Aspects of Nonrenormalizable Terms in a Superstring derived Standard–like Models [†]

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ABSTRACT

I investigate the role of nonrenormalizable terms, up to order N=8, in a superstring derived standard–like model. I argue that nonrenormalizable terms restrict the gauge symmetry, at the Planck scale, to be $SU(3) \times SU(2) \times U(1)_{B-L} \times U(1)_{T_{3_R}}$ rather than $SU(3) \times SU(2) \times U(1)_Y$. I show that breaking the gauge symmetry directly to the Standard Model leads to breaking of supersymmetry at the Planck scale, or to dimension four, baryon and lepton violating, operators. I show that if the gauge symmetry is broken directly to the Standard Model the cubic level solution to the F and D flatness constraints is violated by higher order terms, while if $U(1)_{Z'}$ remains unbroken at the Planck scale, the cubic level solution is valid to all orders of nonrenormalizable terms. I discuss the Higgs and fermion mass spectrum. I demonstrate that realistic, hierarchical, fermion mass spectrum can be generated in this model.

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1. Introduction

Superstring theories [1] are believed to provide a consistent framework for the unification of all the known fundamental interactions. The superstring unification scale is at the Planck scale. At the electroweak scale the Standard Model is in good agreement with experimental observations. However, the Standard Model, and point field theories in general, leave many problems unresolved. Among them, the origin of the number of generations, the origin of Yukawa couplings and their hierarchy, quantum gravity, etc. These problems find natural solutions in superstring theories. Thus, an extremely important task is to connect the superstring with the Standard Model.

Two approaches can be pursued to derive the Standard Model from the superstring. One is to use a GUT symmetry at an intermediate energy scale. Many attempts have been made in this direction and most notable are the flipped $SU(5)$ [2,3] and the $SU(3)^3$ models [4]. The second approach is to derive the Standard Model directly from the superstring without any non–abelian gauge symmetry at an intermediate energy scale [5,6,7,8,9]. In refs. [7,8,9] realistic standard–like models were constructed in the free fermionic formulation [10], with the following properties:

1. Three and only three generations of chiral fermions. There are no additional generations and mirror generations which presumably get massive at a high scale. This property of the standard–like models leads to an unambiguous identification of the different generations.

2. The gauge group is $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{T_{3_R}} \times U(1)^n \times hidden$. n reduces to one or zero after application of the Dine–Seiberg–Witten (DSW) mechanism. The $U(1)_{Z'} = \frac{1}{2}U(1)_{B-L} - \frac{2}{3}U(1)_{T_{3_R}}$ combination may be broken at the Planck scale, by the DSW mechanism. If it remains unbroken down to low energies, it results in a gauged mechanism to suppress proton decay from dimension four operators [11,12].

3. There are enough scalar doublets and singlets to break the symmetry in a realistic

way and to generate realistic fermion mass hierarchy [8,9].

4. Proton decay from dimension four and dimension five operators is suppressed due to gauged $U(1)$ symmetries [9].

5. These models suggest an explanation for the top-bottom mass hierarchy. At the trilinear level of the superpotential, only the top quark gets a non vanishing mass term. The mass terms for the bottom quark and for the lighter quarks and leptons are obtained from nonrenormalizable terms. Thus, only the top quark mass is characterized by the electroweak scale and the masses of the lighter quarks and leptons are naturally suppressed [8,9]. The top–bottom mass hierarchy is correlated with the requirement of a supersymmetric vacuum at the Planck scale [7,8,9].

In this paper I examine the role of nonrenormalizable terms in these models. For finiteness, I focus on the model of Ref. [7]. Nonrenormalizable terms are expected to play an important role in the low energy phenomenology of these models. I show that because of nonrenormalizable terms the favored observable gauge symmetry at the Planck scale is $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{T_{3R}}$. I show that in this case the solution to the cubic level F and D flatness constraints is obeyed to all orders. In contrast if the gauge symmetry is broken directly to the Standard Model, at the Planck scale, the cubic level constraints are violated by higher order terms. Moreover, I illustrate that breaking of the gauge symmetry directly to the Standard Model may induce dimension four operators which mediate rapid proton decay. I suggest that these considerations restrict the possible gauge symmetry at the Planck scale to be $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{T_{3R}}$. Furthermore, they may nessecitate the existence of an additional neutral gauge boson at low energies, with $U(1)_{Z'} = \frac{1}{2} U(1)_{B-L} - \frac{2}{3} U(1)_{T_{3R}}$. I discuss the Higgs and fermion mass matrices in this model. I show that this model can generate realistic, hierarchical fermion mass spectrum.

The paper is organized as follows. In section 2, I review the model and its symmetries. I discuss the rules for obtaining the non vanishing nonrenormalizable terms and emphasize the special properties of the standard–like model which

simplify the analysis. In section 3, I discuss the F and D flatness constraints. In sections 4 and 5, I discuss the implications of nonrenormalizable terms on proton decay and on the fractionally charged states. In sections 6 and 7, I discuss the Higgs and fermion mass matrices. Section 8 concludes the paper.

2. The superstring model

The superstring model is constructed in the free fermionic formulation [10]. The model is generated by a basis of eight boundary condition vectors. The first five vectors in the basis consist of the NAHE[†] set, $\{1, S, b_1, b_2, b_3\}$ [2,16,9]. This set is common to all the realistic models in the free fermionic formulation [2,6,14,7,8,9]. The important functions of the NAHE set are emphasized in Ref. [16,9]. The three vectors that extend the NAHE set and the choice of generalized GSO coefficients are given in table 1. The notation in the table emphasizes the division of the internal fermions according to their division by the NAHE set. In particular, it emphasizes the division and assignment of boundary conditions to the set of real fermions $\{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\}$ $(i = 1, \cdots, 6)$. The boundary conditions for this set of internal fermions determine many of the properties of the low energy spectrum [9].

The gauge group after application of the generalized GSO projections is

Observable[*] : $SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_L \times U(1)^6$

Hidden[#] : $SU(5)_H \times SU(3)_H \times U(1)^2$.

The weak hypercharge is uniquely given by $U(1)_Y = \frac{1}{3}U(1)_C + \frac{1}{2}U(1)_L$. The orthogonal combination is given by $U(1)_{Z'} = U(1)_C - U(1)_L$. In the observable sector there are six horizontal $U(1)$ symmetries. The first three, $U(1)_j$ $(j = 1, 2, 3)$, correspond to the right–moving world–sheet currents $\bar{\eta}_1\bar{\eta}_1^*$, $\bar{\eta}_2\bar{\eta}_2^*$ and $\bar{\eta}_3\bar{\eta}_3^*$. The last three, $U(1)_{r_{j+3}}$ $(j = 1, 2, 3)$, correspond to the right–moving world–sheet currents,

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† This set was first constructed by Nanopoulos, Antoniadis, Hagelin and Ellis, in the construction of the flipped $SU(5)$, *nahe* =pretty in Hebrew.

* $U(1)_C = \frac{1}{2}U(1)_{B-L}$, $U(1)_L = \frac{1}{2}U(1)_{T_{3_R}}$.

Hidden here means that the states which are identified with the chiral generations do not transform under the hidden gauge group [16].

$\bar{y}^3\bar{y}^6$, $\bar{y}^1\bar{\omega}^5$ and $\bar{\omega}^2\bar{\omega}^4$, respectively. For every right–moving $U(1)$ symmetry correspond a left–moving global $U(1)$ symmetry. The first three correspond to the charges of the supersymmetry generator χ^{12}, χ^{34} and χ^{56}. The last three, $U(1)_{\ell_{j+3}}$ ($j = 1, 2, 3$), correspond to the complexified left–moving fermions $y^3 y^6$, $y^1 \omega^5$ and $\omega^2 \omega^4$. Finally the model contains six Ising model sigma operators which are obtained by pairing a left–moving real fermion with a right–moving real fermion,
$$\sigma_\pm^i = \{\omega^1 \bar{\omega}^1, y^2 \bar{y}^2, \omega^3 \bar{\omega}^3, y^4 \bar{y}^4, y^5 \bar{y}^5, \omega^6 \bar{\omega}^6\}_\pm.$$

The full massless spectrum is analyzed by using a FORTRAN program. The program takes as input the basis vectors $B = \{b_1, \cdots, b_8\}$, and the GSO coefficients $c\begin{pmatrix} b_i \\ b_j \end{pmatrix}$, $(i, j = 1, \cdots, 8)$. The program checks the modular invariance rules, spans the additive group $\Xi = \sum_j n_j b_j$; $(j = 1, \cdots, 8)$, selects the sectors in Ξ which lead to massless states and performs the GSO projections. It calculates the traces of the of the $U(1)$ symmetries and evaluates the quantum numbers of the massless states under all the symmetries in the model. This output is read by subsequent programs which can analyze the superpotential up to any order (the limit being a sensible CPU time limit). This program enables a thorough exploration of a wider range of models rather than specific isolated examples. Combined with the conformal field theory techniques for evaluating corralators between vertex operators, and the Renormalization Group Equations (RGE), it provides powerful machinery for studying the phenomenology of the superstring models.

