

Baryogenesis from Unstable Domain Walls

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ABSTRACT

There exists a class of cosmic strings that turn matter into antimatter (Alice strings). In a GUT where the unbroken gauge group contains charge conjugation (C), such strings form when a phase transition renders C a discrete symmetry. They become boundaries of domain walls at a later, C -breaking transition. These 'Alice walls' are cosmologically harmless, but can play an important role in baryogenesis. We present a three-generation toy model with scalar baryons, where a quasi-static Alice wall (or a gas of such walls) temporarily gives rise to net baryogenesis of uniform sign everywhere in space. This becomes a permanent baryon excess if the wall shrinks away early enough. We comment on the possible relevance of a similar mechanism to baryogenesis in a realistic $SO(10)$ unification model, where Alice walls would form at the scale of left-right symmetry breaking.

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1. Introduction

Alice strings [1,2,3] are a class of cosmic strings with the remarkable property that a particle traveling around one will come back as its own antiparticle. In a Grand Unified Theory in which the charge conjugation operator C is contained in the original gauge group, such strings form via the Kibble mechanism when the symmetry is broken to a smaller group having C as a discrete (i.e. not deformable to 1) symmetry. This occurs, for example, for certain SO(10) GUTs.

In the vacuum we inhabit, of course, C is not a symmetry. This means it must have been spontaneously broken at some later phase transition. At that epoch, the Alice strings would have become boundaries of domain walls. We shall henceforth refer to the latter as ‘Alice walls’. The dynamics of cosmological networks of string-bounded walls has been studied [6]. The walls eventually shrink via surface tension, string intercommutation and nucleation of new string loops. Thus they never dominate the energy density of the Universe, and can have interesting cosmological effects while they last.

Until recently it was believed that Alice strings could only have existed in pre-inflationary epochs, since a phase transition giving rise to Alice strings also tends to form magnetic monopoles in abundances ruled out by experiment [4]. However, in ref.7 we gave an example of a natural symmetry-breaking scheme in which this problem is avoided. The models considered in ref.7 were variants of the original Alice-string model of ref.1 in which an SO(3) gauge symmetry is spontaneously broken to O(2), and then (optionally) to smaller groups.

The present paper is dedicated to the study, in such a toy model, of a novel baryogenesis mechanism involving Alice walls. The original gauge group is SO(3), and the (scalar) baryons belong to SO(3) triplets. We define the *baryon number*, B , to be the electric charge for members of these ‘baryonic triplets’, and zero for all other fields. Here ‘electric charge’ refers to the $U(1)_{\text{em}}$ subgroup of SO(3) generated by T_3 . Baryon number is violated by perturbative processes, such as exchange of the charged[†] vector bosons W_{\pm} of SO(3), although no net baryogenesis can occur while $C \equiv \exp(\pi iT_2)$ is still conserved. When SO(3) is broken to O(2) (which is the semidirect product $U(1)_{\text{em}} \times \{1, C\}$), the W_{\pm} bosons become massive and the perturbative violations of baryon number become suppressed as the temperature continues to fall. However, this phase transition also creates Alice strings, through the Kibble mechanism. In the presence of such a string C is still conserved (it is an element of

[†] By ‘charged’ we shall always mean $U(1)_{\text{em}}$ charge.

$O(2)$), but B is globally ill-defined since when a ‘baryon’ is taken once around a string it becomes an ‘antibaryon’. Locally, however, B may be sensibly defined and is still weakly violated by perturbative processes[‡].

At a lower temperature, we spontaneously break C by allowing a (non-baryonic) $SO(3)$ Higgs triplet to acquire a VEV[§]. A closed Alice string loop now becomes the boundary of an Alice wall, where the triplet Higgs develops a kink. *Outside* such a wall, it is clear that B may be globally defined. In this case, the passage of a baryon through the wall will be seen as a B -violating process — it will emerge on the other side as an antibaryon (and vice versa). If we could arrange for baryons to bounce off the wall more frequently than antibaryons, say, then we would have a means of driving baryogenesis. In practice, we find that the model must be complicated slightly to achieve this: it must contain at least three copies (‘generations’) of baryonic triplets, and also at least three Higgs triplets. The transmission rates of baryons and antibaryons through the wall are generically different in the extended model.

If we now impose that (prior to wall formation) the ambient baryon-antibaryon plasma has $B = 0$ and is out of thermal equilibrium, then baryogenesis will initially occur. This result holds even for a static Alice wall — a novel result so far as we are aware[¶]. In a cosmological setting there will be a gas of such Alice walls, each driving baryogenesis. It is important to stress that the *sign* of the baryon excess will be uniform throughout space. However, if the walls are left intact indefinitely, B will execute damped oscillations and return asymptotically to zero. We must therefore allow the walls to shrink away sufficiently quickly that a net B remains.

This baryogenesis mechanism occurs at the tree level. It is completely classical if we think of the scalar baryons as wave packets. It is more useful, however, to think of the relevant transmission (and reflection) rates as squares of moduli of quantum amplitudes, in a formalism where the baryons are first-quantized and evolve in the classical background of the Alice wall.

In our specific model, net baryon number implies also a net electric charge. Thus the Alice wall must accumulate charge (Cheshire or otherwise) to offset the baryon number it

[‡] We thank J. Preskill for a discussion concerning this point.

[§] Parity is conserved in our model, and so CP and C are not independent symmetries. This circumstance will of course change in the realistic $SO(10)$ model.

[¶] Note that even in this case Sakharov’s conditions are met since the ambient plasma is not in thermal equilibrium.

creates around it; in the parameter regime we shall focus on, only Cheshire (i.e. nonlocalized) charge accumulates. This charge continuously decays as it forms, by electrically polarizing the surrounding plasma and also by the quantum-mechanical emission of charged particles. Since the plasma is composed of other charged particles besides (anti)baryons, the decay of Cheshire charge will only partially cancel the accumulated B , not eliminate it.

The Alice walls in our model may or may not exhibit the Meissner effect. This depends on whether the charged Higgs components acquire VEVs in the wall core. We refer to these two cases as the *superconducting* and *normal* cases, respectively. In the version where we find baryogenesis, the normal phase is chosen for all three Higgs triplets, since this is simplest. The case of superconducting walls, however, has its own points of interest, quite apart from baryogenesis. We shall return to this case in a future publication [8].

So far we have been discussing a toy model. Some viable GUT schemes can also form Alice strings and walls; we wish to raise the question of whether a variant of our mechanism can play an important role in baryogenesis for such GUT's—e.g. for the group $SO(10)$. This question is currently under investigation; we discuss it briefly in the concluding section.

The remainder of the paper is organized as follows. In section 2 we set up the toy model. We specialize to a Higgs potential such that both an Alice string and the wall it bounds are formed, and such that the wall is normal (non-superconducting). In this regime, the wall is described as kinks in the three neutral Higgs fields. In section 3, we analyze the spectrum and discrete symmetries of the model; it is seen that C (and thus CP) are broken in the space surrounding the wall. Section 4 is the main part of the paper. It is shown there that the discrete symmetries are not capable of constraining the baryon and antibaryon transmission rates sufficiently to rule out baryogenesis. A parameter regime is chosen which simplifies the physics. In particular, the heaviest among the three mass-eigenstate baryons is decoupled. We next proceed to derive the momentum- and position-averaged rate equations for the concentrations of the two lightest baryon species, and their antibaryons^b. The asymmetry between the transmission rates of baryons and antibaryons through the wall is computed in the Born approximation. It is thus shown that an out-of-equilibrium plasma with $B = 0$ initially accumulates a net baryon number, which becomes permanent if the wall disappears in time. This baryon number has a uniform, physically-meaningful sign throughout space.

^b The fact that we are able to completely decouple one of the three mass-eigenstate baryon species yet still obtain baryogenesis, does not contradict our previous statement that at least three generations are needed. This is explained in section 4.

This remains true in the presence of a gas of such walls. In section 5 we summarize our conclusions, and briefly discuss ongoing work on baryogenesis in SO(10).

2. The Model

Our model is an SO(3) gauge theory with the following matter content: An isospin-2 Higgs field[†] Φ_{ab} (a 3×3 traceless symmetric matrix); three scalar ‘baryonic triplets’ $\psi_a^{(I)}$, with I ranging from 1 to 3; and three Higgs triplets, $v_a^{(I)}$. We shall refer to I, J , etc. as *generation* indices. The lower-case a, b etc. will denote gauge indices; repeated gauge or Lorentz indices will be summed over except when stated otherwise, but not generation indices. We employ the West-Coast metric $(1, -1, -1, -1)$. We also include one additional nonbaryonic isotriplet, u_a ; its sole interaction is a minimal coupling to the gauge fields. The only role u will play in the model is to cancel the Alice wall’s accumulated electric Cheshire charge. We will not need to explicitly write the u -dependent terms in the action.

