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Coupling Scale of the Three-Gluon Vertex*

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Abstract

We apply the multi-momentum renormalization group equation to the gauge-invariant gluon two-point and three-point functions and obtain the effective coupling constant for the quark-gluon and three-gluon vertices. For the three-gluon vertex, we show that the effective coupling scale is essentially given by $\mu^2 \sim Q^2_{\rm min} Q^2_{\rm med}/Q^2_{\rm max}$, where $Q^2_{\rm min}$, $Q^2_{\rm med}$ and $Q^2_{\rm max}$ are respectively the smallest, the next-to-smallest and the largest scale among the three gluon virtualities. This functional form suggests that the three-gluon vertex becomes non-perturbative at highly asymmetric momentum configurations. Implication for the coupling scale in four-jet physics is discussed.

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1. Introduction

The scale-ambiguity problem remains as one of the major cornerstones impeding precise QCD predictions. The relevance of this problem is often obscured by the consideration of only anomalous-dimension-free single-scale processes, where the coupling scales can be easily "guessed", since they should lie around the mass scale of each process.

For multiple-scale processes, however, the scale ambiguity problem becomes unavoidable. Although there are various scale-setting methods based on mathematical principles [1], their reliability in multiple-scale processes remains an open issue. Also, the application of these methods generally requires the full calculation of all the Feynman diagrams to one-loop order. The choice of scale in this context becomes merely a mechanical problem.

It is desirable to have a prescription for coupling scales from simply considering the Feynman diagrams of a given process. For instance, in Fig. 1(a) we have the elastic scattering of two quarks. We clearly have to assign $\mu^2 \sim q^2$ for the coupling scales at the quark-gluon vertices a and b. Similarly, in the case of the elastic scattering of three quarks via a three-gluon vertex as indicated in Fig. 1(b), we would intuitively assign $\mu^2 \sim p^2, q^2, r^2$ for the vertices a, b and c. However, there is a priori no clear prescription for the coupling scale for the three-gluon vertex d.

The assignment of different coupling scale to different vertices cannot be done in an arbitrary fashion, though. The gauge invariance has to be observed; otherwise, the final result would be physically meaningless. The tree-level Feynman diagrams in Fig. 1 are gauge-invariant; hence, the assignment of different coupling constants for the various vertices is allowed to this order.

Recently the author has pointed out that the dressed skeleton expansion [2] offers a perturbative calculation method without scale ambiguity. This method has been applied to a variety of field theoretical models [3]. The extension of dressed-skeleton method to gauge theories is not straightforward, since the skeleton graphs in these theories are in general not gauge invariant. Unlike QED, where the dressed-photon expansion provides a gauge-invariant way of clustering Feynman diagrams, in QCD we lack of a systematic method of obtaining gauge-invariant skeletons.

Sometime ago Cornwall and Papavassiliou obtained a gauge-invariant gluon propagator and three-gluon vertex function [4] to one-loop order through the application of the "pinch" technique. Essentially, these functions correspond to the gauge-invariant skeletons of QCD to one-loop level. In this paper, we apply the multi-momentum renormalization group equation of the dress skeleton method to the gauge-invariant gluon two- and three-point functions and obtain their effective coupling scales.

In Section 2 we study the case of the quark-gluon vertex and recover the well known result of one-loop QCD running coupling constant.

In Section 3 we analyze the case of the three-gluon vertex. We obtain a somewhat more involved expression. However, the effective coupling scale is roughly given by

$$\mu^2 \sim \frac{Q^2_{\rm min} Q^2_{\rm med}}{Q^2_{\rm max}} \ , \tag{1}$$

being $Q^2_{\rm min}$, $Q^2_{\rm med}$ and $Q^2_{\rm max}$ respectively the smallest, the next-to-smallest and the largest gluon virtuality of the three-gluon vertex. We show that the functional form for the effective coupling supports BLM's ansatz [5] of using fermion loops as probes of coupling scales.

2. Quark-Gluon Coupling

The gauge-invariant gluon propagator is calculated by using the pinch technique in Ref. [4]. The one-loop Feynman diagrams are indicated in Fig. 1. The interpretation of the pinched diagrams is explained in Ref. [4]. The expression for the full propagator can be parametrized as

$$-i\Delta_{\mu\nu}(q^2) = -\frac{i}{q^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) Z(q^2) + i(1-\eta)\frac{q_\mu q_\nu}{q^2} \ , \tag{2}$$

where $Z(q^2)$ is the gauge-invariant gluon wavefunction renormalization constant and η the gauge parameter.