The following massless states are produced by the sectors $b_{1,2,3}$, $S + b_1 + b_2 + \alpha + \beta$, O and their superpartners in the observable sector:

(a) The massless spectrum contains three generations of chiral fermions from the sectors b_1, b_2 and b_3: $G_\alpha = e^c_{L_\alpha} + u^c_{L_\alpha} + N^c_{L_\alpha} + d^c_{L_\alpha} + Q_\alpha + L_\alpha$ ($\alpha = 1, \cdots, 3$), where

$$e^c_L \equiv [(1, \tfrac{3}{2}); (1, 1)]; \quad u^c_L \equiv [(\bar{3}, -\tfrac{1}{2}); (1, -1)]; \quad Q \equiv [(3, \tfrac{1}{2}); (2, 0)] \qquad (1a, b, c)$$

$$N^c_L \equiv [(1, \tfrac{3}{2}); (1, -1)]; \quad d^c_L \equiv [(\bar{3}, -\tfrac{1}{2}); (1, 1)]; \quad L \equiv [(1, -\tfrac{3}{2}); (2, 0)] \qquad (1d, e, f)$$

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of $SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_L$, with charges under the six horizontal $U(1)$s. From the sector b_1 we obtain

$$(e_L^c + u_L^c)_{\frac{1}{2},0,0,\frac{1}{2},0,0} + (d_L^c + N_L^c)_{\frac{1}{2},0,0,-\frac{1}{2},0,0} + (L)_{\frac{1}{2},0,0,\frac{1}{2},0,0} + (Q)_{\frac{1}{2},0,0,-\frac{1}{2},0,0}, \quad (2a)$$

from the sector b_2

$$(e_L^c + u_L^c)_{0,\frac{1}{2},0,0,\frac{1}{2},0} + (N_L^c + d_L^c)_{0,\frac{1}{2},0,0,-\frac{1}{2},0} + (L)_{0,\frac{1}{2},0,0,\frac{1}{2},0} + (Q)_{0,\frac{1}{2},0,0,-\frac{1}{2},0}, \quad (2b)$$

and from the sector b_3

$$(e_L^c + u_L^c)_{0,0,\frac{1}{2},0,0,\frac{1}{2}} + (N_L^c + d_L^c)_{0,0,\frac{1}{2},0,0,-\frac{1}{2}} + (L)_{0,0,\frac{1}{2},0,0,\frac{1}{2}} + (Q)_{0,0,\frac{1}{2},0,0,-\frac{1}{2}}. \quad (2c)$$

The vectors b_1, b_2 and b_3 are the only vectors in the additive group Ξ that produce spinorial 16 of $SO(10)$. This is in contrast to the case in which the $SO(10)$ symmetry is broken to $SU(5) \times U(1)$ [2] or to $SO(6) \times SO(4)$ [14]. There the massless spectrum contains additional 16 and $\overline{16}$ multiplets. The fact that there are exactly three generations, without any extra generations and mirror generations, is unique to the choice of $SU(3) \times SU(2) \times U(1)_C \times U(1)_L$ as the observable gauge symmetry at the level of the spin structure. This property of the standard–like models leads to an unambiguous identification of the hierarchical generations.

(b) The $S + b_1 + b_2 + \alpha + \beta$ sector gives

$$h_{45} \equiv [(1,0);(2,1)]_{-\frac{1}{2},-\frac{1}{2},0,0,0,0} \qquad D_{45} \equiv [(3,-1);(1,0)]_{-\frac{1}{2},-\frac{1}{2},0,0,0,0} \quad (3a,b)$$

$$\Phi_{45} \equiv [(1,0);(1,0)]_{-\frac{1}{2},-\frac{1}{2},-1,0,0,0} \qquad \Phi_1^{\pm} \equiv [(1,0);(1,0)]_{-\frac{1}{2},\frac{1}{2},0,\pm1,0,0} \quad (3c,d)$$

$$\Phi_2^{\pm} \equiv [(1,0);(1,0)]_{-\frac{1}{2},\frac{1}{2},0,0,\pm1,0} \qquad \Phi_3^{\pm} \equiv [(1,0);(1,0)]_{-\frac{1}{2},\frac{1}{2},0,0,0,\pm1} \quad (3e,f)$$

(and their conjugates $\bar{h}_{45}$, etc.). The states are obtained by acting on the vacuum with the fermionic oscillators $\bar{\psi}^{4,5}, \bar{\psi}^{1,\cdots,3}, \bar{\eta}^3, \bar{y}^3 \pm i\bar{y}^6, \bar{y}^1 \pm i\bar{\omega}^5, \bar{\omega}^2 \pm i\bar{\omega}^4$, respectively (and their complex conjugates for $\bar{h}_{45}$, etc.).

(c) The Neveu–Schwarz O sector gives, in addition to the graviton, dilaton, antisymmetric tensor and spin 1 gauge bosons, the following scalar representations:

Electroweak doublets and singlets:

$$h_1 \equiv [(1,0);(2,-1)]_{1,0,0,0,0,0} \qquad\qquad \Phi_{23} \equiv [(1,0);(1,0)]_{0,1,-1,0,0,0} \quad (4a)$$

$$h_2 \equiv [(1,0);(2,-1)]_{0,1,0,0,0,0} \qquad\qquad \Phi_{13} \equiv [(1,0);(1,0)]_{1,0,-1,0,0,0} \quad (4b)$$

$$h_3 \equiv [(1,0);(2,-1)]_{0,0,1,0,0,0} \qquad\qquad \Phi_{12} \equiv [(1,0);(1,0)]_{1,-1,0,0,0,0} \quad (4c)$$

(and their conjugates $\bar{h}_1$, etc.). Finally, the Neveu–Schwarz sector gives rise to three singlet states that are neutral under all the U(1) symmetries. $\xi_{1,2,3}$: $\chi_{\frac{1}{2}}^{12} \bar{\omega}_{\frac{1}{2}}^{3} \bar{\omega}_{\frac{1}{2}}^{6} |0\rangle_0$, $\chi_{\frac{1}{2}}^{34} \bar{y}_{\frac{1}{2}}^{5} \bar{\omega}_{\frac{1}{2}}^{1} |0\rangle_0$, $\chi_{\frac{1}{2}}^{56} \bar{y}_{\frac{1}{2}}^{2} \bar{y}_{\frac{1}{2}}^{4} |0\rangle_0$.

The sectors $b_i + 2\gamma + (I)$ $(i = 1,..,3)$ give vector representations which are $SU(3)_C \times SU(2)_L \times U(1)_L \times U(1)_C$ singlets (see Table 1). The vectors with some combination of $(b_1, b_2, b_3, \alpha, \beta)$ plus $\gamma + (I)$ (see Table 2) give representations which transform under $SU(3)_C \times SU(2)_L \times U(1)_L \times U(1)_C$, most of them singlets, but carry either $U(1)_Y$ or $U(1)_{Z'}$ charges. Some of these states carry fractional charges $\pm\frac{1}{2}$ or $\pm\frac{1}{3}$. There are no representations that transform nontrivially both under the observable and hidden sectors. The only mixing which occurs is of states that transform nontrivially under the observable or hidden sectors and carry U(1) charges under the hidden or observable sectors, respectively.