The Lagrangian density is as follows:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4e^2} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2} \text{tr}(D_\mu \Phi)^2 + \frac{1}{2} \sum_I (D_\mu v^{(I)})^2 + \frac{1}{2} \sum_I (D_\mu \psi^{(I)})^2 \\ & - V_1(\Phi) - \sum_I V_2^{(I)}(v^{(I)}, \Phi) - V_3(v) - V_4(\psi, \Phi) - V_5(\psi, v) \end{aligned} \quad (2.1)$$

with (the superscript T denotes isovector transposition):

$$\begin{aligned} V_2^{(I)}(v, \Phi) = & \lambda_I v^T \Phi v + a_I v^2 \\ & + \rho_I (v^2)^2 \end{aligned} \quad (2.2)$$

$$V_3(v) = -\epsilon \sum_{I < J} v^{(I)} \cdot v^{(J)} \quad (2.3)$$

$$V_4(\psi, \Phi) = \frac{1}{2} \sum_{IJ} \nu_{IJ} \psi^{(I)T} \psi^{(J)} - \frac{\kappa}{2} \sum_I \psi^{(I)T} \Phi \psi^{(I)} \quad (2.4)$$

$$V_5(\psi, v) = -\frac{b}{2} \sum_{IJK} \epsilon_{IJK} \epsilon_{abc} \psi_a^{(I)} \psi_b^{(J)} v_c^{(K)} \quad (2.5)$$

where e is the gauge coupling, and $\lambda_I, \rho_I, \epsilon, \kappa$ and b are all chosen to be positive. The gauge fields are A_μ^a , and their field strengths $F_{\mu\nu}^a$.

[†] ‘Isospin’ refers to the gauge-group transformation properties.

We now explain the motivation of the various potential terms. V_1 is some polynomial in traces of powers of the matrix Φ , chosen to have a minimum (unique up to a gauge transformation) at

$$\Phi = \alpha \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & -2 \end{pmatrix}, \quad \alpha > 0; \quad (2.6)$$

we shall not require a specific form for V_1 here. The role of the $V_2^{(I)}$ terms is to give the triplet Higgs fields vacuum expectation values (VEVs); apart from the standard Mexican-hat potentials, they contain alignment terms (with couplings λ_I) which tend to align $\langle v_a^{(I)} \rangle$, in isospin space, with respect to Φ . This alignment is required to preserve the $U(1)_{\text{em}}$ symmetry, as explained below. The neutral members of the Higgs triplets must acquire VEVs in order to break C . As we shall see, these VEVs are constant in the bulk of space, and have kinks at the wall since the Alice twist requires them to vanish on some surface within the wall.

The Higgs couplings are generation-dependent, and in a cosmological setting they are also temperature- (and thus time-) dependent. Thus, a_I should decrease with temperature, and at some epoch they will be in a regime such that all the Higgs triplets acquire VEV's, and C is broken.

The signs of $\langle v_3^{(I)} \rangle$ for different I are *a priori* uncorrelated. The purpose of the term V_3 is to correlate them, thus ensuring that there are only two true vacua after C is broken—and as one is the C -reflection of the other, these are gauge equivalent. The parameter ϵ can be chosen as small as desired. Together with V_2 , it constitutes the mass-squared matrix of the v -fields in the C -symmetric vacuum. The V_3 term correlates not only the bulk VEVs of the Higgs triplets, but also the locations of the kinks for different values of I .

The potential V_4 contains the ψ mass term and a $\psi\psi\Phi$ Yukawa interaction. The latter is introduced in order to break the degeneracy between the charged and neutral members of the baryonic multiplets. By (2.4),(2.6) we have

$$m_A^2 = \mu_A^2 - \kappa\alpha, \quad M_A^2 = \mu_A^2 + 2\kappa\alpha \quad (A = 1, 2, 3) \quad (2.7)$$

where μ_A^2 are the eigenvalues of the symmetric matrix ν_{IJ} , and m_A, M_A are the masses of the charged and neutral ψ fields, respectively, before C is broken. This mass splitting between charged and neutral baryons is not really necessary, but it is convenient, since we can now

proceed to make the neutrals very massive:

$$M_A = O(\alpha), \quad m_A \ll \alpha \quad (2.8)$$

where α is the scale of the Φ VEV. This is accomplished by choosing $\mu_A = O(\alpha)$, $|\mu_A - \mu_B| \ll \alpha$ and tuning κ to produce a hierarchy between M_A , which are at the $SO(3)$ -breaking scale, and m_A . Thus neutral baryonic particles can be assumed to have already decoupled at the temperatures where the Alice wall forms. This suits our purpose, since these particles have no baryon number—we wish to concentrate on the baryons and antibaryons.

Finally, there is the potential V_5 . This consists of Yukawa couplings of the form $\psi\psi v$, and serves to couple the baryons to the wall. In the bulk (outside the wall), it changes the mass-squared matrix of the baryons in the C -broken phase, so the bulk definition of the three baryons (and their antibaryons) changes. In this phase a mass-eigenstate baryon is, in general, no longer in the same gauge multiplet as its CPT -conjugated antiparticle. Note that despite the breaking of C and CP , each baryon-antibaryon pair is still degenerate, as required by the CPT theorem. In addition to its effect in the bulk, we shall see that the V_5 potential gives rise to different transmission probabilities for incoming baryons and antibaryons of the same energy. Since each transmitted baryon (antibaryon) is turned into an antibaryon (baryon)[◇], this asymmetry enables baryogenesis. This will be explained in detail in section 4.

The VEV eq.(2.6) breaks the gauge group $SO(3)$ down to $O(2)$, which has two connected components and is the semidirect product of $U(1)_{\text{em}}$ and $\mathbf{Z}_2 \equiv \{1, C\}$. The group $U(1)_{\text{em}}$ consists of isospin rotations about the internal 3-axis; C is charge conjugation, and we choose it to be a rotation by π about the internal 2-axis. Stable Alice strings exist in the $O(2)$ phase. We wish to consider processes in the background of a closed Alice string, so we must begin by twisting all charged fields by C around curves linking the Alice-string loop^{*}. Thus, the physical gauge defined by eq.(2.6) is valid in all of space excluding some branch-cut surface. This surface is arbitrary, except that its boundary must be the string. Any given field is

◇ In general a baryon is sometimes transmitted while staying a baryon. However, we shall choose the parameters of the model so this does not happen. This choice will also ensure that a *reflected* baryon (antibaryon) remains unflipped.

* We shall not need to consider the structure of the string core in this paper. Since we assume a hierarchy between the two symmetry-breaking scales, it is natural for the wall thickness (and of course its size) to be much larger than the string core. We *shall*, however, require details of the kink configuration at the core of the Alice *wall*.

matched to itself across the cut via the C operation. Thus $\langle v_3^{(I)} \rangle$ flip their signs at the cut, $\langle v_2^{(I)} \rangle$ are continuous, etc.

It is straightforward to check that the $vv\Phi$ alignment term in V_2 forces $\langle v^{(I)} \rangle$ in the bulk to lie along the internal 3 direction. The term was chosen for precisely that reason: we wish to break C , but not $U(1)_{\text{em}}$, in the bulk of space[†].

We next investigate the structure of a static Alice wall. We shall assume the string loop to be a smooth curve (e.g. a circle), statically held in place, with the pancake-shaped wall centered at the planar disk bounded by the string[‡].

As the temperature decreases, we assume that all three a_i parameters become small enough so that the C -symmetric vacuum $\langle v^{(I)} \rangle = 0$ becomes unstable. Let us begin by considering a single Higgs triplet, say $I = 1$. Since we would first like to understand the structure of the wall itself, without ambient matter, the ψ fields will be set to zero at this stage. The relevant single-generation potential is then:

$$V_{\text{wall}} = V_1(\Phi) + \lambda v^T \Phi v + av^2 + \rho(v^2)^2 \quad (2.9)$$

In addition, the Higgs mechanism causes the two charged gauge bosons, W_{\pm} , to eat two of the components of Φ , and acquire a mass

$$M_w = 3\sqrt{2}\alpha\epsilon \quad (2.10)$$

The VEV α defines the scale of the Alice *strings*.

The $v = 0$ vacuum is unstable, but $\langle v_a \rangle$ cannot be uniform throughout space, due to the Alice twist. Since v_1, v_3 are twisted by -1 around a closed curve linking the string, each of these two components must vanish on some surface having the string as boundary. As explained above, however, of the three isospin components only v_3 will develop a kink, i.e. a soliton configuration interpolating between two VEVs of opposite sign. The deviations of

† Except during a brief Langacker-Pi phase, which is desirable in order to get rid of magnetic monopoles [7]. Such a phase does not arise in the model described here, but we shall see below that there is a parameter range for which the kink that defines the wall is superconducting. In this regime, $U(1)_{\text{em}}$ is broken *in the wall*, though it is still a good symmetry in the bulk. However, in this paper we concentrate on the regime where $\langle v_a^{(I)} \rangle = 0$ everywhere for $a = 1, 2$.

