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ELECTROMAGNETISM FROM COUNTING*

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ABSTRACT

The fact that experimental accuracy is finite makes the measurement of particle positions and velocities non-local and often non-commutative even in a scale invariant theory. Applied to electromagnetic and gravitational phenomena, we argue that this leads to a relativistic action at a distance theory in which "fields" are simply a quasi-local interpolating concept extrapolated from macroscopic conservation laws. We sketch how this analysis could lead to classical field equations as a macroscopic approximation to relativistic quantum mechanics, but do not construct a formal proof.

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1. MEASUREMENT AS COUNTING

1.1 SCALE INVARIANCE

Physics is often characterized as a science of *measurement.* In practice the measurements are always subject to experimental error whether or not we are willing to recognize that fact. Unfortunately, as Segré often remarked, "You can't measure errors." When a physicist wants to make the most accurate measurement of some quantity that he can, he often tries to reduce it to a measurement of *time* recorded as the *number* of ticks of a standard clock. This is because in any situation where the numbered tick corresponding to the measurement is unambiguous, the accuracy of this part of the measurement is known. In the fortunate situation when measurements can be put into correspondence with the ordered integers, Segré's dictum refers to the laboratory protocol rather than to the numerical data.

The decision of the physics community to pick an *integer* as the reference *speed* by the convention

$$c \equiv 299\ 792\ 458\ m\ sec^{-1} \tag{1.1}$$

allows us to make contact with any system of units in which the ratio of any length to any time is a rational fraction. So long as there is no maximum or minimum quantity which theory and experiment in combination fix unambiguously, all such systems of units are freely interconvertible; they are *scale invariant.* Although the speed c serves as the upper limit for the transfer of *information* in both classical theory and any version of relativistic quantum mechanics for which the test is empirically meaningful, in dispersive media or for relativistic free particle deBroglie waves this unique speed is simply the geometric mean between phase and group velocity:

$$v_{ph}v_{gr} = c^2 \tag{1.2}$$

Hence, if we take the conventional step of setting $c = 1$, and call the rational fraction velocities measured in this system of units $\beta = v/c$, we have that

$$\beta_{ph}\beta_{gr} = 1 \tag{1.3}$$

1.2 THE COUNTER PARADIGM

We take as our basic paradigm for two distinct events the sequential firing of counters named "1" and "2" which are separated by a finite distance. Associated with each counter is a ticking clock. The clocks are synchronized by the Einstein or "radar distance" convention that the distance between any two counters is half the time it takes a light signal to go from one to the other and return. Assume counter 2 fires after counter 1 and that we wish to attribute these correlated firings to the passage of a "particle". Further assume that when 1 fires, a light signal is sent from 1 through the position of 2 and on to a third counter 3 at position 3' on that line; it is reflected back from 3' to 2 and arrives at the same time as the particle. Then, if the (radar) distance from 1 to 3' is called $x_{13'}$ and from 3' back to 2 is called $x_{3'2}$, the velocity of the particle in units of c is given by

$$\beta_{12;3'} = \frac{x_{12;3'}}{t_{12;3'}} = \frac{x_{13'} - x_{3'2}}{x_{13'} + x_{3'2}} \tag{1.4}$$

In terms of the two integers $r_1 = x_{13'}, r_2 = x_{3'2}$, the square of the interval τ_{12} which is invariant under Lorentz boosts and the square of the dilation factor γ_{12} are given by

$$\tau_{12}^2 = t_{12}^2 - x_{12}^2 = 4r_1r_2;\ \gamma_{12}^2 = \frac{1}{1-\beta_{12}^2} = \frac{(r_1+r_2)^2}{4r_1r_2} \tag{1.5}$$

If counter 3 is not at rest with respect to counter's 1 and 2 but is moving along the same line, moving from position 3 to position 4 in the time it takes a light signal to move from 3 to 3' and back from 3' to 4, using the same notation

$$\beta_{34;3'} = \frac{x_{34;3'}}{t_{34;3'}} = \frac{x_{33'} - x_{3'4}}{x_{33'} + x_{3'4}} = \frac{r_3 - r_4}{r_3 + r_4} \tag{1.6}$$

and

$$\tau_{34}^2 = t_{34}^2 - x_{34}^2 = 4r_3r_4; \;\; \gamma_{34}^2 = \frac{1}{1-\beta_{34}^2} = \frac{(r_3+r_4)^2}{4r_3r_4} \tag{1.7}$$

It follows immediately that the velocity of 3 relative to 1, called β_{13}, is given by

$$\beta_{13} = \frac{\beta_{12} + \beta_{23}}{1 + \beta_{12}\beta_{23}} \tag{1.8}$$

and the usual Lorentz boost transformations can be easily constructed.