To one-loop order we have

$$Z(q^2) = 1 - \frac{g_o^2}{(4\pi)^2}\left[\left(11 - \frac{2}{3}N_f\right)\left(\frac{1}{\hat{\epsilon}} + \ln(-q^2)\right) - 22 + \frac{10}{9}N_f\right] \ , \tag{3}$$

where we have employed the dimensional regularization with $D = 4 + 2\epsilon$ and $1/\hat{\epsilon} = 1/\epsilon + \gamma_{\rm E} - 4\pi$. N_f is the number of light quark flavors. In analogy with QED, we define the effective quark-gluon running coupling constant to be

$$g_2^2(q^2) \equiv g_o^2 Z(q^2) \ . \tag{4}$$

Thus

$$\frac{1}{g_2^2(q^2)} = \frac{1}{g_o^2} + \frac{1}{(4\pi)^2}\left[\left(11 - \frac{2}{3}N_f\right)\left(\frac{1}{\hat{\epsilon}} + \ln(-q^2)\right) - 22 + \frac{10}{9}N_f\right] . \tag{5}$$

Upon solving this renormalization group equation we obtain the familiar expression

$$\alpha_2(q^2) \equiv \frac{g_2^2(q^2)}{4\pi} = \frac{4\pi}{(11 - \frac{2}{3}N_f)\ln(-q^2/\Lambda_2^2)} . \tag{6}$$

The scale Λ_2 is formally an integration constant to be fixed by experimental measurement. We observe that the gauge-invariant gluon propagator effectively introduces a renormalization scheme with itself. To this order, the relationship between Λ_2 and the more conventional $\Lambda_{\overline{\rm MS}}$ can be obtained by noting that in the $\overline{\rm MS}$ scheme

$$\frac{1}{g_{\overline{\rm MS}}^2(\mu^2)} = \frac{1}{g_o^2} + \frac{1}{(4\pi)^2}\left[\left(11 - \frac{2}{3}N_f\right)\left(\frac{1}{\hat{\epsilon}} + \ln(-\mu^2)\right)\right] . \tag{7}$$

By comparing equations (5) and (7) at $-q^2 = \Lambda_2^2$ and $\mu^2 = \Lambda_{\overline{\rm MS}}^2$, and noting that the left-hand sides of both equations vanish, we obtain the relationship

$$\Lambda_2 = \exp\left(\frac{99 - 5N_f}{99 - 6N_f}\right)\Lambda_{\overline{\rm MS}} . \tag{8}$$

For $N_f = 4$ and $N_f = 5$ we have respectively $\Lambda_2 = 2.867\Lambda_{\overline{\rm MS}}$ and $\Lambda_2 = 2.923\Lambda_{\overline{\rm MS}}$.

3. Three-Gluon Coupling

The effective coupling of the three-gluon vertex has been studied previously by a number of authors [7]. However, previous studies have been focused on the gauge-dependent three-gluon vertex. The presence of the gauge parameter impeded a reliable physical interpretation of the effective charge.

The gauge-invariant three-gluon vertex to one-loop order was first obtained by Cornwall and Papavassiliou [4]. The renormalized version of this vertex function is given below, where we have added the quark-loop contribution absent in Ref. [4].

