SLAC–PUB–5826
June 1992
T/E

Subleading Isgur-Wise Form Factors from QCD Sum Rules*

MATTHIAS NEUBERT†

Stanford Linear Accelerator Center

Stanford University, Stanford, California 94309

ABSTRACT

In the heavy-quark effective theory, current matrix elements between two heavy pseudoscalar or vector mesons are parametrized by a set of universal form factors. These functions are calculated to subleading order in the $1/m_Q$ expansion using QCD sum rules. The equations of motion and Ward identities of the effective theory are incorporated in the analysis. Within this approach, parameter-free predictions are obtained for all form factors at zero recoil. The results allow for an almost model-independent analysis of current-induced transitions between heavy mesons. As an application, the $1/m_c$ and $1/m_b$ corrections to the hadronic form factors describing semileptonic $B \to D\,\ell\,\nu$ and $B \to D^*\ell\,\nu$ decays are computed. The possibility of extracting V_{cb} from these processes is discussed, and the importance of a measurement of symmetry-violating effects in ratios of factors is pointed out.

Submitted to *Physical Review D*

⋆ Work supported by the Department of Energy, contract DE–AC03–76SF00515.

† Research fellow of the BASF Aktiengesellschaft and the German National Scholarship Foundation.

I. INTRODUCTION

The theoretical description of hadronic processes involving the decay of a heavy quark Q has recently experienced great simplification due to the discovery of new symmetries of QCD in the limit where $m_Q \to \infty$ [1, 2]. The properties of a hadron containing the heavy quark become then independent of its mass and spin, and the complexity of the hadronic dynamics results from the strong interactions among the light degrees of freedom only. A covariant effective-field-theory approach provides an elegant framework to analyze such processes. It allows an expansion of decay amplitudes in powers of $1/m_Q$ in such a way that the spin-flavor symmetry relations become explicit [3–8]. Hadronic matrix elements in the effective theory are parametrized in terms of form factors which characterize the properties of the light degrees of freedom. They are universal in the sense that they do not depend on the properties of the heavy quark itself.

The heavy-quark symmetries impose restrictive constraints on weak decay amplitudes. In particular, the description of semileptonic transitions between two heavy mesons or baryons becomes very simple in the formal limit of infinite heavy-quark masses. Both for mesonic and baryonic processes, the large set of hadronic form factors is then reduced to a small number of universal functions, which depend on the quantum numbers of the light degrees of freedom but not on the heavy-quark masses and spins [2, 9–11]. These so-called Isgur-Wise form factors are functions of the kinematical variable $v \cdot v'$, which measures the change of velocities that the heavy hadrons undergo during the transition.

The reduction of form factors greatly simplify the phenomenology of heavy-quark decays in the limit where the the heavy-quark masses can be considered very large compared to any other hadronic scale in the process. But clearly, a careful analysis of at least the leading symmetry-breaking corrections is essential for any phenomenological application. Much attention has been devoted to this subject. Already in leading order in the heavy-quark expansion the spin-flavor symmetries are violated by hard-gluon exchange. The corresponding perturbative corrections have been calculated first in leading logarithmic approximation [12–14], and more recently in next-to-leading order in renormalization-group improved perturbation theory [15–18]. At order $1/m_Q$, one is generally forced to introduce a larger set of universal form factors. The equations of motion and the Ward identities of the effective theory impose constraints on some of these form factors. The structures that arise have been worked out for matrix elements between two heavy mesons [19] or Λ-baryons [20], as well as for the decay constants of heavy mesons [21].

Generally, the effective theory allows one to derived in a concise way the various symmetry relations among form factors to a given order in the $1/m_Q$ expansion. Having established these relations, the non-trivial dynamical information is parametrized in terms of a set of universal functions, which characterize the properties of the light degrees of freedom in the background of the static color source provided by the heavy quark. An understanding of these functions is at the heart of nonperturbative QCD.

Ultimately, they may be computable using a formulation of the effective theory on a space-time lattice [22–25]. However, any other consistent analysis is interesting in its on right, and several model-calculations have been discussed in the literature [26–29]. QCD sum rules are particularly suited for this purpose. They have been recently adapted to study matrix elements in the effective theory and have been employed to calculate the asymptotic value of the scaled pseudoscalar decay constant $f_P\sqrt{m_P}$ [31, 32], the $1/m_Q$ corrections to this quantity [21], and the universal Isgur-Wise form factor [30, 31].

In this paper we extend the sum rule analysis to the subleading form factors which appear at order $1/m_Q$ in the desciption of matrix elements between two heavy mesons. In Sec. II we review the formalism of deriving the structure of $1/m_Q$ corrections in the effective theory, define a minimal set of universal functions and discuss their properties under renormalization. The QCD sum rule analysis of the subleading form factors is presented in Secs. III and IV. We show how to satisfy the equations of motion and the Ward identities of the effective theory, which lead to relations among certain form factors and require others to vanish at zero recoil. Most importantly, under the approximations usually made in QCD sum rules we obtain parameter-free predictions at zero recoil for all form factors not constrained by symmetries. These predictions are expected to be quite accurate. When combined with the rather elaborate computations of the perturbative corrections to the heavy-quark form factors that have been performed recently, our results form a solid basis for a detailled analysis of semileptonic B decays to subleading order in the $1/m_Q$ expansion. Some specific applications, as well as a summary of the results, are presented in Sec. V. We emphasize that a measurement of symmetry-breaking corrections to the infinite quark-mass limit does not only test the heavy-quark expansion at next-to-leading order, but also provides valuable information about strong interaction dynamics.

II. THE HEAVY-QUARK EXPANSION

The construction of the so-called heavy-quark effective theory (HQET) is based on the observation that, in the limit $m_Q \gg \Lambda_{QCD}$, the velocity v of a heavy quark is conserved with respect to soft processes [5]. It is then possible to remove the mass-dependent piece of the momentum operator by a field redefinition. To this end, one introduces a field $h_Q(v,x)$, which annihilates a heavy quark with velocity v ($v^2=1, v^0\ge 1$), by

$$h_Q(v,x) = \frac{(1+\not v)}{2}\,\exp(i m_Q v\cdot x)\,Q(x)\ . \tag{2.1}$$

Then if P is the total momentum of the heavy quark, the new field carries only the residual momentum $k = P - m_Q v$, which is of order Λ_{QCD}. In the limit $m_Q\to\infty$, the effective Lagrangian for the strong interactions of the heavy quark becomes [5–7]

$$\mathcal{L}_{eff} = \bar h_Q\, v\cdot(iD - \delta m\, v)\, h_Q\ . \tag{2.2}$$

where $D_\mu = \partial_\mu - ig_s A_\mu^a t^a$ is the gauge-covariant derivative, and δm is the residual mass of the heavy quark in the effective theory [33].

Note that there is some ambiguity associated with the construction of HQET since the heavy-quark mass used in the definition of the fields h_Q is not uniquely defined. In fact, for HQET to be consistent it is only necessary that k and δm be of order Λ_{QCD}, *i.e.*, stay finite in the limit $m_Q \to \infty$. A redefinition of m_Q by a small amount Δ induces changes in these quantities

$$m_Q \to m_Q + \Delta \quad \Rightarrow \quad \begin{cases} k \to k - \Delta\, v & , \\ \delta m \to \delta m - \Delta & , \end{cases} \tag{2.3}$$

such that only the combinations $(m_Q + \delta m)$ and $(k - \delta m\, v)$ remain unchanged. This suffices, however, to guarantee that physical quantities computed in HQET are independent of the choice of the expansion paramater. The reason is that the heavy-quark expansion can be organized as an expansion in powers of $1/(m_Q + \delta m)$ with coefficients being the matrix elements of operators containing the covariant derivative acting on the heavy-quark fields only in the combination $(iD - \delta m\, v)$ [33].

It is clear from this discussion that there exist a unique choice m_Q^* for the heavy-quark mass in the construction of the effective theory such that the residual mass vanishes, $\delta m = 0$, and the heavy-quark expansion becomes an expansion in powers of iD/m_Q^*. This prescription provides a nonperturbative definition of the heavy-quark mass, which has been implicitly adopted in most previous analyses based on HQET. Yet it is important to notice the mass m_Q^* is a nontrivial parameter of the theory. For instance, one can associate the difference $\bar\Lambda$ between this mass and the mass of a meson M (or baryon) containing the heavy quark with the energy carried by the light constituents in the restframe of the hadron. That $\bar\Lambda$ is in fact a parameter characterizing the properties of the light degrees of freedom becomes explicit in the relation

$$\bar\Lambda = m_M - m_Q^* = \frac{\langle\, 0\,|\, \bar q\, (iv\cdot \overleftarrow{D}\,)\, \Gamma\, h_Q\, | M(v) \rangle}{\langle\, 0\,|\, \bar q\, \Gamma\, h_Q\, | M(v) \rangle} \,, \tag{2.4}$$

which can be derived from the equations of motion of HQET [21, 33]. Here Γ is an appropriate Dirac matrix such that the currents interpolate the heavy meson M.

The scale $\bar\Lambda$ determines the canonical size of power corrections to the infinite quark-mass limit [19, 20]. QCD sum rules predict $\bar\Lambda \simeq 0.50$ GeV [32, 21], corresponding to the quark masses $m_b^* \simeq 4.8$ GeV and $m_c^* \simeq 1.4$ GeV. For the leading power corrections relevant to processes involving B and/or D mesons one thus expects $\bar\Lambda/2m_b^* \simeq 5\%$ and $\bar\Lambda/2m_c^* \simeq 20\%$, respectively. It is the aim of this paper to put this estimate on a more quantitative basis.

Let us now review the construction of the heavy-quark expansion for current matrix elements between two heavy mesons. The heavy-quark current $\bar Q'\,\Gamma\, Q$, where $\Gamma = \gamma_\mu$ or $\Gamma = \gamma_\mu\gamma_5$ for the vector or axial vector current, has a short-distance expansion in terms of operators in the effective theory. In leading logarithmic approximation it reads

$$
\begin{aligned}
\bar{Q}'\,\Gamma\,Q \to\; & C_0(\mu)\,\bar{h}_{Q'}\,\Gamma\,h_Q \\
& + \frac{1}{2m_Q^*}\Big[C_1(\mu)\,\bar{h}_{Q'}\,\Gamma\,i\not{D}\,h_Q + C_2(\mu)\,\bar{h}_{Q'}\,\Gamma\,iv'\!\cdot\! D\,h_Q\Big] \\
& + \frac{1}{2m_{Q'}^*}\Big[C_1'(\mu)\,\bar{h}_{Q'}\,(-i\overleftarrow{\not{D}}\,)\,\Gamma\,h_Q + C_2'(\mu)\,\bar{h}_{Q'}\,(-iv\!\cdot\!\overleftarrow{D}\,)\,\Gamma\,h_Q\Big] \\
& + \dots
\end{aligned}
\tag{2.5}
$$

The effective current operators renormalize differently from their QCD counterparts. In particular, they have non-zero anomalous dimensions, such that matrix elements in the effective theory depend on the renormalization scheme. The short-distance coefficients $C_i(\mu)$ ensure that the final result for any physical quantity is independent of the renormalization procedure. If, for simplicity, QCD is matched onto the effective theory at a scale $\mu = \bar{m}$, which is some average of the heavy-quark masses, the coefficients are given by [13, 8, 33]

$$
\begin{aligned}
C_0(\mu) = C_1(\mu) &= C_1'(\mu) = \left[\frac{\alpha_s(\bar{m})}{\alpha_s(\mu)}\right]^{a_L} , \\
C_2(\mu) = C_2'(\mu) &= -\frac{16}{\beta}\,C_0(\mu)\,\frac{r(y)-y}{y^2-1}\,\ln\left[\frac{\alpha_s(\bar{m})}{\alpha_s(\mu)}\right] ,
\end{aligned}
\tag{2.6}
$$

where $y = v\cdot v'$ denotes the product of the heavy-quark velocities, and

$$
\begin{aligned}
r(y) &= \frac{1}{\sqrt{y^2-1}}\,\ln\,(y+\sqrt{y^2-1}) , \\
a_L &= \frac{8}{\beta}\,[y\,r(y)-1] , \quad \beta = 33 - 2n_f .
\end{aligned}
\tag{2.7}
$$

Here n_f is the number of light quark flavors in the low-energy theory. More sophisticated expressions for the short-distance coefficients, which include the full one-loop matching conditions, a summation of logarithms between m_Q^* and $m_{Q'}^*$ to resolve the ambiguity in $\bar{m}$, or higher-order corrections in perturbation theory, are discussed in the literature [14–18]. However, the structure of the operators in the effective theory remains the same as in (2.5).