‡ The wall is unstable against nucleating virtual Alice string-loops on its surface, which will eventually destroy it even if we hold the string fixed. But the rate of this nucleation is strongly suppressed for even a modest hierarchy between the string-forming and C -breaking scales [5], so this is not a problem.

v_a from their bulk VEVs will be limited to a wall, enclosing the $v_3 = 0$ surface and having a thickness of order

$$\text{wall thickness} = O(1/\sqrt{a}). \quad (2.11)$$

This is precisely the Alice wall (figure 1). We let the string lie along a fixed circle in the $z = 0$ plane; the $v_3 = 0$ surface is then taken to be the $z = 0$ disk bounded by this circle. The v -field configuration of the wall reacts back, through the equations of motion, on the $\{\Phi, A_\mu^a\}$ fields of its preexisting string boundary. Our approach shall be to choose the parameters so as to make this back-reaction small, and then to study the effect of the wall on a dilute ambient plasma, which in turn has negligible back-reaction on the Alice wall.

We shall find it convenient to employ *two* distinct physical gauges in what follows; each of them has its advantages. One of them we call the ‘disk’ gauge; it has its branch-cut inside the wall, at the surface where $\langle v_3^{(I)} \rangle$ vanish[§]. For our static wall, this surface lies in the symmetry plane $z = 0$. In this gauge, electric charge and baryon number are well defined outside the wall, and there is no difficulty in describing multi-wall configurations. The other physical gauge is the ‘transmission’ gauge. Here the cut surface is chosen to extend from the string loop *outwards*, to infinity, along the plane of the string. In this gauge charge and baryon number are not globally defined in the bulk, but it is easier to analyze baryon transmission since the cut lies away from the wall. The branch-cut surfaces for the two physical gauges are shown in figure 2.

In either of these physical gauges, the surviving components of the Φ field after the Higgs mechanism are:

$$\Phi = \begin{pmatrix} \alpha + \varphi_1 + \varphi_2 & \varphi_3 & 0 \\ \varphi_3 & \alpha + \varphi_1 - \varphi_2 & 0 \\ 0 & 0 & -2\alpha - 2\varphi_1 \end{pmatrix} \quad (2.12)$$

Here φ_1 is neutral, whereas the two real fields φ_2, φ_3 together constitute a charged field. The masses of these Φ Higgs bosons are of order α . Due to the assumed hierarchy, the wall thickness is much larger than the $1/\alpha$ scale, and also larger than the related W -boson (inverse) mass scale:

$$a \ll \alpha^2, \quad a \ll e^2 \alpha^2 \quad (2.13)$$

We now require that the excitation of the Φ -Higgs modes $\varphi_1, \varphi_2, \varphi_3$ in the vicinity of the

§ As pointed out above, the inter-generation alignment potential V_3 makes it energetically favorable for the kinks of the three generations to be centered about the same surface

wall[†] be small relative to the bulk VEV, α . Upon consulting eqs.(2.9-13), we find this to hold for the following parameter range:

$$\lambda(a/\rho) \ll \alpha^3 \quad (2.14)$$

Next, consider the back-reaction of the wall on the gauge fields; in the absence of the wall, A_μ^a vanish in a physical gauge. For a normal (non-superconducting) wall, it is easy to check that the gauge fields remain zero even near the wall. In the superconducting case, the gauge fields are excited in the wall, but again one can choose a regime where this back-reaction is small [8].

Having dealt with the issue of the wall's back-reaction on the Alice-string fields, it remains to study the wall configuration itself—that is, the v kink. We find there are three pertinent Higgs-parameter ranges [8]:

- $a > 2\lambda\alpha$: the $v_a = 0$ vacuum is stable, no wall forms. We call this regime I.
- Regime II: $-\zeta_0\lambda\alpha < a < 2\lambda\alpha$. The $v = 0$ vacuum is unstable, but a v_3 kink with $v_1 = v_2 = 0$ is stable. Here ζ_0 is a pure number; we do not know it, but the relevant point is that $-2 < -\zeta_0 \leq 11/2$, so regime II exists. The wall formed by this kink is normal (non-superconducting).
- Finally, regime III is $a < -\zeta_0\lambda\alpha$. In it the above kink is unstable, and there exists another, stable kink with v_2 nonvanishing in the kink core and vanishing outside it. This kink is the superconducting wall referred to above.

From here on we shall restrict attention to normal kinks. For our quartic potential, and in the *transmission* gauge, this kink is the well-known *tanh* soliton centered at $z = 0$ [¶]:

$$v_1 = v_2 = 0, \quad v_3 = \sqrt{\frac{2\lambda\alpha - a}{2\rho}} \tanh(z\sqrt{2\lambda\alpha - a}) \quad (2.15)$$

Thus far, we have been discussing the structure of the Alice wall for a single Higgs triplet. When there are several generations, as in our model, we need to separately consider the three potentials $V_2^{(I)}$. Assuming the temperature has fallen sufficiently so that all three a_i have entered regime II, three kinks of type (2.15) will form, with different widths and heights;

[†] Actually, the neutral component φ_1 acquires a small VEV throughout space, but this serves only to modify α slightly.

[¶] This expression for the soliton requires corrections near the edge of the wall.

all are bounded by the same Alice string. As discussed above (2.7), the role of the inter-generation alignment potential V_3 is to break the degeneracy between the vacua with different relative signs of $\langle v_3^{(j)} \rangle$, so that only two degenerate vacua are left. In these true vacua, all three VEVs have the same sign. Furthermore, since C belongs to the original gauge group, these two vacua are gauge-equivalent and are thus physically identical. The spatial regions between kinks of different generations consist of false vacua, and the pressure differences between these regions and the true vacuum will force them to shrink. It is thus energetically favorable for the three kinks to coalesce. We therefore assume from here on that all three are centered at $z = 0^*$ (figure 3). Finally, note that in addition to kinks bounded by the string, the C -breaking transition will also form kink bubbles with no boundaries. Since we are assuming a mass hierarchy, those bubbles which interpolate between the two equivalent vacua are metastable, while the other ones shrink and disappear due to pressure differences. The baryogenesis mechanism, to be demonstrated below, applies to the metastable bubbles as well as to Alice walls[‡]. According to refs.6, however, bubbles tend to be shredded into bounded walls in an evolving string-wall network.

- 3. The Spectrum and Discrete Symmetries

At temperatures below the $\langle \Phi \rangle$ scale, $SO(3)$ is broken down to $O(2)$. The discrete symmetries of the vacuum are then C, P and T . The action of C on the various fields is straightforward: for each gauge index $a \neq 2$ appearing in a field, it gets multiplied by (-1) . Parity simply reverses the sign of spatial coordinates, and multiplies a field by (-1) for every spatial index appearing in it. Time-reversal reverses the sign of time, changes c-numbers into their complex conjugates[†] and, as for C and P , multiplies the various fields by some signs. These signs are *almost* determined by the requirements that all couplings in the action are C, P and T invariant, and that CT remains a symmetry when the Alice wall forms. The only freedom left in choosing these signs reflects the following two symmetries:

* Realistically their relative positions will fluctuate, at least for small ϵ . However, they are more likely to be found near each other than far apart, so our assumption is reasonable even in a true finite-temperature environment.

‡ As long as such a bubble does not begin to decay via virtual-string nucleation, however, a global definition of baryon number is not physically relevant. Once it decays it becomes an Alice wall, to which our analysis applies.

† As mentioned above the baryogenesis mechanism explicated here is classical, although we choose to interpret it in particle language. At the classical level, then, one may work with the real components of all fields, and then no complex numbers are involved.

(I) $\psi \rightarrow -\psi$, other fields unchanged; this symmetry is just baryon-number conservation modulo 2, if perturbative baryon-violating processes are neglected.

(II) A global $U(1)_{\text{em}}$ rotation by π .

Modulo (I) and (II), one finds that T acts on the real fields precisely as C does, except that $t \rightarrow -t$. In other words, CT (which must be conserved by the CPT theorem, since P is) acts as follows in our model:

$$(CT)F(\vec{x}, t)(CT)^{-1} = F(\vec{x}, -t) \quad (3.1)$$

for any field F . When $v^{(I)}$ acquire VEVs, C and T are broken as separate symmetries. Analogously to what happens for CP violation in the Standard Model, the mass terms by themselves obey a modified C , but the interaction terms violate it, so in fact *no* good charge-conjugation symmetry can be found once the wall has formed. However, CT remains a good symmetry, since the VEVs are left invariant under it (eq.(3.1)).

Next, let us take stock of the spectrum after the formation of the Alice wall. The surviving components of the Φ Higgs are φ_1, φ_2 and φ_3 , which make up one real neutral field and one complex, charged field. Their masses are of order α , if we make the quartic couplings in $V_1(\Phi)$ of order 1. The charged gauge bosons W_{\pm} have a mass given by (2.10), and the three neutral components of the $\psi^{(I)}$ triplets also have masses $O(\alpha)$ (see (2.7-8)). We shall refer to these particles (with masses of order α) as *superheavy*; they will decouple before the wall-forming epoch, so we henceforth ignore them and the quantum processes they mediate.