$$
\begin{aligned}
-g_o f^{abc}\Gamma_{\lambda\mu\nu}(p,q,r) =& Z^{1/2}(p^2)Z^{1/2}(q^2)Z^{1/2}(r^2)f^{abc}\Bigg\{ \\
& -g_o\left[(p-q)_\nu g_{\lambda\mu}+(q-r)_\lambda g_{\mu\nu}+(r-p)_\mu g_{\nu\lambda}\right] \\
& -\frac{3}{2}ig_o^3\int\frac{d^4k}{(2\pi)^4}\frac{1}{k_1^2k_2^2k_3^2}\times \\
& \qquad\left[\Gamma^{\mathrm{F}}_{1\,\lambda}\Gamma^{\mathrm{F}}_{2\,\mu}\Gamma^{\mathrm{F}}_{3\,\nu}+2\left(k_2+k_3\right)_\lambda\left(k_3+k_1\right)_\mu\left(k_1+k_2\right)_\nu\right] \\
& -12ig_o^3\left(p_\nu g_{\lambda\mu}-p_\mu g_{\nu\lambda}\right)\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2(k+p)^2} \\
& -12ig_o^3\left(q_\lambda g_{\mu\nu}-q_\nu g_{\lambda\mu}\right)\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2(k+q)^2} \\
& -12ig_o^3\left(r_\mu g_{\nu\lambda}-r_\lambda g_{\mu\nu}\right)\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2(k+r)^2} \\
& -\frac{N_f}{2}ig_o^3\int\frac{d^4k}{(2\pi)^4}\frac{\mathrm{Tr}\left[\not{k}_1\gamma_\nu\not{k}_2\gamma_\lambda\not{k}_3\gamma_\mu\right]}{k_1^2k_2^2k_3^2}\Bigg\},
\end{aligned}
\tag{9}
$$

where $Z(p^2)$ is the gauge-invariant gluon wavefunction renormalization as given in

Eq. (3), and the Feynman parts [4] of the three-gluon vertex are given by

$$\begin{aligned}
\Gamma^{F}_{1\ \beta\lambda\gamma} &= 2p_\gamma g_{\lambda\beta} - 2p_\beta g_{\gamma\lambda} - (k_2 + k_3)_\lambda g_{\beta\gamma} \ , \\
\Gamma^{F}_{2\ \gamma\mu\alpha} &= 2q_\alpha g_{\mu\gamma} - 2q_\gamma g_{\alpha\mu} - (k_3 + k_1)_\mu g_{\gamma\alpha} \ , \\
\Gamma^{F}_{3\ \alpha\nu\beta} &= 2r_\beta g_{\nu\alpha} - 2r_\alpha g_{\beta\nu} - (k_1 + k_2)_\nu g_{\alpha\beta} \ .
\end{aligned} \tag{10}$$

The definition of the various momenta and indices is given in Fig. 3.

The gluon vertex has a complicated tensor structure. We can classify the various tensor components of this vertex into

$$\begin{aligned}
\Gamma_{\lambda\mu\nu} =& \Gamma^1 g_{\lambda\mu}(p-q)_\nu + \Gamma^2 g_{\mu\nu}(q-r)_\lambda + \Gamma^3 g_{\nu\lambda}(r-p)_\mu \\
&+ \Gamma^4 (q-r)_\lambda (r-p)_\mu (p-q)_\nu + \Gamma^{\rm long}_{\lambda\mu\nu}(p,q,r) \ ,
\end{aligned} \tag{11}$$

where the longitudinal part $\Gamma^{\rm long}_{\lambda\mu\nu}$ contains all the terms that vanish upon contracting with the projector operator

$$\Pi^{\lambda'\mu'\nu'}_{\lambda\mu\nu}(p,q,r) = \left(g_{\lambda\lambda'} - \frac{p_\lambda p_{\lambda'}}{p^2}\right)\left(g_{\mu\mu'} - \frac{q_\mu q_{\mu'}}{q^2}\right)\left(g_{\nu\nu'} - \frac{r_\nu r_{\nu'}}{r^2}\right) \ . \tag{12}$$

That is

$$\Pi \cdot \Gamma^{\rm long} = \Pi^{\lambda'\mu'\nu'}_{\lambda\mu\nu}(p,q,r)\Gamma^{\rm long}_{\lambda'\mu'\nu'}(p,q,r) = 0 \ . \tag{13}$$

The Born component, i.e., the component proportional to the tree-level tensor, is given by

$$\Gamma^0 = \frac{1}{3}\left(\Gamma^1 + \Gamma^2 + \Gamma^3\right) \ . \tag{14}$$