Similar to the appearance of higher-dimensional operators in the expansion of the current, there are also additional terms in the effective Lagrangian at order $1/m_Q^*$ [8]

$$
\delta\mathcal{L}_{eff} = \frac{1}{2m_Q^*}\Big[\bar{h}_Q\,(iD)^2\,h_Q + \frac{g_s}{2}\,Z(\mu)\,\bar{h}_Q\,\sigma_{\mu\nu}G^{\mu\nu}\,h_Q\Big] . \tag{2.8}
$$

In leading logarithmic approximation, the renormalization constant for the "color-magnetic moment" operator is

$$
Z(\mu) = \left[\frac{\alpha_s(m_Q)}{\alpha_s(\mu)}\right]^{9/\beta} . \tag{2.9}
$$

At subleading order in the heavy-quark expansion, matrix elements in HQET receive power corrections from both the higher-dimensional effective current operators in (2.5), and from insertions of $\delta\mathcal{L}_{eff}$ into diagrams involving the leading-order current [19]. These matrix elements are constrained by the heavy-quark symmetries and can be parametrized in terms of few universal functions. The number of independent form factors and the relations among matrix elements become most transparent in a compact trace-formalism [13, 34]. In HQET, a heavy meson M is represented by a spin wave-function

$$\mathcal{M}(v) = \sqrt{m_M}\,\frac{(1+\not v)}{2}\begin{cases} -\gamma_5 & ;\ J^P = 0^- \ , \\ \not\epsilon & ;\ J^P = 1^- \ , \end{cases} \tag{2.10}$$

which satisfies $\not v\,\mathcal{M}(v) = \mathcal{M}(v) = -\mathcal{M}(v)\,\not v$. Hadronic matrix elements of the leading-order current are written as

$$\langle M'(v')|\,\bar h_{Q'}\,\Gamma\,h_Q\,|M(v)\rangle = -\xi(y,\mu)\,\mathrm{Tr}\,\{\,\overline{\mathcal{M}}'(v')\,\Gamma\,\mathcal{M}(v)\,\} \tag{2.11}$$

with $\xi(y,\mu)$ being the universal Isgur-Wise form factor [2]. It is the "reduced matrix element" of the transition and describes the overlap of the wave functions of the light degrees of freedom in the two mesons moving at velocities v and v'. The fact that there is a single reduced matrix element in this case is a consequence of the projection properties of the spin wave-functions, which in turn reflect the heavy-quark spin symmetry.

At order $1/m_Q^*$ one encounters matrix elements of higher-dimensional operators. The corrections to the current in (2.5) have the structure [1]

$$\langle M'(v')|\,\bar h_{Q'}\,\Gamma\,iD_\mu\,h_Q\,|M(v)\rangle = -\bar\Lambda\,\mathrm{Tr}\,\{\,\xi_\mu(v,v',\mu)\,\overline{\mathcal{M}}'(v')\,\Gamma\,\mathcal{M}(v)\,\}\ . \tag{2.12}$$

The most general decomposition of the form factor is

$$\xi_\mu(v,v',\mu) = \xi_+(y,\mu)\,(v+v')_\mu + \xi_-(y,\mu)\,(v-v')_\mu - \xi_3(y,\mu)\,\gamma_\mu\ , \tag{2.13}$$

and because of T-invariance of the strong interactions the functions $\xi_i(y,\mu)$ are real. An expressions for the corresponding matrix element with the derivative acting to the left can be obtained from (2.12) by complex conjugation. Imposing the equation of motion $iv\!\cdot\! D\,h_Q$ to the matrix element and to its conjugate, one derives the constraints [19]

$$\begin{aligned}(y+1)\,\xi_+(y,\mu) - (y-1)\,\xi_-(y,\mu) + \xi_3(y,\mu) &= 0\ ,\\ \xi_-(y,\mu) &= \frac{1}{2}\,\xi(y,\mu)\ .\end{aligned} \tag{2.14}$$

---

[1] In contrast to Ref. [19] we have inserted a power of $\bar\Lambda$ in order for the universal functions $\xi_\mu(y,\mu)$ to be dimensionless.

Thus only one of these funtions, say $\xi_3(y,\mu)$, is independent. Insertions of $\delta\mathcal{L}_{eff}$ into diagrams involving the leading-order current yield to matrix elements of the operators

$$O_1 = i\int \mathrm{d}y\, T\,\{\,(\bar h_{Q'}\,\Gamma\, h_Q)_0, (\bar h_Q\,(iD)^2\, h_Q)_y\,\}\ , \tag{2.15}$$
$$O_2 = i\int \mathrm{d}y\, T\,\{\,(\bar h_{Q'}\,\Gamma\, h_Q)_0, (\bar h_Q\, g_s\sigma_{\mu\nu}G^{\mu\nu}\, h_Q)_y\,\}\ .$$

Those give rise to new functions defined by [19]

$$\langle M'(v')|\,O_1\,|M(v)\rangle = -2\bar\Lambda\,\chi_1(y,\mu)\,\mathrm{Tr}\,\{\,\overline{\mathcal{M}}'(v')\,\Gamma\,\mathcal{M}(v)\,\}\ ,$$
$$\langle M'(v')|\,O_2\,|M(v)\rangle = -2\bar\Lambda\,\mathrm{Tr}\,\{\,\chi_{\mu\nu}(v,v',\mu)\,\overline{\mathcal{M}}'(v')\,\Gamma\,P_+(v)\,\sigma^{\mu\nu}\,\mathcal{M}(v)\,\}\ , \tag{2.16}$$

where $P_+(v) = (1+\not v)/2$. The most general decomposition of the form factor $\chi_{\mu\nu}$ is

$$\chi_{\mu\nu}(v,v',\mu) = i\chi_2(y,\mu)\,(v'_\mu\gamma_\nu - v'_\nu\gamma_\mu) + 2\chi_3(y,\mu)\,\sigma_{\mu\nu}\ . \tag{2.17}$$

The universal functions $\xi_i(y,\mu)$ and $\chi_i(y,\mu)$ can be interpreted as higher-dimensional structure functions of the light constituents in an infinitely heavy meson. As the Isgur-Wise form factor itself they are fundamental quantities of QCD.

At subleading order in the heavy-quark expansion, any current matrix element between two ground-state heavy mesons can be expressed in terms of the Isgur-Wise functions and the four subleading form factors $\xi_3(y,\mu)$ and $\chi_i(y,\mu)$. Using the conservation of the vector current $\bar Q\gamma_\mu Q$ it is then possible to derive normalization conditions for some of these functions at zero recoil. They are [2, 19]

$$\xi(1,\mu) = 1\ ,$$
$$\chi_1(1,\mu) = \chi_3(1,\mu) = 0\ . \tag{2.18}$$

The normalization of the Isgur-Wise function allows model-independent predictions for decay rates close to the kinematical endpoint region. The fact that two of the subleading form factors vanish at $y=1$ implies that some of these predictions are protected against leading power corrections. This fact plays an important role in the determination of the weak mixing angle V_{cb} from semileptonic decays [35].

Before we present the calculation of the universal form factors from QCD sum rules, let us discuss their behavior under renormalization. The scale dependence of matrix elements in HQET is such that it combines with that of the short-distance coefficients $C_i(\mu)$ to give scale independent results for physical matrix elements. Thus, *e.g.*, the μ-dependence of the Isgur-Wise function $\xi(y,\mu)$ is opposite to that of $C_0(\mu)$ in (2.5). This is the content of the renormalization group equation which $C_0(\mu)$ is the solution of. In general, it can be shown that the combinations

$$\xi(y,\bar m) = C_0(\mu)\,\xi(y,\mu)\ ,$$
$$\xi_3(y,\bar m) = C_1(\mu)\,\xi_3(y,\mu)\ ,$$
$$\chi_1(y,\bar m) = C_0(\mu)\,\chi_1(y,\mu) + (y-1)\,C_2(\mu)\,\xi(y,\mu)\ ,$$
$$\chi_i(y,\bar m) = C_0(\mu)\,Z(\mu)\,\chi_i(y,\mu)\ ;\ i=2,3 \tag{2.19}$$

are renormalization-group invariant quantities [33]. It is convenient to split these functions into a mass dependent part and renormalized form factors which are independent of μ and $\bar{m}$. We thus define

$$
\begin{aligned}
&\xi^{ren}(y) = [\alpha_s(\mu)]^{-a_L}\, \xi(y,\mu) \;, \\
&\xi_3^{ren}(y) = [\alpha_s(\mu)]^{-a_L}\, \xi_3(y,\mu) \;, \\
&\chi_1^{ren}(y) = [\alpha_s(\mu)]^{-a_L} \Big\{ \chi_1(y,\mu) + \frac{16}{\beta}\, \frac{r(y)-y}{y+1}\, \ln[\alpha_s(\mu)]\, \xi(y,\mu) \Big\} \;, \\
&\chi_i^{ren}(y) = [\alpha_s(\mu)]^{-a_L-9/\beta}\, \chi_i(y,\mu) \;;\; i=2,3.
\end{aligned} \tag{2.20}
$$

In this way the renormalized form factors are still universal functions with respect to heavy-quark symmetry transformations. Their relation to the physical form factors evaluated at the scale $\mu = \bar{m}$ is

$$
\begin{aligned}
&\xi(y,\bar{m}) = [\alpha_s(\bar{m})]^{a_L}\, \xi^{ren}(y) \;, \\
&\xi_3(y,\bar{m}) = [\alpha_s(\bar{m})]^{a_L}\, \xi_3^{ren}(y) \;, \\
&\chi_1(y,\bar{m}) = [\alpha_s(\bar{m})]^{a_L} \Big\{ \chi_1^{ren}(y) - \frac{16}{\beta}\, \frac{r(y)-y}{y+1}\, \ln[\alpha_s(\bar{m})]\, \xi^{ren}(y) \Big\} \;, \\
&\chi_i(y,\bar{m}) = [\alpha_s(\bar{m})]^{a_L+9/\beta}\, \chi_i^{ren}(y) \;;\; i=2,3.
\end{aligned} \tag{2.21}
$$

Note that at zero recoil $a_L = 0$, such that the renormalized form factors still obey the normalization conditions (2.18). If more elaborated expressions for the short-distance functions $C_i(\mu)$ are used, the renormalized universal functions (in leading logarithmic approximation) stay the same as in (2.20). However, in this case eqs. (2.21) become more complicated.

III. QCD SUM RULES FOR ξ AND ξ_μ

The application of the QCD sum rules developed by Shifman, Vainshtein and Zakharov [36] to the calculation of universal heavy-quark form factors has been recently worked out and is described in detail in Refs. [30–32, 21]. Here we shall only briefly outline the procedure by reviewing the analysis of the Isgur-Wise function. The idea is to study the analytic properties of correlators of heavy-quark currents in the effective theory. Specifically, consider the three-point function

$$
\Xi = \int \mathrm{d}x\, \mathrm{d}y\, e^{i(k'\cdot x - k\cdot y)}\, \langle\, 0\,|\, \mathcal{T}\{ [\bar{q}\,\bar{\Gamma}_{M'} h_{Q'}(v')]_x, [\bar{h}_{Q'}(v')\,\Gamma\, h_Q(v)]_0, [\bar{h}_Q(v)\,\Gamma_M\, q]_y \}\,|\, 0\,\rangle \;.
\tag{3.1}
$$

The heavy-light currents interpolate heavy pseudoscalar or vector mesons $M(v)$ and $M'(v')$, which is achieved by choosing respectively

$$
\Gamma_M = \begin{cases} -\gamma_5 & ;\; J^P = 0^- \;, \\ \gamma_\mu - v_\mu & ;\; J^P = 1^- \;. \end{cases} \tag{3.2}
$$

In leading order in HQET the correlator Ξ is an analytic function in $\omega = 2v \cdot k$ and $\omega' = 2v' \cdot k'$ with discontinuities for positive values of these variables. It can be written as a double dispersion integral over physical intermediate states. Separating the double-pole from the resonance contributions one obtains the phenomenological representation

$$\Xi_{phen} = \Xi_{pole} + \int \mathrm{d}\nu\, \mathrm{d}\nu' \frac{\rho_{res}(\nu,\nu')}{(\nu - \omega - i\epsilon)\,(\nu' - \omega' - i\epsilon)} + \text{subtractions}. \tag{3.3}$$