The non vanishing trilevel terms in the superpotential of the model are

$$W = \{(u_{L_1}^c Q_1 \bar{h}_1 + N_{L_1}^c L_1 \bar{h}_1 + u_{L_2}^c Q_2 \bar{h}_2 + N_{L_2}^c L_2 \bar{h}_2 + u_{L_3}^c Q_3 \bar{h}_3 + N_{L_3}^c L_3 \bar{h}_3) + h_1 h_2 \bar{\Phi}_{12} + h_1 h_3 \bar{\Phi}_{13}$$
$$+ h_2 h_3 \bar{\Phi}_{23} + \bar{h}_1 h_2 \Phi_{12} + \bar{h}_1 h_3 \Phi_{13} + \bar{h}_2 h_3 \Phi_{23} + \Phi_{23} \bar{\Phi}_{13} \Phi_{12} + \bar{\Phi}_{23} \Phi_{13} \bar{\Phi}_{12} + \Phi_{12}(\bar{\Phi}_1^+ \bar{\Phi}_1^-$$
$$+ \bar{\Phi}_2^+ \bar{\Phi}_2^- + \bar{\Phi}_3^+ \bar{\Phi}_3^-) + \Phi_{12}(\Phi_1^- \Phi_1^+ + \Phi_2^- \Phi_2^+ + \Phi_3^- \Phi_3^+) + \frac{1}{2}\xi_3(\Phi_{45}\bar{\Phi}_{45} + h_{45}\bar{h}_{45} + D_{45}\bar{D}_{45}$$
$$+ \Phi_1^+ \bar{\Phi}_1^+ + \Phi_1^- \bar{\Phi}_1^- + \Phi_2^+ \bar{\Phi}_2^+ + \Phi_2^- \bar{\Phi}_2^- + \Phi_3^+ \bar{\Phi}_3^+ + \Phi_3^- \bar{\Phi}_3^-) + h_3 \bar{h}_{45}\Phi_{45} + \bar{h}_3 h_{45}\bar{\Phi}_{45}\}$$
$$+ \{\frac{1}{2}[\xi_1(H_{19}H_{20} + H_{21}H_{22} + H_{23}H_{24} + H_{25}H_{26}) + \xi_2(H_{13}H_{14} + H_{15}H_{16} + H_{17}H_{18})]$$
$$+ \bar{\Phi}_{23}H_{24}H_{25} + \Phi_{23}H_{23}H_{26} + h_2 H_{16}H_{17} + \bar{h}_2 H_{15}H_{18} + e_{L_1}^c H_{10}H_{27} + e_{L_2}^c H_8 H_{29} + (V_1 H_9$$

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$$+ V_2 H_{11})H_{27} + V_6 H_5 H_{29} + \bar{\Phi}_{45} H_{17} H_{24} + D_{45} H_{18} H_{21} + h_{45} H_{16} H_{25}\} \qquad (5)$$

where a common normalization constant $\sqrt{2}g$ is assumed.

Nonrenormalizable contributions to the superpotential are obtained by calculating corralators between vertex operators

$$A_N \sim \langle V_1^f V_2^f V_3^b \cdots V_N^b \rangle, \qquad (6)$$

where V_i^f (V_i^b) are the fermionic (scalar) components of the vertex operators. The non vanishing terms are obtained by applying the rules of Ref. [15]. To obtain the correct ghost charge some of the vertex operators are picture changed by taking

$$V_{q+1}(z) = \lim_{w \to z} exp(c)(w) T_F(w) V_q(z), \qquad (7)$$

where T_F is the super current and in the fermionic construction is given by

$$T_F = \psi^\mu \partial_\mu X + i\sum_{I=1}^{6} \chi_I y_I \omega_I = T_F^0 + T_F^{-1} + T_F^{+1} \qquad (8)$$

with

$$T_F^{-1} = e^{-i\chi^{12}}\tau_{12} + e^{-i\chi^{34}}\tau_{34} + e^{-i\chi^{56}}\tau_{56} \quad ; \quad T_F^{-1} = (T_F^{+1})^* \qquad (9)$$

where $\tau_{ij} = \frac{i}{\sqrt{2}}(y^i \omega^i + y^j \omega^j)$ and $e^{\chi^{ij}} = \frac{1}{\sqrt{2}}(\chi^i + i\chi^j)$.

Several observations simplify the analysis of the potential non vanishing terms. First, it is observed that only the T_F^{+1} piece of T_F contributes to A_N [15]. Second, in the standard–like model the pairing of left–moving fermions is $y^1\omega^5$, $\omega^2\omega^4$ and y^3y^6. One of the fermionic states in every term $y^i\omega^i$ ($i = 1, ..., 6$) is complexified and therefore can be written, for example for y^3 and y^6, as

$$y^3 = \frac{1}{\sqrt{2}}(e^{iy^3y^6} + e^{-iy^3y^6}), y^6 = \frac{1}{\sqrt{2}}(e^{iy^3y^6} - e^{-iy^3y^6}). \qquad (10)$$

Consequently, every picture changing operation changes the total $U(1)_\ell = U(1)_{\ell_4} + U(1)_{\ell_5} + U(1)_{\ell_6}$ charge by ± 1. An odd (even) order term requires an even (odd)

number of picture changing operations to get the correct ghost number [15]. Thus, for A_N to be non vanishing, the total $U(1)_\ell$ charge, before picture changing, has to be an odd (even) number, for even (odd) order terms, respectively. Similarly, in every pair $y_i\omega_i$, one real fermion, either y_i or ω_i, remains real and is paired with the corresponding right–moving real fermion to produce an Ising model sigma operator. Every picture changing operation changes the number of left–moving real fermions by one. This property of the standard–like model significantly reduces the number of potential non vanishing terms.

3. F and D constraints

The massless spectrum of the superstring model contains six anomalous $U(1)$ symmetries. Of the six anomalous $U(1)$s only five can be rotated by an orthogonal transformation and one combination remains anomalous. The six combinations can be taken as [7]

$$U'_1 = U_1 - U_2 \quad , \quad U'_2 = U_1 + U_2 - 2U_3, \qquad (11a,b)$$

$$U'_3 = U_4 - U_5 \quad , \quad U'_4 = U_4 + U_5 - 2U_6, \qquad (11c,d)$$

$$U'_5 = U_1 + U_2 + U_3 + 2U_4 + 2U_5 + 2U_6, \qquad (11e)$$

$$U_A = 2U_1 + 2U_2 + 2U_3 - U_4 - U_5 - U_6, \qquad (11f)$$

with $Tr(Q_A) = 180$.

The anomalous $U(1)$ generates a Fayet–Iliopoulos D–term by the VEV of the dilaton field. Such a D–term, in general, breaks supersymmetry. Supersymmetry is restored if there exist a direction in the scalar potential $\phi = \sum_i \alpha_i \phi_i$ which is F flat and also D flat with respect to the non anomalous gauge symmetries and in which $\sum_i Q_i^A |\alpha_i|^2 < 0$. If such a direction exists, it will acquire a VEV, canceling the anomalous D–term, restoring supersymmetry and stabilizing the vacuum [17]. Since the fields corresponding to such a flat direction typically also carry charges for the non anomalous D–terms, a non trivial set of constraints on the possible choices

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of VEVs is imposed. It is, in general, a non trivial problem to find solutions to the set of constraints.

The set of constraints is summarized in the following set of equations,

$$D_A = \sum_k Q_k^A |\chi_k|^2 = \frac{-g^2 e^{\phi_D}}{192\pi^2} Tr(Q_A) \qquad (12a)$$

$$D'_j = \sum_k Q'^j_k |\chi_k|^2 = 0 \quad j = 1 \cdots 5 \qquad (12b)$$

$$D_j = \sum_k Q_k^j |\chi_k|^2 = 0 \quad j = C, L, 7, 8 \qquad (12c)$$

$$W = \frac{\partial W}{\partial \eta_i} = 0 \qquad (12d)$$

where χ_k are the fields that get a VEV and Q_k^j is their charge under the $U(1)_j$ symmetry. The set $\{\eta_i\}$ is the set of fields with vanishing VEV.

