Strong Decays of Excited Heavy Mesons In Chiral Perturbation Theory*

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Abstract

We construct an effective Lagrangian describing the interaction of soft pions and kaons with mesons containing a heavy quark and light degrees of freedom in an orbital p wave. The formalism is easily extended to heavy mesons and baryons in arbitrary excited states. We calculate the leading contributions to the strong decays $D_2^* \to D\pi$, $D_2^* \to D^*\pi$ and $D_1 \to D^*\pi$. We confirm the relations between the rates previously obtained by Isgur and Wise using heavy quark symmetry, and find that the absolute widths are consistent with naïve power counting. We also estimate the branching ratios for the two pion decays $D_2^* \to D^*\pi\pi$, $D_1 \to D^*\pi\pi$ and $D_1 \to D\pi\pi$, which are dominated by pole graphs. Our predictions depend on the masses and widths of the as yet unseen scalar-pseudovector p-wave doublet. Heavy quark spin symmetry predicts $\Gamma(D_2^* \to D^*\pi\pi) : \Gamma(D_1 \to D^*\pi\pi) : \Gamma(D_1 \to D\pi\pi) = 3 : 1 : 2$, but this relation is badly violated in practice because $1/M$ effects arising purely from kinematics are large.

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1. Introduction

The interactions of the octet of pseudogoldstone bosons with hadrons containing a single heavy quark are constrained by two independent symmetries: spontaneously broken chiral $SU(3)_L \times SU(3)_R$ and heavy quark spin-flavour $SU(2N_h)$ [1]. One may implement both of these symmetries by constructing a "heavy-light" chiral lagrangian, in which one performs a simultaneous expansion in the momenta of the pseudogoldstone bosons and the inverse masses of the heavy hadrons. Such a lagrangian has been described in refs. [2]–[5] for heavy hadrons with the light degrees of freedom in the ground state. We begin by briefly reviewing this construction.

The lagrangian is written in terms of the usual exponentiated matrix of pseudogoldstone bosons,

$$\xi = \exp\left(\mathrm{i}\mathcal{M}/f_\pi\right), \qquad \Sigma \equiv \xi^2 \tag{1.1}$$

where

$$\mathcal{M} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi_0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} \tag{1.2}$$

and $f_\pi \approx 135\,\mathrm{MeV}$. Under chiral $SU(3)_L \times SU(3)_R$, the field ξ transforms as $\xi \to L\xi U^\dagger = U\xi R^\dagger$, where U is a matrix which depends on the fields $\mathcal{M}$, while Σ transforms more simply as $\Sigma \to L\Sigma R^\dagger$. The ground state heavy mesons consist of a doublet under heavy quark spin symmetry, containing the pseudoscalar meson P and the vector meson P^*; these also transform under the unbroken flavour $SU(3)$ as an antitriplet. (We take our heavy mesons always to contain a heavy quark rather than an antiquark.) We represent these fields in the usual way by a 4×4 Dirac matrix,

$$H_a = \frac{(1+\not{v})}{2\sqrt{2}}\left[P_a^{*\mu}\gamma_\mu - P_a\gamma_5\right]. \tag{1.3}$$

We have absorbed factors of $\sqrt{2M_P}$ and $\sqrt{2M_P^*}$ into the definition of the heavy fields, so they have mass dimension 3/2 (our normalisation differs slightly from that of ref. [2]; our fields are normalised to 1, not to 2). To recover the correct relativistic normalisation, we multiply amplitudes by $\sqrt{2M}$ for each external heavy meson.