Using the fact that the total external momenta are $P = m_Q^* v + k$ and $P' = m_{Q'}^* v' + k'$, as well as the definition of $\bar\Lambda$ in (2.4), one finds for the double-pole contribution in the infinite quark-mass limit

$$\begin{aligned}\Xi_{pole} &= -\Big(\sum_{\text{pol.}}\Big) \frac{\langle\, 0\,|\,\bar q\,\bar\Gamma_{M'} h_{Q'}\,|M'(v')\rangle\,\langle M'(v')|\,\bar h_{Q'}\,\Gamma\, h_Q\,|M(v)\rangle\,\langle M(v)|\,\bar h_Q\,\Gamma_M\, q\,|\,0\,\rangle}{(P^2 - m_M^2 + i\epsilon)\,(P'^2 - m_{M'}^2 + i\epsilon)} \\ &= \frac{F^2\,\xi(y)}{(\omega - 2\bar\Lambda + i\epsilon)\,(\omega' - 2\bar\Lambda + i\epsilon)}\,\mathrm{Tr}\,\{\,\bar\Gamma_{M'} P_+(v')\,\Gamma\, P_+(v)\,\Gamma_M\,\}\,,\end{aligned} \tag{3.4}$$

where again $P_+(v) = (1 + \not{v})/2$. The sum over polarizations applies if M or M' is a vector meson. For the evaluation of the hadronic matrix elements one uses (2.11) as well as

$$\langle\, 0\,|\,\bar q\,\Gamma\, h_Q\,|M(v)\rangle = \frac{iF}{2}\,\mathrm{Tr}\,\{\,\Gamma\,\mathcal{M}(v)\,\}\,, \tag{3.5}$$

where F denotes the asymptotic value of the scaled decay constant of M, $F = f_M\sqrt{m_M}$ [31]. We suppress, for the moment, the μ-dependence of $\xi(y)$ and F. It will be discussed later. The traces in the numerator in (3.4) can be combined by use of the relation

$$\begin{aligned}&\Big(\sum_{\text{pol.}}\Big)\mathrm{Tr}\,\{\,\bar\Gamma_{M'}\mathcal{M}'(v')\,\}\,\mathrm{Tr}\,\{\,\Lambda\,\overline{\mathcal{M}}'(v')\,\Gamma\,\mathcal{M}(v)\,\}\,\mathrm{Tr}\,\{\,\overline{\mathcal{M}}(v)\,\Gamma_M\,\} \\ &\quad = 4 m_M m_{M'}\,\mathrm{Tr}\,\{\,\Lambda\,\bar\Gamma_{M'} P_+(v')\,\Gamma\, P_+(v)\,\Gamma_M\,\}\,,\end{aligned} \tag{3.6}$$

which is valid for arbitrary matrices Λ and Γ. Note that the product $P_+(v)\,\Gamma_M$ has the same projection properties as $\mathcal{M}(v)$.

For large negative values of ω and ω' (*i.e.*, $\Lambda_{QCD} \ll -\omega^{(\prime)} \ll m^*_{Q^{(\prime)}}$) the three-point function can be calculated in perturbation theory using the Feynman rules of HQET. The idea of QCD sum rules is that, at the transition from the perturbative to the nonperturbative regime, nonperturbative effects can be accounted for by including the leading power corrections in the operator product expansion of the correlator. They are proportional to vacuum expectation values of local quark-gluon operators, the so-called condensates [36]. Hence one approximates

$$\Xi_{theo} \simeq \int d\nu\, d\nu' \frac{\rho_{pert}(\nu,\nu')}{(\nu-\omega-i\epsilon)(\nu'-\omega'-i\epsilon)} + \text{subtractions} + \Xi_{cond} \ . \tag{3.7}$$

For our purposes it is sufficient to consider the corrections proportional to the quark condensate (dimension $d = 3$) and the mixed quark-gluon condensate ($d = 5$), which have values

$$\begin{aligned} \langle \bar{q}q \rangle &\simeq -(230 \text{ MeV})^3 \ , \\ \langle \bar{q}\, g_s \sigma_{\mu\nu} G^{\mu\nu}\, q \rangle &= m_0^2 \langle \bar{q}q \rangle \ , \quad m_0^2 \simeq 0.8 \text{ GeV}^2 \ . \end{aligned} \tag{3.8}$$

The contribution involving the gluon condensate ($d = 4$) is tiny and can safely be neglected [31].

The QCD sum rule is obtained by matching the phenomenological and theoretical expressions for Ξ. In doing this, one assumes quark-hadron duality to model the contributions of higher-resonance states described by ρ_{res} in (3.3) by the perturbative continuum above some threshold ω_0. Furthermore, in order to reduce the importance of higher-resonance states a double Borel transformation $\omega^{(\prime)} \to \tau^{(\prime)}$ is applied to both sides of the sum rule. This yields to an exponential damping factor in the dispersion integrals and also eliminates possible subtraction terms. Because of the heavy-quark symmetries the Borel paramters are equal, and we set $\tau = \tau' = 2T$. After the Borel transformation it is convenient to change variables in the dispersion integral according to

$$\nu_+ = \frac{(\nu+\nu')}{2} \ , \quad \nu_- = \left(\frac{y+1}{y-1}\right)^{1/2} \frac{(\nu-\nu')}{2} \ . \tag{3.9}$$

To one-loop order in perturbation theory the double discontinuities of the correlator are confined to the region $2y\nu\nu' - \nu^2 - \nu'^2 \geq 0$ and $\nu, \nu' \geq 0$, which then transforms into $\nu_+^2 \geq \nu_-^2$ and $\nu_+ \geq 0$, such that [31]

$$\begin{aligned} &\int d\nu\, d\nu' \frac{\rho_{pert}(\nu,\nu') - \rho_{res}(\nu,\nu')}{(\nu-\omega-i\epsilon)(\nu'-\omega'-i\epsilon)} + \text{subtractions} \\ &\xrightarrow{\text{B.T.}} \frac{1}{2T^2} \left(\frac{y-1}{y+1}\right)^{1/2} \int\limits_0^{\omega_0(y)} d\nu_+\, e^{-\nu_+/T} \int\limits_{-\nu_+}^{\nu_+} d\nu_-\, \rho_{pert}(\nu_+,\nu_-) \ . \end{aligned} \tag{3.10}$$

Facing the lack of information on the structure of resonance contributions to the three-point function Ξ, the separation between pole and continuum states in the sum rule has an unavoidable arbitrariness which leads to the dominant uncertainty in the prediction for the Isgur-Wise function. We shall, therefore, explore two simple models for the continuum threshold $\omega_0(y)$ in (3.10), namely

$$\omega_0(y) = f(y)\,\omega_0 \quad \text{with} \quad f(y) = \begin{cases} \dfrac{(y+1)}{2y} & ; \text{model 1}, \\ 1 & ; \text{model 2}. \end{cases} \tag{3.11}$$

It has been argued in Ref. [31] that the second choice leads to a conservative upper bound for the form factor, while the first one might be more realistic. Both forms of $f(y)$ have the non-trivial property that the slope of the Isgur-Wise function at zero recoil is finite.

Putting everything together one obtains the Laplace sum rule [30, 31]

$$F^2\,\xi(y)\,e^{-2\bar\Lambda/T} = \frac{3}{8\pi^2}\left(\frac{2}{y+1}\right)^2 \int\limits_0^{\omega_0(y)} \mathrm{d}\nu_+\,\nu_+^2\,e^{-\nu_+/T} - \langle\bar qq\rangle + \frac{(2y+1)}{3}\,\frac{m_0^2\,\langle\bar qq\rangle}{4T^2} \equiv K_\infty(T^{-1},\omega_0;y)\,. \qquad (3.12)$$

The first term on the right-hand side arises from the perturbative triangle diagram (bare quark loop), while the remaining terms are the leading nonperturbative corrections. A remark is in order concerning the large-recoil behavior of the Isgur-Wise function. For $y \gg 1$ the form factor should tend to zero, whereas the power corrections in the above sum rule stay finite or even increase. It is then necessary to sum the series of higher-dimensional condensates. This can be simulated by using so-called soft condensates which exponentially decrease for $y \gg 1$ [30, 31]. However, the corresponding effects are very small for values of y accessible in semileptonic B decays, and we shall neglect them here.

At zero recoil (3.12) reduces to the sum rule $F^2\,e^{-2\bar\Lambda/T} = K_\infty(T^{-1},\omega_0;1)$, from which the parameters $\bar\Lambda, \omega_0$ and F can be extracted in a self-consistent way by requiring optimal stability against variations of the Borel parameter T inside the "sum rule window" $0.6 < T < 1.0$ GeV, where the theoretical calculation of Ξ is reliable [21]. One finds good stability for

$$\begin{aligned} \bar\Lambda &\simeq 0.50 \pm 0.07~\text{GeV}, \\ \omega_0 &\simeq 2.00 \pm 0.30~\text{GeV}, \\ F &\simeq 0.30 \pm 0.05~\text{GeV}^{3/2}, \end{aligned} \qquad (3.13)$$

with correlated errors. Once the value of ω_0 is determined, one can compute the Isgur-Wise function from the ratio $\xi(y) = K_\infty(T^{-1},\omega_0;y)/K_\infty(T^{-1},\omega_0;1)$, which is independent of $\bar\Lambda$ and F and explicitly exhibits the zero-recoil normalization $\xi(1) = 1$. Before we present the result let us discuss the renormalization-group improvement of the sum rule analysis. In leading logarithmic approximation this is accomplished in a trivial way, since there are no large ratios of mass parameters that enter the sum rule calculation. Both $\bar\Lambda$ and the Borel parameter T are low-energy parameters. If the subtraction point μ is identified with one of them it is guaranteed that the sum rule is free of large logarithms even if radiative corrections were included. To be specific we choose $\mu = 2\bar\Lambda \simeq 1$ GeV, which still allows for a perturbative treatment. Hence in leading logarithmic approximation it is the function $\xi(y, 2\bar\Lambda)$ which can be extracted from the sum rule analysis, and the renormalized form factor defined in (2.20) is obtained from

$$\xi^{ren}(y) = [\alpha_s(2\bar{\Lambda})]^{-a_L} \, \frac{K_\infty(T^{-1},\omega_0;y)}{K_\infty(T^{-1},\omega_0;1)} \; . \tag{3.14}$$

This is in fact consistent with the result of a more detailled calculation of radiative corrections [31].

In Fig. 1(a) we show the sum rule predictions for the renromalized Isgur-Wise function for the two continuum models defined in (3.11). Inside the "sum rule window" the dependence on the precise values of T,ω_0 and the vacuum condensates is rather weak, as indicated by the width of the bands. The largest uncertainty arises from the arbitrariness in the choice of $f(y)$.

Let us now turn to the derivation of the QCD sum rules for the subleading form factor $\xi_\mu(v,v')$ defined in (2.12). To this end, we study the correlator Ξ_μ that is obtained from Ξ in (3.1) by replacing the heavy-quark current by $[\bar{h}_{Q'}(v')\,\Gamma\, iD_\mu\, h_Q(v)]_0$. The double-pole contribution to Ξ_μ is of the same form as in (3.4), but with $\xi(y)$ replaced by $\bar{\Lambda}\,\xi_\mu(v,v')$, *i.e.*

$$\Xi_\mu^{pole} = \frac{\bar{\Lambda}\,F^2}{(\omega - 2\bar{\Lambda} + i\epsilon)\,(\omega' - 2\bar{\Lambda} + i\epsilon)}\,\mathrm{Tr}\,\{\,\xi_\mu(v,v')\,\overline{\Gamma}_{M'}P_+(v')\,\Gamma\,P_+(v)\,\Gamma_M\,\}\;. \tag{3.15}$$

In the theoretical calculation of the correlator it is convenient to choose the external momentum $P = m_Q^* v + k$ parallel to v, such that $k_\mu = (k\cdot v)v_\mu$ (and similar for k'). In the analysis of the triangle diagram one encounters tensor one-loop integrals in HQET, which are collected in Appendix A. After the double Borel transformation the dispersion integrals are evaluated according to (3.10). Decomposing $\xi_\mu(v,v')$ as in (2.13) we find the sum rules

$$\begin{aligned} \bar{\Lambda}\,F^2\,\xi_-(y)\,e^{-2\bar{\Lambda}/T} &= -\frac{1}{4}\,\frac{\partial}{\partial T^{-1}}\,K_\infty(T^{-1},\omega_0;y)\;, \\ \bar{\Lambda}\,F^2\,\xi_3(y)\,e^{-2\bar{\Lambda}/T} &= -\frac{1}{6}\,\frac{\partial}{\partial T^{-1}}\,K_\infty(T^{-1},\omega_0;y) + \frac{m_0^2\,\langle\bar{q}q\rangle}{18T}\,(y-1)\;, \end{aligned} \tag{3.16}$$

which are expressed in terms of the derivative of the function K_∞ with respect to the inverse Borel parameter. The form factor $\xi_+(y)$ is related to $\xi_-(y)$ and $\xi_3(y)$ as shown in the first equation in (2.14), in accordance with the equations of motion of HQET. Using the sum rule (3.12) for the Isgur-Wise function we identify

$$\frac{\partial}{\partial T^{-1}}\,K_\infty(T^{-1},\omega_0;y) = -2\bar{\Lambda}\,F^2\,\xi(y)\,e^{-2\bar{\Lambda}/T}\;, \tag{3.17}$$

which leads to

$$\begin{aligned} \xi_-(y) &= \frac{1}{2}\,\xi(y)\;, \\ \xi_3(y) &= \frac{1}{3}\,\Big[\xi(y) - \kappa(T)\,(y-1)\Big]\;, \end{aligned} \tag{3.18}$$

where

$$\kappa(T) = -\frac{m_0^2\,\langle\bar q q\rangle}{6\bar\Lambda T F^2}\,e^{2\bar\Lambda/T}\,. \tag{3.19}$$

We have recovered the second relation in (2.14), which states that the equations of motion require that $\xi_-(y)$ be a multiple of the Isgur-Wise function. In good approximation this is also true for the form factor $\xi_3(y)$, since $\kappa(T)$ is small. For $T = T_0 = 0.8$ GeV, corresponding to the center of the "sum rule window", one finds $\kappa(T_0) \simeq 0.16 \mp 0.04$ with errors anticorrelated with those in (3.13).

