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ANTI-HYDROGEN:

The cusp between Quantum Mechanics and General Relativity*

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ABSTRACT

We argue that the crossing (CPT) symmetry of relativistic quantum mechanics requires that both the Coulombic and the Newtonian force between pairs of particles will reverse when one is replaced by its anti-particle. For consistency, this requires a theory in which both the equivalence principle and gauge invariance are abandoned. Thus whether anti-hydrogen "falls" up or down will provide an *experiment crusis* separating general relativity and gauge invariance from this version of quantum mechanics.

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1. Introduction

C.J.Isham, in his lectures at Schladming in 1991 and in a forthcoming review, finds a fundamental *conceptual* conflict between a continuum metric theory of gravitation such as General Relativity (GR) and second quantized relativistic field theory (RFT). One way of bringing out this conflict is to note that the CPT theorem requires all additive quantum numbers to change sign when changing particles to anti-particles. Hence a RFT of gravitation in which there is *gravitational charge* and anti-particles "fall" up in a gravitation situation where particles fall "down" is at least conceivable. GR does not allow this possibility to exist.

In a conventional RFT spin 1 fields cause like charges to repel and unlike charges to attract, while spin 0 and spin 2 fields are always attractive. This leads Nieto and Goldman[1] to examine only spin 1 fields as possible sources of small violations of the equivalence principle. However, if one examines the situation in more detail, the preliminary conclusion is not so straightforward. Weinberg[2] finds that for massless fields of spin j the second quantized interaction must vanish like p^j as $p \to 0$. Consequently, in order to introduce into the theory the Coulombic and Newtonian interactions which are dominant at low energy, he insists that the interactions must be gauge invariant and not just Lorentz invariant. In contrast to the fields, the gauge potentials do not lead directly to the change in the momenta of particles or objects on which any direct experimental test of the theory must rely. One is therefore free to question the *necessity* of requiring gauge invariance in either relativistic quantum mechanics (RQM) or its correspondence limit in classical physics. Granted this, the basic argument against anti-gravity is no longer compelling.

2. Relativistic Quantum Mechanics without Gauge Invariance.

Kuhn has remarked[3] that a theory is rarely challenged or submitted to critical tests until an alternative has appeared on the scene. The alternative theory we consider is sometimes called discrete physics[4] or bit-string physics or combinatorial physics. It started with the combinatorial hierarchy, $(3,10,137,2^{127}+136)$, discovered by A.F. Parker-Rhodes in 1961[5] and was put on a firmer mathematical and physical foundation in 1979.[6] In discrete physics the interaction between particles and quanta leads to finite and discrete changes in momenta, invariant under finite and discrete Lorentz transformations and rotations. This discrete formulation leads immediately to the commutation relations of relativistic quantum mechanics. Consequently Feynman's derivation of the Maxwell equations from Newton's second law and the commutation relations[7] is no longer conceptually ambiguous.[8] This establishes a correspondence limit for the theory in the Maxwell Equations, including the charge and current sources. The same argument applied to gravity leads to the Newtonian interaction and spin 2 gravitons.[9]

We have in hand a theory which makes no use of gauge potentials, and hence removes the most serious barrier to constructing a theory of anti-gravity. The theory makes contact with experiment by the axioms that events can take place only an integral number of "de Broglie wavelengths" h/p apart and that the separation of the positions of the two events as measured by the Einstein "radar" distance is an integral number of "Compton wavelengths" h/mc apart. Particulate double slit interference and the definition of mass ratios as inverse to the ratio of spacing between interference maxima in the same kinematic circumstances follows. Further, this implies relativistic 3-momentum conservation for particles with mass ratios so defined.

3. Bound and Scattering States

The next step is to consider how this theory describes bound states stabilized by "massless quanta". Assuming that the probability of an event which provides the momentum change that keeps the hydrogen atom together is $1/137n$ compared to a background of random events, and that the "orbit" closes, one arrives at the relativistic energy level formula of Bohr.[10] Here n is the principle quantum number and $1/137$ is the first approximation for $e^2/\hbar c$ computed from the combinatorial hierarchy. By noting that including a second degree of freedom requires us to make a second and overlapping construction of the combinatorial hierarchy, we extend the calculation to include the fine structure splitting. This gives us both the Sommerfeld formula and a combinatorial correction to our initial value of $e^2/\hbar c$ which improves agreement with experiment by four significant figures.[11] Why Sommerfeld arrived at the correct result before spin was discovered is a complicated story,[12] and will need extension when viewed from our point of view. Gravitationally bound states are described by the same formula with $Ze^2/n\hbar c$ replaced by $GmM/n\hbar c$; the fine structure splitting becomes important only when considering gravitational radiation.