The remaining particles are the charged ψ -particles and the (charged and neutral) v -particles. Since we wish to study the regime II (in the classification above (2.15)), we must choose λ_I small enough so that

$$\lambda_I = O(a_I/\alpha). \quad (3.2)$$

We shall collectively denote the scales a_I by a where convenient.

From (2.2),(3.2) we find that each generation of Higgs triplet has one real neutral field and one complex charged field* with masses of order \sqrt{a} :

$$(m_v^n)^{(I)} = O(\sqrt{a}), \quad (m_v^{ch})^{(I)} = O(\sqrt{a}) \quad (3.3)$$

where the superscripts n, ch refer to neutral and charged components, respectively.

* We assume that the coupling ϵ is small, so the inter-generation mixing it engenders is negligible.

Finally there are the three charged ψ fields, which describe the three baryons and their antibaryons. Before C breaking their masses were m_A (eqs.(2.7-8)). When $v^{(I)}$ acquire VEVs, the Yukawa potential V_5 modifies the ψ mass-squared matrix. The new mass term for the baryons is z -dependent; neglecting wall-edge effects, the baryonic action in the transmission gauge is

$$S_{baryonic} = \int d^4x \left(\sum_I \partial_\mu \psi_-^{(I)} \partial^\mu \psi_+^{(I)} - \sum_{IJ} \psi_-^{(I)} M_{IJ}^2(z) \psi_+^{(J)} \right) \quad (3.4)$$

where

$$\psi_\pm^{(I)} \equiv \frac{1}{\sqrt{2}} (\psi_1^{(I)} \pm i\psi_2^{(I)}) \quad (3.5)$$

are the (anti)baryon fields, and the position-dependent, hermitian mass-squared matrix for the $\psi_+^{(I)}$ fields is:

$$M_{IJ}^2(z) = \tilde{\nu}_{IJ} - ib \sum_K \epsilon_{IJK} v_3^{(K)}(z) \quad (3.6)$$

where $\tilde{\nu}_{IJ} \equiv \nu_{IJ} - \kappa\alpha\delta_{IJ}$ has eigenvalues m_A^2 (eqs.(2.4),(2.7)). The $v_3^{(I)}$ kinks appearing in (3.6) are given by (2.15), with (λ, a, ρ) replaced by (λ_I, a_I, ρ_I) . It is crucial for the baryogenesis effect that the widths of the three kinks all be distinct from one another, as will be seen in section 4.

Let us denote

$$r_I \equiv v_3^{(I)}(+\infty) \quad (3.7)$$

These three numbers are positive**, and determine the mass-squared matrix in the bulk. Switching to the disk gauge and choosing it so that $v_3^{(I)} = r_I$ away from the wall, we obtain from (3.6) the mass-squared matrix in the bulk:

$$(M_{IJ}^2)_{\text{bulk}} = \tilde{\nu}_{IJ} - ib \sum_K \epsilon_{IJK} r_K \quad (\text{disk gauge}) \quad (3.8)$$

At this point, it is useful to change basis in generation space. We do this by acting with an $\text{SO}(3)$ matrix on the I index, in such a way that in the *new* basis (with indices denoted by

** Their relative signs are determined by the potential V_3 , as discussed in section 2. Their absolute sign, however, can be changed by a global C transformation, to which the physics is invariant.

the letters A, B , etc.)

$$(M_{AB}^2)_{\text{bulk}} = (\tilde{\nu} + \omega t_3)_{AB} \quad (3.9)$$

where:

$$\omega = +b\sqrt{\sum_I (r_I)^2}, \quad (3.10)$$

$$(t_C)_{AB} = -i\epsilon_{ABC} \quad (3.11)$$

are the generators of the generation-space rotation group, and $\tilde{\nu}_{AB}$ is the matrix $\tilde{\nu}$ in the new basis. We now conveniently choose $\tilde{\nu}$ as follows:

$$\tilde{\nu} = m_h^2 + (\omega + m_l^2 - m_h^2)(t_3)^2 \quad (3.12)$$

It follows from (3.9),(3.12) that the mass-squared spectrum of baryons in the bulk is: m_l^2, m_h^2 and

$$m_H^2 \equiv m_l^2 + 2\omega, \quad (3.13)$$

and we choose $m_H > m_h > m_l$. Each baryon is degenerate with its antibaryon, as required by CPT invariance. The ‘light’ (l) and ‘medium heavy’ (h) baryons in the bulk are described (in disk gauge) by the $t_3 = -1$ and $t_3 = 0$ components of the ψ_+ field, respectively, whereas the ‘heavy’ (H) baryon is described by the $t_3 = +1$ component. Thus, upon decomposing the baryon field in the t_3 basis,

$$\psi_{\pm}^{(A)} = \frac{1}{\sqrt{2}} \sum_{\eta \in \{1, -1\}} \psi_{\pm}^{(\eta)} \begin{pmatrix} 1 \\ i\eta \\ 0 \end{pmatrix} + \psi_{\pm}^{(0)} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad (3.14)$$

we find the following disk-gauge field assignments in the bulk:

$$\text{baryons} : (l, h, H) = (\psi_+^{(-)}, \psi_+^{(0)}, \psi_+^{(+)}) \quad (3.15)$$

$$\text{antibaryons} : (\bar{l}, \bar{h}, \bar{H}) = (\psi_-^{(+)}, \psi_-^{(0)}, \psi_-^{(-)}) \quad (3.16)$$

In what follows, we shall choose to decouple the H baryon,

$$m_l < m_h \ll m_H \quad (3.17)$$

so that only the l and h baryons and their antibaryons will play a role in the plasma^b.

^b All three baryonic masses are still kept much lower than the superheavy scale.

So far we have discussed the mass-squared matrix in the bulk, and in disk gauge. Next, we return to (3.6) in the vicinity of the wall, and in the transmission gauge (in which it holds). The bulk assignments (3.15-16) now become:

$$\pm z > 0 \text{ baryons} : (l, h, H) = (\psi_{\pm}^{(-)}, \psi_{\pm}^{(0)}, \psi_{\pm}^{(+)}) \quad (3.18)$$

$$\pm z > 0 \text{ antibaryons} : (\bar{l}, \bar{h}, \bar{H}) = (\psi_{\mp}^{(+)}, \psi_{\mp}^{(0)}, \psi_{\mp}^{(-)}) \quad (3.19)$$

It is important to note that although eqs.(3.18-19) hold in transmission gauge, our usage of the labels ‘baryons’, ‘antibaryons’, l, \bar{l} etc. still refers to the *global* (disk gauge) definition. Thus an l baryon impinging on the Alice wall from the $z < 0$ side, for instance, may be transmitted through the wall to the $z > 0$ side. In this case, the transmission-gauge description is that the ψ_- field is transmitted from a $t_3 = -1$ to a $t_3 = +1$ component, and the outgoing particle is called \bar{l} . In the disk gauge, however, we would say that the $\psi_+^{(-)}$ field (l) acquires upon transmission a $\psi_-^{(+)}$ component (\bar{l}).

The mass-squared matrix throughout space, including the wall core, is

$$-M^2(z) = \tilde{\nu} + \sum_A \omega_A(z) t_A \quad (\text{transmission gauge}) \quad (3.20)$$

where $\omega_A(z)$ are related to $v_3^{(I)}$ by the $SO(3)$ matrix which relates the two generation-space bases. The functions ω_1, ω_2 are thus only defined modulo an arbitrary $SO(2)$ rotation, but the physics we shall be interested in is unaffected by this ambiguity. The functions $\omega_A(z)$ satisfy (see (3.9))

$$\omega_3(\pm\infty) = \pm\omega, \quad (3.21a)$$

$$\omega_A(\pm\infty) = 0 \quad \text{for } A = 1, 2. \quad (3.21b)$$

All the information on transmission and reflection of baryons and antibaryons at the wall is encoded in the following Klein-Gordon equation ((3.4),(3.20)):

$$\{\partial^2 + \tilde{\nu} + \sum_A \omega_A(z) t_A\} \psi_{\pm} = 0 \quad (3.22)$$

with $\tilde{\nu}$ given by (3.12).

4. Baryogenesis via Asymmetric Transmission

Let us consider a temperature low enough so that the Alice wall has formed. The wall is embedded in a plasma of baryons and antibaryons (and other particles) having an initially-zero net baryon number, and we are interested in baryon-number violating processes capable of giving rise to baryogenesis.

In the *bulk* of the plasma one has the usual kinds of baryon-violating processes, familiar from realistic GUTs. There are perturbative quantum processes, all of them suppressed by inverse powers of the superheavy scale; A few such processes are depicted in figure 4 (In general one also expects sphaleron-induced baryogenesis, but that requires chiral fermions and hence cannot arise in our model). The wall-catalyzed baryogenesis mechanism, however, is independent of the superheavy scale^{*}. Furthermore, we shall see that it can be catalyzed even by a static wall, if the wall disappears at finite time. We now investigate this mechanism in detail, for a single static wall.