The pseudogoldstone bosons couple to the heavy fields through the covariant derivative

$$D^\mu_{ab} \equiv \delta_{ab}\partial^\mu + V^\mu_{ab} = \delta_{ab}\partial^\mu + \tfrac{1}{2}\left(\xi^\dagger\partial^\mu\xi + \xi\partial^\mu\xi^\dagger\right)_{ab} \tag{1.4}$$

and the axial vector field

$$A^{\mu}_{ab} = \tfrac{\mathrm{i}}{2}\left(\xi^{\dagger}\partial^{\mu}\xi - \xi\partial^{\mu}\xi^{\dagger}\right)_{ab} . \tag{1.5}$$

Under $SU(3)_L \times SU(3)_R$,

$$H_a \to U_{ab}H_b\,, \qquad (D_\mu H)_a \to U_{ab}(D_\mu H)_b\,, \qquad A^{\mu}_{ab} \to U_{ac}A^{\mu}_{cd}U^{\dagger}_{db}\,. \tag{1.6}$$

At leading order in the momentum expansion, the lagrangian is written in terms of these fields as

$$\begin{aligned}\mathcal{L} = \frac{f_\pi^2}{8}\partial^{\mu}\Sigma_{ab}\partial_{\mu}\Sigma^{\dagger}_{ba} - \mathrm{Tr}\left[\overline{H}_a \mathrm{i}v\cdot D_{ba}H_b\right] + g\,\mathrm{Tr}\left[\overline{H}_a H_b \not{A}_{ba}\gamma_5\right] \\ + \lambda_0\left[m_{\mathrm{q}}\Sigma + m_{\mathrm{q}}\Sigma^{\dagger}\right]_{aa} + \cdots\,,\end{aligned} \tag{1.7}$$

where the traces are over Dirac indices and we keep the $SU(3)$ flavour indices a, b explicit. The ellipses denote terms higher order in the derivative expansion, terms suppressed by powers of $1/M$, and additional explicit $SU(3)_L \times SU(3)_R$ violating terms proportional to the quark mass matrix

$$m_{\mathrm{q}} = \begin{pmatrix} m_{\mathrm{u}} & 0 & 0 \\ 0 & m_{\mathrm{d}} & 0 \\ 0 & 0 & m_{\mathrm{s}} \end{pmatrix}. \tag{1.8}$$

2. Excited States and Reparameterisation Invariance

We would now like to consider the form of such a lagrangian for heavy mesons in an excited state. In the limit that the heavy quark mass M is taken to infinity the light degrees of freedom carry a well-defined angular momentum, flavour and spectrum of excitations. In general, the light degrees of freedom in a heavy meson are in a state with half-integral angular momentum j and parity P, corresponding to two degenerate heavy mesons of spin $j \pm \frac{1}{2}$ and parity $-P$ (since quarks and antiquarks have opposite parity). We may describe both states by a more complicated analogue of the H_a matrix (1.3), the traceless, symmetric Lorentz tensor

$$H_a^{\mu_1\cdots\mu_k}\,, \quad k = j - 1/2\,, \tag{2.1}$$

satisfying the conditions

$$v_{\mu_1}H_a^{\mu_1\cdots\mu_k} = \gamma_{\mu_1}H_a^{\mu_1\cdots\mu_k} = 0\,. \tag{2.2}$$

Under Lorentz transformations,

$$H_a^{\mu_1\cdots\mu_k} \to D(\Lambda)\Lambda^{\mu_1}_{\nu_1}\ldots\Lambda^{\mu_k}_{\nu_k}H_a^{\nu_1\cdots\nu_k}D^{\dagger}(\Lambda)\,, \tag{2.3}$$

where $D(\Lambda)$ is an element of the 4×4 matrix representation of the Lorentz group, while under spatial rotations $\widetilde{\Lambda}$ of the heavy quark,

$$H_a^{\mu_1\cdots\mu_k} \to D(\widetilde{\Lambda}) H_a^{\mu_1\cdots\mu_k} \,. \tag{2.4}$$

The general form for $H_a^{\mu_1\cdots\mu_k}$ has been derived in ref. [6]; for light degrees of freedom with parity $(-1)^{j-1/2}$ we have the doublet of states* $Q^*_{j+1/2}$ and $Q_{j-1/2}$,

