{d\cos\theta} - \frac{d\sigma(t_R\bar{t}_R)}{d\cos\theta} = \frac{9\pi\alpha_s^2\beta_t\,\delta}{64s} \sum_{\alpha=1}^{2}(-1)^\alpha\theta(\sqrt{s}-2m_\lambda)\Bigg\{ \\ \frac{1}{1-\beta_t\cos\theta}\Bigg[\frac{\beta_t}{\beta_\lambda}(1-\beta_\lambda^2)K_0^- - (1-\beta_t^2)K_1^- \\ + \beta_t\beta_\lambda\sin^2\theta(K_2^- - K_3^-)\Bigg] \\ + \frac{1}{1+\beta_t\cos\theta}\Bigg[\frac{\beta_t}{\beta_\lambda}(1-\beta_\lambda^2)K_0^+ - (1-\beta_t^2)K_1^+ \\ + \beta_t\beta_\lambda\sin^2\theta(K_2^+ - K_3^+)\Bigg]\Bigg\} . \end{aligned} \tag{9}$$

Here, θ is the center-of-mass production angle of the top quark, $\beta_i = (1-4m_i^2/s)^{1/2}$, the index α refers to the two squark mass eigenstates, and

$$\delta = \alpha_s \frac{m_\lambda m_t}{s\beta_t} \sin 2\alpha \sin 2\phi \ . \tag{10}$$

The form factors are given by

$$\begin{aligned}
K_0^\pm &= \frac{\beta}{2V^\pm} \ln\left(\frac{U^\pm + V^\pm}{U^\pm - V^\pm}\right) \\
K_1^\pm &= yK_0^\pm - \frac{1}{2}\ln\left(\frac{1+\beta}{1-\beta}\right) \\
K_2^\pm &= W^\pm K_0^\pm - \frac{1}{2}\ln\left(\frac{1+\beta}{1-\beta}\right) \mp \frac{1}{2}\cos\theta \ln\left(\frac{y+1}{y-1}\right) \\
K_3^\pm &= \left[\frac{2W^{\pm 2}}{\sin^2\theta} + \frac{(1-\beta^2)}{\beta^2}\right] K_0^\pm - \frac{W^\pm}{\sin^2\theta}\ln\left(\frac{1+\beta}{1-\beta}\right) \\
&\quad - \left[\frac{1}{2\beta} \pm \frac{1}{2} y\cos\theta \pm \frac{W^\pm \cos\theta}{\sin^2\theta}\right] \ln\left(\frac{y+1}{y-1}\right) \pm \cos\theta \ ,
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
U^\pm &= y \pm \beta\cos\theta \\
V^\pm &= \left(U^{\pm 2} + (1-\beta^2)(1-y)^2\right)^{1/2} \\
W^\pm &= y \pm \frac{1}{\beta}\cos\theta \ ,
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
y &= \frac{s + 2m_\alpha^2 - 2m_t^2 - 2m_\lambda^2}{s\beta_t\beta_\lambda} \\
\beta &= \beta_\lambda \ .
\end{aligned} \tag{13}$$

The diagrams of Fig. 1(b) contribute if the center-of-mass energy is large enough to produce on-shell top squark pairs. We obtain

$$\frac{d\sigma(t_L\bar{t}_L)}{d\cos\theta} - \frac{d\sigma(t_R\bar{t}_R)}{d\cos\theta} = \frac{9\pi\alpha_s^2\beta_t\,\delta}{64s}\sum_{\alpha=1}^{2}(-1)^\alpha\theta(\sqrt{s}-2m_\alpha)\Bigg\{ \left(\frac{1/81}{1-\beta_t\cos\theta}+\frac{10/81}{1+\beta_t\cos\theta}\right)\left[\frac{\beta_t}{\beta_\alpha}(1-\beta_\alpha^2)\overline{K}_0^- - \beta_t\beta_\alpha\sin^2\theta\overline{K}_3^-\right] + \left(\frac{1/81}{1+\beta_t\cos\theta}+\frac{10/81}{1-\beta_t\cos\theta}\right)\left[\frac{\beta_t}{\beta_\alpha}(1-\beta_\alpha^2)\overline{K}_0^+ - \beta_t\beta_\alpha\sin^2\theta\overline{K}_3^+\right]\Bigg\}\,. \tag{14}$$