Note that the position 3' to which we referred our velocities has dropped out. Note also that the accuracy to which we can check the Lorentz invariance of our results depends on the implicit assumption that we can compare our clocks to the nearest tick using some standard clock at rest with respect to some standard reference system such as a laboratory at rest with respect to the 2.7^oK cosmic background radiation. This puts an upper limit on the velocities we can measure because at high velocities this background is Doppler shifted to become hard gamma rays which will destroy any macroscopic clock. In principle, at least, these should also be included in any second quantized field theory calculation involving sufficiently high energies.

The lower limit on the velocities we can discuss is set by the fact that, if r_i^0, r_j^0 are integers with no common factor, then

$$\beta_{ij} = \frac{r_i - r_j}{r_i + r_j}; \;\; r_i = N_{ij}r_i^0; \;\; r_j = N_{ij}r_j^0 \tag{1.9}$$

is invariant to any *upward* change in scale, but scale invariance is broken once we try to go below $N_{ij} = 1$ for any pair of events we consider. The velocity *resolution* implied is

$$\Delta\beta = \frac{1}{r_i^0 + r_j^0} \tag{1.10}$$

We can only consider the finite set of rational fraction velocities this increment allows and Lorentz boosts constructed from them allowed by this constraint as *experimentally meaningful* no matter how this limit is postulated or determined.

1.3 MASS RATIOS FROM ELASTIC SCATTERING

In particle physics, we need to measure not only the velocity of a particle, but also its velocity change in scattering processes. The paradigmatic case is when we collimate a particle beam by two counters 1,2 where 2 is the *entrance counter* into the scattering region and detect a scattered beam at an angle θ from this direction in a second counter telescope 1'2' with 1' the *exit counter* from the scattering region. To some known or estimated experimental accuracy all 4 counters lie in the same plane, which is called the *scattering plane. Elastic scattering* is defined as the case when the entrance and exit velocities, β, are the same to some velocity resolution $\Delta\beta$. If we assume that we can measure the length of the entrance telescope x_{12} to some accuracy Δx and the time interval between the two firings t_{12} to some accuracy Δt, we must *also* know the correlation error between these two measurements in order to calculate the velocity resolution. We do not attempt to give a general formula, but simply assume that our *context sensitive* uncertainty is summarized by saying that we know two integers b_1, b_2 with an uncertainty of 1. Then

$$\beta = \frac{x_{12}}{t_{12}} = \frac{x_{1'2'}}{t_{1'2'}} = \frac{b_1 - b_2}{b_1 + b_2} = (b_1 - b_2)\Delta\beta \tag{1.11}$$

The two lines which define the angle θ also specify a region which we will call a *vertex* that lies in the scattering volume, and which is a distance L_V from either counter with an uncertainty ΔL_V. Let $N_b(b_1 + b_2)$ be the nearest integer to $L_V/\Delta L_V$. Since we have assumed that we know b_1 and b_2 to the nearest integer, we can define the integral distance from 2 or 1' to the vertex along the entrance and exit lines as $R_b = N_b(b_1 + b_2)$.

A second distance we can measure using our beam of velocity β as the integer calibration is the distance from the entrance counter 2 to the exit counter 1'. We call this $R_c = N_c(b_1 + b_2)$. We have defined an isosceles triangle with vertex angle $\pi - \theta$, sides N_b and base N_c, assuming that the distances are long enough in terms of our beam resolution to define the two scale factors to the nearest integer. These two distances determine the scattering angle because

$$N_b^2 sin^2\frac{\theta}{2} = N_b^2 - \frac{1}{4}N_c^2 \tag{1.12}$$

The velocity change $\beta_\perp$ is in the direction perpendicular to the base of the triangle and is of magnitude

$$\beta_\perp = \beta(1 - cos\ \theta) = 2\beta sin^2\frac{\theta}{2} \tag{1.13}$$

So far we have not given an explanation for the scattering. We now add to our paradigm two additional counter telescopes 34 and 3'4' set at scattering angle θ' with the bisectors of the two scattering angles pointed to the same vertex. Assuming this to be at the center of the scattering volume, 21'43' lie on a circle of radius $N_b' = N_b$. We assume that both counter telescopes have the same velocity resolution $\Delta\beta$. We further assume that the beam defined by 34 and 3'4' has the same speed as that defined by 12 and 1'2'; note that N_c' is parallel to N_c. If we find that particles are scattered *in coincidence* and that $N_c' = N_c$ we say that the two beams contain particles with the same *mass.* If $N_c' \neq N_c$, we say that the ratio of the masses is inverse to these distances, i.e.

$$if\ \ \beta_{12} = \beta_{1'2'} = \beta_{34} = \beta_{3'4'} \quad then\ m'N_c' \equiv mN_c \tag{1.14}$$

Empirically, we find that for two beams of the same or different composition this ratio is independent of the magnitude of their common speed, justifying our assumption that "mass" is an invariant property of the particles and not of their state of motion.