In order to facilitate a clear presentation, the remainder of this section is divided into subsections mirroring its main conceptual components, which are:

- 1) Choice of \bar{a} convenient *parameter regime*.
- 2) The *wall-catalyzed baryonic S-matrix*; *CPT* constraints.
- 3) *Averaged rate equations*: approximate, linear Boltzmann equations for the plasma populations of the particle species $\{l, h, \bar{l}, \bar{h}\}$ in the presence of the wall, averaged over particle positions and momenta.
- 4) *Calculation of the microscopic baryon-antibaryon asymmetries* (reflection and transmission), in a Born approximation.
- 5) *Demonstration of finite-time baryogenesis* $B(t)$ for a plasma which, at $t = 0$, is out of thermal equilibrium and has $B(0) = 0$.
- 6) *Different kinds of Alice walls*.

4.1 Parameter Regime.—We choose the range of model parameters and plasma temperature so as to simplify the analysis and suppress uninteresting effects. Let m_u be the mass of the (charged and neutral) u -bosons, which are implicitly present in our model (see

^{*} Except in that the Alice string itself was formed at that scale.

discussion above (2.1)). Let T be the temperature, and define $\eta_I \equiv 2\lambda_I\alpha - a_I > 0$. The regime we work in is defined in the appendix. It implies the following:

- (I) The wall is normal (not superconducting).
- (II) The plasma is populated mainly by nonbaryonic u -particles, and its baryonic component is dilute and non-relativistic. The back-reaction of the plasma upon the wall is negligible.
- (III) The populations of H, \bar{H} baryons in the plasma, as well as those of the v -particles and the superheavies, can be approximated as zero.
- (IV) A baryon (l or h) can only disappear by annihilating an antibaryon. Such annihilations occur at negligible rates. In general, since u -particles are far more abundant than baryons we may ignore reactions involving more than one baryon (or antibaryon) in the initial state. Collisions (predominantly electromagnetic) between baryonic particles and u -particles serve to equilibrate the momenta of the former, but the probability of such a collision changing the *species* of a baryonic particle (from l to h , etc.) is suppressed[†].
- (V) The ω_1, ω_2 terms in (3.22) are small enough to be treated in a Born approximation (but not ω_3); this will prove useful in subsection 4.4.

4.2 Wall Catalyzed S -Matrix.— The plasma particles exert a negligible back-reaction on the Alice wall; hence they can exchange momentum, but not energy, with it. The creation of baryon-antibaryon pairs at the wall is thus kinematically suppressed[†]. Consulting point (IV), we see that the only species-changing microscopic baryonic processes occurring in the plasma are the reflection or transmission of an l, \bar{l}, h or \bar{h} particle at the Alice wall. Therefore the baryonic rate (or Boltzmann) equations, to be considered in subsection 4.3, are linear in the concentrations of these four particle species and determined by the reflection and transmission rates. Since the normal Alice wall considered here has no charged zero modes, impinging particles cannot exchange localized charge with it. The *wall-catalyzed S -matrix* thus has nonzero entries only for the following processes:

- $\Delta B = 0$ reflection: S_{ij} , with i the initial-particle label, j the final; $i, j \in \{l, \bar{l}, h, \bar{h}\}$; and (i, j) both baryons or both antibaryons.

† Because gauge interactions are diagonal in generation space.

‡ We shall ignore electromagnetic bremsstrahlung at the wall, since it does not change the (anti)baryon species (see (IV) above).

- $\Delta B = \pm 2$ transmission: S_{ij} , with $i \in \{l, h\}$; $j \in \{\bar{l}, \bar{h}\}$ or vice versa. For transmission processes, the wall picks up an electric Cheshire charge $Q = -\Delta B$. Some reflection and transmission processes are depicted in figure 5.

The S -matrix depends on the incoming momentum, but the *outgoing* (j -particle) momentum is uniquely determined by conservation of energy and \vec{k}_{\parallel} (momentum parallel to wall):

$$S_{ij} = S_{ij}(\vec{k}_i). \quad (4.2.1)$$

As before we are ignoring wall-edge effects, which would slightly violate \vec{k}_{\parallel} conservation. For particles that miss the wall altogether S_{ij} is, of course, the unit matrix δ_{ij} . For kinematical reasons,

$$S_{ij}(\vec{k}) = 0 \quad \text{if } \vec{k}^2 < m_j^2 - m_i^2. \quad (4.2.2)$$

Since \vec{k}_i determines \vec{k}_j , unitarity simply means that the 2×2 complex matrix $S_{ij}(\vec{k})$ is unitary. The wall is physically symmetric under $z \rightarrow -z$, as is manifest in disk gauge; thus $S_{ij}(\vec{k})$ does not depend on the sign of k_z —the particle i may be incident on either side of the wall. The CPT symmetry implies that

$$S_{ij}(\vec{k}_i) = S_{\bar{j}\bar{i}}(\vec{k}_j) \quad (4.2.3a)$$

when

$$E_i(\vec{k}_i) = E_j(\vec{k}_j), \quad \vec{k}_{\parallel}^{(i)} = \vec{k}_{\parallel}^{(j)}. \quad (4.2.3b)$$

Here $E_i(\vec{k}) \equiv \sqrt{m_i^2 + \vec{k}^2}$.

This constraint relates, for instance, $S_{ll}(\vec{k})$ to $S_{\bar{l}\bar{l}}(\vec{k})$. Thus, had we chosen a parameter range such that only the lightest baryon is present in the plasma, no baryogenesis could occur[§]. However, since we have h -particles as well, the CPT symmetry cannot rule out asymmetries enabling the production of a net baryon number.

4.3 Averaged Rate Equations.— In order to simplify the four-species Boltzmann equations into a set of four first-order, linear, ordinary differential equations, we assume that the initial plasma has Boltzmann distributions in all particle energies— but only within

§ We thank J. Preskill for a discussion on this point.

each species. In other words, denoting by $N_i(\vec{k}, t)d^3k$ the momentum-space distribution of particle momenta at time t , we assume

$$N_i(\vec{k}, 0) = n_i(0)e^{-E_i(\vec{k})/T} / \int d^3k e^{-E_i(\vec{k})/T} \quad (4.3.1)$$

where $n_i(t)$ is the total number of i -particles. The exact rate equations do *not* preserve this Boltzmann momentum distribution within a given species, unless n_i are also in the correct Boltzmann ratios at $t = 0$. However, the baryonic particles in the plasma are constantly undergoing electromagnetic collisions with u_{\pm} particles, and this tends to restore their Boltzmann \vec{k} distribution *without* affecting the ratios n_i/n_j (point (IV)). Since the mean free path between such collisions is much shorter than a baryon's mean path between consecutive encounters with the wall, we will assume that the momentum distribution within a species stays frozen at its initial, Boltzmann shape[¶] at all times $t > 0$. However, the total population of the i -th species, $n_i(t)$, does evolve with time. The non-equilibrium nature of the initial plasma, necessary for baryogenesis, will then manifest itself only through non-Boltzmann initial population ratios, $n_h(0)/n_l(0)$ and $n_{\bar{h}}(0)/n_{\bar{l}}(0)$ *. Since the wall interconverts light and medium-heavy particles, the rate equations will tend to restore these ratios to their Boltzmann values.

A note on spatial dependence: the generated baryon number will actually diffuse outwards from the Alice wall; we assume a very large diffusion coefficient, so the species populations $n_i(t)$ can be taken to be spatially homogeneous. We have not attempted to translate this simplifying assumption into an additional quantitative condition on our model parameters. In our thought experiment, we also stipulate that the wall is formed (or externally introduced) at $t = 0$ into a plasma box of finite volume V . This is reasonable: in a cosmological setting, a gas of Alice walls will form, so V may be thought of as the average volume per wall.

Given a plasma with such properties, then, we now derive the averaged (over space and momenta) rate equation satisfied by $n_i(t)$. Let A denote the area of the wall, which is assumed much larger than all baryonic Compton wavelengths; we are then justified in neglecting wall-edge effects. We define P_{ij} to be the probability per unit time that a given

¶ The baryonic plasma is dilute, so the Bose-Einstein and Boltzmann distributions agree.