$$\begin{aligned} H_a^{\mu_1\cdots\mu_k} =& \frac{(1+\not v)}{2\sqrt{2}} \Big\{ (Q^*_{j+1/2})_a^{\mu_1\cdots\mu_{k+1}} \gamma_{\mu_{k+1}} - \sqrt{\tfrac{2k+1}{k+1}} \gamma_5 (Q_{j-1/2})_a^{\nu_1\cdots\nu_k} \\ & \Big[g^{\mu_1}_{\nu_1} \cdots g^{\mu_k}_{\nu_k} - \tfrac{1}{2k+1} \gamma_{\nu_1} \left(\gamma^{\mu_1} - v^{\mu_1}\right) g^{\mu_2}_{\nu_2} \cdots g^{\mu_k}_{\nu_k} - \cdots \\ & - \tfrac{1}{2k+1} g^{\mu_1}_{\nu_1} \cdots g^{\mu_{k-1}}_{\nu_{k-1}} \gamma_{\nu_k} \left(\gamma^{\mu_k} - v^{\mu_k}\right) \Big] \Big\} \,, \end{aligned} \tag{2.5}$$

while for parity $(-1)^{j+1/2}$ we have $Q_{j+1/2}$ and $Q^*_{j-1/2}$,

$$\begin{aligned} H_a^{\mu_1\cdots\mu_k} =& \frac{(1+\not v)}{2\sqrt{2}} \Big\{ (Q_{j+1/2})_a^{\mu_1\cdots\mu_{k+1}} \gamma_5 \gamma_{\mu_{k+1}} - \sqrt{\tfrac{2k+1}{k+1}} (Q^*_{j-1/2})_a^{\nu_1\cdots\nu_k} \\ & \Big[g^{\mu_1}_{\nu_1} \cdots g^{\mu_k}_{\nu_k} - \tfrac{1}{2k+1} \gamma_{\nu_1} \left(\gamma^{\mu_1} + v^{\mu_1}\right) g^{\mu_2}_{\nu_2} \cdots g^{\mu_k}_{\nu_k} - \cdots \\ & - \tfrac{1}{2k+1} g^{\mu_1}_{\nu_1} \cdots g^{\mu_{k-1}}_{\nu_{k-1}} \gamma_{\nu_k} \left(\gamma^{\mu_k} + v^{\mu_k}\right) \Big] \Big\} \,. \end{aligned} \tag{2.6}$$

For simplicity, we will restrict ourselves in the rest of this paper to the lowest lying p-wave excitations; we have included the complete expressions (2.5) and (2.6) to make it clear that the extension of this formalism to arbitrary excited heavy mesons is cumbersome but straightforward. (Using the formalism of ref. [6], one could include excited heavy baryons as well.) In the quark model, these p-wave states correspond to light degrees of freedom with orbital angular momentum $\ell = 1$, and hence with total spin $j=\frac{1}{2}$ or $j=\frac{3}{2}$. For the D system, these are the (as-yet unobserved) $J^P = 0^+, 1^+$ doublet D_0^* and $D_1{}'$,

$$S_a = \frac{(1+\not v)}{2\sqrt{2}} \left(D_1{}'^{\mu} \gamma_\mu \gamma_5 - D_0^* \right) \,, \tag{2.7}$$

and the $J^P = 1^+, 2^+$ doublet D_1 and D_2^*,

$$T_a^\mu = \frac{(1+\not v)}{2\sqrt{2}} \left\{ D_2^{*\mu\nu} \gamma_\nu - \sqrt{\tfrac{3}{2}} D_1{}^\nu \gamma_5 \left[g^\mu_\nu - \tfrac{1}{3} \gamma_\nu \left(\gamma^\mu - v^\mu \right) \right] \right\} \tag{2.8}$$

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* We use here the particle data book convention of labeling states with a subscript for their spin, and adding a superscript "*" if the spin-parity is in the series $J^P = 0^+, 1^-, 2^+, \ldots$.

(we add a prime to distinguish the two pseudovector states). We identify the neutral members of this multiplet as the $D_1(2420)^0$ and the $D_2^*(2460)^0$ [7]. Including these states along with the ground state mesons, the kinetic piece of the chiral Lagrangian is given by

$$\begin{aligned}\mathcal{L}_{\rm kin} =& -\operatorname{Tr}\left[\overline{H}_a \mathrm{i} v\cdot D_{ba} H_b\right] + \operatorname{Tr}\left[\overline{S}_a \left(\mathrm{i}v\cdot D_{ba} - \delta m_S \delta_{ba}\right) S_b\right] \\ &+ \operatorname{Tr}\left[\overline{T}^\mu_a \left(\mathrm{i}v\cdot D_{ba} - \delta m_T \delta_{ba}\right) T_{\mu b}\right] ,\end{aligned} \tag{2.9}$$

where the residual masses $\delta m_S = M_{D_0^*} - M_D = M_{D_1{}'} - M_D$ and $\delta m_T = M_{D_1} - M_D = M_{D_2^*} - M_D$ are defined in the heavy quark limit, where the doublets are degenerate [8].