1.4 QUANTIZED LINEAR AND ANGULAR MOMENTUM

In our elastic scattering paradigm, the velocity resolution $\Delta\beta = 1/(b_1 + b_2)$, was set by the temporal and geometrical resolution of our entrance and exit counter telescopes, independent of the dimensionless radius N_b of our scattering chamber referred to the velocity $\beta = (b_1 - b_2)\Delta\beta$. We have measured the dimensionless distance N_c of the straight line segment between 2 and 1' in the same units. This is equivalent to saying that if we send a light signal from 2 to the vertex and back to 1', it will arrive at the same time as a particle which moves from 2 to 1' with velocity β, giving us the relationship:

$$\beta = \frac{b_1 - b_2}{b_1 + b_2} = N_c/2N_b = \frac{k_1 - k_2}{k_1 + k_2} = \frac{(N_b + \frac{1}{2}N_c) - (N_b - \frac{1}{2}N_c)}{2N_b} \tag{1.15}$$

where we have introduced the integers

$$k_1 = N_k(N_B + \frac{1}{2}N_c); \quad k_2 = N_k(N_B - \frac{1}{2}N_c) \tag{1.16}$$

We see that our measurement of mass implies a second set of integer parameters k_1, k_2 defined in terms of velocity *change* which are related to the linear beam velocity parameters b_1, b_2 by equating the two velocities $\beta(b_1, b_2) = \beta(k_1, k_2)$. This is equivalent to the constraint on these four integers

$$b_1 k_2 = b_2 k_1 \tag{1.17}$$

Note that the velocities $\beta(b_1, b_2)$ and $\beta(k_1, k_2)$. could be defined on independent scales, but that our definition of mass ratios ties these two scales together.

Once we have realized that there is a distinct experimental resolution implied by the invariance of mass ratios in elastic scattering, we can use it to quantize *mass* in terms of experimental resolution by defining $\Delta m = 1/(k_1 + k_2)$. Note that we have not defined mass in *absolute* terms but only relative to some standard particulate mass. To relate it to macroscopic mass standards requires Avogadro's number, or some conceptual equivalent, which we will not discuss here. However this is done, once we have quantized mass, we can relate it to energy, momentum and velocity in a Lorentz invariant way by

$$E = (k_1 + k_2)\Delta m; \; P = (k_1 - k_2)\Delta m; E = \beta_{12} P$$

$$E^2 - P^2 = s_{12}\Delta m^2 = 4k_1 k_2 \Delta m^2; \; \gamma_{12}^2 = \frac{1}{1-\beta_{12}^2} = \frac{(k_1+k_2)^2}{4k_1 k_2} \tag{1.18}$$

Then the mass ratio measurement we have described can be derived from the translational invariance of the formalism and amounts to a relativistic version of Mach's definition of mass ratios from Newton's Third Law.

Once we have introduced the concept of mass, energy and momentum, we see that what "happens" in our paradigmatic scattering experiment is momentum transfer between two particles which occurs somewhere in the scattering volume. So far as the measurement process goes, this is "relativistic action at a distance" —*relativistic* because we have established the Lorentz invariance of the description of the measuring process. Momentum is conserved when the process is completed, but whether it is localizable in the intermediate steps is not under direct operational control. We emphasize that, once we have "quantized" the macroscopic measurement process by introducing the limitations on experimental accuracy in terms of integers, this "non-locality" is inevitable.

Our paradigmatic scattering experiment allows us to introduce a second conserved quantity, which is the angular momentum about the vertex defined by the intersection of our beam lines. Consider the motion of a particle which goes directly from the entrance to the exit counter along the line 21' taking N_c potentially observable steps. The perpendicular distance from the vertex is fixed and its square is given by $N_b^2 - \frac{1}{4}N_c^2$. Since the point takes the same time to move from n_c to $n_c + 1$ (where $0 \leq n_c < N_c$), the area swept out per step is the area of a triangle with this height and base 1 and is constant. This is a special instance of our quantized version of Kepler's Second Law for a particle moving past a center with constant velocity.