* These ratios could have realistically deviated from equilibrium as T first decreased through m_h and then through m_l .

particle of species i , anywhere in the box, is converted by the wall into a j -particle. We have:

$$f_i P_{ij} = \frac{A}{V} \int \frac{|k_z|}{E_i(\vec{k})} d^3 k e^{-E_i(\vec{k})/T} |S_{ij}(\vec{k})|^2, \quad j \neq i \quad (4.3.2)$$

where:

$$f_i = f_{\bar{i}} \equiv \int d^3 k e^{-E_i(\vec{k})/T}. \quad (4.3.3)$$

In (4.3.2), $|k_z|/E_i$ is the normal velocity of the incoming baryon or antibaryon relative to the wall, and no summation over i is implied. By S -matrix unitarity,

$$f_i \sum_{j \neq i} P_{ij} = \frac{A}{V} \int \frac{|k_z|}{E_i(\vec{k})} d^3 k e^{-E_i(\vec{k})/T} (1 - |S_{ii}(\vec{k})|^2). \quad (4.3.4)$$

The CPT invariance yields the relation

$$f_i P_{ij} = f_j P_{\bar{j}\bar{i}}. \quad (4.3.5)$$

where we have used the fact that, for a transition $i \rightarrow j$,

$$|k_z^{(i)}| d^3 k_i = |k_z^{(j)}| d^3 k_j \quad (4.3.6)$$

This follows from the conservation laws (4.2.3b) and is simply a manifestation of the conservation of phase-space volume in hamiltonian mechanics (Liouville's theorem).

The averaged rate equations now assume the form, familiar from statistical mechanics:

$$\frac{dn_j(t)}{dt} = \sum_{i \neq j} n_i P_{ij} - n_j \sum_{i \neq j} P_{ji} \quad i, j \in \{l, h, \bar{l}, \bar{h}\}. \quad (4.3.7)$$

4.4 Calculation of Microscopic Asymmetries.—As noted at the end of section 3, the wall-catalyzed S_{ij} is encoded in the Klein-Gordon equation (3.22). We can work with this single equation, and yet describe (in transmission gauge) both baryons and antibaryons. This can be seen from the assignments (3.18-19), which tell us that eq.(3.22) describes baryons for $z > 0$ and antibaryons for $z < 0$.

We shall assume the ψ_+ wave is monochromatic, so the methods of time-independent scattering theory can be used.

The idea of the Born approximation we shall employ, is to perturb ψ_+ (to first order in ω_1, ω_2) around the solution of (3.22) with only the $A = 3$ term. The parameter regime chosen in 4.1 ensures that this Born approximation is valid, and that the limit $m_H \rightarrow \infty$ may be taken with the other two baryon masses remaining finite. The truncated space of wave functions ('Hilbert space' in a first-quantized picture) consists, in this limit, of functions with only the two components $t_3 = -\text{sgn}z, 0$ at position (x, y, z) . In addition, the $t_3 \neq 0$ components vanish at $z = 0$, whereas the $t_3 = 0$ component and its first derivative are continuous there.

The parameter range further ensures that the width of the $I = 3$ kink is very small compared with those of $I = 1, 2$, and these latter widths are in turn much smaller than the de Broglie and Compton wavelengths of the baryons. The unperturbed ψ_+ wave for an incoming l is thus the wave function of a plane wave, impinging on a 'brick wall' (at $z = 0$) from the $z > 0$ side and totally reflecting from it. It is therefore the sum of the incident and reflected waves. The unperturbed ψ_+ wave for an incoming \bar{l} , on the other hand, is the sum of a plane wave incident from the $z < 0$ side, and a totally-reflected wave. For an incoming h or \bar{h} ; the potential is flat and the zeroth-order ψ_+ is just the incoming plane wave, from the appropriate direction.

The perturbing potentials $\omega_1(z), \omega_2(z)$ are finite in the $m_H \rightarrow \infty$ limit, and are given by eq.(A.10) of the appendix. They are also depicted in figure 6.

Using the zeroth-order waves described above, it is straightforward to compute the various entries in the wall-catalyzed S -matrix. For instance, the element S_{lh} in the Born approximation reads up to a phase (where kinematically allowed by (4.2.2))^{*}

$$S_{lh}(\vec{k}) = S_{\bar{h}\bar{l}}(\vec{k}') \approx \frac{1}{\sqrt{|k_z||k'_z|}} \int_0^\infty dz e^{-i|k'_z|z} \sin(|k_z|z) \langle 0 | \sum_{A=1}^2 \omega_A(z) t_A | -1 \rangle, \quad (4.4.2)$$

where $\vec{k} = \vec{k}_l$, $\vec{k}' = \vec{k}_h$ and $|\zeta\rangle$ is a generation-space state having $t_3|\zeta\rangle = \zeta|\zeta\rangle$. As will be seen in 4.5, all the relevant baryon-antibaryon transmission asymmetries are expressible in terms of a single *reflection* asymmetry, by use of the CPT constraints and unitarity. This

* For typical (thermal) momenta, $|S_{lh}|^2 \ll 1$ thanks to (A.7). For the kinematical endpoint $k'_z \rightarrow 0$, the expression (4.4.2) is singular. This is an artifact of the Born approximation, and has a negligible effect on the momentum-averaged asymmetry ((4.4.4a) below).

asymmetry is

$$\mathcal{A}_{hl} \equiv P_{h\bar{l}} - P_{hl}, \quad (4.4.3)$$

And in the Born approximation it is:

$$\mathcal{A}_{hl} \approx -2 \frac{A}{V} \left\langle \frac{1}{|k'_z| E_h(k)} \text{Im} \{ D_1(k) D_2^*(k) \} \right\rangle, \quad (4.4.4a)$$

where now \vec{k} is the incoming (h or \bar{h}) momentum, \vec{k}' the outgoing (l or \bar{l}) momentum, the average is performed over incoming momenta with Boltzmann measure, and the complex amplitudes D_A are defined as follows:

$$D_A(k) \equiv \int_0^\infty dz e^{-i|k_z|z} \sin(|k'_z|z) \omega_A(z) \quad (4.4.4b)$$

with ω_A given by eq.(A.10). This asymmetry is, in general, nonzero. One easily checks that the minimal numbers of baryonic and Higgs generations for which \mathcal{A}_{hl} does not vanish identically, are three and three; this statement holds to all orders in perturbation theory.

In subsection 4.5 we will show that the plasma, which starts at $B(0) = 0$ and out of equilibrium, develops a finite $B(t)$ thanks to the asymmetry \mathcal{A}_{hl} . Note that this baryogenesis results even though we chose to decouple one of the three baryon mass-eigenstates (by making its mass m_H high). This does not contradict our above statement, which implied that two baryonic generations cannot yield baryogenesis. This is because, of the surviving baryons and antibaryons, h and \bar{h} are in the same gauge multiplet, *but l and \bar{l} are not*— they are not related by C (see (3.15-16)). The dynamics of the Alice wall still involves all three baryon generations, even when H is decoupled.

4.5 Finite-Time Baryogenesis.— From unitarity and CPT -invariance (eqs.(4.3.3-5)) it can be shown that the rate equation, (4.3.7), admits a unique equilibrium state:

$$(n_i)_{\text{equil}} \propto f_i. \quad (4.5.1)$$

Now in general, the net plasma baryon number is

$$B(t) = n_l + n_h - n_{\bar{l}} - n_{\bar{h}} \quad (4.5.2)$$

so from (4.5.1), $B_{\text{equil}} = 0$; by assumption, the initial baryon number is also zero, $B(0) = 0$. However, at finite t , $B(t) \neq 0$, provided the initial ratio n_h/n_l differs from the equilibrium

ratio f_h/f_l . To see this, let us compute $\dot{B}(0)$. From (4.3.2-7),

$$\dot{B}(0) = -2\mathcal{A}_{hl}(n_h - f_h n_l/f_l), \quad (4.5.3)$$

where \mathcal{A}_{hl} is the asymmetry computed in subsection 4.4. If the wall remains in place, then $B(t)$ must eventually approach zero again. If the wall disappears at some finite time, however, then a baryon asymmetry will be left behind.