We can calculate this component by using the tensor method. Namely, we first obtain a set of linearly-independent equations by contracting the three-gluon vertex in Eq. (11) with a complete set of basis tensors, and then we solve for Γ^0 from this set of equations. Fortunately, the outcome of this lengthy analysis can be expressed in a rather compact form,

$$\begin{aligned}\Gamma^0 &= \frac{1}{48\mathcal{R}} S \cdot \Pi \cdot \Gamma \\ &= \frac{1}{48\mathcal{R}} S_{\lambda'\mu'\nu'} \Pi^{\lambda'\mu'\nu'}_{\lambda\mu\nu} \Gamma^{\lambda\mu\nu} \ ,\end{aligned} \tag{15}$$

with S the projection tensor given by

$$\begin{aligned}S_{\lambda\mu\nu} =& 2p^2(q-r)_\lambda g_{\mu\nu} + 2q^2(r-p)_\mu g_{\nu\lambda} + 2r^2(p-q)_\nu g_{\lambda\mu} \\ &+ (q-r)_\lambda (r-p)_\mu (p-q)_\nu \ ,\end{aligned} \tag{16}$$

and

$$\mathcal{R} = \frac{1}{4}\left(2p^2q^2 + 2q^2r^2 + 2r^2p^2 - p^4 - q^4 - r^4\right) \ . \tag{17}$$

The effective three-gluon coupling is defined in term of the Born component by

$$g_3(p^2, q^2, r^2) \equiv g_o \Gamma^0 \ . \tag{18}$$

This is a natural choice since at the perturbative regimen the Born structure dominates. All the non-Born components are formally higher-order in g_o and hence are subleading. Also notice that, to one-loop order, all the ultraviolet divergences are contained within the Born component; therefore, it is the only component responsible for the coupling constant renormalization.

Upon inverting and squaring the previous equation,

$$\begin{aligned}\frac{1}{g_3^2(p^2,q^2,r^2)} = \frac{1}{g_o^2} + \frac{1}{(4\pi)^2}\Bigg[&\left(11 - \frac{2}{3}N_f\right) \times \\ &\left(\frac{1}{\hat{\epsilon}} + \mathrm{L}(-p^2,-q^2,-r^2) - \frac{16}{3\sqrt{3}}\mathrm{Lsin}_2\left(\frac{\pi}{3}\right)\right) - 22 + \frac{2}{3}N_f\Bigg] \ .\end{aligned} \tag{19}$$

where

$$
\begin{aligned}
\mathrm{L}(-p^2,-q^2,-r^2) &= \frac{r\cdot p\; p\cdot q}{\mathcal{R}}\ln(-p^2) + \frac{p\cdot q\; q\cdot r}{\mathcal{R}}\ln(-q^2) \\
&+ \frac{q\cdot r\; r\cdot p}{\mathcal{R}}\ln(-r^2) + \frac{p^2q^2r^2}{\mathcal{R}}\mathrm{F}(p^2,q^2,r^2) + \frac{16}{3\sqrt{3}}\mathrm{Lsin}_2\left(\frac{\pi}{3}\right) \ .
\end{aligned} \tag{20}
$$

The various dot products are expressible in terms of the gluon virtualities, e.g., $p\cdot q = (r^2-p^2-q^2)/2$. The functions $\mathrm{F}(p^2,q^2,r^2)$ and $\mathrm{Lsin}_2(x)$ are fully described in Ref. [6]. For completeness, we reproduce a summary here.

$$
\begin{aligned}
\mathrm{F}(p^2,q^2,r^2) &= \frac{i}{\pi^2}\int d^4k \frac{1}{k_1^2k_2^2k_3^2} \ , \\
&= \frac{1}{\rho}\left[\mathrm{Lsin}_2(2\phi_1) + \mathrm{Lsin}_2(2\phi_2) + \mathrm{Lsin}_2(2\phi_3)\right] \ , \\
\rho &= \sqrt{\mathcal{R}} \ , \\
\phi_1 &= \arctan\left(\frac{\rho}{q\cdot r - i\epsilon}\right) = \frac{i}{2}\ln\left(\frac{q\cdot r - i\rho - i\epsilon}{q\cdot r + i\rho - i\epsilon}\right) \ , \\
\phi_2 &= \arctan\left(\frac{\rho}{r\cdot p - i\epsilon}\right) = \frac{i}{2}\ln\left(\frac{r\cdot p - i\rho - i\epsilon}{r\cdot p + i\rho - i\epsilon}\right) \ , \\
\phi_3 &= \arctan\left(\frac{\rho}{p\cdot q - i\epsilon}\right) = \frac{i}{2}\ln\left(\frac{p\cdot q - i\rho - i\epsilon}{p\cdot q + i\rho - i\epsilon}\right) \ , \\
\mathrm{Lsin}_2(z) &= \frac{1}{2i}\left[\mathrm{Li}_2\left(e^{iz}\right) - \mathrm{Li}_2\left(e^{-iz}\right)\right] = \sum_1 \frac{\sin nz}{n^2} \ , \\
\mathrm{Lsin}_2\left(\frac{\pi}{3}\right) &= 1.01494160\ldots \ .
\end{aligned} \tag{21}
$$