In leading logarithmic approximation the renormalization-group improvement of the universal functions is again accomplished by writing $\xi_i^{ren}(y) = [\alpha_s(2\bar\Lambda)]^{-a_L}\,\xi_i(y)$. For the two continuum models specified in (3.11) the renormalized form factors are shown in Fig. 1(b). The sensitivity of these curves to changes in the sum rule parameters is similar as shown in Fig. 1(a). The most important observation is that, independent of all sum rule parameters, we obtain the zero-recoil normalization

$$\xi_3^{ren}(1) = \frac{1}{3}\,. \tag{3.20}$$

Since there is no restriction on the value of this form factor from heavy-quark symmetries (in contrast to the exact relation $\xi_-^{ren}(1) = 1/2$) one expects corrections to a simple result like (3.20). In the context of QCD sum rules, however, these could only come from next-to-leading-logarithmic radiative corrections to the triangle diagram, or from higher-dimensional condensates not included in our analysis. They are expected to be small.

IV. QCD SUM RULES FOR χ_i

In order to derive sum rules for the subleading universal functions $\chi_i(y)$ one has to repeat the analysis of the three-point correlator Ξ in (3.1) taking into account insertions of vertices from the higher-order effective Lagrangian $\delta\mathcal{L}_{eff}$ in (2.8). Because of the spin-flavor symmetry it is sufficient to consider the special case of equal heavy mesons ($m_{Q'}^* = m_Q^*$ and $M' = M$), thereby simplifying the presentation. In the calculation it is of advantage to sum the insertions of the operator $(1/2m_Q^*)\,\bar h_Q(i\partial)^2 h_Q$ in $\delta\mathcal{L}_{eff}$ to all orders by using $i\,(\not v+1)/\omega_Q$ for the heavy-quark propagator in momentum space, and $\omega_Q = 2v\cdot k + k^2/m_Q^*$ instead of $\omega = 2v\cdot k$ as dispersive variable.[2]

The spin-symmetry violating operator $(g_s/4m_Q^*)\,\bar h_Q\sigma_{\mu\nu}G^{\mu\nu}h_Q$ in $\delta\mathcal{L}_{eff}$ needs some special consideration. Since we do not consider radiative corrections (beyond the leading logarithmic approximation), insertions of this operator only contribute to diagrams involving gluonic condensates. The leading ones are proportional to the mixed quark-gluon condensate, and noting that

---

[2] Since $\omega_Q = (P^2 - m_Q^{*2})/m_Q^*$ this treatment insures that there is no left-hand cut in the complex ω_Q-plane.

$$\langle \bar{q}_\alpha \, i g_s G^{\mu\nu} \, q_\beta \rangle = \frac{i}{48} \, m_0^2 \, \langle \bar{q} q \rangle \, (\sigma^{\mu\nu})_{\beta\alpha} \tag{4.1}$$

it can be readily seen from (2.17) that there is no such contribution to $\chi_2(y)$. Thus within the standard approximations made in QCD sum rules we find that

$$\chi_2(y) = 0 \ . \tag{4.2}$$

Corrections to this result are again expected to be small. The more complicated trace structure associated with the χ_3-term in (2.16) can be reduced to that of the remaining terms in the sum rule by use of the identity

$$P_+(v) \, \sigma_{\mu\nu} \, \mathcal{M}(v) \, \sigma^{\mu\nu} = 2 d_M \, \mathcal{M}(v) \ , \tag{4.3}$$

where $d_M = 3$ for a pseudoscalar meson, and $d_M = -1$ for a vector meson.

After these remarks we present the expression for the theoretical side of the sum rule (3.12) which includes the $1/m_Q^*$ corrections. Using the tensor integrals collected in Appendix A we find

$$\begin{aligned} K_{m_Q^*}(T^{-1}, \tilde{\omega}_0; y) = & \frac{3}{8\pi^2} \left(\frac{2}{y+1} \right)^2 \int\limits_0^{\tilde{\omega}_0(y)} \mathrm{d}\nu_+ \, \nu_+^2 \, e^{-\nu_+/T} \left[1 - \frac{\nu_+}{m_Q^*} \left(1 + \frac{2}{3} \frac{y-1}{y+1} \right) \right] \\ & - \langle \bar{q} q \rangle + \frac{m_0^2 \, \langle \bar{q} q \rangle}{4T^2} \left[\frac{2y+1}{3} - \frac{T}{3 m_Q^*} \left(4y - 1 + d_M \right) \right] . \end{aligned} \tag{4.4}$$

On the phenomenological side one now has to include the $1/m_Q^*$ corrections to the Isgur-Wise function as well as to the "decay constant" F. Using the fact that $\chi_2(y) = 0$ to the order we are working, the left-hand side of (3.12) is replaced by

$$\left(\frac{m_M}{m_Q^*} \right)^2 e^{-2\tilde{\Lambda}/T} \, F^2 \left\{ 1 + \frac{2}{m_Q^*} \left[G_1 + 2 d_M \, G_2 \right] \right\} \left\{ \xi(y) + \frac{2\bar{\Lambda}}{m_Q^*} \left[\chi_1(y) + 2 d_M \, \chi_3(y) \right] \right\} , \tag{4.5}$$

where the mass ratio arises from the factor m_M^2 in (3.6) and $1/m_Q^{*2}$ from the meson propagators. The constants G_1 and G_2 have been defined in Ref. [21]. They are the analogues of $\chi_1(y)$ and $\chi_3(y)$ for the case of meson decay constants. In the above expressions it is important to realize that, in addition to the explicit $1/m_Q^*$ corrections, also the sum rule paramters $\tilde{\Lambda} = (m_M^2 - m_Q^{*2})/2m_Q^*$ and $\tilde{\omega}_0(y) = f(y) \, \tilde{\omega}_0$ contain both spin-symmetry conserving and violating corrections, *i.e.* [21]

$$\begin{aligned} \tilde{\Lambda} &= \bar{\Lambda} \left\{ 1 + \frac{1}{m_Q^*} \left[\delta\Lambda_1 + d_M \, \delta\Lambda_2 \right] \right\} , \\ \tilde{\omega}_0 &= \omega_0 \left\{ 1 + \frac{1}{m_Q^*} \left[\delta\omega_1 + d_M \, \delta\omega_2 \right] \right\} . \end{aligned} \tag{4.6}$$

Before evaluating the sum rule it is convenient to eliminate the explicit $1/m_Q^*$ correction in the dispersion integral by a redefinition of the Borel parameter,

$$\frac{1}{T} \to \frac{1}{T} - \left(1 + \frac{2}{3}\,\frac{y-1}{y+1}\right) \frac{1}{m_Q^*} \,. \tag{4.7}$$

Using $m_M = m_Q^* + \bar\Lambda$ in (4.5) we then obtain

$$\begin{aligned}
&F^2\, e^{-2\tilde\Lambda/T} \left\{1 + \frac{2}{m_Q^*}\left[G_1 + 2\bar\Lambda + 2d_M\, G_2\right]\right\} \\
&\times\left\{\xi(y) + \frac{2\bar\Lambda}{m_Q^*}\left[\chi_1(y) + \frac{2}{3}\,\frac{y-1}{y+1}\,\xi(y) + 2d_M\,\chi_3(y)\right]\right\} \\
&= \frac{3}{8\pi^2}\left(\frac{2}{y+1}\right)^2 \int\limits_0^{\tilde\omega_0(y)} \mathrm{d}\nu_+\, \nu_+^2\, e^{-\nu_+/T} - \langle\bar q q\rangle \\
&+ \frac{m_0^2\,\langle\bar q q\rangle}{4T^2}\left[\frac{2y+1}{3} - \frac{T}{9m_Q^*}\left(32y - 5 - 4\,\frac{y-1}{y+1} + 3d_M\right)\right] .
\end{aligned} \tag{4.8}$$

The next step is to expand this sum rule in inverse powers of the heavy-quark mass. In leading order one immediately recovers (3.12). At order $1/m_Q^*$ we separate the spin-symmetry conserving and violating terms to obtain the two sum rules

$$\begin{aligned}
&\chi_1(y) + \frac{2}{3}\,\frac{y-1}{y+1}\,\xi(y) + \left[\frac{G_1}{\bar\Lambda} + 2 - \frac{\delta\Lambda_1}{T}\right]\xi(y) \\
&\quad = \frac{\kappa(T)}{12}\left(32y - 5 - 4\,\frac{y-1}{y+1}\right) - \frac{9\,r}{4}\,\varepsilon(T)\left(\frac{2}{y+1}\right)^2 f^3(y)\, e^{[1-f(y)]\,\omega_0/T} \,, \\
&\chi_3(y) + \left[\frac{G_2}{\bar\Lambda} - \frac{\delta\Lambda_2}{2T}\right]\xi(y) = \frac{\kappa(T)}{8} - \frac{\varepsilon(T)}{8}\left(\frac{2}{y+1}\right)^2 f^3(y)\, e^{[1-f(y)]\,\omega_0/T} \,,
\end{aligned} \tag{4.9}$$

where

$$\varepsilon(T) = -\frac{3}{4\pi^2}\,\frac{\delta\omega_2\,\omega_0^3}{\bar\Lambda\,F^2}\, e^{(2\bar\Lambda-\omega_0)/T} \,, \quad r = \frac{\delta\omega_1}{9\delta\omega_2} \,. \tag{4.10}$$

One can use the zero-recoil normalization conditions (2.18) for the universal form factors to obtain sum rules for the parameters G_1 and G_2. Setting $y = 1$ in the above equations gives

$$\begin{aligned}
\frac{G_1}{\bar\Lambda} + 2 - \frac{\delta\Lambda_1}{T} &= \frac{9}{4}\left[\kappa(T) - r\,\varepsilon(T)\right] , \\
\frac{G_2}{\bar\Lambda} - \frac{\delta\Lambda_2}{2T} &= \frac{1}{8}\left[\kappa(T) - \varepsilon(T)\right] .
\end{aligned} \tag{4.11}$$

The same sum rules have recently been derived from the study of a two-point correlator of heavy-light currents in HQET [21], and the agreement of the results provides a check of our calculation. From the structure of (4.11) one can deduce simple relations between the spin-symmetry conserving and violating parameters, namely [21]

$$\begin{aligned} r = 1 \quad &\Leftrightarrow \quad \delta\omega_1 = 9\delta\omega_2 \ , \\ \delta\Lambda_1 &= 9\delta\Lambda_2 \ , \\ G_1 &= 18 G_2 - 2\bar\Lambda \ . \end{aligned} \tag{4.12}$$

It thus suffices to analyze the second sum rule in (4.11) and its derivative with respect to T^{-1} to determine the parameters $\delta\omega_i, \delta\Lambda_i$ and G_i. In particular, one finds excellent stability inside the "sum rule window" for $\delta\omega_2 \simeq -(0.10 \mp 0.02)$ GeV [21], corresponding to $\varepsilon(T_0) \simeq 0.40^{-0.12}_{+0.24}$ evaluated at the center of the "sum rule window", $T_0 = 0.8$ GeV. As the parameter $\kappa(T)$, both $\delta\omega_2$ and $\varepsilon(T)$ are proportional to the mixed quark-gluon condensate.