In general, one must include all terms in a chiral Lagrangian which are not forbidden by symmetries of the effective theory. Hence one might be tempted to write down a mixing term of the form

$$\operatorname{Tr}\left[\overline{H}_a \left(\mathrm{i} D_\mu T^\mu\right)_a\right] \tag{2.10}$$

(recall that in the effective theory, $D_\mu T^\mu \neq 0$; the transversality condition is $v_\mu T^\mu = 0$). However, such a term is forbidden because it is not invariant under redefinitions of the velocity v^μ [9]. Recall that the definition of the velocity v^μ of a heavy field of mass M is somewhat arbitrary, in that we could equally well choose a slightly different velocity $v'^\mu = v^\mu - q^\mu/M$, where $q\cdot v = q^2/2M$ to ensure $v'^2 = 1$, and shift the residual momentum by q^μ:

$$P^\mu = M v^\mu + k^\mu = M v'^\mu + k^\mu + q^\mu. \tag{2.11}$$

For a heavy scalar ϕ or vector field A^μ, this corresponds to the transformation

$$\begin{aligned} v^\mu &\to v^\mu - \tfrac{1}{M} q^\mu , \\ \phi &\to e^{iq\cdot x}\phi , \\ A^\mu &\to \left[g^{\mu\nu} + \tfrac{1}{M} v^\mu q^\nu + \mathcal{O}\left(\tfrac{1}{M^2}\right)\right] e^{iq\cdot x} A_\nu . \end{aligned} \tag{2.12}$$

Under the shift (2.12),

$$\operatorname{Tr}\left[\overline{H}_a \left(\mathrm{i} D_\mu T^\mu\right)_a\right] \to \operatorname{Tr}\left[\overline{H}_a \left((\mathrm{i} D_\mu - q_\mu) T^\mu\right)_a\right] + \mathcal{O}(1/M) , \tag{2.13}$$

so the term (2.10) is forbidden.

The single pion transitions between states in the same heavy spin doublet are given by terms in the effective lagrangian analogous to the g coupling in eq. (1.7):

$$\mathcal{L}_{1\pi} = g \operatorname{Tr}\left[\overline{H}_a H_b \not{A}_{ba}\gamma_5\right] + g' \operatorname{Tr}\left[\overline{S}_a S_b \not{A}_{ba}\gamma_5\right] + g'' \operatorname{Tr}\left[\overline{T}^\mu_a T_{\mu b} \not{A}_{ba}\gamma_5\right] , \tag{2.14}$$

while the single pion transitions between doublets, again to lowest order in the derivative expansion, are given by

$$\mathcal{L}_s = f' \operatorname{Tr}\left[\overline{S}_a T^{\mu}_b A_{\mu ba}\gamma_5\right] + f'' \operatorname{Tr}\left[\overline{H}_a S_b \not{A}_{ba}\gamma_5\right] + \text{h.c.}\,. \tag{2.15}$$

These correspond to s-wave transitions; however the analogous s-wave transitions $T^{\mu} \to H\pi$ are forbidden by heavy quark spin symmetry [10], and indeed the term $\operatorname{Tr}\left[\overline{H}_a T^{\mu}_b A_{\mu ba}\gamma_5\right]$ vanishes. These decays must then proceed through d-waves, which are suppressed by one derivative in the chiral lagrangian:

$$\mathcal{L}_d = \frac{h_1}{\Lambda_\chi} \operatorname{Tr}\left[\overline{H}_a T^{\mu}_b \left(\mathrm{i}D_\mu \not{A}\right)_{ba}\gamma_5\right] + \frac{h_2}{\Lambda_\chi} \operatorname{Tr}\left[\overline{H}_a T^{\mu}_b \left(\mathrm{i}\not{D} A_\mu\right)_{ba}\gamma_5\right] + \text{h.c.}\,, \tag{2.16}$$