The function $\mathrm{L}(x,y,z)$ can be considered as a three-variable extension of the logarithmic function. In fact, on the symmetric axis $x=y=z$, the function $\mathrm{L}(x,y,z)$ reduces to

$$
\mathrm{L}(x,x,x) = \ln(x) \ . \tag{22}
$$

The function $\mathrm{L}(x,y,z)$ also satisfies the simple scaling property

$$
\mathrm{L}(\lambda x,\lambda y,\lambda z) = \ln\lambda + \mathrm{L}(x,y,z) \ , \quad \text{for } \lambda > 0 \ . \tag{23}
$$

We can interpret Eq. (19) as a multi-momentum renormalization group equation (see Refs. [2-3]). Its solution is given by

$$\begin{aligned}\alpha_3(p^2,q^2,r^2) &= \frac{g_3^2(p^2,q^2,r^2)}{4\pi} \\ &= \frac{4\pi}{\left(11-\frac{2}{3}N_f\right)\mathrm{L}(-p^2/\Lambda_3^2,-q^2/\Lambda_3^2,-r^2/\Lambda_3^2)}\ ,\end{aligned} \tag{24}$$

where the function Λ_3 is a quantity to be fixed by experimental measurement. Notice the similarity between this formula and the familiar form of the strong coupling constant as given in (6). In both cases, the factor $11-\frac{2}{3}N_f$ multiplies a single function. The functional form of the fermion contribution thus is identical to the pure-gluon contribution. In the three-gluon vertex, this feature is a surprise given the complicated form of the integrals in Eq. (9). This strongly supports BLM's proposal [5] of using fermion loops as probes of QCD coupling scales, since the scale obtained via fermion-loop analysis is identical to the one obtained by a more complete analysis.

The scale Λ_3 can be expressed in terms of Λ_2 or $\Lambda_{\overline{\mathrm{MS}}}$ since the bare coupling-constant of QCD is unique. By comparing Eqs. (5), (7) and (19), we obtain

$$\begin{aligned}\Lambda_3 &= \exp\left(\frac{33-N_f}{33-2N_f}+\frac{8}{3\sqrt{3}}\mathrm{Lsin}_2\left(\frac{\pi}{3}\right)\right)\Lambda_{\overline{\mathrm{MS}}} \\ &= \exp\left(\frac{2N_f}{99-6N_f}+\frac{8}{3\sqrt{3}}\mathrm{Lsin}_2\left(\frac{\pi}{3}\right)\right)\Lambda_2\ .\end{aligned} \tag{25}$$

For $N_f=4$ and $N_f=5$ we have respectively $\Lambda_3 = 15.22\Lambda_{\overline{\mathrm{MS}}} = 5.308\Lambda_2$ and $\Lambda_3 = 16.12\Lambda_{\overline{\mathrm{MS}}} = 5.515\Lambda_2$.

In what follows we will consider only the case where p^2, q^2 and r^2 are all spacelike [8]. In Fig. 4 we plot the equal-coupling surfaces of $\alpha_3(p^2,q^2,r^2)$ in this kinematic region.