The form factors $\overline{K}$ are the same as those given in (11) and (12), but with the roles of the squark mass and gluino mass interchanged. The practical effect of this is that the parameters (13) are replaced by

$$\begin{aligned}\overline{y} &= \frac{s+2m_\lambda^2-2m_t^2-2m_\alpha^2}{s\beta_t\beta_\alpha} \\ \overline{\beta} &= \beta_\alpha\,.\end{aligned} \tag{15}$$

Using these formulae we obtain the asymmetry ΔN_{LR} shown in Fig. 2 for $m_t = 150$ GeV and several different choices of the sparticle masses. In this figure, we choose the maximum value for the CP violating angle, $\sin 2\alpha \sin 2\phi = -1$, and we choose values of m_2 which are in accordance with Eq. (5). The asymmetry is generally dominated by the amplitudes of Fig. 1(a), which contain intermediate gluinos. This is easily seen in the dotdash curve with $m_\lambda = 210$ GeV. Below $E_{\rm cm} = 2m_\lambda$, only the diagrams of Fig. 1(b) contribute, while above $E_{\rm cm} = 2m_\lambda$, both sets of diagrams contribute. Essentially, the amplitudes of Fig. 1(b), with intermediate squarks, are suppressed by $1/N_c^2$. We also see that the asymmetry is reduced for increasing m_1 or decreasing m_2. It is possible to have an asymmetry in the production of $t_L\bar{t}_L$ and $t_R\bar{t}_R$ at the percent level.

4. Lepton asymmetry

As explained in the introduction, the CP violating polarization asymmetry gives rise to an asymmetry in the energy spectra of the charged leptons from top decay. At tree level in the top quark center of mass, the decay distribution of the charged lepton is simply

$$\frac{d^2\Gamma}{dE_l\, d\cos\psi} = \frac{d\Gamma}{dE_l}\frac{(1+\cos\psi)}{2}, \tag{16}$$

where ψ is the angle between the top spin and the lepton momentum, and $d\Gamma/dE_l$ is the unpolarized energy distribution.[14] When the top quark is boosted, (16) gives a correlation between the lepton energy and the top helicity. In Ref. 8 it was shown that the lepton energy spectrum can effectively analyze the top spin.

We are now prepared to calculate the observable asymmetry due to CP violation in the Higgs sector. To remove some effects of the longitudinal boost of the parton-parton collision, we present the asymmetry in the distribution of lepton transverse energy. To compute this, we fold the production cross sections for $t\bar{t}$ pairs of each helicity combination, including the asymmetric contributions of (9) and (14), with the decay distribution (16). In Fig. 3(a), we plot the average lepton transverse energy distribution at the SSC. We use a top mass of 150 GeV, $\sqrt{s} = 40$ TeV, and the "average" parton density functions of Diemoz *et al.*[15] The bulk of the top decay leptons have transverse energy (and also total energy) below 100 GeV and rapidity $|y| < 2.5$.

In Fig. 3(b) we present the CP violating lepton asymmetry,

$$\Delta N(E_T) = \frac{d\sigma/dE_{T,\ell^+} - d\sigma/dE_{T,\ell^-}}{d\sigma/dE_{T,\ell^+} + d\sigma/dE_{T,\ell^-}}, \tag{17}$$

at the SSC for a top mass of 150 GeV, $\sin 2\alpha \sin 2\phi = -1$, and for several different sparticle masses, satisfying Eq. (5). The asymmetry is typically of order a few times 10^{-3}. Note that the asymmetry is concentrated at higher lepton energies for larger gluino masses. We also present on the same plot an estimate of the perturbative non-CP-violating background asymmetry calculated in Ref. 8. It is considerably smaller than the CP violating effect and is essentially independent of lepton energy. Unfortunately, in models where (6) applies, the maximum CP violating effect is reduced by a factor of about 100, and the asymmetry would be very difficult to observe.