In the standard–like models the solutions to the set of F and D constraints divide into two kinds of solutions. Solutions which break $U(1)_{Z'}$ and those which do not. Only the Neveu–Schwarz sector and the $b_1 + b_2 + \alpha + \beta$ sector produce $SO(10)$ singlets with negative Q_A. Therefore, only these sectors contribute to solutions which keep both $U(1)_Y$ and $U(1)_{Z'}$ unbroken at the Plank scale. For solutions which break $U(1)_{Z'}$, the states from the sectors $b_{1,2} + b_3 + \alpha + \gamma \pm (I)$, and the states $\{N_1, N_2, N_3\}$ from the sectors b_1, b_2 and b_3, can obtain a VEV as well. These states have vanishing weak hypercharge but non vanishing $U(1)_{Z'}$ charge.

The F flatness conditions derived from the cubic superpotential are

$$\bar{\Phi}_{13}\Phi_{12} + H_{23}H_{26} = \Phi_{13}\bar{\Phi}_{12} + H_{24}H_{25} = \Phi_{23}\Phi_{12} = \bar{\Phi}_{23}\bar{\Phi}_{12} = 0 \qquad (13a)$$

$$\bar{\Phi}_{23}\Phi_{13} + \bar{\Phi}_i^+ \bar{\Phi}_i^- = 0 \qquad (13b)$$

$$\bar{\Phi}_i^+ \bar{\Phi}_{12} + \Phi_i^- \xi_3 = 0 \qquad (13c)$$

$$\bar{\Phi}_i^- \bar{\Phi}_{12} + \Phi_i^+ \xi_3 = 0 \qquad (13d)$$

$$\Phi_{45}\bar{\Phi}_{45} + \Phi_i^+ \bar{\Phi}_i^+ + \Phi_i^- \bar{\Phi}_i^- = 0 \qquad (13e)$$

$$\Phi_{45}\xi_3 + H_{17}H_{24} = 0 \qquad (13f)$$

$$\bar{\Phi}_{45}\xi_3 = 0 \tag{13g}$$

$$H_{19}H_{20} + H_{23}H_{24} + H_{25}H_{26} = 0 \tag{13h}$$

$$H_{13}H_{14} + H_{17}H_{18} = 0. \tag{13i}$$

For equations $(13b) - (13d)$ the barred equations have to be taken as well. In addition to these equations we have 12 constraints of the form $H\xi$. The total number of F flatness constraints results in 35 equations.

I focus first on solutions which do not break $U(1)_{Z'}$. I show that in this case the cubic level solution is obeyed to all orders of nonrenormalizable terms. I demonstrate that solutions which break $U(1)_{Z'}$ do not hold to all orders.

For solutions which do not break $U(1)_{Z'}$, $\langle H \rangle = 0$. Therefore, the choice

$$\langle \Phi_{12}, \bar{\Phi}_{12}, \xi_3 \rangle = 0, \tag{14}$$

satisfies the cubic level F constraints. I also impose $\langle \Phi_{23}, \bar{\Phi}_{45} \rangle = 0$. In this case the set of cubic level F constraints reduces to

$$\frac{\partial W}{\partial \Phi_{12}} = \bar{\Phi}_{23}\Phi_{13} + \bar{\Phi}_i^+ \bar{\Phi}_i^- = 0 \tag{15a}$$

$$\frac{\partial W}{\partial \bar{\Phi}_{12}} = \bar{\Phi}_{13}\Phi_{23} + \Phi_i^+ \Phi_i^- = \Phi_i^+ \Phi_i^- = 0 \tag{15b}$$

$$\frac{\partial W}{\partial \xi_3} = \Phi_{45}\bar{\Phi}_{45} + \Phi_i^+ \bar{\Phi}_i^+ + \Phi_i^- \bar{\Phi}_i^- = 0 \tag{15c}$$

where W is the cubic superpotential and summation on repeated indices is implied.

I now turn to discuss the implication of nonrenormalizable terms on the cubic level F flatness constraints. The order N terms that have to be investigated are of the form

$$\langle (\alpha\beta)^j (NS)^{N-j} \rangle \qquad (j = 4, \cdots, N) \tag{16}$$

where (NS) denotes fields which belong to the Neveu–Schwarz sector and $(\alpha\beta)$ denotes fields that belong to the sector $b_1 + b_2 + \alpha + \beta$. Without loss of generality

we can choose two of the $(\alpha\beta)$ fields to be the two space–time fermions in these corralators. The $N = 2$ world–sheet global $U(1)$ charges, $(\chi_{12}, \chi_{34}, \chi_{56})$, for the $(\alpha\beta)$ fields are $(0, 0, \frac{1}{2})$ for fermions and $(-\frac{1}{2}, -\frac{1}{2}, 0)$ for scalars. All the Neveu–Schwarz fields in Eq. (16) are scalar fields, with charges $\chi_{ij} = 0$ or -1. Of the Neveu–Schwarz singlets, only Φ_{12}, $\bar{\Phi}_{12}$ and ξ_3 carry $U(1)_{\ell_3}$ charges. We can always choose a basis in which the χ_{56} charge of these fields is picture changed to zero. The picture changing operation on the $(\alpha\beta)$ scalars can only change them to $(\pm\frac{1}{2}, \pm\frac{1}{2}, 0)$. Therefore, all the terms of the form of Eq. (16) are not invariant under $U(1)_{\ell_3}$. The conclusion is that all these terms vanish identically to all orders. Thus, in models with $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{T_{3_R}}$ gauge symmetry at the Planck scale the cubic level F flatness solution is valid to all orders of nonrenormalizable terms.

I now turn to show that in models with broken $U(1)_{Z'}$, at the Planck scale, the cubic level solution is violated by higher order terms. As an illustrative example I take the solution that was found in Ref. [7]. With the set of non vanishing VEVs, $\{H_{23}, H_{18}, \bar{\Phi}_{13}, \Phi_{45}, \bar{\Phi}_{23}, \Phi_2^+, \bar{\Phi}_3^-\}$, Eqs. (12) have the solution,

$$|H_{23}|^2 = |H_{18}|^2 = \frac{1}{3}|\Phi_{45}|^2 = \frac{3}{2}|\bar{\Phi}_{13}|^2 = \frac{3}{2}|\bar{\Phi}_{23}|^2 = \frac{1}{2}|\Phi_2^+|^2 = |\bar{\Phi}_3^-|^2 = \frac{g^2}{16\pi^2}. \quad (18)$$

This set breaks the observable gauge symmetry to $SU(3)_C \times SU(2)_L \times U(1)_Y$. This solution obeys the cubic level F and D flatness constraints. At order seven we find the following non vanishing term,

$$H_{23}^2 H_{18}^2 \Phi_{45}^2 \xi_2.$$

Thus the cubic level constraint $\frac{\partial W}{\partial \xi_2} \equiv 0$ is violated. Moreover, if ξ_2 gets a Planck scale VEV the superpotential receives a contribution of $O(M_{Pl})$ and $W \neq 0$. Therefore, in models with broken $U(1)_{Z'}$ the cubic level solution is violated by higher order terms, while in models with unbroken $U(1)_{Z'}$ the cubic level F flatness solution is valid to all orders. I would like to emphasize that giving a VEV to any pair

of singlets from the sectors $b_{1,2} + b_3 + \alpha + \gamma \pm (I)$ leads to a violation of the cubic level F flatness solution at the quintic or $N = 7$ orders. We could contemplate giving a VEV to one of the three Standard Model singlets in the 16 of $SO(10)$, N_1, N_2 or N_3 and, for example, to H_{23}. In this case F violating terms do not appear up to $N = 7$, but may appear at orders higher than $N = 7$. However, as I show in the next section giving a Planck scale VEV to N_1, N_2 or N_3 leads to problems with proton decay.