In the limit when one of the momentum scales is much larger than the other two, we have

$$\begin{aligned} \mathrm{L}(-p^2/\Lambda_3^2, -q^2/\Lambda_3^2, -r^2/\Lambda_3^2) &\to \ln\left(\frac{Q_{\rm min}^2 Q_{\rm med}^2}{Q_{\rm max}^2 \Lambda_3^2}\right) + \frac{16}{3\sqrt{3}}\mathrm{Lsin}_2\left(\frac{\pi}{3}\right) \\ &\equiv \ln\left(\frac{Q_{\rm min}^2 Q_{\rm med}^2}{Q_{\rm max}^2 \tilde{\Lambda}_3^2}\right) \ . \end{aligned} \tag{26}$$

with $Q_{\rm min}^2$, $Q_{\rm med}^2$ and $Q_{\rm max}^2$ respectively the smallest, the next-to-smallest and the largest scales among $-p^2$, $-q^2$ and $-r^2$, and

$$\begin{aligned} \tilde{\Lambda}_3 &= \exp\left(-\frac{8}{3\sqrt{3}}\mathrm{Lsin}_2\left(\frac{\pi}{3}\right)\right)\Lambda_3 \\ &= \exp\left(\frac{33-N_f}{33-2N_f}\right)\Lambda_{\overline{\rm MS}} \\ &= \exp\left(\frac{2N_f}{99-6N_f}\right)\Lambda_2 \ . \end{aligned} \tag{27}$$

For $N_f = 4$ and $N_f = 5$ we have respectively $\tilde{\Lambda}_3 = 3.190\Lambda_{\overline{\rm MS}} = 1.113\Lambda_2$ and $\tilde{\Lambda}_3 = 3.378\Lambda_{\overline{\rm MS}} = 1.156\Lambda_2$. From Eq. (26) we see that the effective coupling scale of the three-gluon vertex is essentially given by

$$Q_{\rm eff}^2 \sim \frac{Q_{\rm min}^2 Q_{\rm med}^2}{Q_{\rm max}^2} \ . \tag{28}$$

Next, we define the scale correction factor K through the relation

$$\mathrm{L}(-p^2/\Lambda_3^2, -q^2/\Lambda_3^2, -r^2/\Lambda_3^2) \equiv \ln\left(K^2 \frac{Q_{\rm min}^2 Q_{\rm med}^2}{Q_{\rm max}^2 \tilde{\Lambda}_3^2}\right) \ . \tag{29}$$

In Fig. 5 we plot $K(x,y)$ as function of the ratios $x = Q_1^2/Q_{\rm max}^2$ and $y = Q_2^2/Q_{\rm max}^2$, where $Q_{\rm max}^2$ is the maximum scale among $-p^2, -q^2, -r^2$, and Q_1^2 and Q_2^2 are the two remaining scales. From the figure we see that the actual coupling scale is in general within a factor $0.2096 \sim 1/5$ of the simple expression given in Eq. (28).

Note that formula (28) indicates that the coupling scale in general will be small when there is one scale disproportionately larger than the other two scales. Consider for instance the jet-production process indicated in Fig. 6. Formula (28) implies that, for fixed gluon-jet invariant masses M_1^2 and M_2^2, the three-gluon vertex becomes non-perturbative at high values of Q^2. That is, the three-gluon vertex is perturbative only if the invariant-masses of the gluon jets are allowed to increase simultaneously with Q^2.

To conclude, we make the following observations.

1. The large values of Λ_3 and $\tilde{\Lambda}_3$ with respect to $\Lambda_{\overline{\rm MS}}$ (see Eqs. (25) and (27)) indicate that in general one should choose a smaller-than-expected scale for the coupling constant $\alpha_{\overline{\rm MS}}(\mu^2)$ in four-jet physics, where the three-gluon vertex plays an essential role. This, together with the fact that the effective scale for the three-gluon vertex as given by (28) is always smaller than the smallest scale, might help to explain the surprising smallness of the effective-coupling observed in four-jet cross-sections [9]

$$\mu^2_{\rm exp} \sim 0.002 \; s \; , \qquad (30)$$

with s the squared total center-of-mass energy.

2. The running of the three-gluon coupling can be studies by the detailed measurement of four-jet events in e^+e^- annihilation [10]. In particular, these measurements allow us to test the validity of the functional dependence of effective-coupling scale as given in Eqs. (28) and (29).

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FIGURE CAPTIONS

1) (a) Two-quark scattering process via one-gluon exchange. (b) The non-Abelian part of three-gluon scattering process. The coupling scale at the vertex d lacks a prescription.

2) Diagrams involved in the gauge-invariant gluon propagator calculation to one-loop order. The definition of the pinched diagrams are given in Ref. [4].

3) The definition of the various momenta, Lorentz indices and color indices involved in the one-loop three-gluon vertex calculation.

