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The Accuracy of Beam-Beam Diagnostics for Circular Colliders*

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ABSTRACT

We investigate the potential of beam-beam deflection techniques for the determination of spot sizes, tilt angle, centering, and angular divergence for circular colliders. Achievable accuracies for all measured quantities are estimated.

1 Introduction

In asymmetric colliders [1] electrons and positrons circulate independently in two different storage rings, which intersect only at the interaction point (IP). Therefore, the beams are not necessarily centered, and the beam sizes can be different. Due to the large aspect ratios, beam tilt angles in the x-y plane can seriously degrade the luminosity [2]. To achieve high integrated luminosities, efficient tuning tools and automatic feedback systems are a necessity.

The success of diagnostic tools based on the mutual deflection of charged particle beams in the final focus of the Stanford Linear Collider (SLC) [3] suggests the investigation of the applicability of these methods for future high-luminosity circular colliders [4]. Here we will estimate the sensitivity of the measured quantities in the framework of a rigid bunch model. Systematic errors from beam blowup due to the nonlinear character of the beam-beam interaction are investigated in Ref. [4], and are shown to be small for the parameters used (see Table 1). Here we will ignore this effect. A more complete analysis should include the dynamic beam blowup.

At the SLC deflection curves are reconstructed from beam position monitor (BPM) signals near the IP from a single beam traversal. In a circular machine a localized kick perturbs the closed orbit [4], which allows us to use BPMs throughout the storage ring to reconstruct beam-beam deflection angles.

In this report we will investigate the accuracy to which deflection curves can be reconstructed from a multitude of BPM data throughout the storage ring, and to what accuracy physically relevant parameters, such as spot sizes and angular divergences, can be inferred from these curves.

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Table 1: *Parameters used in the simulations.*

	HER Beam	LER Beam
Energy	9 GeV	3.1 GeV
Particles per bunch	3.878×10^{10}	5.630×10^{10}
Horizontal spot size	$185.60\,\mu$m	$185.60\,\mu$m
Vertical spot size	$7.45\,\mu$m	$7.45\,\mu$m
Vertical anglular divergence	$250\,\mu$rad	$500\,\mu$rad

2 Deflection Angle

The residual closed orbit distortion resulting from the beam-beam kick angle ε at the IP leads to an offset x_j in the BPM j of

$$x_j = \frac{1}{2}\sqrt{\beta_0\beta_j}\,\frac{\cos(\Delta\psi_j - \pi\nu)}{\sin\pi\nu}\,\varepsilon\,. \qquad (1)$$

where β_0, β_j are the beta functions at the IP and the BPM j, respectively. The $\Delta\psi_j$ is the phase advance between the IP and the BPM. The deflection angle ε can now be found from fitting $\Delta\psi_j$ to measured BPM readings x_j. Thus we obtain a linear least-square fit problem for which the solution, and the corresponding errors, are easily calculated [5].

Assuming equal BPM errors σ_{BPM} throughout the ring, the following scaling relation for the error on the deflection angle can be deduced [6]:

$$\sigma(\varepsilon) \approx \frac{2\sqrt{2}\sin\pi\nu}{\sqrt{\beta_0\beta}}\,\frac{\sigma_{BPM}}{\sqrt{N}}\,. \qquad (2)$$

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We see that increasing the number of BPMs has the expected $1/\sqrt{N}$ behavior, and that the errors are inversely proportional to the root of the beta function at the BPMs, $\hat{\beta}$.

In order to estimate the magnitude of the error for the vertical deflection angle, we make the following assumptions: $\nu_y = n.57$, $\beta_0 = 1.5\,\text{cm}$, $\hat{\beta} = 20\,\text{m}$, and $N = 100$, which lead to $\sigma(\varepsilon)[\mu\text{rad}] \approx 0.5\,\sigma_{BPM}[\mu\text{m}]$. Consequently, a $5\,\mu$m BPM error leads to an accuracy of about $3\,\mu$rad error for the deflection angle. Note, however, that this analysis does not take additional noise sources in the ring into account, and therefore is a lower limit on the achievable error.