In our scattering paradigm, the maximum distance the particle can go in the scattering volume on its way from 2 to 1' is $2N_b(b_1 + b_2)$ while the minimum distance it can go is $N_c(b_1 + b_2)$, so the ratio $N_b/N_c \equiv J$ defines a scale invariant parameter independent of our velocity resolution. Then the total area swept out at constant velocity by the line to the particle as it moves the distance N_c is

$$N_c^2(J^2 - \frac{1}{4}) = N_c^2 L(L+1); \; L \equiv J - \frac{1}{2} \tag{1.19}$$

where we have introduced the parameter L to facilitate contact with relativistic quantum mechanics.[1] The area per step is just $L(L+1)$ and vanishes when the scattering angle is zero, corresponding to $N_c = 2N_b$. For N_b fixed, we can define $2L+1$ values of an integer parameter ℓ_c in the range $-L \leq \ell_c \leq +L$ and the same number of scattering angles $\theta(n_c)$ defined by

$$\ell_c = [L(L+1)]^{\frac{1}{2}} sin(\theta(n_c) + \frac{\pi}{2}) \tag{1.20}$$

We see from these familiar relationships that we have gone from our elastic scattering and mass ratio paradigm to a scale invariant definition of angular momentum per unit mass independent of the mass scale, using only the geometrical arrangement of counter telescopes and the correlated velocity resolutions. We could go on from this to a definition of quantized Mandelstam invariants and relativistic particle kinematics to replace the continuum version[2] and will do so elsewhere. All we need note here that is until we break scale invariance by an absolute definition of the unit of mass *or* of angular momentum, we still can talk about a quantized, but scale invariant, theory. Quantized linear and angular momenta are conserved in elastic collisions between any pair of particles in any Lorentz frame provided only we specify their states in terms of three of the four parameters b_1, b_2, k_1, k_2 for one particle and three of the four parameters b_3, b_4, k_3, k_4 for the other particle defined and constrained as we have spelled out above.

2. CLASSICAL ELECTROMAGNETISM AND WEAK GRAVITY

The scale invariant quantization of linear and angular momentum based on velocity resolution is, so far as we are aware, novel. Unfortunately, working it out used up the time available for preparing this paper, so the actual application to classical electromagnetism and weak gravitation will have to be deferred to another occasion. We intend to extend our description of scattering to piecewise continuous straight line trajectories obeying the quantized but scale invariant conservation laws we developed in chapter 1. We could then *define* force per unit mass as impulsive change of momentum and *define* "fields" as a theoretical construct which provide a quasi-local model for that momentum change. Fields *interpolate* conservation laws for linear and angular momentum that, from an operational point of view, can only be given empirical content by measuring changes in momentum of the "sources" and "sinks" of the radiation. Since these measurements necessarily involve *finite* space and time intervals they are necessarily non-local and often non-commutative.

Historically, our approach is related to that of Bohr and Rosenfeld, who showed that the uncertainty in the measurement of electromagnetic fields due to the quantum uncertainty in their detection can be used to derive the commutation relations between $\mathcal{E}$ and $\mathcal{H}$ that are usually obtained by second quantizing the fields. It is also related to the "relativistic action at a distance" theory that Feynman developed for classical fields in his graduate work with Wheeler, but unfortunately did not extend to the quantum case in the way we propose here. He came close in an unpublished proof of the Maxwell Equations starting from Newton's Second Law and the non-relativistic commutation relations for position and velocity which was reconstructed by Dyson after his death[3]. We have discussed elsewhere why this proof is valid in bit-string physics.[4,5]

What became apparent in the course of writing the paper you are reading is that there are four different types of *non-locality* involved in the measurement of the momentum changes usually attributed to the classical fields. They are operationally distinct, but require more than one type of measurement before they can be assigned properly to one or another of the various possible fields.

1) The first type of non-local interaction is an acceleration of one particle toward a second along the line of centers; the three examples which concern us are attraction or repulsion between two electric charges (Coulomb interaction) or attraction between two masses (Newtonian gravitation). These are "local" because the change in position and velocity of a particle starting from rest measures an average force. Newton attributed this acceleration to gravity. If we measure *only* momentum change in a straight line starting from rest, we cannot tell the gravitostatic field from an electrostatic field. In the case of gravitation, Galileo also used circular motion by measuring the time it takes a body to fall some distance from rest compared to the time it takes a pendulum of the same length to swing to the vertical through a small arc.[6] His measurement provides a geometric and scale invariant definition of local acceleration that, subsequent to Newton, can be viewed as a dynamical measurement of π.[7]

2) We can measure a *velocity dependent* interaction which defines a radius of curvature proportional to momentum. This can be circular motion about a gravitating or charged center, or motion of a charged particle in a magnetostatic field. Magnetostatic and gravitostatic cases can be distinguished once we have test charges and can correlate attraction and repulsion in the electrostatic case and with the right or left handed curvature of the circle with respect to the direction of the magnetic field. The measurement of a radius of curvature requires at least three counters not in a line, and can be velocity dependent, so is less "local" than the first case. We can extend the discussion to elliptical orbits with these three pieces of information, if the context allows.