4) Equal-coupling surfaces for the effective three-gluon coupling constant in the completely spacelike region.

5) Scale-correction factor function, as defined in text. Note that this function takes values between $K_{\rm min} = 0.2096$ and $K_{\rm max} = 1$.

6) A four-jet process involving a three-gluon vertex. The three-gluon coupling is expected to be large at large values of Q^2 and fixed values of invariant masses M_1^2 and M_2^2.

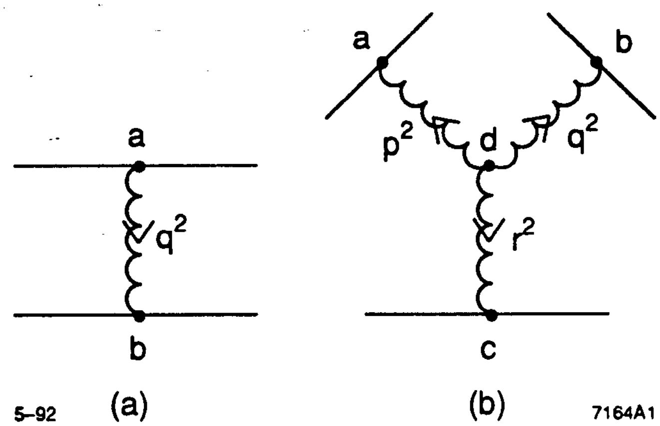

Fig. 1

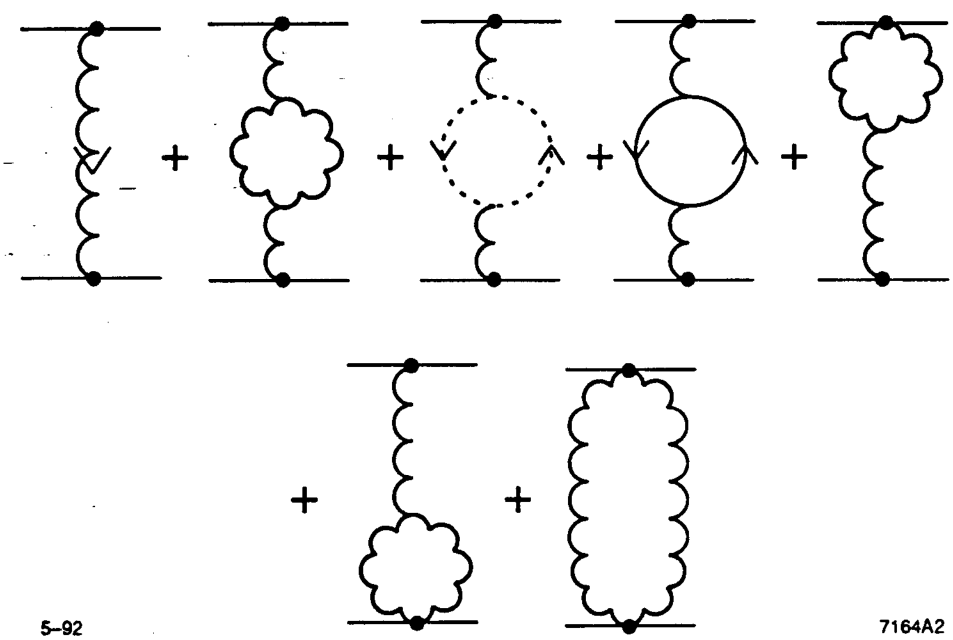