The accuracy in the determination of the separation between the beams can be calculated from Eq. 1, if we set $j = 0$. Reference. 6 shows that this obeys the scaling relation

$$\sigma(x_0) \approx \sqrt{2}\cos(\pi\nu)\sqrt{\frac{\beta_0}{\hat{\beta}}}\,\frac{\sigma_{BPM}}{\sqrt{N}} \quad . \qquad (3)$$

Using the above assumptions, this leads to $\sigma(x_0) \approx 8 \times 10^{-4}\sigma_{BPM}$, which is truly negligible.

3 Spot Size and Centering

We now deduce the initial separation between the beams and the spot size by fitting an approximation for large aspect ratios of the well-known Bassetti & Erskine formula [7] to the deflection curve. The approximation for the horizontal deflection angle for a horizontal scan reads

$$\varepsilon_{x,h} \approx -\frac{\sqrt{2\pi}Nr_e}{\gamma\Sigma_x}\mathcal{D}\left(\frac{x-x_0}{\sqrt{2}\Sigma_x}\right) \quad , \qquad (4)$$

where $\mathcal{D}(z)$ is Dawson's integral and can efficiently be evaluated using a Padé approximation. The approximation for the vertical deflection angle for a vertical scan is given by

$$\varepsilon_{y,v} \approx -\frac{2Nr_e}{\gamma\Sigma_x}\left\{\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{y-y_0}{\sqrt{2}\Sigma_y}\right) - \frac{y-y_0}{\Sigma_x}\right\} \quad , \qquad (5)$$

where N is the number of particles in the field producing bunch, r_e is the classical electron radius, γ is the normalized energy of the deflected beam, and $\Sigma_{x,y} = \sqrt{\sigma^2_{x,y}(e^-) + \sigma^2_{x,y}(e^+)}$ are the convoluted or overlap spot sizes. Equations 4 and 5 can each be cast into 4–parameter fits for the spot sizes $\Sigma_{x,y}$, the initial offset between the beam x_0, y_0, the constant $-2Nr_e/\gamma\Sigma_x$, and an offset to take BPM offsets into account. In the vertical scan, we have chosen to explicitly enter the horizontal spot size Σ_x as a parameter in order to make the fitting numerically more stable.

The actual fitting is done with a χ^2–minimization technique, using Nelder & Mead's downhill simplex method [5]. Errors in the fitted parameters are estimated by changing the data points by the amount of the errors and redoing the fit. The rms of the fitted parameters then yields their errors. The first quoted error in Fig. 1 is due to the $3\,\mu$rad angle error; the second is due to the $1\,\mu$m separation error.

Figure 1 shows the result of fits to a horizontal and a vertical scan. The error bars indicate a $3\,\mu$rad