In general, (4.3.7) is solved by rescaling $n_i = \sqrt{f_i}\tilde{n}_i$ and expanding the initial distribution in the eigenvectors of $\tilde{P}_{ij} - \delta_{ij}q_i$, with $\tilde{P}_{ij} = \sqrt{f_j}P_{ji}/\sqrt{f_i}$, $q_i \equiv \sum_k P_{ik}$ and $P_{ii} \equiv 0$. Since \tilde{P} is real and asymmetric^{*}, the corresponding eigenvalues are pairs of complex-conjugate complex numbers. Thus $n_i(t)$, and therefore $B(t)$ as well, are linear combinations of exponentially damped sine functions with various periods, phases and decay constants. This asymptotic behavior of $B(t)$ (for a wall held in place indefinitely) is more general than the particular model considered here. To see a particularly elegant example of this behavior, let us concoct a simple-minded P matrix which realizes the necessary baryon-antibaryon asymmetry in a ‘maximal’ way. The P matrix we choose is deterministic—the label of the incoming particle determines that of the outgoing one, and thus whether the incoming particle is reflected or transmitted. The rules we choose are encapsulated in figure 7: an l is always reflected (into an h); an \bar{h} reflects into an \bar{l} ; an \bar{l} is always transmitted into an l , whereas h is always transmitted to become an \bar{h} . It is readily seen that the asymmetry is then nonzero. For simplicity we set $m_h = m_l$, so that all nonzero entries in P_{ij} are equal to $\tau^{-1} \equiv \frac{A}{V} \langle |v_z| \rangle$, with $\langle |v_z| \rangle$ the average z -velocity of any incident particle. It is then straightforward to show that, as long as the wall is present,

$$B(t) = -2(n_h(0) - n_l(0))e^{-t/\tau} \sin(t/\tau). \quad (4.5.4)$$

4.6 Different Kinds of Walls.—As discussed in section 2, the inter-generation alignment term V_3 correlates the relative signs of the three bulk VEVs $\langle v_3^{(I)} \rangle$, and also causes the three types of kinks to be preferentially centered at the same surface. This is important for baryogenesis: if the alignment coupling ϵ is set to zero there are degenerate, nonequivalent vacua with, say, the same VEV sign for $I = 1, 3$, but opposite signs of $\langle v_3^{(2)} \rangle$. Therefore, by eqs. (3.7), (4.4.4) and (A.10), Alice walls embedded in these two vacua will yield different

* Its asymmetry is solely due to reflection and transmission asymmetries, such as \mathcal{A}_{hl} .

asymmetries \mathcal{A}_{hl} — in fact, in the decoupled- H limit the asymmetries are equal and opposite. When $\epsilon \neq 0$, however, the bubbles enclosing one type of vacuum tend to collapse due to the pressure difference between the vacua, and so only one \mathcal{A}_{hl} value is relevant.

For topological and dynamical reasons, there is only one kind of Alice wall (bounded by one type of Alice string) in the true vacuum, since an Alice wall must be a solution of the equations of motion. However, suppose our plasma is inhabited by capable engineers who are able to change the *shapes* of the $v_3^{(I)}$ kinks, without changing their VEVs in the bulk of the plasma. Assume further that they are able to hold these altered kinks static by some means. Such an *artificial wall* is still bounded by an Alice string. If an artificial wall is adjusted so that its ω_1, ω_2 functions are interchanged, we see from (4.4.4), (4.5.3) that the baryogenesis it causes will be precisely opposite to that engendered by a genuine (‘natural’) Alice wall of the same area. One can easily show that this conclusion holds for the exact Klein-Gordon equation, and is thus not limited to the Born approximation. Fortunately, however, even the full transport equations are unlikely to give rise to engineers — so our conclusions are safe.

Although there is only one kind of Alice wall (in the true vacuum), two otherwise identical Alice walls can still be different—they may carry different amounts of electromagnetic charges. More specifically, there are two such charges [2]: Cheshire electric charge and Cheshire magnetic charge. The electric Cheshire charge (caused e.g. by transmission of baryons) decays rapidly, mostly by attracting oppositely charged u -particles from the plasma, which are almost always transmitted and thereby cancel the Cheshire charge. We have ignored magnetic charges in this paper; in ref.7 a scenario was presented in which they disappear. But even if they are present, their effect on the wall S -matrix is not expected to cancel the baryogenesis found here—especially since in a gas of Alice walls, the sign of a given wall’s magnetic charge is random.

5. Summary and Conclusions

In a Grand Unified Theory in which the charge conjugation operator is contained in the original gauge group, Alice strings form when the symmetry is broken to a smaller group having C as a discrete symmetry. Magnetic monopoles usually form as well, but (as shown in ref.7) these can be eliminated by a Langacker-Pi mechanism, at least in a toy model. Thus magnetic monopole bounds need not rule out cosmological, post-inflationary Alice strings.

In this paper we have studied, also in a toy model, a novel baryogenesis mechanism involving Alice walls. The original gauge group is $SO(3)$; baryons are scalar and belong to $SO(3)$ triplets. Baryon number B is violated by perturbative processes, which become suppressed after $SO(3)$ is spontaneously broken to $O(2)$. However, this same transition forms Alice strings, in the presence of which B is globally ill-defined. At a lower temperature, C too is spontaneously broken, through VEVs of non-baryonic $SO(3)$ Higgs triplets. A closed Alice string loop now becomes the boundary of a domain wall—an ‘Alice wall’, where the Higgs triplets develop kinks. In a cosmological network of string-bounded walls, the walls eventually shrink and decay, and never dominate the energy density of the Universe; C and CP remain broken. While the walls exist, the symmetries C , CP and B are all violated, the latter mainly by the transmission of baryons/antibaryons through the wall. This enables baryogenesis, which indeed occurs (even for static walls) when there are three baryonic ‘generations’ and three Higgs triplets, and when the initial plasma (with $B = 0$) is out of thermal equilibrium. The baryogenesis occurs due to different transmission rates of baryons and antibaryons through the wall. Such a transmission changes a baryon into an antibaryon and vice versa, in the ‘disk’ gauge where baryon number is well-defined in the region excluding the disk. The Alice walls must be removed at finite time, or else the net baryon number equilibrates back to zero.

This baryogenesis mechanism is classical in nature. As baryon number increases the Alice wall tends to accumulate electric charge; in the parameter regime that we have focused on in this paper, this charge is purely of the ‘Cheshire’ (unlocalized) variety, and decays rapidly, predominantly by attracting oppositely-charged particles from the ambient plasma and flipping their charges. These charge-decay processes have a negligible effect on the produced baryon number for the parameters we choose. They merely serve to electrically screen the net baryon excess by a corresponding excess of oppositely charged, non-baryonic particles.

The main conceptual difference between our mechanism and electroweak scale, sphaleron-induced baryogenesis [9] is as follows. Electroweak baryogenesis assumes a preexisting CP -violation, uses sphalerons to supply B -violation, and requires moving bubble-walls. The Alice-catalyzed mechanism, on the other hand, relies on preexisting deviations from equilibrium, and uses unstable domain walls to supply the other Sakharov conditions^{*}.

Some realistic GUT schemes can form Alice strings and walls, and these might catalyze cosmologically significant baryogenesis of the type found in the toy model. This possibility, for the unification group $SO(10)$, is currently under investigation in a scenario in which C, P and CP are spontaneously broken at the left-right symmetry breaking scale. Both in the toy model and $SO(10)$ GUTs, topologically stable bubbles may form, in addition to Alice walls. In the toy model, such bubbles shrink away due to pressure differences and do not affect the wall-catalyzed baryogenesis. We do not yet know how non-Alice walls affect similar baryogenesis mechanisms for $SO(10)$, but the different nature of C and CP violations is likely to play an important role in that (or any other realistic) model. Finally, it remains to be seen to what extent the baryon number produced at the left-right symmetry breaking epoch can survive the above-mentioned electroweak-sphaleron processes.

APPENDIX

The constraints we choose to impose on the temperature and the parameters of the toy model, are as follows (apart from the condition that regime II of section 2 holds):

$$\lambda_I = O(a/\alpha) \quad (A.1)$$

$$\rho_I \gg a^2/\alpha^4 \quad (I = 1, 2, 3) \quad (A.2)$$

$$\rho_3 = O\left(\frac{ab^2}{m_H^4}\right), \quad \rho_I = O(1) \quad (I = 1, 2) \quad (A.3)$$

$$m_u \ll T \ll m_l < m_h, \quad (A.4)$$

$$m_h \ll m_\nu^{(n, ch)} \ll m_H \ll M_W \quad (A.5)$$

^{*} The phase transition that gives rise to Alice walls is also likely to increase the deviations from equilibrium, thus enhancing baryogenesis.

$$(m_H/\alpha)^4 \ll b^2/a \quad (\text{A.6})$$

$$b^2 \ll m_I T \quad (\text{A.7})$$

$$\sqrt{\eta_I} \ll \sqrt{a}/\ln(m_H^4/(b^2\eta_I)) \quad (I = 1, 2) \quad (\text{A.8})$$

$$\eta_3 = O(a) \quad (\text{A.9})$$

Condition (A.1) is eq.(3.2), and follows from the requirement that the three kinks are normal (regime II of section 2). Condition (A.2) follows from (2.14) (small back-reaction of the wall on the string) together with (A.1). The choices (A.3-9) are consistent with (2.15),(3.10),(3.13). Eq.(A.7) ensures the validity of the Born approximation in section 4.4, and (A.6) follows from (A.2-3).

For the purposes of the wall S -matrix calculations of 4.4, the large- m_H limit results in the following approximation for the perturbing potentials in (3.22):

$$\omega_A(z) \approx br_A(\tanh(\sqrt{\eta_A}z) - \text{sgn}z), \quad A = 1, 2. \quad (\text{A.10})$$

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FIGURE CAPTIONS

- 1) (1a) The Alice wall. (1b) An edge-on, cross-sectional view of the wall. The small circles represent sections of the Alice-string core.
- 2) Edge-on view of the branch cut surfaces in two physical gauges: (2a) disk gauge (2b) transmission gauge.
- 3) Profiles of the three Higgs-triplet kinks.
- 4) Some perturbative baryon-violating processes. Diagrams (4a-c) are suppressed by superheavy propagators. The plasma contains a negligible W_{\pm} population, so (4d) can be ignored as well.
- 5) Some reflection and transmission processes in the presence of the Alice wall.
- 6) The potentials of the Klein-Gordon eq.(3.22) in the $m_H \rightarrow \infty$ limit. In this limit, the amplitude of the $I = 3$ kink of figure 3 becomes infinitely large, and this kink becomes infinitely narrow. The two solid curves are profiles of $\omega_1(z)$ and $\omega_2(z)$, while the dashed horizontal lines are the two surviving mass-squared levels.
- 7) A set of deterministic reflection/transmission rules with maximal transmission asymmetry.