3) When we go to scattering, we go from elliptical to hyperbolic trajectories and the type of non-local measurements we discussed in the first chapter. In themselves these do not distinguish electrostatic from gravitostatic cases without further ex-

perimentation, which can be quasi-local. In general, for the electromagnetic case, the trajectories we encounter are no longer confined to simple momentum transfer but can include "emission" and "absorption" of radiation. These effects can be measured quasi-locally and used to derive Maxwell's equations for a hypothetical field connecting sources and sinks. The corresponding detection of gravitational radiation remains a hard sought goal.

4) In the case of Mercury, we find that the gravitostatic orbit analysis fails to predict the observed perihelion precession. The direction of precession compared to the direction of orbital motion defines a chirality and hence indicates that angular momentum is being transferred. That this precession is a factor of 6 greater than that predicted by the relativistic mass increase near perihelion shows that the quanta of the weak gravitational field have spin 2, as we observed in our presentation at PIRT I. This detection system is not only "non-local" and macroscopic but literally astronomical in size.

We have found that these effects are somewhat difficult to disentangle by operational analysis. Nevertheless, we are confident that we can extend the approach used in Chapter 1 to a general treatment of conic section trajectories in that scale invariant but quantized context. We can characterize the strength of the attraction or repulsion by the velocity at perihelion for either closed orbits or hyperbolic trajectories (scattering). Our treatment of differs from the conventional discussion because it is *relativistic* as well as quantized. Consequently we must restrict our interaction strengths to those which do not produce perihelion and/or asymptotic velocities greater than c. This still does not break scale invariance.

Whatever the phenomenon we choose to use to quantize momentum and angular momentum measurements we find that position and momentum measurements in the same direction and that three angular momentum measurements not in the same plane do not commute. Therefore the Feynman-Dyson derivation of the Maxwell equations can be carried through, showing that it can be thought of arising just from limitations in the accuracy of measurement without invoking quantum mechanics explicitly. The corresponding derivation of the weak field Einstein equations should follow in the same way, but we have not been able to prove it in time for this paper.

The study of Rutherford scattering allows us to discover that *electric charge* is quantized, using chemically identified ions and counter measurements. This relates charge quantization to angular momentum quantization by the scale breaking length $r_{Bohr} = \hbar^2/me^2$, even in a non-relativistic theory. Because we are precluded from discussing orbital or escape velocities which exceed c, this also defines the dimensionless scale constant $\alpha = e^2/\hbar c \approx 1/137$, the QED scale length $r_{Compton} = \alpha r_{Bohr} = \hbar/mc$ and the nuclear scale length $r_{nuc} = h/m_\pi c \approx \frac{1}{2}\alpha r_{Compton} = e^2/2mc^2$. Hence, in any relativistic theory specified by c, the quantization of charge e, or the absolute quantization of angular momentum $\hbar$ or pair creation specified by $h/2mc$, or nuclear size specified by $e^2/2mc^2$, or the Planck mass $[\hbar c/G]^{\frac{1}{2}}$ *break scale invariance.*

3. BREAKING SCALE INVARIANCE BY DEBROGLIE WAVE INTERFERENCE

3.1 QUANTUM MECHANICAL MASS QUANTIZATION

Bastin once remarked[8] that the basic quantization is the quantization of mass. As we saw in the first two chapters, it takes some work to see how this scale breaking is accomplished in terms of a realistic analysis of laboratory measurements starting from the counter paradigm and classical field experiments. But by making use of a specific quantum phenomenon, we can get there more quickly. A detailed discussion lies outside the scope of a paper of this length, but will be presented elsewhere.[9]

Assume that some source of moving particles with uniform mass illuminates a collimator and counter telescope which restricts the emerging beam to some measured velocity β with appropriate accuracy. Let the beam be incident on a double slit with spacing w followed by a detector array a distance D behind the screen. We find a double slit interference pattern. The spacing s between the interference maxima *measures* the deBroglie wavelength $h/P = \Lambda$ in terms of laboratory length standards thanks to the relation

$$\Lambda = \frac{ws}{D} \tag{3.1}$$

We abstract from this experience the postulate that counters recording particulate events due to particles with uniform velocity can produce correlated firings only when separated by an integral number of deBroglie wavelengths. The positions of the interference maxima then follow from the assumption that the paths from the two slits to the detector differ by an integral number of deBroglie wavelengths. Of course a more detailed discussion is required to explain the line shape and other diffraction phenomena.

We are now able to extend our measurement of deBroglie wavelength to a measurement of mass ratios. Assume we have a second source producing another type of particle which, using the same geometrical arrangement, produces the same velocity but a different spacing s' between the interference maxima. Then a measurement of the distance s' to the same maximum amounts to a *measurement* of the mass m' of the particles in this beam relative to the mass m because, in our theory,

$$m' = [\frac{s}{s'}]m \tag{3.2}$$

Note that neither the concept of mass (m) nor the concept of momentum (P) have acquired *absolute* significance as yet; note that "h" also remains undefined. Note also the equivalence in form of the defining paradigm to the operational definition of mass ratios developed in Chapter 1. Both use comparison at the *same* velocity so that relativistic kinematics is avoided at the level of the paradigm.