5. Discussion and conclusions

We have shown that supersymmetric models with CP violating couplings can produce an asymmetry in the charged lepton energy spectra occuring in $t\bar{t}$ production. This asymmetry can be of the order a few times 10^{-3}, and it is well above any perturbative background asymmetries. Thus, this CP violating effect might be observable in hadron supercolliders, which are expected to produce of order 10^8 top quark pairs per year. All of our plots have been for SSC energies, but the effect is comparable for the LHC.

In order to estimate the level to which this CP asymmetry can be measured, we must consider the effects of the top quark leptonic branching ratio and non-$t\bar{t}$ background events. The top quark decays about 1/9 of the time each to electrons and muons. However, for muons, a misalignment of the tracking system can produce a charge bias in the energy measurement, which could mimic the asymmetry we are looking for.[16] It remains to be seen whether this systematic error can be overcome. Fortunately, the energy measurement in a calorimeter depends negligibly on the charge, so this important systematic error cancels if one measures the asymmetry in the calorimeter response for electrons versus positrons.

The effects of non-$t\bar{t}$ backgrounds at the LHC have been studied by Cavanna, Denegri, and Rodrigo.[17] Presumably, a lepton isolation cut and a cut on the total transverse mass of jets can bring the backgrounds from $gg \to b\bar{b}$, $q\bar{q} \to W+$jets, and $W^{\pm}g \to t\bar{b}/b\bar{t}$ down to acceptable levels. Note that this last process is a potential source of a non-CP-violating lepton energy asymmetry. More detailed investigations are needed to predict precise limits, but it is reasonable to expect that the next generation of hadron colliders may be able to study the charge-dependent energy asymmetry of leptons at the 10^{-3} level.

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FIGURE CAPTIONS

1) Feynman graphs which produce CP violation in the process $gg \to t\bar{t}$.

2) The CP-violating asymmetry ΔN_{LR} in $gg \to t\bar{t}$. The asymmetry is computed for a top mass of 150 GeV, $\sin 2\alpha \sin 2\phi = -1$, and $m_\lambda = 150$ GeV, $m_1 = 100$ GeV, $m_2 = 500$ GeV (solid); $m_\lambda = 150$ GeV, $m_1 = 100$ GeV, $m_2 = 200$ GeV (dashes); $m_\lambda = 210$ GeV, $m_1 = 100$ GeV, $m_2 = 500$ GeV (dotdash).

3) (a) The transverse energy distribution (in GeV^{-1}) of leptons from the decay of top quark pairs produced at the SSC; (b) the charge asymmetry in this energy distribution due to CP violation, and due to non-CP-violating effects (dots). In this calculation, $m_t = 150$ GeV, $\sin 2\alpha \sin 2\phi = -1$, $m_1 = 100$ GeV, and $m_\lambda = 150$ GeV, $m_2 = 500$ GeV (solid); $m_\lambda = 150$ GeV, $m_2 = 200$ GeV (dashes); $m_\lambda = 210$ GeV, $m_2 = 500$ GeV (dotdash). In (a), the top curve contains all leptons, and the lower curves show the effects of cuts on the lepton rapidity, $|y| < 2.5$ and $|y| < 1$; (b) is computed with a rapidity cut $|y| < 2.5$.