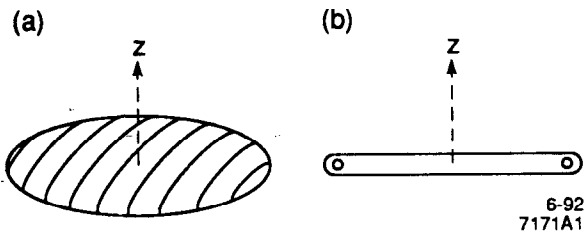
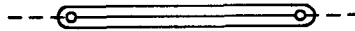


Fig. 1

(a)

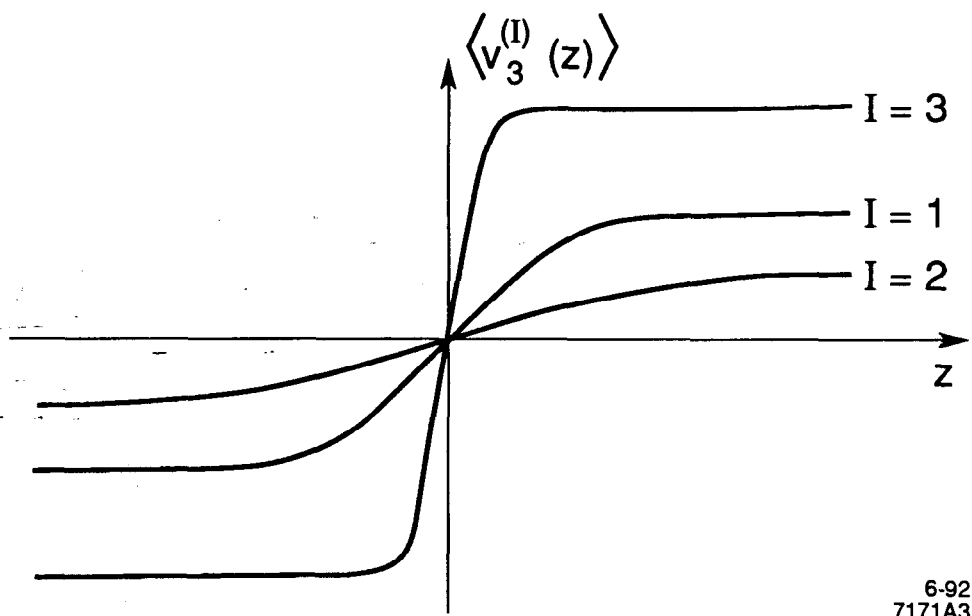


(b)



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Fig. 2



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Fig. 3

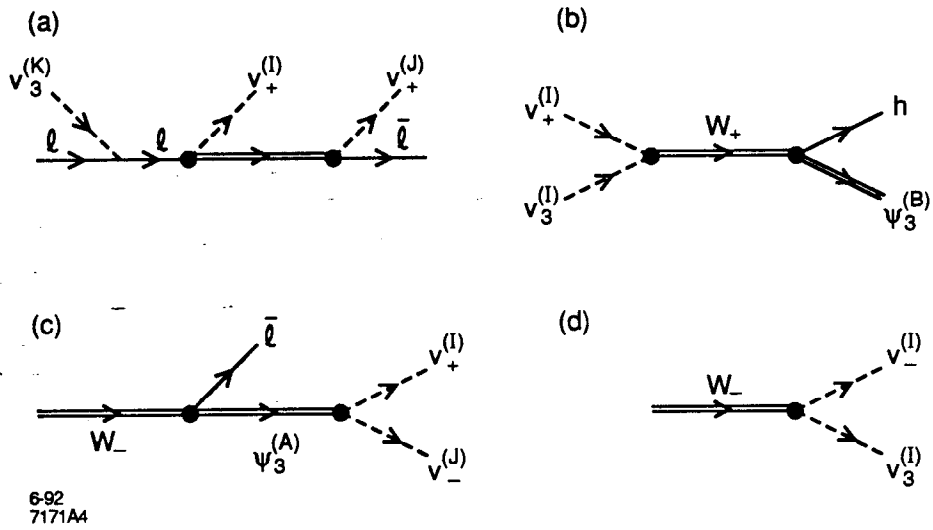


Fig. 4

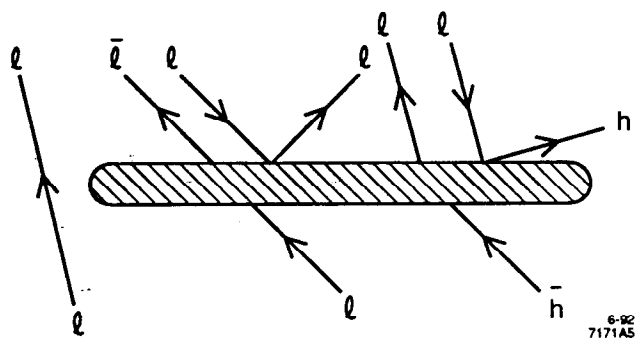
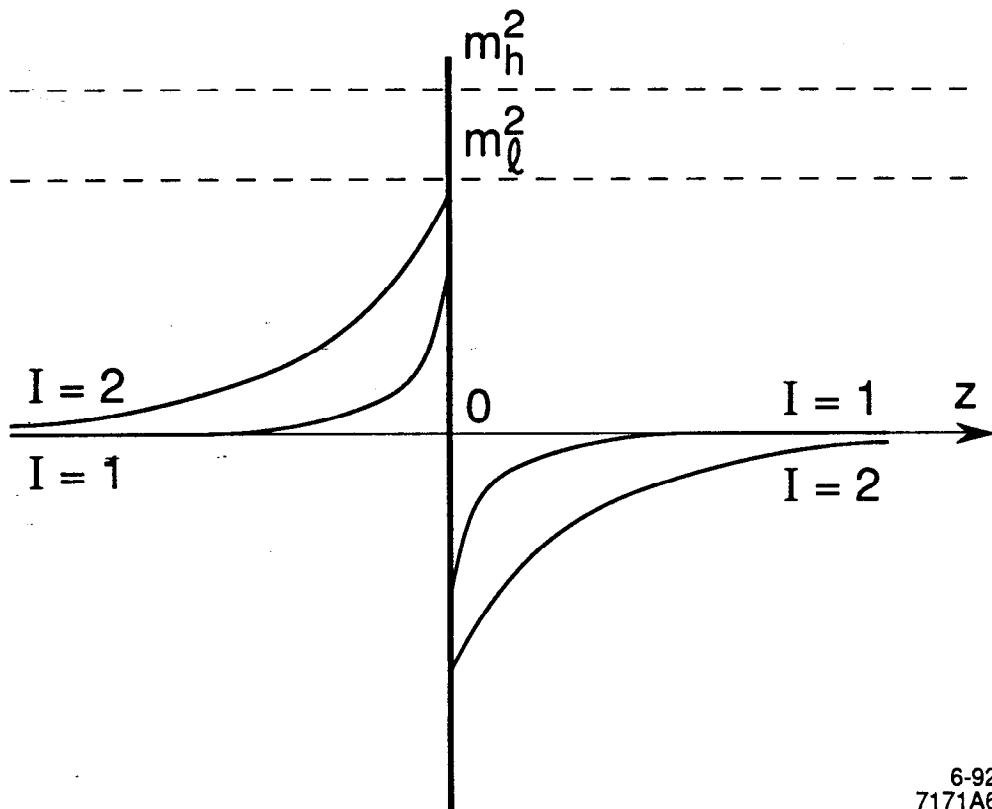


Fig. 5



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Fig. 6

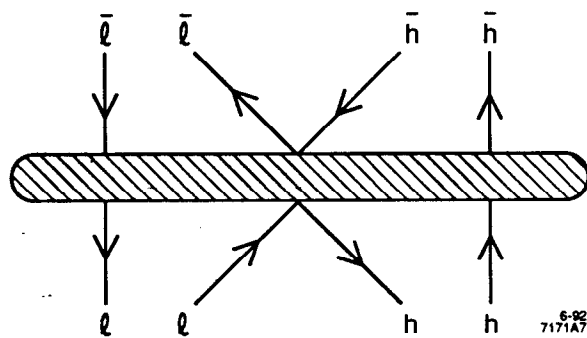


Fig. 7