We now insert (4.11) into (4.9) to eliminate the parameters G_i and $\delta\Lambda_i$ from the final result

$$\begin{aligned} \chi_1(y) &= \frac{2}{3}\,\frac{y-1}{y+1}\left[\left(4y+\frac{7}{2}\right)\kappa(T) - \xi(y)\right] + 18\,\chi_3(y) \ , \\ \chi_3(y) &= \frac{\kappa(T)}{8}\Big[1-\xi(y)\Big] - \frac{\varepsilon(T)}{8}\left[\left(\frac{2}{y+1}\right)^2 f^3(y)\, e^{[1-f(y)]\omega_0/T} - \xi(y)\right] \ , \end{aligned} \tag{4.13}$$

which explicitly exhibits the normalization conditions $\chi_1(1) = \chi_3(1) = 0$. As discussed in Sec. III, the renormalized functions $\chi_i^{ren}(y)$ can be simply obtained from the sum rule results by multiplying with appropriate powers of $\alpha_s(2\bar\Lambda)$ as shown in (2.20), for instance $\chi_3^{ren}(y) = [\alpha_s(2\bar\Lambda)]^{-a_L - 9/\beta}\,\chi_3(y)$. The resulting curves are shown in Fig. 2. We note that in this case the results are rather insensitive to the continuum model employed. The function $\chi_1^{ren}(y)$ induces sizeable corrections to the infinite quark-mass limit for large recoil. However, these corrections respect the spin symmetry and thus affect all $\bar B \to D$ and $\bar B \to D^*$ form factors in the same way. They are therefore irrelevant. The spin-symmetry violating corrections described by $\chi_3^{ren}(y)$, on the other hand, are much smaller, typically $\chi_3^{ren}(y) \lesssim 0.1\,\chi_1^{ren}(y)$. This, together with the sum rule prediction $\chi_2^{ren}(y) \simeq 0$, indicates that the heavy-quark spin symmetry is predominantly broken by the higher-dimensional current operators in (2.5), *i.e.* by the universal functions $\xi_i^{ren}(y)$.

It has been pointed out in Ref. [21] that the relation $\delta\omega_1 = 9\delta\omega_2$ is subject to large higher-order corrections in the $1/m_Q^*$ expansion, leading to an *effective* value $r_{eff} \neq 1$. The difference $(1-r_{eff})$ is formally of order $1/m_Q^*$, but numerically of order unity for the case of charmed and beauty mesons. This induces large higher-order corrections (of order $1/m_Q^{*2}$) to the decay constants of heavy mesons. Let us show that there is no such effect in the case of heavy-meson form factors. If $r_{eff} \neq 1$ one has to replace $\chi_1(y)$ in (4.13) by $\chi_1(y) + (1-r_{eff})\,\delta\chi_1(y)$ with

$$\delta\chi_1(y) = \frac{9\,\kappa(T)}{4}\Big[1-\xi(y)\Big] - 18\,\chi_3(y) \ . \tag{4.14}$$

Numerically one finds that $|\delta\chi_1(y)| < 0.1\,\chi_1(y)$, such that even for an effective value $(1 - r_{eff})$ of order unity the higher-order correction is very small and can safely be neglected. This is in fact not a coincidence. Consider, for simplicity, the continuum model 2 in (3.11), *i.e.* $f(y) = 1$. It then follows from (3.12) that $\xi(y) \simeq [2/(y+1)]^2$ up to corrections from vacuum condensates. Therefore the contribution involving $\varepsilon(T)$ in (4.13) is formally proportional to a product of condensates and can as well be neglected. In fact, this term is much smaller than the contribution involving $\kappa(T)$. In this approximation, however,

$$\chi_3(y) = \frac{\kappa(T)}{8}\Big[1 - \xi(y)\Big]\,, \tag{4.15}$$

and $\delta\chi_1(y) = 0$, such that the value of r_{eff} becomes irrelevant.

V. SUMMARY AND PHENOMENOLOGICAL APPLICATIONS

In the previous sections we have presented an analysis of the universal functions that appear in leading and subleading order in the heavy quark expansion of current matrix elements between two heavy mesons, using QCD sum rules in HQET. The results for the subleading form factors $\xi_i(v\cdot v')$ and $\chi_i(v\cdot v')$ given in (3.18) and (4.13) involve the Isgur-Wise function $\xi(v\cdot v')$ and two nonperturbative parameters, κ and ε, which are proportional to the mixed quark-gluon condensate. It is worthwhile to summarize the advantages of such an approach over previous sum rule calculations for heavy meson form factors.

- The most important distinction is that our approach incorporates the Ward identities of HQET in the sum rule analysis, *i.e.*, the zero-recoil conditions (2.18) are *exactly* reproduced. In the standard formulation of QCD sum rules, on the other hand, these relations would only be satisfied *approximately* as a result of the self-consistent numerical analysis.[3]
- By relating the sum rules for the subleading form factors to that for the Isgur-Wise function we derived the *paramter-free* predictions $\xi_3(1) = 1/3$ and $\chi_2(y) = 0$, which could only receive corrections from diagrams usually not included in the sum rule analysis of a three-point function. These predictions should have a higher accuracy than sum rule results in general, which suffer from uncertainties in various parameters and in the numerical analysis. In particular, since the remaining two subleading form factors, $\chi_1(v\cdot v')$ and $\chi_3(v\cdot v')$, are known to vanish at zero recoil, we conclude that for $v = v'$ the leading power corrections

[3]Therefore, the conclusion of Ref. [37] that the ratio of a standard sum rule for the axial form factor $A_1(q^2)$ over the sum rule for the Isgur-Wise function would provide a measure of $1/m_Q^{*2}$ corrections has no foundation.

to the infinite quark-mass limit can be predicted with good accuracy, and in an almost model-independent way.

- By constructing separate sum rules for the universal functions which appear in different orders of the heavy-quark expansion one increases the accuracy in the description of symmetry-breaking corrections to quantities which become equal in the infinite quark-mass limit. Examples are the very accurate calculation of the B^*-B mass difference in Ref. [21], or ratios of the various form factors describing $\bar{B}\to D^*\ell\,\bar{\nu}$ decays, which will be discussed below. For instance, even a 30% uncertainty in the sum rule analysis of a subleading universal function corresponds to an uncertainty of only a few percent once this function is multiplied by $\bar{\Lambda}/2m_Q^*$.

- Finally, it is an appealing feature of our approach that certain universal functions are related to a particular type of diagrams. For instance, the leading contribution to the spin-symmetry violating form factor $\chi_3(v\cdot v')$ comes from diagrams involving the mixed quark-gluon condensate, and it was immediate to find that $\chi_2(v\cdot v')=0$ when higher-dimensional condensates and radiative corrections are neglected.

Let us now discuss the application of our results to the theoretical description of the semileptonic processes $\bar{B}\to D^{(*)}\ell\,\bar{\nu}$. Following Refs. [18, 26, 38] we define heavy-meson form factors $h_i(v\cdot v')$ by

$$
\begin{aligned}
\langle D(v')|\,V_\mu\,|\bar{B}(v)\rangle &= \sqrt{m_B\,m_D}\,\Big[h_+(v\cdot v')\,(v+v')_\mu + h_-(v\cdot v')\,(v-v')_\mu\Big]\,,\\
\langle D^*(v')|\,V_\mu\,|\bar{B}(v)\rangle &= i\sqrt{m_B\,m_{D^*}}\;h_V(v\cdot v')\,\epsilon_{\mu\nu\alpha\beta}\,\epsilon^{*\nu}\,v'^\alpha\,v^\beta\,,\\
\langle D^*(v')|\,A_\mu\,|\bar{B}(v)\rangle &= \sqrt{m_B\,m_{D^*}}\,\Big[h_{A_1}(v\cdot v')\,(v\cdot v'+1)\,\epsilon_\mu^*\\
&\qquad - h_{A_2}(v\cdot v')\,\epsilon^*\!\cdot v\,v_\mu - h_{A_3}(v\cdot v')\,\epsilon^*\!\cdot v\,v'_\mu\Big]\,,
\end{aligned}
\tag{5.1}
$$

where $V_\mu=\bar{c}\,\gamma_\mu\,b$ and $A_\mu=\bar{c}\,\gamma_\mu\gamma_5\,b$. In order to make the heavy-quark symmetry-limit and the leading symmetry-breaking corrections to it explicit we write $(y=v\cdot v')$

$$
h_i(y)=\Big[\alpha_i+\beta_i(y)+\gamma_i(y)+\dots\Big]\,\xi^{ren}(y)\,,
\tag{5.2}
$$

where, according to (2.11), $\alpha_+=\alpha_V=\alpha_{A_1}=\alpha_{A_3}=1$ and $\alpha_-=\alpha_{A_2}=0$ [2]. The functions $\beta_i(y)$ are short-distance perturbative corrections, and $\gamma_i(y)$ contain the $1/m_c^*$ and $1/m_b^*$ corrections. The ellipses represent terms of order $1/m_Q^{*2}$.

In leading order in the heavy-quark expansion the renormalization of the form factors is known in next-to-leading order in renormalization-group-improved perturbation theory. Explicit expressions for the functions $\beta_i(y)$ are given in Refs. [18]. For the numerical evaluation we use the quark masses $m_c^*=1.44$ GeV and $m_b^*=4.80$ GeV

($m_c^*/m_b^* = 0.3$), as well as $\Lambda_{\overline{\rm MS}} = 0.2$ GeV for $n_f = 4$. Over the kinematical range accessible in semileptonic B decays ($y_{max} \simeq 1.59$ for $\bar{B} \to D$ and $y_{max} \simeq 1.50$ for $\bar{B} \to D^*$ transitions, respectively), the resulting coefficients are compiled in Table I. They are accurate up to terms of order $[\alpha_s(m_c^*)/\pi]^2 \simeq 1\%$.

In this paper we are mainly interested in the leading power corrections $\gamma_i(y)$. By evaluating the traces in (2.12) and (2.16) one can relate these functions to the subleading form factors $\xi_3^{ren}(y,\bar{m})$ and $\chi_i(y,\bar{m})$, which we renormalize in leading logarithmic approximation at the scale $\bar{m} = \frac{2m_b^* m_c^*}{m_b^*+m_c^*} \simeq 2.2$ GeV. The explicit expressions are given in Appendix B. In Table II we present the numerical results obtained from the QCD sum rule analysis. The numbers refer to continuum model 1 in (3.11), but the results are not very sensitive to this choice. The theoretical uncertainty is estimated for zero recoil, assuming a 15% accuracy of the prediction (3.20) and $|\chi_2^{ren}(1)| < 2.5\%$. At maximum recoil, on the other hand, the sum rule results should have an accuracy of better than 30%.

The theoretical results summarized in these tables form a solid basis for a comprehensive analysis of semileptonic B decays to subleading order in HQET. We shall restrict ourselves to some specific examples here and perform a more complete analysis elsewhere. As a first application, let us focus on the extraction of the quark-mixing paramter V_{cb} from an extrapolation of the semileptonic $\bar{B}$ decay rates to zero recoil. This subject has been discussed in detail in Ref. [35]. In general, one finds that

$$\begin{aligned}
\lim_{v\cdot v'\to 1} \frac{1}{[(v\cdot v')^2-1]^{1/2}}\,\frac{\mathrm{d}\Gamma(\bar{B}\to D^*\ell\,\bar{\nu})}{\mathrm{d}(v\cdot v')} &= \frac{G_F^2}{4\pi^3}\,|\,V_{cb}\,|^2\,(m_B-m_{D^*})^2\,m_{D^*}^3\,\eta^{*2}\ ,\\
\lim_{v\cdot v'\to 1} \frac{1}{[(v\cdot v')^2-1]^{3/2}}\,\frac{\mathrm{d}\Gamma(\bar{B}\to D\,\ell\,\bar{\nu})}{\mathrm{d}(v\cdot v')} &= \frac{G_F^2}{48\pi^3}\,|\,V_{cb}\,|^2\,(m_B+m_D)^2\,m_D^3\,\eta^2\ ,
\end{aligned} \qquad (5.3)$$

with $\eta^* = \eta = 1$ in the infinite quark-mass limit. Because of Luke's theorem [19] the decay rate for $\bar{B} \to D^*\ell\,\bar{\nu}$ is protected against $1/m_Q^*$ corrections at zero recoil (see Appendix B). Thus to subleading order in HQET the coefficient η^* deviates from unity only because of perturbative QCD corrections. One finds that [35, 18]

$$\eta^* = 1 + \delta_{QCD}^* + \mathcal{O}\Big(1/m_Q^{*2}\Big)\ ,\quad \delta_{QCD}^* = \beta_{A_1}(1) \simeq -0.01\ . \qquad (5.4)$$

On the other hand, Luke's theorem does not apply for $\bar{B} \to D\,\ell\,\bar{\nu}$ decays because the decay rate is helicity-suppressed at zero recoil [26, 35]. In this case

$$\eta = 1 + \delta_{QCD} + \delta_{1/m_Q^*} + \mathcal{O}\Big(1/m_Q^{*2}\Big) \qquad (5.5)$$

with

$$\begin{aligned}
\delta_{QCD} &= \beta_+(1) - \frac{m_B-m_D}{m_B+m_D}\,\beta_-(1) \simeq 0.05\ ,\\
\delta_{1/m_Q^*} &= \frac{\bar{\Lambda}}{2}\left(\frac{1}{m_c^*}+\frac{1}{m_b^*}\right)\left(\frac{m_B-m_D}{m_B+m_D}\right)^2\,[1-2\,\xi_3^{ren}(1)] \simeq 0.02\ .
\end{aligned} \qquad (5.6)$$

Note that, as pointed out by Voloshin and Shifman, the $1/m_Q^*$ corrections are suppressed by the factor $[(m_B - m_D)/(m_B + m_D)]^2 \simeq 0.23$ [1], and that the corrections to the sum rule prediction $\xi_3^{ren}(1) = 1/3$ are expected to be small. Since the canonical size of $1/m_Q^{*2}$ corrections is $1 - 5\%$, we thus conclude that the theoretical uncertainty in η is comparable to that in η^*. Hence one should extract V_{cb} from both decay modes, using the theoretical numbers

$$\eta^* \simeq 0.99 \ , \quad \eta \simeq 1.07 \ , \tag{5.7}$$

which are expected to have an accuracy of better than 5%.