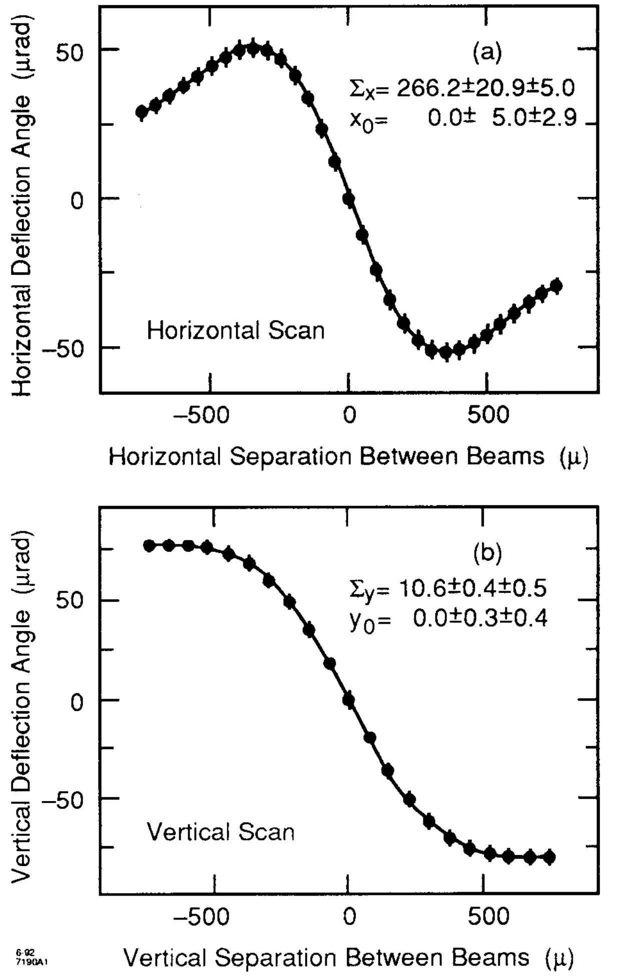

Figure 1. *Horizontal and vertical beam-beam deflection curves for scan ranges of $\pm 750\,\mu m$ and $\pm 30\,\mu m$, respectively. The asterisks are the points from the full Bassetti & Erskine formula and the solid lines from the fits according to Eqs. 4 and 5, with fitted spot size and center in the top right corner. The error bars are $3\,\mu rad$ in angle and $1\,\mu m$ in separation.*

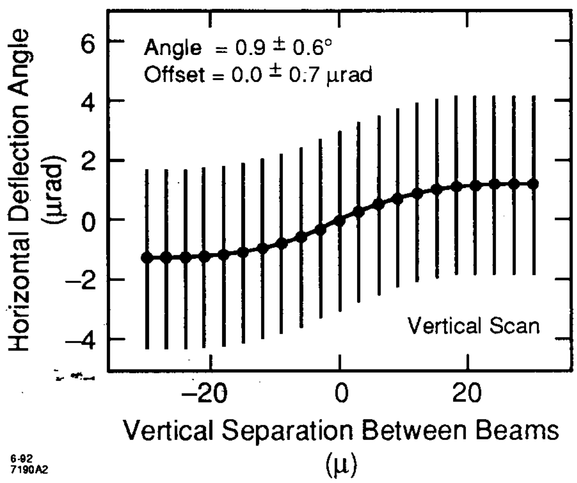

Figure 2 *Horizontal beam-beam deflection angle for a vertical scan range of* $\pm 30\,\mu m$. *The tilt angle of the electron beam is* $2°$. *The asterisks are points from the full Bassetti & Erskine formula, and the solid line comes from the fit according to Eq. 6.*

error in the deflection angle and a $1\,\mu$m error on the beam separation. We see that we can determine Σ_x and x_0 to the order of $20\,\mu$m or better, which is less than 10 % of the horizontal spot size. Similarly, in the vertical scan Σ_y and y_0 can be determined to better than 10 % of the vertical spot size. Note that the fitted spot sizes are convoluted spot sizes that are $\sqrt{2}$ bigger than those reported in Table 1.

4 Tilt Angle

In order to assess the signatures of tilt angle ϕ in the horizontal deflection angle $\varepsilon_{x,v}$ in a vertical scan, we expand the generalized Bassetti & Erskine formula [6] in ϕ. For large aspect ratios Σ_x/Σ_y, we obtain

$$\varepsilon_{x,v} \approx -\phi \varepsilon_{y,v} \tag{6}$$

where $\varepsilon_{y,v}$ is given by Eq. 4.

Clearly the horizontal deflection angle $\varepsilon_{x,v}$ is given by a projection of the vertical deflection angle $\varepsilon_{y,v}$, and thus its shape is entirely determined by $\varepsilon_{y,v}$. Therefore, we can use fitted parameters like Σ_y, y_0, and just perform a linear fit for ϕ and a possible constant offset, which can come from BPM offsets or horizontal miscentering. Figure 2 shows the horizontal beam-beam deflection curve for a vertical scan range of $\pm 30\,\mu$m, in which the electron beam is tilted by $2°$ versus the horizontal axis. In the top left corner the calculated values for the fitted offset and tilt angle are given.

Note that the value reported in Fig. 2 is only half of the tilt angle of the electron beam, because the deflection curve only depends on the sum of the covariance matrices of electron and positron beam; therefore, only average tilt angles can be deduced.

Despite the inability to detect individual tilt angles, the possibility of resolving average tilt angles to accuracies on the order of one degree makes it possible to control the luminosity degradation to a few percent [2].