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Fig. 2

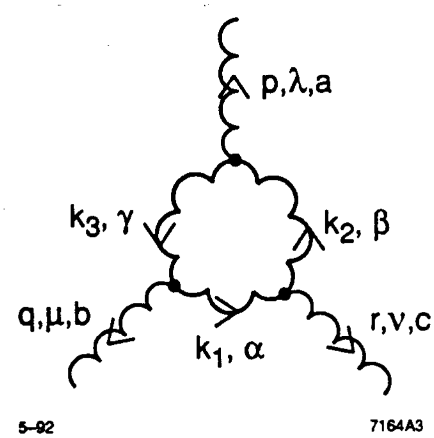
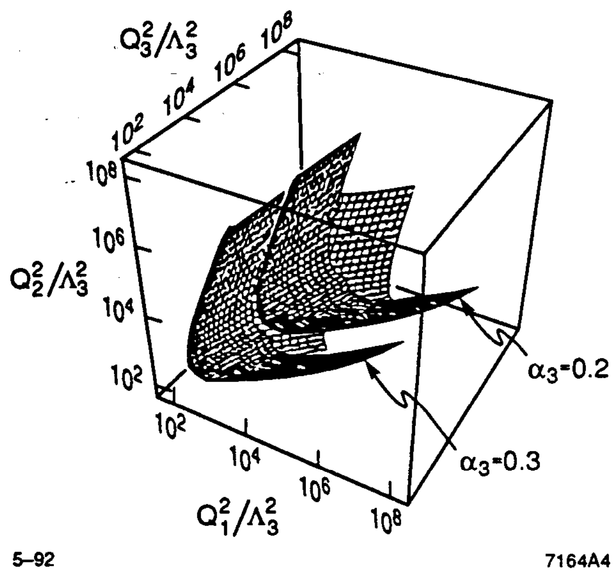


Fig. 3



Q_3^2/Λ_3^2
10^2 10^4 10^6 10^8
10^8
10^6
Q_2^2/Λ_3^2
10^4
10^2
10^2
10^4
10^6
10^8
Q_1^2/Λ_3^2
α_3=0.2
α_3=0.3
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Fig. 4

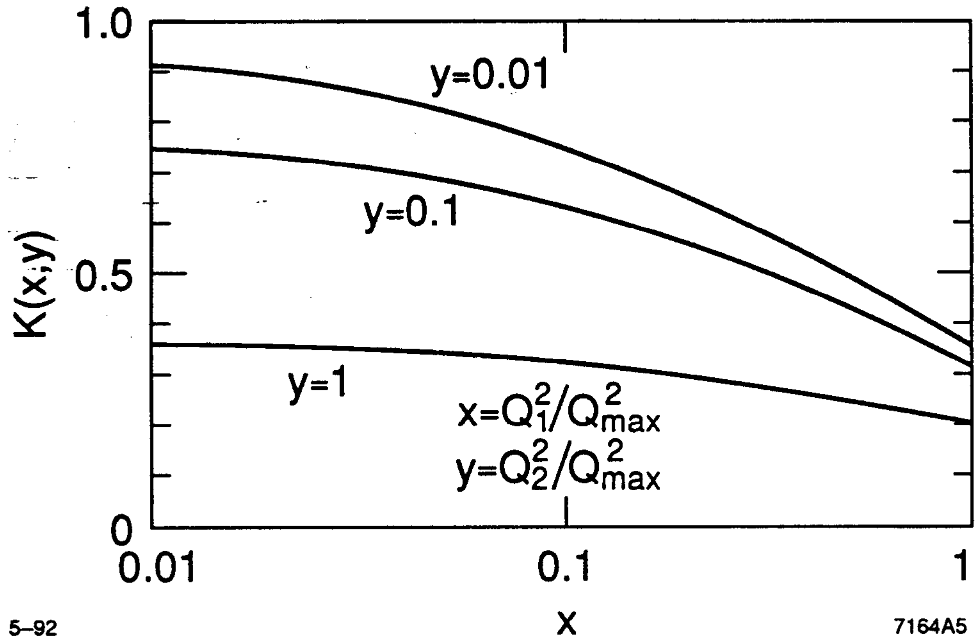
1.0
0.5
0
K(x;y)
y=0.01
y=0.1
y=1
x=Q_1^2/Q_{max}^2
y=Q_2^2/Q_{max}^2
0.01
0.1
1
x
5–92
7164A5

Fig. 5

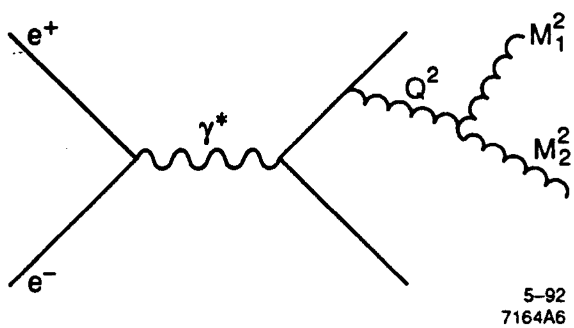

Fig. 6