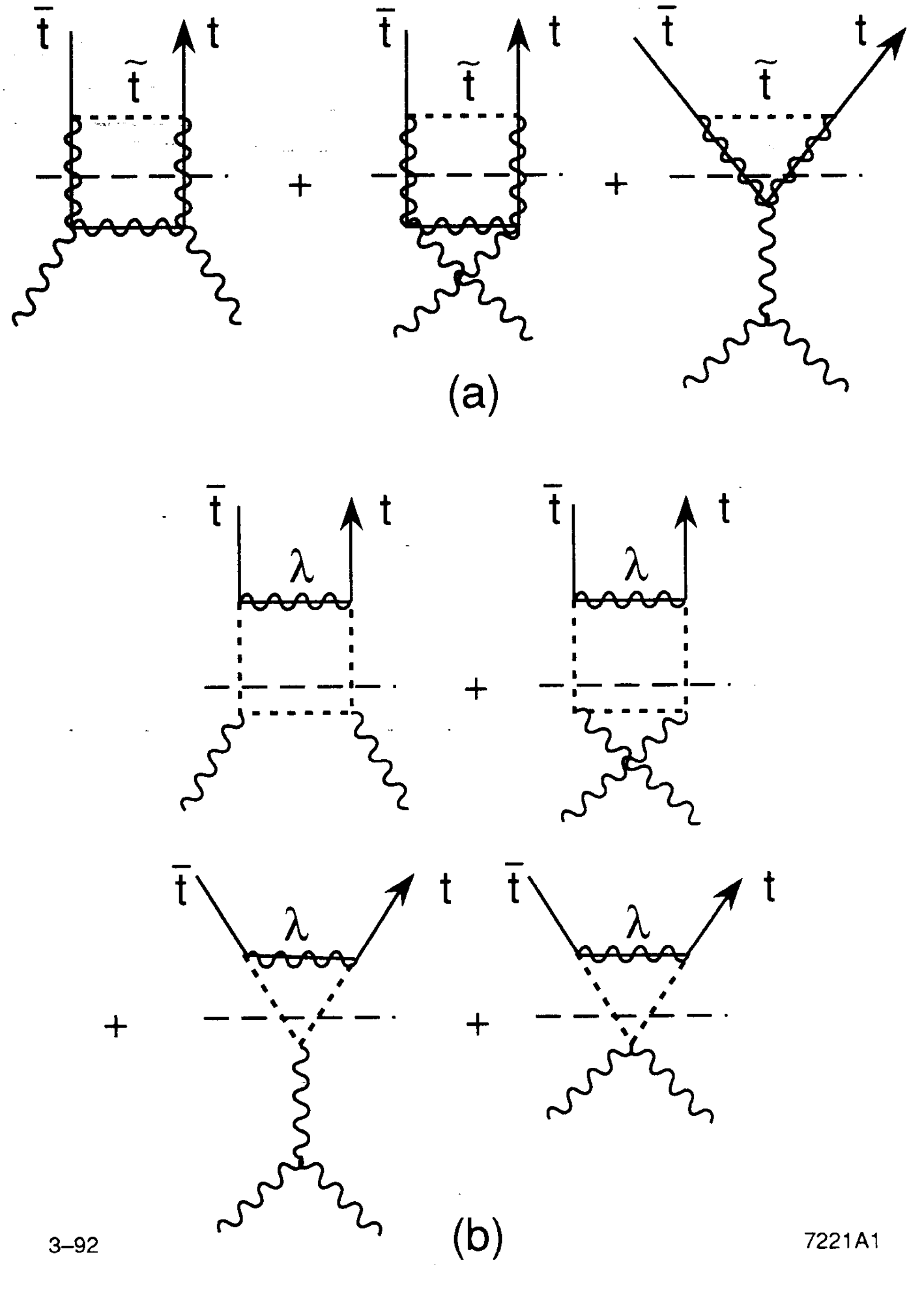

Fig. 1

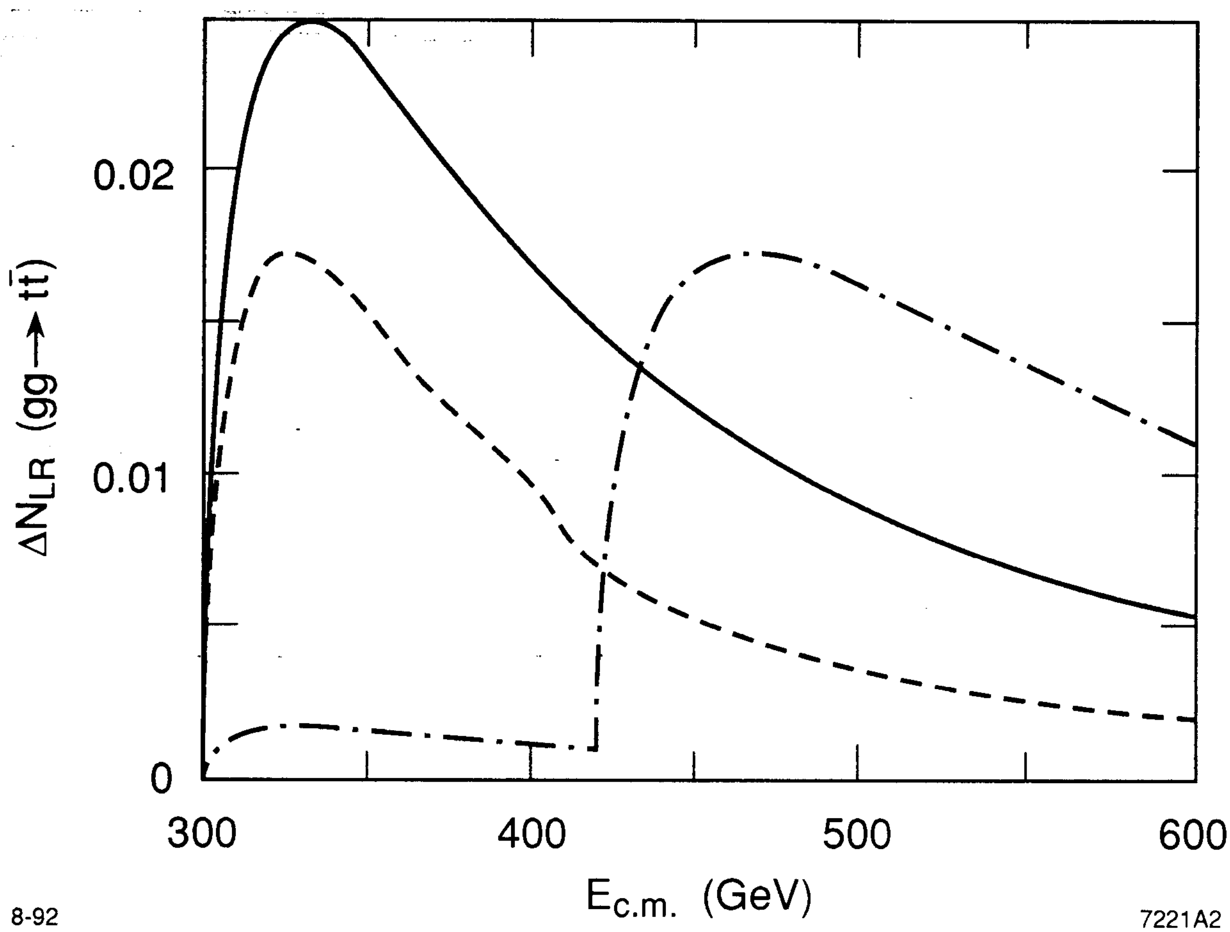