5 Angular Divergence and Waist Position

In order to measure the angular divergence of one beam we have to move the minimum of the beta function, the so-called waist, by changing quadrupole excitations in the vicinity of the IP. Making such a local change of the optics transparent to the rest of the ring is part of a demanding optical matching problem that has to be investigated independently.

Performing deflection scans as described in Secs. 2 and 3 we can plot the squared measured spot sizes versus the waist position. This results in a parabola, shown in Fig. 3. The angular divergence σ' can be easily extracted from a linear least square fit to a parabola of the type

$$\Sigma^2(z) = \Sigma_0^2 + \sigma'^2 (z - z_0)^2 \; . \tag{7}$$

Here, Σ is the convoluted spot size from both beams, and z_0 is the position of the waist with respect to the starting point of the waist scan. Note that this method allows the determination of the angular divergence of a single beam, but only allows the determination of the convoluted spot sizes for both beams.

In order to calculate the accuracy to which the angular divergence can be resolved, we assume an angular divergence of $500\,\mu$rad and calculate the spot sizes, using Eq. 7. We then use these spot sizes as input data for single scan simulations with a scan range of $\pm 30\,\mu m$ in steps of $3\,\mu m$, which leads to a collection of scans similar to those shown in Fig. 1.

The results of the parabola fit are shown in Fig. 3. The angular divergence can be reconstructed to about 10 % or $\pm 50\,\mu$rad, and the waist position can be found to about 0.1 cm.

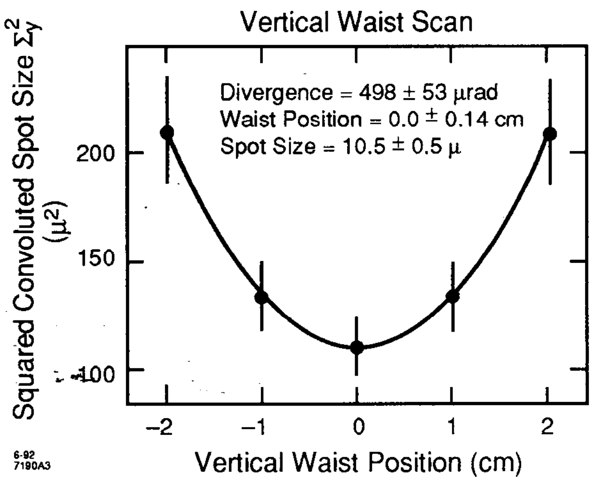

Figure 3. *A waist scan in the range ±2 cm in steps of 1 cm.*

6 Conclusion and Outlook

We showed that IP deflection angles from the beam-beam interaction can be measured by BPMs throughout the storage ring to an accuracy of a few micro-radians. In this way, beam-beam deflection curves can be traced out and used to determine the spot size and centering of the beams. We found that the spot size and the centering of both beams can be determined to about $1\,\mu$m, or to about 10 %. These accuracies will make a feedback system utilizing this signal attractive.

Furthermore, the average of the tilt angle will be measurable down to 1 degree. This allows control of luminosity degradations due to tilted beams down to a few percent.

Moving the minimum of the beta function by ±2 cm will allow the determination of the angular divergence to an accuracy of about 10 %. The determination of the emittance is hampered by the impossibility of determining individual spot sizes in beam-beam deflection scans.

The difficulty of measuring individual deflection angles of the large number of circulating bunches (1658 in Ref. 1) can be overcome by utilizing the pickup system and electronics of the transverse multibunch feedback system [1], which will be capable of detecting betatron oscillations of individual bunches. In order to determine the closed-orbit distortion referred to in Sec. 2, we can average and filter out the DC signal instead of filtering out the betatron component. Interference with the feedback system can be made small by making the feedback respond to, and damp only, oscillations at the betatron frequency, and by making it insensitive to the DC offset. In this way the average signal gives the required closed orbit information with little extra effort in computing power. On the other hand, the hardware investment is large, because fast BPM electronics would be needed for a large number of BPMs.

We conclude that diagnostic methods based on beam-beam deflections will provide powerful and accurate measurements of various beam dynamical quantities in circular colliders. More work, especially on the hardware requirements. is definitely needed.

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