The number of flat directions is larger than the number of constraints. Therefore, the solution to the F and D constraints is not unique. However, once a specific choice has been made, the phenomenology of the model is determined. In what follows bellow I focus on one illustrative example. An explicit solution which satisfies all the F and D constraints is given by the following set of non vanishing VEVs

$$\{\Phi_{45}, \Phi_{1,2}^{\pm}, \bar{\Phi}_{1,2,3}^{-}, \bar{\Phi}_1^{+}, \bar{\Phi}_{23}, \bar{\Phi}_{13}, \Phi_{13}, \xi_1, \xi_2\} \tag{19}$$

with

$$\frac{1}{3}|\Phi_{45}|^2 = |\Phi_{23}|^2 = \frac{4}{3}|\Phi_{13}|^2 = \frac{4}{3}|\bar{\Phi}_{13}|^2 = \frac{g^2}{16\pi^2} \tag{20a}$$

$$\frac{1}{2}|\bar{\Phi}_1^{-}|^2 = |\bar{\Phi}_1^{+}|^2 = |\Phi_2^{+}|^2 = \frac{1}{2}|\Phi_2^{-}|^2 = |\Phi_3^{-}|^2 = \frac{g^2}{16\pi^2} \tag{20b}$$

$$|\Phi_1^{+}|^2 = |\Phi_1^{-}|^2 = (\frac{7}{8})^{\frac{1}{2}}\frac{g^2}{16\pi^2} \tag{20c}$$

$$|\bar{\Phi}_2^{-}|^2 = \sqrt{\frac{7}{2}}\frac{1+\sqrt{2}}{60}\frac{g^2}{16\pi^2}. \tag{20d}$$

4. Dimension four operators

In this section I show that nonrenormalizable terms induce effective dimension four operators which may result in rapid proton decay. It is well known that the most general supersymmetric standard model gives rise to dimension four opera-

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tors, which induce rapid proton decay,

$$\eta_1 u_L^C d_L^C d_L^C + \eta_2 d_L^C Q L$$

where generations indices are suppressed. If η_1, η_2 are of $O(1)$, the proton will decay instantly. These dimension four operators are forbidden if the gauge symmetry of the Standard Model is extended by an additional $U(1)$ gauge symmetry which is a combination of $B-L$, baryon number minus lepton number, and T_{3_R} [2]. This $U(1)$ symmetry is exactly the $U(1)_{Z'}$ which is derived in the superstring standard–like models. The dimension four operators may still appear from the nonrenormalizable terms,

$$\eta_1 (u_L^C d_L^C d_L^C N_L^C)\Phi + \eta_2 (d_L^C Q L N_L^C)\Phi$$

where Φ is a combination of fields that fixes the string selection rules [3] and gets a VEV of $O(m_{pl})$, and N_L^C is the Standard Model singlet in the 16 of $SO(10)$. Thus, the ratio $\frac{\langle N_L^C \rangle}{M_{Pl}}$ controls the rate of proton decay. In the standard–like model, the following non vanishing terms appear at order $N = 6$,

$$(u_3 d_3 + Q_3 L_3) d_2 N_2 \Phi_{45} \bar{\Phi}_2^- \tag{21a}$$

$$+(u_3 d_3 + Q_3 L_3) d_1 N_1 \Phi_{45} \Phi_1^+ \tag{21b}$$

$$+u_3 d_2 d_2 N_3 \Phi_{45} \bar{\Phi}_2^- + u_3 d_1 d_1 N_3 \Phi_{45} \Phi_1^+ \tag{21c}$$

$$+Q_3 L_1 d_3 N_1 \Phi_{45} \Phi_3^+ + Q_3 L_1 d_1 N_3 \Phi_{45} \Phi_3^+ \tag{21d}$$

$$+Q_3 L_2 d_3 N_2 \Phi_{45} \bar{\Phi}_3^- + Q_3 L_2 d_2 N_3 \Phi_{45} \bar{\Phi}_3^- . \tag{21e}$$

In section 7, I will show that the states in G_3 have to be identified with the lightest generation. From Eqs. (20) and (21) it is evident that if any of N_1, N_2 or N_3 gets a Planck scale VEV, dimension four operators are induced, which result in rapid proton decay. Thus, we conclude that $\langle N_1, N_2, N_3 \rangle \equiv 0$ at the Planck scale.

Moreover, since the coefficients in front of the terms in Eqs. (21) are expected to be of order one [15], a possible VEV for $N_{1,2,3}$ has to be well below the GUT scale. This result, combined with the result of the previous section, show that models with an $SU(3) \times SU(2) \times U(1)_{B-L} \times U(1)_{T_{3_R}}$ observable gauge symmetry, at the Planck scale, are favored over models with $SU(3) \times SU(2) \times U(1)_Y$.

5. Fractionally charged states

The massless spectrum of the superstring model contains the following singlet states with fractional charge $\pm\frac{1}{2}$,

$$H_3, H_4, H_7, H_8, H_{11}, H_{12}, H_{29}, H_{30}. \qquad (22)$$

These states do not transform under any of the non abelian gauge groups in the model. Therefore, they are not confined by any non abelian gauge symmetry. While many experimental searches for fractional charges have been conducted, no reported observation of a fractionally charged state has ever been confirmed and there are upper bounds on the abundance of any such particle in the range of 10^{-19} to 10^{-26} [18] of the nucleon abundance for charges between $\frac{1}{3}$ and 1. This may be a fundamental property of nature or merely an accidental property of the low energy spectrum that we have been able to observe so far. Indeed, fractionally charged particles may exist provided they are sufficiently heavy or sufficiently rare.

In the superstring standard–like model the following mass terms for the fractionally charged states are obtained from nonrenormalizable terms,

$$H_3 H_4 (\Phi_{12} \Phi_2^+ \Phi_1^+ + \Phi_{12} \bar{\Phi}_2^- \bar{\Phi}_1^- + \xi_3 \Phi_2^+ \bar{\Phi}_1^- + \xi_3 \bar{\Phi}_2^- \Phi_1^+) \qquad (23a)$$

$$H_7 H_8 (\xi_1 \bar{\Phi}_1^- \Phi_3^+ + \xi_1 \bar{\Phi}_3^- \phi_1^+) \qquad (23b)$$

$$H_{11} H_{12} (\xi_2 \Phi_2^+ \bar{\Phi}_3^- + \xi_2 \bar{\Phi}_2^- \Phi_3^+) \qquad (23c)$$

$$H_{29} H_{30} (\xi_3 \frac{\partial W_3}{\partial \xi_3} + \Phi_{12} (\bar{\Phi}_i^+ \bar{\Phi}_i^-) + \bar{\Phi}_{12} (\Phi_i^+ \Phi_i^-)). \qquad (23d)$$

From Eq. (23b) and Eq. (23c) we learn that $H_7 H_8$ and $H_{11} H_{12}$ acquire a large mass by the non vanishing VEV of the fields $\{\xi_1, \xi_2, \bar{\Phi}_{1,2,3}^-, \Phi_{1,2}^+\}$. Since $\bar{\Phi}_{1,2,3}^-, \Phi_{1,2}^+$

obtain a Planck scale VEV the mass scale of these fractionally charged singlets is determined by the VEV of ξ_1, ξ_2.

The term $H_3 H_4 \Phi_1^+ \Phi_2^+ \Phi_{12}$ induces an effective mass term $H_3 H_4 \Phi_{12} \left(\frac{\langle \Phi_1^+ \rangle \langle \Phi_2^+ \rangle}{M^2} \right)$, where M, $\langle \Phi_1^+ \rangle$ and $\langle \Phi_2^+ \rangle$ are $O(M_{Pl})$. This term will give a heavy mass term to $H_3 H_4$ by the VEV of Φ_{12}. According to the F and D flatness solution, this VEV vanishes at the Planck scale, and is constrained by the yet unknown mechanism for supersymmetry breaking. Thus, Φ_{12} may obtain a VEV which is still tolerated by the requirement of $N = 1$ space–time supersymmetry, giving a superheavy mass to $H_3 H_4$, which is beyond the reach of present accelerators. Similarly, the term $H_{29} H_{30} \bar{\Phi}_i^+ \bar{\Phi}_i^- \Phi_{12}$ induces an effective mass term $H_{29} H_{30} \frac{\langle \bar{\Phi}_i^+ \bar{\Phi}_i^- \rangle}{M^2} \Phi_{12}$. From Eq. (15a) $\langle \bar{\Phi}_i^+ \bar{\Phi}_i^- \rangle$ is $O(M_{Pl}^2)$. Therefore this term is an effective mass term for $H_{29} H_{30}$ by the VEV of Φ_{12}.