where Λ_χ is some momentum scale characterising the convergence of the derivative expansion. From previous experience with chiral Lagrangians, we expect $\Lambda_\chi \simeq 1$ GeV [11], and so we expect the T^{μ} states to be much narrower than the S states, simply from power counting. Note that the symmetry (2.12) also forbids couplings such as $\operatorname{Tr}\left[(\mathrm{i}D_\mu \overline{H})_a T^{\mu}_b \not{A}_{ba}\gamma_5\right]$, with derivatives acting on the heavy fields, at this order in $1/M$.

Following the authors of ref. [4], who obtained an estimate of g, we may estimate the couplings g', g'' and f' in the nonrelativistic quark model by evaluating matrix elements of the axial current between the appropriate states. This requires the assumption that the pseudogoldstone bosons couple only to the spin of the brown muck, and not to the orbital angular momentum. We note that the nonrelativistic quark model may not provide a very appropriate description of these excited states, as the mass splitting from the ground state is of the order of several hundred MeV, comparable to the mass of the constituent light quark. Hence we should probably regard our estimates of the couplings primarily as an indication of what are likely to be reasonable values for these parameters.

In the nonrelativistic quark model, the S_a and T^{μ}_a mesons have the light degrees of freedom in the same excited radial wavefunction, and we may decompose physical states into the eigenstates $|s_H, m_\ell, s_\ell\rangle$ of the z components of heavy quark spin s_H, angular momentum of the light degrees of freedom m_ℓ and light quark spin s_ℓ. In particular, we decompose the $m=0$ states of the D_2^* and the $D_1{}'$ as

$$\begin{aligned} |D_2^*(m=0)\rangle =& \sqrt{\tfrac{1}{3}}\left|\tfrac{1}{2}, 0, -\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{6}}\left|\tfrac{1}{2}, -1, \tfrac{1}{2}\right\rangle \\ &+ \sqrt{\tfrac{1}{6}}\left|-\tfrac{1}{2}, 1, -\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\left|-\tfrac{1}{2}, 0, \tfrac{1}{2}\right\rangle \end{aligned} \tag{2.17}$$

and

$$\left|D_1{}'(m=0)\right\rangle = \sqrt{\tfrac{1}{6}}\left|\tfrac{1}{2},0,-\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{1}{3}}\left|\tfrac{1}{2},-1,\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\left|-\tfrac{1}{2},1,-\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{1}{6}}\left|-\tfrac{1}{2},0,\tfrac{1}{2}\right\rangle . \tag{2.18}$$

Consider the matrix element of the axial current $(j_5^i)^\mu$ between these states. In the non-relativistic quark model,

$$(j_5^{1+\mathrm{i}2})^3 = -g_A u^\dagger \sigma^3 d, \tag{2.19}$$

where we take $g_A = 0.75$ as suggested by the chiral quark model [12] (this reproduces the correct value of g_A in the nucleon). Thus we obtain

$$\left\langle D_2^*(m=0)\right| \int d^3x\,(j_5^{1+\mathrm{i}2})^3 \left|D_1{}'(m=0)\right\rangle = \frac{2\sqrt{2}}{3} g_A\,. \tag{2.20}$$

In the chiral lagrangian, the f' coupling in (2.15) gives a contribution to the axial current of

$$(j_5^i)^\mu = -f' \operatorname{Tr}\left[\overline{S}_a T_b^\mu \gamma_5 T_{ba}^i\right] + \dots\,. \tag{2.21}$$