As a second example we study symmetry-breaking effects in the form factors which describe $\bar{B} \to D^* \ell \bar{\nu}$ transitions. In the limit where the lepton mass is neglected, two axial form factors $A_1(q^2)$ and $A_2(q^2)$ as well as one vector form factor $V(q^2)$ are observable in these decays. They are related to the heavy-quark form factors defined in (5.1) by [26]

$$\begin{aligned} A_1(q^2) &= \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] \frac{(m_B + m_{D^*})}{2\sqrt{m_B m_{D^*}}}\, h_{A_1}(v \cdot v') \ , \\ A_2(q^2) &= \frac{(m_B + m_{D^*})}{2\sqrt{m_B m_{D^*}}} \left[h_{A_3}(v \cdot v') + \frac{m_{D^*}}{m_B}\, h_{A_2}(v \cdot v')\right] , \\ V(q^2) &= \frac{(m_B + m_{D^*})}{2\sqrt{m_B m_{D^*}}}\, h_V(v \cdot v') \ , \end{aligned} \tag{5.8}$$

where

$$v \cdot v' = \frac{m_B^2 + m_{D^*}^2 - q^2}{2 m_B m_{D^*}} \ . \tag{5.9}$$

In the infinite quark-mass limit the form factors h_{A_1}, h_{A_3} and h_V become equal to the Isgur-Wise function, whereas h_{A_2} vanishes. The ratios

$$\begin{aligned} R_1 &= \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] \frac{V(q^2)}{A_1(q^2)} = \frac{h_V(v \cdot v')}{h_{A_1}(v \cdot v')} \ , \\ R_2 &= \left[1 - \frac{q^2}{(m_B + m_{D^*})^2}\right] \frac{A_2(q^2)}{A_1(q^2)} = \frac{h_{A_3}(v \cdot v') + \frac{m_{D^*}}{m_B} h_{A_2}(v \cdot v')}{h_{A_1}(v \cdot v')} \end{aligned} \tag{5.10}$$

are therefore sensitive measures of symmetry-breaking effects. To subleading order in HQET we write

$$R_i = 1 + \varepsilon_i^{QCD} + \varepsilon_i^{1/m_Q^*} \ ; \quad i = 1, 2 \tag{5.11}$$

and find, using the expressions given in Appendix B and the results of Ref. [18],

$$\begin{aligned}
\varepsilon_1^{QCD} &= \frac{4\alpha_s(m_c^*)}{3\pi}\, r(y) , \\
\varepsilon_2^{QCD} &= \frac{2\alpha_s(\bar{m})}{3\pi}\, f\Big(y, \frac{m_c^*}{m_b^*}\Big) , \\
\varepsilon_1^{1/m_Q^*} &= \frac{\bar{\Lambda}}{(y+1)} \left[\frac{1}{m_c^*} + \frac{1}{m_b^*}\left(1 - 2\,\frac{\xi_3^{ren}(y)}{\xi^{ren}(y)}\right)\right] , \\
\varepsilon_2^{1/m_Q^*} &= -\frac{\bar{\Lambda}}{(y+1)}\left(\frac{1}{m_c^*} + \frac{3}{m_b^*}\right)\frac{\xi_3^{ren}(y)}{\xi^{ren}(y)} - 2\bar{\Lambda}\left(\frac{1}{m_c^*} - \frac{1}{m_b^*}\right)[\alpha_s(\bar{m})]^{1/3}\,\frac{\chi_2^{ren}(y)}{\xi^{ren}(y)} ,
\end{aligned} \tag{5.12}$$

where again $y = v \cdot v'$. The function $f(y,z)$ is given by

$$f(y,z) = -\frac{z(1-z)}{1-2yz+z^2}\left[\frac{1+z}{1-z}\ln z + (y+1)\, r(y)\right] \tag{5.13}$$

and is very slowly varying with y. At zero recoil and for $z = m_c^*/m_b^* = 0.3$ its value is $f(1, 0.3) \simeq 0.10$.

In Table III we show the theoretical prediction for R_i and ε_i. We propose a measurement of these quantities as an ideal test of the heavy-quark expansion for $b \to c$ transitions. In particular, note that the large values of R_1 result from both large QCD and large $1/m_Q^*$ corrections, and that the latter ones are to a large extent model-independent since the subleading form factor $\xi_3^{ren}(y)$ only appears in the $1/m_b^*$ correction. Thus the sizeable deviation of R_1 from the symmetry-limit $R_1 = 1$ is an unambiguous prediction of HQET. A measurement of this ratio at a leval of 10% can therefore provide valuable information about the size of higher-order corrections. The ratio R_2, on the other hand, receives only very small QCD corrections and is sensitive to the subleading form factors $\xi_3^{ren}(y)$ and $\chi_2^{ren}(y)$. It can be used to test the sum rule predictions (3.20) and (4.2). For the pratical feasibility of such tests it seems welcome that the theoretical predictions for both ratios are almost independent of q^2, such that it suffices to measure the integrated ratios.

ACKNOWLEDGMENTS

It is a pleasure to thank A. Radyushkin, A. Falk and M. Luke for helpful discussions. Financial support from the BASF Aktiengesellschaft and the German National Scholarship Foundation is gratefully acknowledged. This work was also supported by the Department of Energy, contract DE-AC03-76SF00515.

APPENDIX A: ONE-LOOP TENSOR INTEGRALS IN HQET

The master tensor integral for one-loop diagrams involving two heavy quarks in HQET is (in D space-time dimensions)

$$\begin{aligned} I_{\alpha\beta\gamma}^{\mu_1\ldots\mu_n}(\omega,\omega',v,v') &= \int \mathrm{d}^D t\, t^{\mu_1}\ldots t^{\mu_n} \left(-\frac{1}{t^2}\right)^\alpha \left(\frac{1}{\omega+2v\cdot t}\right)^\beta \left(\frac{1}{\omega'+2v'\cdot t}\right)^\gamma \\ &= i\pi^{D/2}\, I_n(\alpha,\beta,\gamma) \int\limits_0^\infty \mathrm{d}u\, \frac{u^{\gamma-1}}{[\Omega(u)]^{\beta+\gamma}} \left[-\frac{\Omega(u)}{V(u)}\right]^{D-2\alpha+n} K^{\mu_1\ldots\mu_n}(u)\,, \end{aligned} \tag{A1}$$

where

$$\begin{aligned} I_n(\alpha,\beta,\gamma) &= \frac{\Gamma(2\alpha+\beta+\gamma-D-n)\,\Gamma(D/2-\alpha+n)}{\Gamma(\alpha)\,\Gamma(\beta)\,\Gamma(\gamma)}\,, \\ \Omega(u) &= \omega + u\,\omega'\,, \\ V(u) &= (1+u^2+2u\,v\cdot v')^{1/2}\,. \end{aligned} \tag{A2}$$

For $n=0,1,2$ the tensors $K^{\mu_1\ldots\mu_n}$ are given by

$$\begin{aligned} K(u) &= 1\,, \\ K^\mu(u) &= -\hat V^\mu(u)\,, \\ K^{\mu\nu}(u) &= \hat V^\mu(u)\,\hat V^\nu(u) - \frac{g^{\mu\nu}}{D-2\alpha+2}\,, \end{aligned} \tag{A3}$$

with

$$\hat V^\mu(u) = \frac{v^\mu + u\,v'^\mu}{V(u)} \tag{A4}$$

being a unit vector. In the special case where $\omega'=\omega$ and $v'=v$ the general expression (A1) reduces to the tensor integral for one-loop diagrams involving a single heavy quark,

$$\begin{aligned} I_{\alpha\beta}^{\mu_1\ldots\mu_n}(\omega,v) &= \int \mathrm{d}^D t\, t^{\mu_1}\ldots t^{\mu_n} \left(-\frac{1}{t^2}\right)^\alpha \left(\frac{\omega}{\omega+2v\cdot t}\right)^\beta \\ &= i\pi^{D/2}\, I_n(\alpha,\beta)\,(-\omega)^{D-2\alpha+n}\, K^{\mu_1\ldots\mu_n}\,, \end{aligned} \tag{A5}$$

where

$$I_n(\alpha,\beta) = \frac{\Gamma(2\alpha+\beta-D-n)\,\Gamma(D/2-\alpha+n)}{\Gamma(\alpha)\,\Gamma(\beta)}\,, \tag{A6}$$

and $K^{\mu_1\ldots\mu_n}$ is obtained from (A4) by replacing $\hat V^\mu(u)$ by v^μ.

In the sum rule calculation one needs the double spectral densities of the tensor integrals, which are defined by

$$I^{\mu_1\ldots\mu_n}_{\alpha\beta\gamma}(\omega,\omega',v,v') = \int d\nu\, d\nu'\, \frac{\rho^{\mu_1\ldots\mu_n}_{\alpha\beta\gamma}(\nu,\nu',v,v')}{(\nu-\omega-i\epsilon)\,(\nu'-\omega'-i\epsilon)} + \text{polynomials in } \omega \text{ or } \omega'. \tag{A7}$$

A convenient way to compute these is by using Borel transformations [39]. Defining the Borel operator with respect to ω by

$$\frac{1}{T}\,\widehat{B}^{(\omega)}_T = \lim_{\substack{n\to\infty \\ -\omega\to\infty \\ T=\frac{-\omega}{n}\text{ fixed}}} \frac{\omega^n}{\Gamma(n)}\left(-\frac{d}{d\omega}\right)^n, \tag{A8}$$

where $T>0$ is the Borel parameter, it is easy to see that

$$\rho^{\mu_1\ldots\mu_n}_{\alpha\beta\gamma}(\omega,\omega',v,v') = \widehat{B}^{(-z')}_{1/\omega'}\,\widehat{B}^{(-z)}_{1/\omega}\,\widehat{B}^{(\omega')}_{1/z'}\,\widehat{B}^{(\omega)}_{1/z}\,I^{\mu_1\ldots\mu_n}_{\alpha\beta\gamma}(\omega,\omega',v,v')\,. \tag{A9}$$

Using

$$\widehat{B}^{(\omega')}_{1/z'}\,\widehat{B}^{(\omega)}_{1/z}\,[-\Omega(u)]^{-a} = \frac{z^{a-2}}{\Gamma(a)}\,\delta\Big(u-\frac{z'}{z}\Big) \tag{A10}$$

one finds that

$$\widehat{B}^{(\omega')}_{1/z'}\,\widehat{B}^{(\omega)}_{1/z}\,I^{\mu_1\ldots\mu_n}_{\alpha\beta\gamma} = i\pi^{D/2}\,\frac{\Gamma(D/2-\alpha+n)}{\Gamma(\alpha)\,\Gamma(\beta)\,\Gamma(\gamma)}\,\frac{(-z)^{\beta-1}\,(-z')^{\gamma-1}}{[z^2+z'^2+2yzz']^{(D-2\alpha+n)/2}}\,K^{\mu_1\ldots\mu_n}\Big(\frac{z'}{z}\Big)\,, \tag{A11}$$

where $y=v\cdot v'$, and

$$\begin{aligned}
K\Big(\frac{z'}{z}\Big) &= 1\,,\\
K^\mu\Big(\frac{z'}{z}\Big) &= -\frac{z\,v^\mu + z'\,v'^\mu}{[z^2+z'^2+2yzz']^{1/2}}\,,\\
K^{\mu\nu}\Big(\frac{z'}{z}\Big) &= \frac{(z\,v^\mu+z'\,v'^\mu)\,(z\,v^\nu+z'\,v'^\nu)}{z^2+z'^2+2yzz'} - \frac{g^{\mu\nu}}{D-2\alpha+2}\,.
\end{aligned} \tag{A12}$$

Let us now specialize to the case $\alpha=\beta=\gamma=1$ and set $D=4$. Introducing a hyperbolic angle θ by $\cosh\theta = v\cdot v' = y$, and noting that

$$\frac{1}{z^2+z'^2+2zz'y} = \frac{1}{(z+z'e^{\theta})(z+z'e^{-\theta})} = \frac{1}{2z'\sinh\theta}\left[\frac{1}{z+z'e^{-\theta}} - \frac{1}{z+z'e^{\theta}}\right], \tag{A13}$$

one can eliminate all powers of z' (or z) in the numerators in (A12) and express the right-hand side of (A11) in terms of the functions

$$F_{mn}(z,z') = \frac{1}{(z+z'e^{\theta})^m (z+z'e^{-\theta})^n} \ , \tag{A14}$$

the double Borel transforms of which are readily computed using ($a > 0$)

$$\frac{1}{a^n} = \frac{1}{\Gamma(n)} \int_0^\infty \mathrm{d}\beta \, \beta^{n-1} \, e^{-\beta a} \ . \tag{A15}$$

This result is ($m, n > 0$)

$$\begin{aligned} \hat{B}^{(-z')}_{1/\omega'} \, \hat{B}^{(-z)}_{1/\omega} \ F_{mn}(z,z') = {} & \Theta(\omega)\, \Theta(\omega)\, \Theta(2y\,\omega\omega' - \omega^2 - \omega'^2) \\ & \times \left(\frac{1}{2\sinh\theta}\right)^{m+n-1} \frac{(\omega' - \omega\, e^{-\theta})^{m-1} (\omega\, e^{\theta} - \omega')^{n-1}}{\Gamma(m)\,\Gamma(n)} \ , \end{aligned} \tag{A16}$$

where $\sinh\theta = \sqrt{y^2 - 1}$.