Although we now have invoked empirical evidence for "quantization" of mass which correlates with the quantized masses of chemistry, we have yet to relate this to "counting" in a fundamental way. To do this we postulate that there *is* a limit to the local accuracy of our measurements set by the shortest length Δl and the longest time T_x which we can, with confidence, *count* to the nearest integer. Then there will be a best velocity resolution given by $\Delta\beta = \Delta l/T_x$. Since our mass measurement paradigm rests on the comparison of velocities with our standard mass m, the maximum mass to which we can give experimental meaning under these circumstances is given by $M_x = m/\Delta\beta$. This in turn allows us to define a minimum unit of mass $\Delta m = m^2/M_x$.

In discussing space-time measurements in Chapter 1, we found that two light-cone coordinates r_1, r_2 sufficed to specify the basic Lorentz invariants. Once we have quantized mass, we can play the same game in momentum-energy space by defining two integers k_1, k_2 in terms of any system of particulate energy and momentum measurements by

$$E = (k_1 + k_2)\Delta m;\ P = (k_1 - k_2)\Delta m; E = \beta_{12} P$$

$$s_{12} = 4k_1k_2;\ \gamma_{12}^2 = \frac{1}{1-\beta_{12}^2} = \frac{(k_1+k_2)^2}{4k_1k_2} \tag{3.3}$$

Our basic postulate that events can take place only when they are separated by an integral number of deBroglie wavelengths has the immediate consequence that two masses, each moving with constant velocity with respect to a common center have equal and opposite momenta when the motion starts from that center. Suppose particle 1 with mass $m_1\Delta m$ is r_1 Compton wavelengths h/mc from the center and has velocity β_1. By hypothesis it took a time $t_1 = r_1/\beta_1$ to reach r_1. If this is the place where an event can occur it will be $n_1 = p_1r_1$ deBroglie wavelengths from the center. Hence $p_1r_1/t_1 = \beta_1p_1 = e_1 = n_1(t)/t$ independent of the time. Consequently if particles 1 and 2 are $n_1(t)$ and $n_2(t)$ deBroglie wavelengths from the center at some time t, the ratio $n_1(t)/n_2(t) = e_1/e_2$ will remain the same until their energies, momenta, or velocities change. But by our definition of mass ratios, the ratio n_1/n_2 will be in inverse ratio to their masses. Hence if we define

$$R_{12} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}; \; p_{12} = \frac{m_2 p_1 - m_1 p_2}{m_1 + m_2} \tag{3.4}$$

the first quantity will move with constant velocity, and the second will remain constant with respect to any arbitrary origin. This allows us to describe a two-body system as a single particle of mass $m_{12} = m_1 m_2/(m_1 + m_2)$ at a distance $r_{12} = r_1 - r_2$ from a fixed center in the usual way. As we extend the theory to more and more phenomena, we may be able to revise the limits defining mass quantization upward or downward by indirect argument, but until we hit some new phenomenon which gives a different kind of maximum or minimum quantity, our quantization remains determined by our technology, and in that sense our theory is *still* scale invariant.

In conventional theories, this momentum conservation law is derived from "translational invariance". We emphasize that in our theory, it follows directly from mass quantization and our definition of mass ratios in terms of deBroglie wave double slit interference. In the conventional theories mass-ratios can be defined *either* from macroscopic 3-momentum conservation, as advocated by Mach, *or* by deBroglie wave interference. Then their equivalence requires a separate postulate. Our approach removes this potential source of ambiguity by making deBroglie wave interference the basic phenomenon.

If the two directions of motion do not pass through a common center and the two speeds are not the same, they obviously define a couple, and hence an angular momentum, *around* some common center. For free particle motion the angular momentum so defined is also conserved. Thus, as in conventional theory, angular momentum conservation follows from linear momentum conservation.

3.2 ANGULAR MOMENTUM QUANTIZATION AND CONSERVATION

Quantizing angular momentum in terms of the absolute unit $\hbar$ rather than the scale invariant angular momentum per unit mass used for Kepler's Second Law, does in fact *break* scale invariance, as we now show. We proved above that the angular momentum of a free particle moving past a center, or the relative angular momentum of two free particles about a center, is a constant. Change in angular momentum can occur when either the direction or the magnitude of the velocity changes, or both.