In the limit of zero momentum transfer, this term dominates the matrix element (2.20) and we find

$$\left\langle D_2^*(m=0)\right| \int d^3x\,(j_5^{1+\mathrm{i}2})^3 \left|D_1{}'(m=0)\right\rangle = -f'\epsilon_\mu^*\eta^{\mu 3} = -\sqrt{\frac{2}{3}} f'\,, \tag{2.22}$$

where ϵ^μ and $\eta^{\mu\nu}$ are respectively the $m=0$ polarisation states of the $D_1{}'$ and D_2^*. Equating the expressions (2.20) and (2.22), we find

$$|f'| = \frac{2}{\sqrt{3}} g_A = 0.87\,. \tag{2.23}$$

The phase of f' is not determined by this procedure; however this will not matter as only the modulus $|f'|^2$ will appear in the widths which we will compute. Similarly, we may obtain estimates of the transition rates within multiplets,

$$g = g_A\,, \qquad g' = \tfrac{1}{3} g_A\,, \qquad g'' = g_A\,, \tag{2.24}$$

where the phases may in this case be fixed by the heavy quark symmetry relation $\mathbf{S}_h^z\,|D^*(m=0)\rangle = \frac{1}{2}\,|D\rangle$, and analogously for the excited doublets. However, the coupling constants g' and g'' are not particularly useful, as the corresponding single pion decays are most probably kinematically forbidden [13].

3. Single Pion Decays

There are four possible single pion transitions between two heavy spin doublets; relations between the amplitudes follow from the heavy quark spin symmetry. These have already been worked out explicitly for D_2^* and D_1 decays [10][14], and those results follow immediately from our formalism. In addition, with the chiral lagrangian we may easily correct for one class of $1/M$ corrections in the widths by using the true particle masses in the phase space integrals. Since the rate for d-wave decays is proportional to the fifth power of the pion momentum, this is likely to be the leading $1/M$ correction. Explicitly, we find

$$\begin{aligned}
\Gamma(D_2^{0*}\to D^+\pi^-) &= \frac{1}{15\pi}\left(\frac{M_D}{M_{D_2^*}}\right)\frac{h^2}{\Lambda_\chi^2}\frac{|\vec{p}_\pi|^5}{f_\pi^2} = 5.51\times10^7\,\frac{h^2}{\Lambda_\chi^2}\,,\\
\Gamma(D_2^{0*}\to D^{*+}\pi^-) &= \frac{1}{10\pi}\left(\frac{M_{D^*}}{M_{D_2^*}}\right)\frac{h^2}{\Lambda_\chi^2}\frac{|\vec{p}_\pi|^5}{f_\pi^2} = 2.03\times10^7\,\frac{h^2}{\Lambda_\chi^2}\,,\\
\Gamma(D_1^0\to D^{*+}\pi^-) &= \frac{1}{6\pi}\left(\frac{M_{D^*}}{M_{D_1}}\right)\frac{h^2}{\Lambda_\chi^2}\frac{|\vec{p}_\pi|^5}{f_\pi^2} = 2.05\times10^7\,\frac{h^2}{\Lambda_\chi^2}\,,
\end{aligned}\tag{3.1}$$

where $\vec{p}_\pi$ is the momentum of the pion emitted in the decay, and $h\equiv|h_1+h_2|$. The full one pion width are $3/2$ times these because of the $D^0\pi^0$ channel. From eq. (3.1) we reproduce the result of Isgur and Wise,

$$\frac{\Gamma(D_2^{0*}\to D^+\pi^-)}{\Gamma(D_2^{0*}\to D^{*+}\pi^-)} = 2.7\,,\tag{3.2}$$

which compares very well with the experimental ratio 2.4 ± 0.7 [7]. We may use these results to gain some confidence in the validity of our derivative expansion. Assuming the total D_2^* width of $19\pm7\,\mathrm{MeV}$ to be saturated by the one pion mode (as we will show in the next section, the two pion width is sufficiently small that this is a reasonable assumption), we find

$$\frac{h^2}{\Lambda_\chi^2}\approx\frac{1}{(2\ \mathrm{GeV})^2}\,,\tag{3.3}$$

which is consistent with our naïve estimate. This also gives us a prediction for the D_1^0 single pion width,

$$\Gamma(D_1^0\to D^{*+}\pi^-+D^{*0}\pi^0)\approx7\,\mathrm{MeV},\tag{3.4}$$

which is significantly smaller than the measured total width of $20^{+9}_{-5}\,\mathrm{MeV}$. As has been suggested [10], this is undoubtedly due to mixing (at order $1/M$) of the D_1 with the substantially broader ${D_1}'$.