Using these techniques it is straightforward to work out the various spectral densities. For $n = 1, 2$ we define scalar invariants by

$$\begin{aligned} \rho^{\mu}_{111} &= G_1\, v^{\mu} + G_2\, v'^{\mu} \ , \\ \rho^{\mu\nu}_{111} &= H_1\, g^{\mu\nu} + H_2\, v^{\mu} v^{\nu} + H_3\, v'^{\mu} v'^{\nu} + H_4\, (v^{\mu} v'^{\nu} + v'^{\mu} v^{\nu}) \ . \end{aligned} \tag{A17}$$

We find that

$$\begin{aligned} \rho_{111} &= \frac{i\pi^2}{2\sqrt{y^2-1}}\, \Theta(\omega)\, \Theta(\omega')\, \Theta(2y\omega\omega' - \omega^2 - \omega'^2) \ , \\ G_1 &= -\frac{\rho_{111}}{2(y^2-1)}\, (y\omega' - \omega) \ , \\ H_1 &= -\frac{\rho_{111}}{8(y^2-1)}\, (2y\omega\omega' - \omega^2 - \omega'^2) \ , \\ H_2 &= \frac{\rho_{111}}{8(y^2-1)^2} \left[3\omega^2 + (2y^2+1)\omega'^2 - 6y\omega\omega'\right] \ , \\ H_4 &= \frac{\rho_{111}}{8(y^2-1)^2} \left[2(2y^2+1)\omega\omega' - 3y(\omega^2 + \omega'^2)\right] \ . \end{aligned} \tag{A18}$$

G_2 and G_1, as well as H_3 and H_2, are related by interchange of ω and ω'.

For $\beta > 1$ or $\gamma > 1$ one can either apply the same technique, or use the recurrence relation

$$I^{\mu_1\ldots\mu_n}_{\alpha\beta\gamma} = \frac{(-\partial_\omega)^{\beta-1} (-\partial_{\omega'})^{\gamma-1}}{\Gamma(\beta)\,\Gamma(\gamma)}\, I^{\mu_1\ldots\mu_n}_{\alpha 11} \ . \tag{A19}$$

For instance, one finds that

$$\rho_{121} = -\frac{i\pi^2}{2\sqrt{y^2-1}}\, \Theta(\omega)\, \Theta(\omega')\, [\delta(\omega' - \omega e^{-\theta}) - \delta(\omega' - \omega e^{\theta})] \ . \tag{A20}$$

APPENDIX B: POWER CORRECTIONS TO HEAVY-MESON FORM FACTORS

In leading logarithmic approximation the power corrections in (5.2) are given by $\gamma_i(y) = [\alpha_s(\bar{m})]^{a_L}\,\hat{\gamma}_i(y)$ with [19, 26]

$$
\begin{aligned}
\hat{\gamma}_+(y) &= \left(\frac{\bar{\Lambda}}{m_c^*} + \frac{\bar{\Lambda}}{m_b^*}\right) \varrho_1(y) \,, \\
\hat{\gamma}_-(y) &= \left(\frac{\bar{\Lambda}}{m_c^*} - \frac{\bar{\Lambda}}{m_b^*}\right) \left[\varrho_4(y) - \frac{1}{2}\right] , \\
\hat{\gamma}_V(y) &= \frac{\bar{\Lambda}}{2}\left(\frac{1}{m_c^*} + \frac{1}{m_b^*}\right) + \frac{\bar{\Lambda}}{m_c^*}\,\varrho_2(y) + \frac{\bar{\Lambda}}{m_b^*}\left[\varrho_1(y) - \varrho_4(y)\right] , \\
\hat{\gamma}_{A_1}(y) &= \frac{\bar{\Lambda}}{2}\,\frac{y-1}{y+1}\left(\frac{1}{m_c^*} + \frac{1}{m_b^*}\right) + \frac{\bar{\Lambda}}{m_c^*}\,\varrho_2(y) + \frac{\bar{\Lambda}}{m_b^*}\left[\varrho_1(y) - \frac{y-1}{y+1}\,\varrho_4(y)\right] , \\
\hat{\gamma}_{A_2}(y) &= \frac{\bar{\Lambda}}{m_c^*}\left[\varrho_3(y) - \frac{\varrho_4(y)+1}{y+1}\right] , \\
\hat{\gamma}_{A_3}(y) &= \frac{\bar{\Lambda}}{2}\left(\frac{y-1}{y+1}\,\frac{1}{m_c^*} + \frac{1}{m_b^*}\right) + \frac{\bar{\Lambda}}{m_c^*}\left[\varrho_2(y) - \varrho_3(y) - \frac{\varrho_4(y)}{y+1}\right] + \frac{\bar{\Lambda}}{m_b^*}\left[\varrho_1(y) - \varrho_4(y)\right] .
\end{aligned}
\tag{B1}
$$

The functions $\varrho_i(y)$ are related to the renormalized universal form factors of HQET [cf. (2.21)] by

$$
\begin{aligned}
\varrho_1(y)\,\xi^{ren}(y) &= \chi_1^{ren}(y) - \frac{16}{27}\,\frac{r(y)-y}{y+1}\,\ln\left[\alpha_s(\bar{m})\right]\,\xi^{ren}(y) \\
&\quad + 2\left[\alpha_s(\bar{m})\right]^{1/3}\left[3\,\chi_3^{ren}(y) - (y-1)\,\chi_2^{ren}(y)\right] , \\
\varrho_2(y)\,\xi^{ren}(y) &= \chi_1^{ren}(y) - \frac{16}{27}\,\frac{r(y)-y}{y+1}\,\ln\left[\alpha_s(\bar{m})\right]\,\xi^{ren}(y) - 2\left[\alpha_s(\bar{m})\right]^{1/3}\chi_3^{ren}(y) \,, \\
\varrho_3(y)\,\xi^{ren}(y) &= 2\left[\alpha_s(\bar{m})\right]^{1/3}\chi_2^{ren}(y) \,, \\
\varrho_4(y)\,\xi^{ren}(y) &= \xi_3^{ren}(y) \,.
\end{aligned}
\tag{B2}
$$

The zero-recoil conditions (2.18), which follow from the conservation of the vector current $\bar{Q}\gamma_\mu Q$, imply

$$
\varrho_1(1) = \varrho_2(1) = 0 \quad \Rightarrow \quad \gamma_+(1) = \gamma_{A_1}(1) = 0 \,. \tag{B3}
$$

From the QCD sum rule analysis we furthermore predict that

$$\varrho_3(y) \simeq 0 \,, \quad \varrho_4(1) \simeq \frac{1}{3} \,, \tag{B4}$$

from which it follows that

$$\begin{aligned}
\gamma_-(1) &\simeq \gamma_{A_3}(1) \simeq -\frac{\bar\Lambda}{6}\left(\frac{1}{m_c^*} - \frac{1}{m_b^*}\right) \simeq -4\% \,, \\
\gamma_V(1) &\simeq \frac{\bar\Lambda}{6}\left(\frac{3}{m_c^*} + \frac{1}{m_b^*}\right) \simeq 19\% \,, \\
\gamma_{A_2}(1) &\simeq -\frac{2\bar\Lambda}{3m_c^*} \simeq -23\% \,.
\end{aligned} \tag{B5}$$

From (B3) it is obvious that the hadronic matrix elements in (5.1) are unaffected from $1/m_Q^*$ corrections at zero recoil, since all form factors other than $h_+(y)$ and $h_{A_1}(y)$ are kinematically suppressed at $v = v'$. This is the content of Luke's theorem [19]. It is important to realize, however, that this does *not* imply that the *observable form factors* do not receive $1/m_Q^*$ corrections. If the lepton mass is neglected, four form factors are measurable in semileptonic B decays, namely $f_+(q^2)$ in $\bar B \to D\,\ell\,\bar\nu$ and $V(q^2), A_1(q^2), A_2(q^2)$ in $\bar B \to D^*\ell\,\bar\nu$ (for the definition of these form factors and their relation to the functions $h_i(v\cdot v')$ defined in (5.1) see Ref. [26]). At zero recoil only one of these, $A_1(q^2_{max})$, is protected by Luke's theorem.

REFERENCES

[1] M.B. Voloshin and M.A. Shifman, Yad. Fiz. **45** (1987) 463 [Sov. J. Nucl. Phys. **45** (1987) 292], Yad. Fiz. **47** (1988) 511 [Sov. J. Nucl. Phys. **47** (1988) 511].

[2] N. Isgur and M.B. Wise, Phys. Lett. **B232** (1989) 113, Phys. Lett. **B237** (1990) 527, Nucl. Phys. **B348** (1991) 276.

[3] E. Eichten and F. Feinberg, Phys. Rev. **D23** (1981) 2724; E. Eichten and B. Hill, Phys. Lett. **B234** (1990) 511, Phys. Lett. **B243** (1990) 427.

[4] W.E. Caswell and G.P. Lepage, Phys. Lett. **B167** (1986) 437; G.P. Lepage and B. Thacker, Nucl. Phys. **B** (Proc. Suppl.) **4** (1988) 119.

[5] H. Georgi, Phys. Lett. **B240** (1990) 447.

[6] J.G. Körner and G. Thompson, Phys. Lett. **B264** (1991) 185.

[7] T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. **B368** (1992) 204.

[8] A.F. Falk, B. Grinstein and M.E. Luke, Nucl. Phys. **B357** (1991) 185.

[9] F. Hussain *et al.*, Phys. Lett. **B249** (1990) 295; F. Hussain, J.G. Körner, M. Krämer and G. Thompson, Z. Phys. **C51** (1991) 321.

[10] T. Mannel, W. Roberts and Z. Ryzak, Phys. Lett. **B255** (1991) 593, Nucl. Phys. **B355** (1991) 38.

[11] A.F. Falk, SLAC preprint SLAC–PUB–5689 (1991), to be published in Nucl. Phys. **B**.

[12] H.D. Politzer and M.B. Wise, Phys. Lett. **B206** (1988) 681, Phys. Lett. **B208** (1988) 504.

[13] A.F. Falk, H. Georgi, B. Grinstein and M.B. Wise, Nucl. Phys. **B343** (1990) 1.

[14] A.F. Falk and B. Grinstein, Phys. Lett. **B247** (1990) 406.

[15] X. Ji and M.J. Musolf, Phys. Lett. **B257** (1991) 409.

[16] D.L. Broadhurst and A.G. Grozin, Phys. Lett. **B267** (1991) 105.

[17] G.P. Korchemsky and A.V. Radyushkin, Nucl. Phys. **B283** (1987) 342, Phys. Lett. **B279** (1992) 359; G.P. Korchemsky, Mod. Phys. Lett. **A4** (1989) 1257.

[18] M. Neubert, Nucl. Phys. **B371** (1992) 149, Heidelberg preprint HD–THEP–91–30 (1991), to be published in Phys. Rev. **D**.

[19] M.E. Luke, Phys. Lett. **B252** (1990) 447.

[20] H. Georgi, B. Grinstein and M.B. Wise, Phys. Lett. **B252** (1990) 456.

[21] M. Neubert, SLAC preprint SLAC–PUB–5770 (1992), to be published in Phys. Rev. **D**.