When going from bound to scattering states, the fine structure parameter $\alpha = e^2/\hbar c$ is replaced by $\eta = \alpha/\beta = e^2/\hbar v$ where the velocity $v = \beta c$ has to be computed using relativistic kinematics. The scattering amplitude for like charges is proportional to $\mathcal{C}^2(\eta)/(1 - cos\ \theta)$ where

$$\mathcal{C}^2(\eta) = 2\pi\eta/(e^{2\pi\eta} - 1)$$

For unlike charges this factor becomes

$$\mathcal{C}^2(-\eta) = 2\pi\eta/(1 - e^{-2\pi\eta})$$

The CT invariance or crossing symmetry of the theory is obvious in that if we reverse both the sign of e^2 and the sign of the velocity, the expression is unaltered.

Although the angular distribution of scattering is the same as that for Rutherford scattering in either case, the absolute magnitude of the cross section can be measured and shown to correspond to one or the other sign of η, thus distinguishing like from unlike pairs of charges. In the correspondence limit the two cases can be distinguished by which focus of the hyperbolic trajectory the attracting or repelling body occupies, or equivalently by whether the trajectory bends toward or away from that body. Since nothing in our treatment, other than the magnitude of the coupling constant, differs when we discuss gravitation, our claim that crossing (or, in general , CPT) requires anti-protons to "fall" up is established.

4. Further Considerations

A number of subsidiary questions, for which we have little space here, should be discussed in order to firm up our prediction. We have considered only the Coulombic or Newtonian term. It is necessary that the spin-dependent terms give the same reversal between particle and antiparticle we have established above. Otherwise we would be in difficulty with well known experimental facts in the case of electromagnetism, and two of the classical tests of general relativity. To see the latter, note that the Newtonian term gives only half the bending of light passing the sun which is observed, and in Sommerfeld's calculation only one-sixth of the observed precession of the perihelion of Mercury. But spin 2 gravitons will flip spin 1 photons, doubling the first effect, and their five states relative to the plane of the orbit bring the total interaction up by the needed a factor of 6 in the case of Mercury. Since our theory gives spin 1 "traveling photons" in addition to the (quantum mechanical) Coulombic interaction, spin 2 "traveling gravitons" in addition to the Newtonian interaction and in both cases the sign of the coupling constant is the same as for the "static" term, we can claim that the empirical tests of GR are correctly and more simply predicted by our theory.

Another major difference between our theory (RQM) and RFT is that we reject the principle of gauge invariance as "unphysical". At first glance, the "reality" of gauge potentials would seem to be supported by the Aharanov-Bohm effect. But this has alternative (topological) descriptions that would fit within our framework. Psychologically more compelling for conventional particle physicists are probably the success of the non-abelian gauge theories in predicting many of the phenomena attributed to weak-electromagnetic unification and the detailed successes of quantum chromodynamics. For us, weak-electromagnetic unification is achieved at the tree level by equating the mass of the electron calculated from its electrostatic interaction with proton-antiproton pairs to the mass calculated in the same way using its Fermi interaction. Note that both are given as finite ratios to the proton mass in our discrete physics. Corrections to the Fermi constant and weak angle predictions similar to that for α are also given in Ref. 11 and in all cases improve agreement with experiment. The first three levels of the combinatorial hierarchy, represented by bit-strings of length 16, give all the states of the standard model for 3 generations of quarks and leptons, including the top quark. We predict all observed states and none (other than the top) which are not observed; only three generations are allowed. Our main difference is that we have no use for a Higgs meson, and that we require lepton number, baryon number, charge, and color to be absolutely conserved. Color is confined. Charge, lepton number, baryon number and the z-component of weak isospin are connected by the extended Gell-Mann Nishijima rule. The stability of the proton is needed for our identification of the proton as a rotating, charged black hole with Bekenstein number $2^{127} + 136 = \hbar c/Gm_p^2$, — the number of bits of information lost in its formation.[13]

Note that the reconciliation between relativistic quantum mechanics and gravitational phenomena achieved by our theory started in 1961 with the identification of the last two terms in the combinatorial hierarchy, $(137, 2^{127+136} \approx 1.7 \times 10^{38})$, with electromagnetic and gravitational interactions. But it has taken some time to put flesh enough on these bare bones to call the result a theory. We have now shown that the appropriate discrete versions of Lorentz invariance, angular momentum, commutation relations, bound and scattering states, weak-electromagnetic unification, the standard model... are all contained. But our version of CPT invariance requires us to abandon gauge invariance and allows us to predict anti-gravity. Once anti-hydrogen is available in a configuration similar to that in which hydrogen atoms were suspended in a magnetic field balancing their gravitational attraction by the earth, the test will be immediate. Hence anti-hydrogen can provide us with the material for an *experiment crusis* that will tell us whether we have to abandon the equivalence principle and gauge invariance or reject my interpretation of CPT in discrete physics.

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