The D_0^* and ${D_1}'$ decay through s-wave pion emission and consequently are very broad; from eq. (2.15) we obtain

$$\begin{aligned}
\Gamma(D_0^* \to D\pi^-) =& \frac{|f''|^2}{2\pi f_\pi^2}\left(\frac{M_D}{M_{D_0^*}}\right)(M_{D_0^*}-M_D)^2\left[(M_{D_0^*}-M_D)^2-m_\pi^2\right]^{1/2} \\
\Gamma({D_1}' \to D^*\pi^-) =& \frac{|f''|^2}{2\pi f_\pi^2}\left(\frac{M_{D^*}}{M_{{D_1}'}}\right)(M_{{D_1}'}-M_{D^*})^2 \\
& \times\left[(M_{{D_1}'}-M_{D^*})^2-m_\pi^2\right]^{1/2}.
\end{aligned} \tag{3.5}$$

Since these states have not been observed, we must use quark model estimates for their masses. Taking $M_{D_0^*} = M_{{D_1}'} = 2.4$ GeV [13], we have

$$\begin{aligned}
\Gamma(D_0^* \to D\pi^-) &= |f''|^2[980\,\mathrm{MeV}] \\
\Gamma({D_1}' \to D^*\pi^-) &= |f''|^2[400\,\mathrm{MeV}].
\end{aligned} \tag{3.6}$$

Again, the full one pion widths are $3/2$ times these because of the π^0 channel. These widths are very sensitive to the value used for the mass of the states; for $M_{D_0^*} = M_{{D_1}'} = 2.3\,\mathrm{GeV}$ we find charged pion widths of $|f''|^2[540\,\mathrm{MeV}]$ and $|f''|^2[160\,\mathrm{MeV}]$, respectively.

4. Two Pion Decays

Like the single pion $T^\mu \to H\pi$ decays, the contact terms mediating $T^\mu \to H\pi\pi$, such as $\mathrm{Tr}\left[\overline{H}_a T_b^\mu A_{\mu bc} \not{A}_{ca}\right]$, are dimension five and are suppressed by one power of Λ_χ in the derivative expansion. We therefore expect that these decays will be dominated by pole graphs in which there is an intermediate ${D_1}'$ or D_0^* which is close to its mass shell. This raises the interesting possibility that the two pion widths could be comparable to the single pion widths (as is observed, for example, in the decay $K_2^*(1430) \to K^*(892)$+pions). The two pion width is given by

$$\Gamma_{2\pi} = \int \frac{1}{(2\pi)^3}\frac{1}{8M'}|\mathcal{A}(E_1,E_2)|^2 dE_1 dE_2\,, \tag{4.1}$$

where the amplitude $\mathcal{A}$ is a function of the energies E_1 and E_2 of the outgoing pions, and the masses $M' = (M_{D_2^*}, M_{D_1})$ and $M = (M_D, M_{D^*})$ are those respectively of the initial and final heavy mesons. Kinematics restricts E_2 to the region

$$\overline{E}_2(E_1) - \frac{g(E_1)}{M} < E_2 < \overline{E}_2(E_1) + \frac{g(E_1)}{M}\,, \tag{4.2}$$

where

$$\begin{aligned}\overline{E}_2(E_1) &\equiv M' - M - E_1\,,\\ g(E_1) &\equiv \sqrt{(E_1^2 - m_\pi^2)[(M' - M - E_1)^2 - m_\pi^2]}\,.\end{aligned} \tag{4.3}$$