This result illustrates that all the fractionally charged states are expected to decouple from the low energy spectrum. Since all the fractionally charged states appear in vector–like representations this result is expected. The exact mass scales can only be determined by resolving the problem of supersymmetry breaking in these models.

6. Higgs mass matrix

The light Higgs spectrum is determined by the massless eigenstates of the doublet Higgs mass matrix. The doublet mass matrix consists of the terms $h_i \bar{h}_j \langle \Phi^n \rangle$, and is defined by $h_i (M_h)_{ij} \bar{h}_j$, $i, j = 1, 2, 3, 4$ where $h_i = (h_1, h_2, h_3, h_{45})$ and $\bar{h}_i = (\bar{h}_1, \bar{h}_2, \bar{h}_3, \bar{h}_{45})$. At the cubic level of the superpotential the Higgs doublets mass matrix is given by,

$$M_h = \begin{pmatrix} 0 & 0 & \bar{\Phi}_{13} & 0 \\ 0 & 0 & \bar{\Phi}_{23} & 0 \\ \Phi_{13} & 0 & 0 & \Phi_{45} \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{24}$$

The matrix M_h is diagonalized by $S M_h T^\dagger$ where S and T are two unitary

matrices and $(SM_hT^\dagger)_{ij} = m_i\delta_{ij}$. It follows that $SMM^\dagger S = TM^\dagger MT = |m|^2$. The h and $\bar{h}$ mass eigenstates are obtained by evaluating the eigenvalues and eigenstates of $MM^\dagger$ and $M^\dagger M$, respectively. The mass eigenvalues are given by

$$m_h; m_{\bar{h}} = (0, 0, \Phi_{13}^2 + \Phi_{45}^2, \Phi_{13}^2 + \Phi_{23}^2). \tag{25}$$

The h mass eigenstates are given by

$$h' = (0, 0, 0, 1); \tag{26a}$$

$$(-\frac{\bar{\Phi}_{23}}{\bar{\Phi}_{13}}, 1, 0, 0); \tag{26b}$$

$$(\frac{\bar{\Phi}_{13}}{\bar{\Phi}_{13}}, 1, 0, 0); \tag{26c}$$

$$(0, 0, 1, 0), \tag{26d}$$

and the $\bar{h}$ mass eigenstates are given by

$$\bar{h}' = (1, 0, 0, -\frac{\bar{\Phi}_{13}}{\bar{\Phi}_{45}}); \tag{27a}$$

$$(0, 1, 0, 0); \tag{27b}$$

$$(0, 0, 1, 0); \tag{27c}$$

$$(1, 0, 0, \frac{\bar{\Phi}_{45}}{\bar{\Phi}_{13}}). \tag{27d}$$

Equations (25), (26) and (27) show that at the cubic level of the superpotential there are two pairs of light Higgs states. The number of light Higgs pairs is reduced by taking into account higher order terms in the superpotential. For example at the quintic level we obtain the following non vanishing terms

$$h_2\bar{h}_{45}\Phi_{45}H_{25}H_{26} \qquad ; \qquad \bar{h}_2 h_{45}\bar{\Phi}_{45}H_{23}H_{27} \tag{28a, b}$$

These additional terms reduce the number of light Higgs pairs to one pair. For example, if $\langle H_{25}\rangle \sim \langle H_{26}\rangle \sim 10^{14} GeV$, one of the light pairs receives a mass of

$O(10^{10}GeV)$. At order $N = 7$ we obtain additional terms which may make the extra pair massive without breaking $U(1)_{Z'}$. The remaining light combinations depend on the specific entries in the Higgs mass matrix which become non zero and is highly model dependent. For example, if the $1\bar{2}$ entry in equation (24) is non zero, the two light Higgs eigenstates will consist of h_{45} and a combination of $\bar{h}_1$ and $\bar{h}_{45}$. Below I assume that only one pair of Higgs doublets remain light. However, I do not make a specific assumption as to what are the exact light eigenstates, but rather assume that the light pairs may contain any of the states that remain light at the cubic level. The purpose in doing so is to try to learn general properties of the light spectrum rather than details which depend on specific choices of flat directions. From equations (26) and (27) it follows that h_3 and $\bar{h}_3$ do not appear in the light eigenstates. Therefore the light eigenstates may contain only (h_1, h_2, h_{45}) and $(\bar{h}_1, \bar{h}_2, \bar{h}_{45})$. The absence of h_3 and $\bar{h}_3$ from the light eigenstates results in G_3 being identified with the lightest generation. As I show in the next section, the states in G_3 do not couple directly to the light Higgs eigenstates. Therefore, diagonal mass terms for G_3 do not appear up to $N = 8$. Consequently, after diagonalization of the mass matrices, the states in G_3 will be the largest component in lightest generation states.

7. Fermion masses

One of the most fundamental problems in high energy physics is the origin and hierarchy of the fermion masses. In this respect the Standard Model, and point field theories in general, can only be considered as successful attempts to parameterize the observed mass spectrum. Superstring theory gives a unique framework to understand the fermion mass hierarchy in terms of symmetries which are derived in specific models, unlike point field theories where the symmetries have to be imposed by hand. Therefore it is important to examine the structure of the fermion mass matrices in specific superstring models [19].

The class of superstring standard–like models is an especially restrictive class of models in which the fermion mass spectrum can be examined. A unique property

of the standard–like models is the possible connection between the requirement of a supersymmetric vacuum at the Planck scale, via the DSW mechanism, and the heaviness of the top quark relative to the lighter quarks and leptons. The only standard–like models which admit a solution to the set of F and D constraints are models in which only $+\frac{2}{3}$ charged quarks obtain trilevel Yukawa couplings. Application of the DSW mechanism leaves a trilevel mass term only to the top quark. The mass terms for the lighter quarks and leptons must come from higher order, nonrenormalizable, terms. These terms become effective mass terms for the lighter quarks and leptons by applying the DSW mechanism, and are naturally suppressed relative to the trilevel top Yukawa coupling. A second property, unique to the standard–like models, is the fact that the massless spectrum contains only three light generations. There are no extra generations and mirror generations which become superheavy at some high scale. This property of the standard–like models eliminates the ambiguity in the identification of the different generations that exist in other realistic superstring models [2,14].

The top quark mass term is obtained from $\lambda_t u_1 Q_1 \bar{h}_1$. At the quartic level there are no potential mass terms for the quarks and leptons. At the quintic level, the following mass terms are obtained

$$d_2 Q_2 h_{45} \bar{\Phi}_2^- \xi_1, \quad e_2 L_2 h_{45} \bar{\Phi}_2^+ \xi_1 \tag{28a}$$

$$d_1 Q_1 h_{45} \Phi_1^+ \xi_2, \quad e_1 L_1 h_{45} \Phi_1^- \xi_2 \tag{28b}$$

$$u_2 Q_2 (\bar{h}_{45} \Phi_{45} \bar{\Phi}_{23} + \bar{h}_1 \bar{\Phi}_i^+ \bar{\Phi}_i^-) \tag{28c}$$

$$u_1 Q_1 (\bar{h}_{45} \Phi_{45} \bar{\Phi}_{13} + \bar{h}_2 \Phi_i^+ \Phi_i^-) \tag{28d}$$

$$(u_2 Q_2 h_2 + u_1 Q_1 h_1) \frac{\partial W}{\partial \xi_3}. \tag{28e}$$

At this level potential mass terms for the heaviest down quark and charged lepton are obtained, $d_1 Q_1 h_{45} \Phi_1^+ \xi_2$, $e_1 L_1 h_{45} \Phi_1^- \xi_2$. From the solution to the F and D constraints $|\Phi_1^+| = |\Phi_1^-|$. Therefore, $\lambda_b = \lambda_\tau$ at the unification scale. However, the VEV of ξ_2 is not determined by the F and D constraints and is left as free parameter.