[22] C.R. Allton *et al.*, Nucl. Phys. **B349** (1991) 598.

[23] C. Alexandrou *et al.*, Phys. Lett. **B256** (1991) 60.

[24] C. Bernard, A.X. El-Khadra and A. Soni, Phys. Rev. **D43** (1991) 2140.

[25] L. Maiani, Helv. Phys. Acta **64** (1991) 853.

[26] M. Neubert and V. Rieckert, Heidelberg preprint HD–THEP–91–6 (1991).

[27] M.A. Ivanov, CEBAF preprint CEBAF–TH–91–08 (1991).

[28] M. Burkardt and E.S. Swanson, MIT preprint CPT#2096 (1992).
[29] B. Grinstein and P.F. Mende, SSCL–Preprint–64 (1992).
[30] A.V. Radyushkin, Phys. Lett. **B271** (1991) 218.
[31] M. Neubert, Phys. Rev. **D45** (1992) 2451.
[32] E. Bagan, P. Ball, V.M. Braun and H.G. Dosch, Phys. Lett. **B278** (1992) 457.
[33] A.F. Falk, M. Neubert and M.E. Luke, SLAC preprint SLAC–PUB–5771 (1992).
[34] J.D. Bjorken, invited talk given at Les Rencontres de la Valle d'Aoste La Thuile, Aosta Valley, Italy, March 1990, SLAC preprint SLAC–PUB–5278 (1990), Lecture at the 1990 SLAC Summer Institute on Particle Physics, SLAC preprint SLAC–PUB–5389 (1990).
[35] M. Neubert, Phys. Lett. **B264** (1991) 455.
[36] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. **B147** (1979) 385, Nucl. Phys. **B147** (1979) 448.
[37] P. Ball, Phys. Lett. **B281** (1992) 133.
[38] M. Neubert, V. Rieckert, B. Stech and Q.P. Xu, Heidelberg preprint HD–THEP–91–28 (1991), to appear in *"Heavy Flavours"*, edited by A.J. Buras and M. Lindner, Advanced Series on Directions in High Energy Physics, World Scientific Publishing Co.; M. Neubert, in Proceedings of the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics, Geneva, Switzerland, July 1991, Heidelberg preprint HD–THEP–91–34 (1991).
[39] A. Radyushkin, private communication about techniques for the calculation of loop integrals.

FIGURES

FIG. 1. (a) Numerical evaluation of the sum rule (3.14) for the renormalized Isgur-Wise form factor. The lower band corresponds to the continuum model 1 in (3.11), the upper one to model 2. We use $\alpha_s(2\bar{\Lambda}) = 0.34$. (b) Sum rule results for the renormalized form factors $\xi_i^{ren}(y)$. The solid lines refer to continuum model 1, the dashed ones to model 2. We use the central values for all sum rule parameters, corresponding to $\kappa = 0.16$.

FIG. 2. Sum rule results for the renormalized form factors $\chi_1^{ren}(y)$ and $10\,\chi_3^{ren}(y)$. The solid lines refer to continuum model 1, the dashed ones to model 2. We use the central values for all sum rule parameters, corresponding to $\kappa = 0.16$ and $\varepsilon = 0.40$. The sensitivity to changes in these parameters is similar as in Fig. 1(a).

TABLES

TABLE I. QCD corrections $\beta_i(v \cdot v')$ in %.

$v \cdot v'$	β_+	β_-	β_V	β_{A_1}	β_{A_2}	β_{A_3}
1.0	2.6	−5.4	11.9	−1.5	−11.0	2.2
1.1	−0.3	−5.4	8.9	−3.8	−10.3	−0.2
1.2	−3.1	−5.3	6.1	−5.9	−9.8	−2.5
1.3	−5.6	−5.3	3.5	−7.9	−9.3	−4.6
1.4	−8.0	−5.2	1.1	−9.7	−8.8	−6.6
1.5	−10.2	−5.2	−1.1	−11.5	−8.4	−8.5
1.59	−12.1	−5.1				

TABLE II. Power corrections $\gamma_i(v \cdot v')$ in %.

$v \cdot v'$	γ_+	γ_-	γ_V	γ_{A_1}	γ_{A_2}	γ_{A_3}
1.0	0.0	−4.1	19.1	0.0	−23.1	−4.1
1.1	2.7	−4.1	20.7	2.9	−21.4	−0.7
1.2	6.2	−4.1	23.1	6.5	−19.8	3.4
1.3	10.5	−4.2	26.3	10.7	−18.3	8.0
1.4	15.3	−4.4	30.0	15.4	−17.0	13.0
1.5	20.6	−4.5	34.3	20.5	−15.8	18.5
1.59	25.7	−4.7				
$\delta\gamma_i(1)$	0.0	1.4	2.9	0.0	4.0	2.1

TABLE III. Theoretical predictions for the ratios R_i and the symmetry-breaking corrections ε_i.

$v \cdot v'$	q^2 [GeV^2]	R_1	ε_1^{QCD} [%]	ε_1^{1/m_Q^*} [%]	R_2	ε_2^{QCD} [%]	ε_2^{1/m_Q^*} [%]
1.0	10.69	1.31	12.0	19.1	0.90	0.5	−11.0
1.1	8.57	1.30	11.7	18.2	0.90	0.5	−10.3
1.2	6.45	1.29	11.3	17.5	0.91	0.5	−9.6
1.3	4.33	1.28	11.0	16.8	0.92	0.5	−8.9
1.4	2.21	1.27	10.7	16.2	0.92	0.5	−8.3
1.5	0.09	1.26	10.4	15.6	0.93	0.5	−7.7

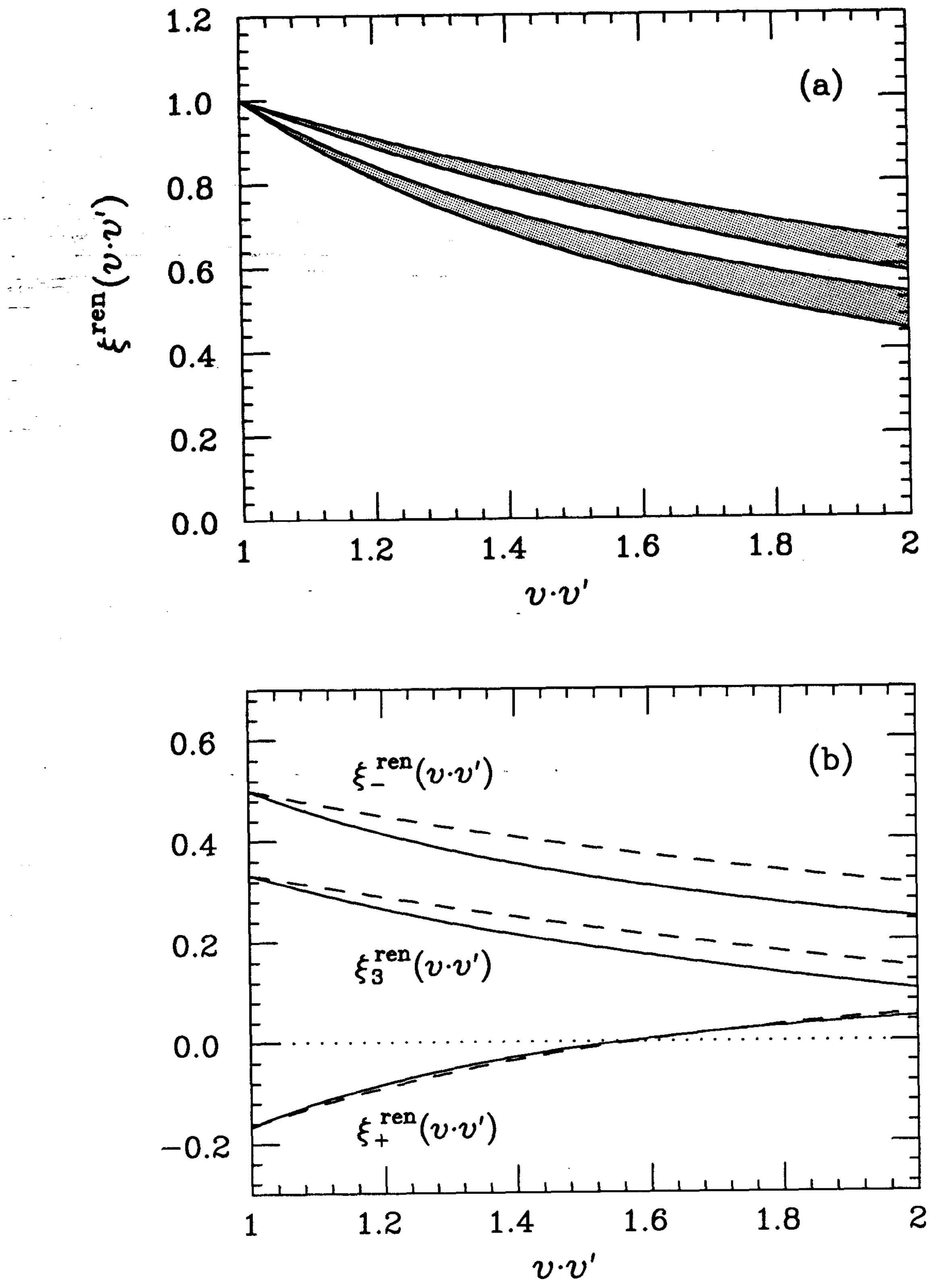
(a)
$\xi^{\rm ren}(v\cdot v')$
$v\cdot v'$
(b)
$\xi_-^{\rm ren}(v\cdot v')$
$\xi_3^{\rm ren}(v\cdot v')$
$\xi_+^{\rm ren}(v\cdot v')$
$v\cdot v'$

Fig. 1

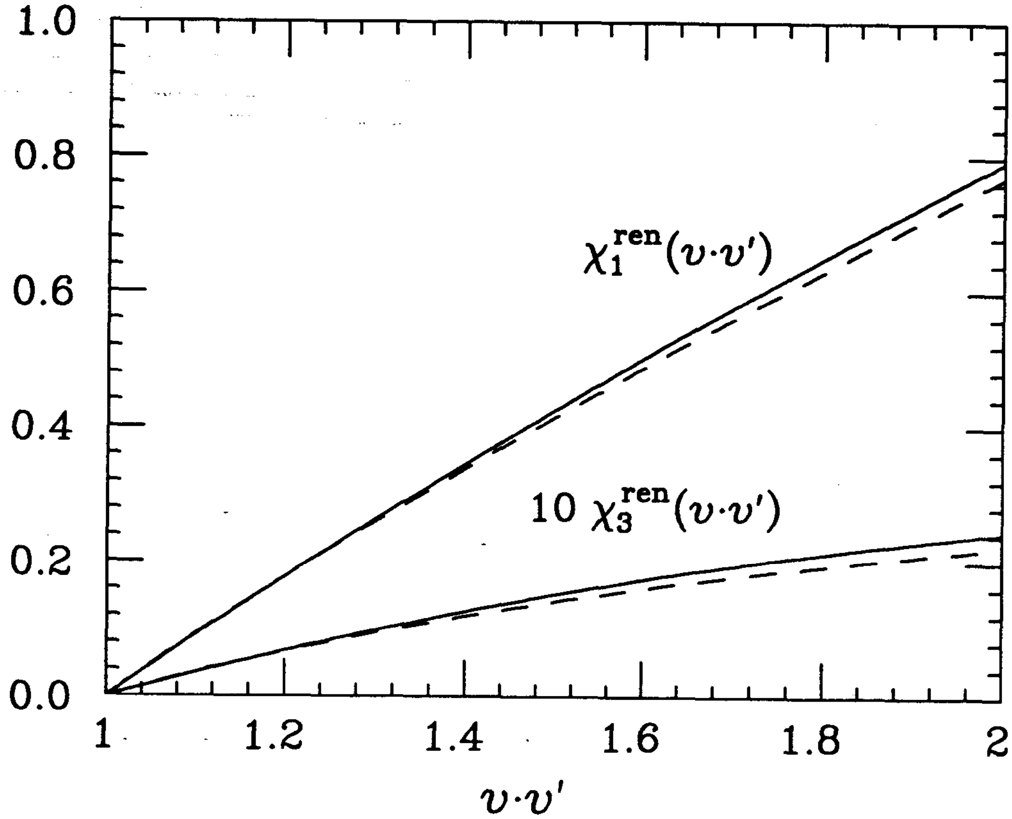
1.0
0.8
0.6
0.4
0.2
0.0
1
1.2
1.4
1.6
1.8
2
$\chi_1^{\mathrm{ren}}(v{\cdot}v')$
$10\ \chi_3^{\mathrm{ren}}(v{\cdot}v')$
$v{\cdot}v'$

Fig. 2