Fig. 2

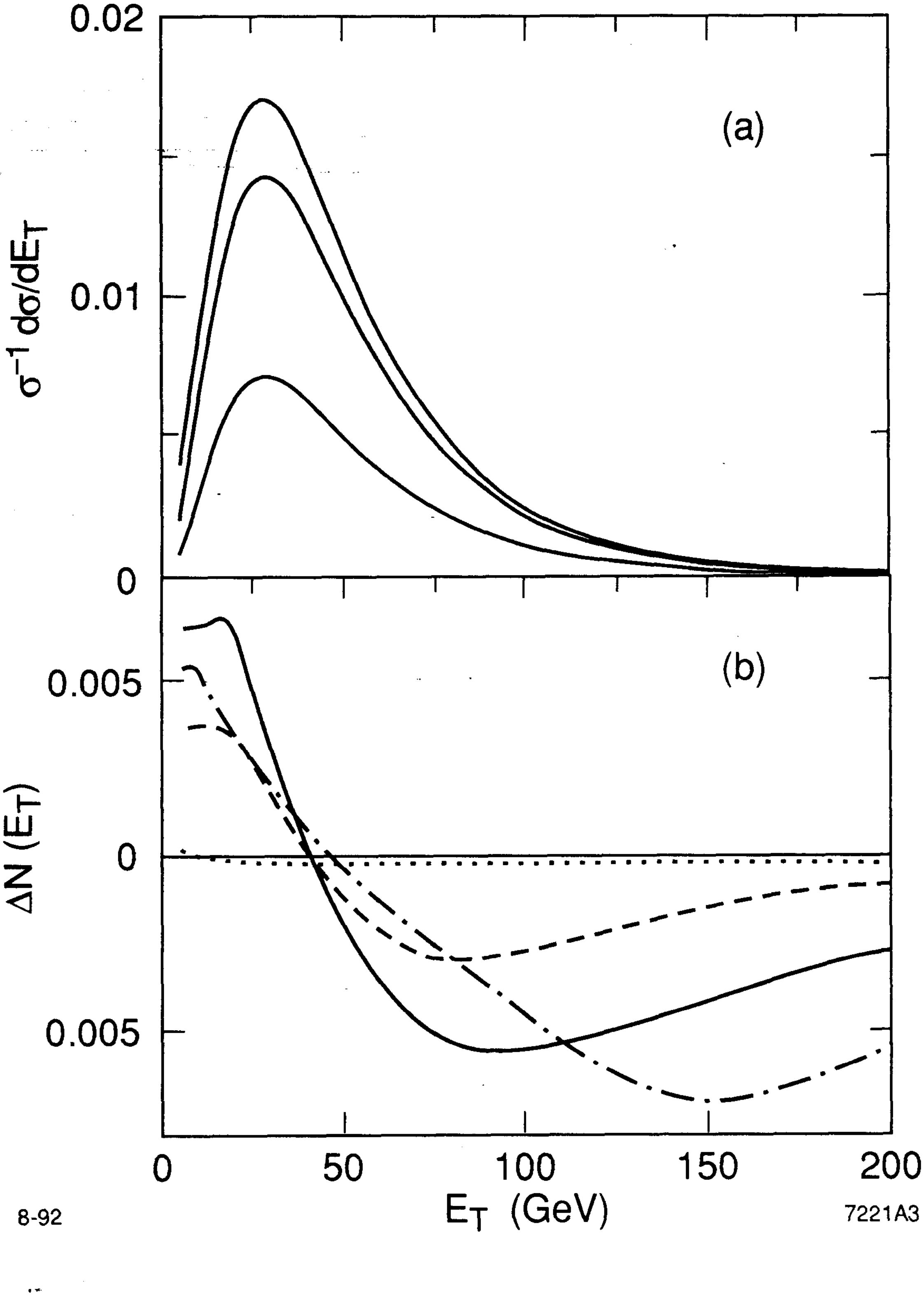
0.02
(a)
σ⁻¹ dσ/dE_T
0.01
0
(b)
0.005
ΔN (E_T)
0
0.005
0
50
100
150
200
E_T (GeV)
8-92
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Fig. 3