The charm quark obtains a mass term from $u_2 Q_2 \bar{h}_1 (\bar{\Phi}_1^+ \bar{\Phi}_1^-)$. The charm quark mass is suppressed by $\frac{(\bar{\Phi}_1^+ \bar{\Phi}_1^-)}{M^2}$ relative to the top quark mass. The suppression factor is expected to be of about two orders of magnitude. If we take $\langle \xi_1 \rangle \neq 0$ at the unification scale, $d_2 Q_2 h_{45} \bar{\Phi}_2^- \xi_1$ and $e_2 L_2 h_{45} \bar{\Phi}_2^+ \xi_1$ can give mass terms to the strange quark and to the muon lepton. According to Eq. (20), $\bar{\Phi}_2^- = (\sqrt{\frac{7}{2}} \frac{1+\sqrt{2}}{60})^{\frac{1}{2}}$ and $\bar{\Phi}_2^+ = 0$. Therefore according to this solution only the strange quark get mass from this term. A modified solution which includes $\bar{\Phi}_2^+ \neq 0$ will give a mass term to the muon lepton as well. At this level the states in G_3 do not receive any mass terms. Therefore, G_3 is identified with the lightest generation.

At every increasing order of nonrenormalizable terms the number of potential non vanishing terms increases exponentially. A search up to $N = 8$ was performed. Several observations simplify the analysis. First, there is no component of h_3 or $\bar{h}_3$ in the light Higgs representations. Second, there are several scales in the model. The leading scale correspond to the VEVs of singlets fields. There are two non abelian hidden gauge groups $SU(5)_H \times SU(3)_H$, with matter in fundamental representations (see tables 2,3). These hidden gauge groups produce two additional scales in the model, which correspond to the scales at which their couplings become strong. I assume that $\Lambda_5 \gg \Lambda_3$.

At order $N = 6$ all the up quark mass terms are suppressed by at least $\frac{\Lambda_3^2}{M^2}$. There are no diagonal mass terms for the states in G_3. In the down quark and charged lepton sectors we obtain the following leading terms,

$$d_3 Q_2 h_{45} \Phi_{45} V_6 V_9, \qquad d_2 Q_3 h_{45} \Phi_{45} V_5 V_{10}, \qquad (29a)$$

$$d_3 Q_1 h_{45} \Phi_{45} V_2 V_9, \qquad d_1 Q_3 h_{45} \Phi_{45} V_1 V_{10}, \qquad (29b)$$

$$e_3 L_2 h_{45} \Phi_{45} V_8 V_{11}, \qquad e_2 L_3 h_{45} \Phi_{45} V_7 V_{12}, \qquad (29c)$$

$$e_3 L_1 h_{45} \Phi_{45} V_4 V_{11}, \qquad e_1 L_3 h_{45} \Phi_{45} V_3 V_{12}. \qquad (29d)$$

At order $N = 6$ we obtain generational mixing in the down quark sector and in the charged lepton sector. In the quark sector the mixing is proportional to

$\frac{\Lambda_3^2 _H}{M^2}$. In the leptonic sector it is proportional to $\frac{\Lambda_5^2 _H}{M^2}$. It may be possible, (and desirable) to reverse this result by changing some of the generalized GSO phases. The importance of this result is to show that generational mixing is obtained. The symmetry between the down quark and charged lepton sectors is broken as the relative magnitude of the mixing is related by $\frac{\Lambda_3^2 _H}{\Lambda_5^2 _H}$.

At order $N = 7$ we obtain the following leading terms. In the down quark sector all the generational mixing terms are proportional to $\frac{\Lambda_3^2 _H}{M^2}$, and are a small correction to the sixth order terms. Similarly, there are small corrections, of order one percent (assuming $\frac{\Phi}{M} \sim \frac{1}{10}$) to the diagonal quintic order terms. There is no diagonal term of the form $d_3 Q_3 h$ or $e_3 L_3 h$.

In the up quark sector we obtain non vanishing generation mixing terms. Here I list only the leading terms which are proportional to $\frac{\Lambda_5^2 _H}{M^2}$.

$$u_3 Q_2 \bar{h}_1 \Phi_{45} \bar{\Phi}_2^- V_7 V_{12} \qquad u_2 Q_3 \bar{h}_1 \Phi_{45} \bar{\Phi}_3^- V_7 V_{12} \qquad (29a)$$

$$u_3 Q_1 \bar{h}_1 \Phi_{45} \bar{\Phi}_1^- V_3 V_{12} \qquad u_1 Q_3 \bar{h}_1 \Phi_{45} \bar{\Phi}_3^- V_3 V_{12} \qquad (29b)$$

$$u_2 Q_1 \bar{h}_1 \Phi_{45} \bar{\Phi}_1^- V_3 V_8 \qquad u_1 Q_2 \bar{h}_1 \Phi_{45} \bar{\Phi}_2^- V_3 V_{12}, \qquad (29c)$$

with additional terms, obtained by replacing $\bar{h}_1$ by $\bar{h}_2$. At order $N = 8$ generational mixing which is proportional to $\frac{\Lambda_5^2 _H}{M^2}$, appears in the down quark sector. Up to level $N = 8$ the diagonal mass terms for the states in G_3 are suppressed by at least $(\frac{M_{SUSY}}{M_{Pl}})^2$. Therefore, these states are identified with the lightest generation states. The identification of G_3 with the lightest generation is unambiguous and completetly general. It is a general characteristic of the class of standard–like models under consideration. It follows from the structure of the boundary condition vectors which characterizes these models. At the level of the NAHE set there is a cyclic symmetry between the vectors b_1, b_2 and b_3. Therefore, there is universality among the generations. This cyclic symmetry is broken by the vectors α and β. The vectors α and β are symmetric with respect to b_1 and b_2. However, the cyclic symmetry between b_1, b_2 and b_3 is broken. The universality

among the three generations with respect to the horizontal $U(1)$ symmetries is still unbroken. The symmetries of the spin structure determine the allowed terms in the cubic superpotential. These symmetries and the requirement of F flatness impose $\langle \Phi_{12}, \bar{\Phi}_{12}, \xi_3 \rangle \equiv 0$. Therefore, requiring D flatness by applying the DSW mechanism removes the degeneracy among the generations and forces h_3 and $\bar{h}_3$ to become superheavy. Since the remaining light doublets are not charged under $U(1)_3$, and because the only NS or $\alpha\beta$ fields with χ_{56} charge are ϕ_{12}, $\bar{\Phi}_{12}$ and ξ_3, diagonal mass terms for the states in G_3 are suppressed.

8. Conclusions

In this paper I examined several aspects of nonrenormalizable terms is a super-string derived standard–like model. This model belongs to a class of standard–like models with unique characteristics. They reproduce most of the properties of the Standard Model and provide explanations to several fundamental puzzles beyond the Standard Model. Among those, the replication of three and only three generations of chiral fermions and the heaviness of the top quark relative to the lighter quarks and leptons.

Nonrenormalizable terms play a pivotal role in the phenomenology of these models. Due to nonrenormalizable terms the preferred vacuum at the Planck scale extends the Standard Model gauge symmetry by an additional, generation independent, $U(1)$ symmetry. This $U(1)$ symmetry is uniquely determined to be, $U(1)_{Z'} = \frac{1}{2} U(1)_{B-L} - \frac{2}{3} U(1)_{T_{3_R}}$. Breaking of the gauge symmetry, at the Planck scale, directly to the Standard Model results in violation of the cubic level F flatness solution or in induction of dimension four operators, which mediate rapid proton decay. Thus, this model predicts the existence of an additional neutral gauge boson below the Planck scale. Nonrenormalizable terms lead to decoupling of the fractionally charged states from the massless spectrum.

The most important function of nonrenormalizable terms is in generating the hierarchy of the fermion mass spectrum. This function of nonrenormalizable terms

is the fingers print of specific superstring models. The origin of the fermion mass spectrum is perhaps the most fundamental problem in physics. The ability of superstring models to generate the observed spectrum is the real challenge facing these models. The standard–like models have the advantage that they explain the mass hierarchy of the top quark relative to the lighter quarks and leptons. In this paper I demonstrated that the superstring standard–like model can in principle account for the observed spectrum, including generational mixing. Resolution of the problem of supersymmetry breaking in these models, better understanding of the dynamics of the hidden sector, and explicit calculation of the coefficients of the higher order terms, will improve our ability to obtain quantitative estimates. Resolving these problems will uniquely determine the singlets VEVs, the hidden sector condensates, and the numerical coefficients of the higher order terms. Thus, yielding a full quantitative confrontation versus the low energy observations. I will come back to the phenomenology extracted from these models in future publications.