As our paradigm, we consider a *scattering region* with diameter $r_s h/mc$ which a free particle enters at counter 1, exits at counter 2, and traverses with velocity β_{12}. If the time it takes a light signal to cross the region is $t_2 - t_1 = r_s$, and the time it takes the particle to cross is $t_1 + t_2$, we define

$$\beta_{12}^t = \frac{t_2 - t_1}{t_1 + t_2} = \frac{r_s}{T_{12,3}} = \frac{r_s}{r_{13} + r_{32}} \tag{3.5}$$

where r_{13} is the distance from counter 1 to reference counter 3, r_{32} is the distance from counter 2 to counter 3, and $T_{12;3}$ is the time it takes a light signal emitted from position 1 when the particle fires counter 1 to go from counter 1 to counter 3 and trigger a light signal from 3 to 2 which arrives at the same time that the particle fires counter 2.

In practice, we must *calibrate* our scattering chamber and counters to insure that the particle also had velocity β_{12} *before* it entered the scattering region and *after* it leaves the scattering region. One way to do this is to add a counter 1' upstream from the entrance counter 1 and 2' downstream from counter 2, all four counters lying on the same line. Then our calibration beam is defined by the requirement

$$\beta_{12}^t = \frac{r_{1'1}}{t_{1'1}} = \frac{r_s}{t_s} = \frac{r_{22'}}{t_{22'}} \tag{3.6}$$

Clearly either 1' or 2' could serve the same function as 3 in the initial description.

Now we have four counts rather than 3, and a consistency check. This is useful for eliminating the false counts usually called "background".

Unfortunately, in high energy physics, life is not so simple. We have guaranteed that the emerging particles have the same velocity as the incident particles, but our initial paradigm relied on the fact that particles with the same velocity can have different masses and hence different quantum interference effects. So we need to make sure that the emerging beam has the same mass as the incident beam. To do that, in addition to our counter telescopes 1'1 an 22', we can add devices that measure energy and momentum. For instance, if the particles are charged the momentum is proportional to the radius of curvature of the trajectory in a constant magnetic field; if they are stopped in a calorimeter, the heat rise is proportional to the energy. We assume all these checks have been made. We summarize the result by assuming that we end up with two space-time integers r_1, r_2 and two momentum-energy integers k_1, k_2 (as defined in Eq. 3.3) which satisfy the constraint

$$\frac{r_1 - r_2}{r_1 + r_2} = \beta_{12} = \frac{k_1 - k_2}{k_1 + k_2} \tag{3.7}$$

and hence

$$(r_1 + r_2)(k_1 - k_2) = (k_1 + k_2)(r_1 - r_2) \Rightarrow r_1 k_2 = r_2 k_1 = j_{12} \tag{3.8}$$

Of the four integers k_1, k_2, r_1, r_2 which describe the connectivity between two particulate events, only three are independent. Since the simplest form of the constraint is the product of a distance times a momentum we give it the conventional symbol j for angular momentum.

So far all we have succeeded in doing is to calibrate our scattering equipment. To measure anything of interest we must provide another exit counter, 4, backed up by a counter telescope 44' which defines the scattering angle θ, backed up by energy and momentum measuring devices which define the mass of the emerging particle. Once we have done this and selected out the appropriate kinematics for elastic scattering, we still have to relate the results to a model for the scattering process that depends only on energy and angle. We shrink down the geometry to the minimum in which the free particle momentum change occurs a some point half-way between two (possible) events a distance λ apart at the apex of an isosceles triangle with this base and angles $\pi - \theta$, whose sides lie along the initial and final directions, i.e similar to the macroscopic counter geometry discussed in Chapter 1. Since the momentum is unchanged in magnitude, the momentum transfer to or from the particle is of magnitude $P(1 - cos\ \theta) = 2Psin^2\frac{\theta}{2}$ along the altitude. This direction passes through a sequence of centers which define a sequence of isosceles triangles with sides r and the common base and altitude a, determined by $a^2 = r^2 - \lambda^2/4$. Note that we have exploited scale invariance down as far as we can. This distance a is called the impact parameter, and since it is parallel to the momentum change, the angular momentum is $Pa = J$ in appropriate units.

Assume that a light signal to the center and back takes the same time as it takes to particle to go the distance λ Then the square of the area A swept out by the line from the center to a point moving along the base is $A^2 = \lambda^2[(a/\lambda)^2 - \frac{1}{4}] = \ell(\ell + 1)$ where $\ell = j - \frac{1}{2} = ph - \frac{1}{2}$. This, eventually, gives us the quantized version of Kepler's second law, as we showed in Chapter 1 without using Planck's constant.