Acknowledgments

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	ψ^μ	$\{\chi^{12}; \chi^{34}; \chi^{56}\}$	$y^3y^6, y^4\bar{y}^4, y^5\bar{y}^5, \bar{y}^3\bar{y}^6$	$y^1\omega^5, y^2\bar{y}^2, \omega^6\bar\omega^6, y^4\bar\omega^5$	$\omega^2\omega^4, \omega^1\bar\omega^1, \omega^3\bar\omega^3, \bar\omega^2\bar\omega^4$	$\bar\psi^1, \bar\psi^2, \bar\psi^3, \bar\psi^4, \bar\psi^5, \bar\eta^1, \bar\eta^2, \bar\eta^3$	$\bar\phi^1, \bar\phi^2, \bar\phi^3, \phi^4, \bar\phi^5, \bar\phi^6, \bar\phi^7, \bar\phi^8$
α	0	$\{0, 0, 0\}$	1, 0, 0, 0	0, 0, 1, 1	0, 0, 1, 1	1, 1, 1, 0, 0, 0, 0, 0	1, 1, 1, 1, 0, 0, 0, 0
β	0	$\{0, 0, 0\}$	0, 0, 1, 1	1, 0, 0, 0	0, 1, 0, 1	1, 1, 1, 0, 0, 0, 0, 0	1, 1, 1, 1, 0, 0, 0, 0
γ	0	$\{0, 0, 0\}$	0, 1, 0, 1	0, 1, 0, 1	1, 0, 0, 0	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, 0, 1, 1, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0$

Table 1. A three generations $SU(3) \times SU(2) \times U(1)^2$ model. The choice of generalized GSO coeficients is: $c\begin{pmatrix} b_j \\ \alpha, \beta, \gamma \end{pmatrix} = -c\begin{pmatrix} \alpha \\ 1 \end{pmatrix} = c\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = -c\begin{pmatrix} \beta \\ 1 \end{pmatrix} = c\begin{pmatrix} \gamma \\ 1, \alpha \end{pmatrix} = -c\begin{pmatrix} \gamma \\ \beta \end{pmatrix} = -1$ (j=1,2,3), with the others specified by modular invariance and space–time supersymmetry. Trilevel Yukawa couplings are obtained only for $+\frac{2}{3}$ charged quarks.

F	SEC	$SU(3)_C \times SU(2)_L$	Q_C	Q_L	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$SU(5) \times SU(3)$	Q_7	Q_8
V_1	$b_1+2\beta+(I)$	$(1,1)$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$(1,3)$	$-\frac{1}{2}$	$\frac{5}{2}$
V_2		$(1,1)$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$(1,\bar{3})$	$\frac{1}{2}$	$-\frac{5}{2}$
V_3		$(1,1)$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$(5,1)$	$-\frac{1}{2}$	$-\frac{3}{2}$
V_4		$(1,1)$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$(\bar{5},1)$	$\frac{1}{2}$	$\frac{3}{2}$
V_5	$b_2+2\beta+(I)$	$(1,1)$	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$(1,3)$	$-\frac{1}{2}$	$\frac{5}{2}$
V_6		$(1,1)$	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$(1,\bar{3})$	$\frac{1}{2}$	$-\frac{5}{2}$
V_7		$(1,1)$	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$(5,1)$	$-\frac{1}{2}$	$-\frac{3}{2}$
V_8		$(1,1)$	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$(\bar{5},1)$	$\frac{1}{2}$	$\frac{3}{2}$
V_9	$b_3+2\beta+(I)$	$(1,1)$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$(1,3)$	$-\frac{1}{2}$	$\frac{5}{2}$
V_{10}		$(1,1)$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$(1,\bar{3})$	$\frac{1}{2}$	$-\frac{5}{2}$
V_{11}		$(1,1)$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$(5,1)$	$-\frac{1}{2}$	$-\frac{3}{2}$
V_{12}		$(1,1)$	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$(\bar{5},1)$	$\frac{1}{2}$	$\frac{3}{2}$

Table 2. Massless states and their quantum numbers. V indicates that these states form vector representations of the hidden group.

F	SEC	$SU(3)_C \times SU(2)_L$	Q_C	Q_L	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$SU(5) \times SU(3)$	Q_7	Q_8
H_1	$b_1+b_2+\alpha$	$(1,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$(1,3)$	$\frac{1}{4}$	$-\frac{5}{4}$
H_2	$\beta \pm \gamma + (I)$	$(1,1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$(1,\bar{3})$	$-\frac{1}{4}$	$\frac{5}{4}$
H_3		$(1,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$(1,1)$	$-\frac{3}{4}$	$\frac{15}{4}$
H_4		$(1,1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	$(1,1)$	$\frac{3}{4}$	$-\frac{15}{4}$
H_5	$b_1+b_3+\alpha$	$(1,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(1,3)$	$\frac{1}{4}$	$-\frac{5}{4}$
H_6	$\beta \pm \gamma + (I)$	$(1,1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(1,\bar{3})$	$-\frac{1}{4}$	$\frac{5}{4}$
H_7		$(1,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(1,1)$	$-\frac{3}{4}$	$\frac{15}{4}$
H_8		$(1,1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$(1,1)$	$\frac{3}{4}$	$-\frac{15}{4}$
H_9	$b_2+b_3+\alpha$	$(1,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$(1,3)$	$\frac{1}{4}$	$-\frac{5}{4}$
H_{10}	$\beta \pm \gamma + (I)$	$(1,1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$(1,\bar{3})$	$-\frac{1}{4}$	$\frac{5}{4}$
H_{11}		$(1,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$(1,1)$	$-\frac{3}{4}$	$\frac{15}{4}$
H_{12}		$(1,1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$(1,1)$	$\frac{3}{4}$	$-\frac{15}{4}$
H_{13}	$b_1+b_3+\alpha$	$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	0	$(1,3)$	$\frac{3}{4}$	$\frac{5}{4}$
H_{14}	$\pm \gamma + (I)$	$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	0	$(1,\bar{3})$	$-\frac{3}{4}$	$-\frac{5}{4}$
H_{15}		$(1,2)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	0	0	0	$(1,1)$	$-\frac{1}{4}$	$-\frac{15}{4}$
H_{16}		$(1,2)$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	0	$(1,1)$	$\frac{1}{4}$	$\frac{15}{4}$
H_{17}		$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{3}{4}$	$-\frac{1}{4}$	0	0	0	$(1,1)$	$-\frac{1}{4}$	$-\frac{15}{4}$
H_{18}		$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	0	0	0	$(1,1)$	$\frac{1}{4}$	$\frac{15}{4}$
H_{19}	$b_2+b_3+\alpha$	$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0	0	$(5,1)$	$-\frac{1}{4}$	$\frac{9}{4}$
H_{20}	$\pm \gamma + (I)$	$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	0	$(\bar{5},1)$	$\frac{1}{4}$	$-\frac{9}{4}$
H_{21}		$(3,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0	0	$(1,1)$	$-\frac{1}{4}$	$-\frac{15}{4}$
H_{22}		$(\bar{3},1)$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	0	$(1,1)$	$\frac{1}{4}$	$\frac{15}{4}$
H_{23}		$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	0	0	0	$(1,1)$	$-\frac{1}{4}$	$-\frac{15}{4}$
H_{24}		$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0	0	$(1,1)$	$\frac{1}{4}$	$\frac{15}{4}$
H_{25}		$(1,1)$	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$-\frac{1}{4}$	0	0	0	$(1,1)$	$-\frac{1}{4}$	$-\frac{15}{4}$
H_{26}		$(1,1)$	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	0	0	0	$(1,1)$	$\frac{1}{4}$	$\frac{15}{4}$
H_{27}		$(1,1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$(1,1)$	$\frac{1}{4}$	$-\frac{5}{4}$
H_{28}		$(1,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$(1,1)$	$-\frac{1}{4}$	$\frac{5}{4}$
H_{29}		$(1,1)$	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$(1,1)$	$-\frac{3}{4}$	$\frac{15}{4}$
H_{30}		$(1,1)$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$(1,1)$	$\frac{3}{4}$	$-\frac{15}{4}$

Table 3. Massless states and their quantum numbers.