3.3 Hydrogenic Bound State Spectra

We now consider the basic orbital situation for two particles of mass m_1 and m_2, reduced mass $m = \frac{m_1 m_2}{m_1 + m_2}$, and bound state mass $\mu = m - \epsilon$, which we discussed from the Kepler-deBroglie point of view in our paper on the fine structure spectrum of hydrogen[10] using the detailed apparatus of our new fundamental theory.[11–13] We present here yet another way to arrive at the same result.

We consider the bound system with mass μ interacting with a larger system which can have a maximum mass-energy $M_x = Nm$ where N is an integer to be fixed by the intent of the modeling exercise. Our first assumption is that the bound state is *stable* against spontaneous decay. However, in a "second quantized" theory[14] virtual transitions up to this maximum can occur. Assuming that "externally" μ is at rest, these fluctuations can be interpreted as massless radiation whose energy and hence whose momentum is $p = N\mu$. However, since $m_1 + m_2$ must have the same non-spacial conserved quantum numbers as μ, a fluctuation leading to this radiation and a system of Nm masses will have energy $E = Nm$ But for the overall system $E^2 - p^2$ is invariant and equal to the square of the rest energy of the bound state with which we started:

$$\mu^2 = (Nm)^2 - (N\mu)^2 \tag{3.9}$$

To recover our previous result, we rewrite this as

$$\mu^2 + (\frac{\mu}{N})^2 = m^2 \tag{3.10}$$

or

$$(\frac{m-\epsilon}{m})^2[1 + (\frac{1}{N})^2] = 1 \tag{3.11}$$

If we take $N = 137n$, with $n = \ell + 1$ the principal quantum number and 137 an approximation for $\hbar c/e^2$, this is Bohr's relativistic generalization of his model for the energy levels of the hydrogen atom.[15] Non-relativistically, or to order $\alpha = e^2/\hbar c \approx 1/137$, the energy levels given by this formula are $\epsilon(n) \approx \frac{me^4}{2\hbar^2 n^2}$. The orbital velocity is $\beta_n = 1/137n = 1/N$ and the radii of the orbits are $R_n = n\hbar^2/me^2 = n\hbar/\alpha mc$.

Sommerfeld added a second quantized degree of freedom by treating the orbits as elliptical as well as circular and explained the *fine structure* of the spectrum of hydrogen, that is the doubling of the Bohr levels (for $0 < \ell \leq n+1$), which is proportional to the *fine structure constant* $\alpha = e^2/\hbar c$. The same formula was obtained by Dirac in an entirely different way a decade and a half later. This paradox is discussed by Biedenharn[16] Our combinatorial derivation[2] should make it clear that a still better understanding of the situation lies deeper than the orbit or the spin pictures of older models.

We can extend our general model immediately to two charges of magnitude $\pm Z_1 e, \mp Z_2 e$ by taking $N = 137n/Z_1 Z_2$. Clearly no "hydrogenic" system can have $Z_1 Z_2 \geq 137$ because this would give an orbital velocity $\geq c$. Similarly, we cannot assemble more than 137 charge particle- anti-particle pairs in a volume whose effective radius is less than $h/2mc$. Following an old paper of Dyson's[17] I used this fact to explain why we cannot count more than 137 charged particle pairs at short distance using electromagnetic measurements in my first paper on the *combinatorial hierarchy*.[18] I also pointed out that this suggests that the neutral pion is 137 electron-positron pairs, predicting a first approximation to its mass as $274m_e$. Adding an positron and a neutrino gives the π^+ while adding an electron and anti-neutrino gives a $\pi-$; either has mass of approximately $275m_e$. The isotriplet character of the pion is explained.

Our version of the relativistic Bohr model explains weak quantum gravity when we take $N = n\hbar c/Gm_1 m_2$, consistent with the Dyson-Noyes argument given above. In our finite and discrete theory this number has to be integral, and allows us a universal particulate mass quantization in terms of the Planck mass.

3.4 "FIELDS"

The extension of the discussion to fields can now follow the lines sketched out in Chapter 2. One advantage of the quantum treatment is that we can now relate the difference between interactions which change angular momentum, and hence are velocity dependent, to the spin of the quanta which are "exchanged". Coulombic and Newtonian forces between two spin $\frac{1}{2}$ particles act *independent* of the spin state and hence only depend on the line of centers. The spin 1 exchanges which we encounter in the fine structure of hydrogen correlate the spin-flip and no spin-flip transitions with the change of orbital angular momentum by zero or 1. This is velocity-dependent but not extremely non-local because the quasi-local radius of curvature can be calibrated using magnetostatic fields, as has long been the practice in particle physics. But the spin 2 quanta involved in the gravitational analog of the fine structure of hydrogen have only so far shown up experimentally in the five states they can have relative to the macroscopic plane defined by Mercury's orbit around the sun, as we discussed at PIRT I. We look forward to putting all of this together in a more coherent way for PIRT IV.

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