Hence the amplitude can be expressed approximately as a function only of E_1,

$$\mathcal{A}(E_1, E_2) \simeq \mathcal{A}\left(E_1, \overline{E}_2(E_1)\right) \tag{4.4}$$

up to corrections of order $1/M$. The integral over E_2 then just brings in a factor of the width of the integral, $2g(E_1)/M$. Because of the poles in the intermediate $D_1{}'$ and D_0^* propagators, their widths must be included in our expressions. The imaginary part of the propagator of this resonance is

$$\Gamma_{\rm int}(p\cdot v) = \frac{|f''|^2}{2\pi f_\pi^2}\frac{M}{M_{\rm res}}(M_{\rm res} - M + p\cdot v)^2\left[(M_{\rm res} - M + p\cdot v)^2 - m_\pi^2\right]^{1/2}\,, \tag{4.5}$$

where p is the residual momentum flowing through the line and $M_{\rm res} = M_{D_0^*}$ or $M_{D_1{}'}$. For $p\cdot v \simeq 0$, this reduces to the usual Breit-Wigner formula. However, because these states are so broad we must include the full momentum dependence of the width in the denominator. It is convenient to extract from the $|\mathcal{A}(E_1, \overline{E}_2)|^2$ the function

$$\begin{aligned}F(E_1) =& \frac{E_1^2[(M_{D_1} - M_D - E_1)^2 - m_\pi^2]}{(E_1 - [M_{D_0^*} - M_D])^2 + \Gamma_{\rm int}(E_1 - [M_{D_0^*} - M_D])^2/4}\\ &+ \frac{(M_{D_1} - M_D - E_1)^2[E_1^2 - m_\pi^2]}{[(M_{D_1} - M_{D_0^*} - E_1)]^2 + \Gamma_{\rm int}(M_{D_1} - M_{D_0^*} - E_1)^2/4}\,,\end{aligned} \tag{4.6}$$

where there are two terms because the pions may be emitted in either order (the cross terms in $|\mathcal{A}|^2$ integrate to zero). Then the partial width is given by

$$\Gamma_{\pi^-\pi^0} = \frac{\alpha}{4(2\pi)^3}\frac{|f'f''|^2}{f_\pi^4}\int F(E_1)g(E_1)dE_1\,, \tag{4.7}$$

where $\alpha = 2/9$ for $D_1^0 \to D^*\pi^-\pi^0$, $\alpha = 4/9$ for $D_1^0 \to D\pi^-\pi^0$ and $\alpha = 2/3$ for $D_2^{0*} \to D^*\pi^-\pi^0$. There are also decays to a neutral charmed hadron and a $\pi^+\pi^-$ pair which occur with the same amplitude (since the final pions are in an antisymmetric wave function, the $I = 0$ $\pi^0\pi^0$ mode is forbidden). Hence the full two pion widths are twice those given in eq. (4.7). Our predictions for the two pion widths depend on several unknown parameters: the masses and widths of the as yet unobserved D_0^* and $D_1{}'$, as well as on the couplings f' and f''. In fig. 1 we plot the total two-pion decay widths for $D_1 \to D^*\pi\pi$, $D_1 \to D\pi\pi$ and

$D_2^* \to D^*\pi\pi$ as functions of $|f''|$, or equivalently as functions of the ${D_1}'$ width, assuming the nonrelativistic quark model prediction (2.23) for f'. Variations in f' just change the overall normalisations, but not the shapes, of the plots. Note that in the heavy quark limit, the widths would satisfy

$$\Gamma(D_2^* \to D^*\pi\pi)_{M\to\infty} : \Gamma(D_1 \to D^*\pi\pi)_{M\to\infty} : \Gamma(D_1 \to D\pi\pi)_{M\to\infty} = 3:1:2\,, \qquad (4.8)$$

but because of the sensitive dependence of eq. (4.7) on the masses, this relation is badly violated. So although in this limit our approach simply reproduces the general results of ref. [10], it has the advantage of being able to take into account the large but calculable $1/M$ symmetry breaking effects which arise purely from kinematics. Since the precise form of the $1/M$ effects depends on the fact that the decay is dominated by pole graphs, its exact form could not be guessed (unlike the $|\vec{p}_\pi|^5$ behaviour for the single pion decays).

One might also think to apply this analysis to the strong transitions of excited strange, charmed mesons. Indeed, at least one such state, the D_{s1}, as already been observed [7]. However, such decays are severely constrained by the combination of phase space and the heavy quark limit. If the outgoing D meson is not strange, there must be a K meson in the final state, but $D_{s1} \to DK$ is prohibited in the heavy quark limit, while $D_1 \to D^*K$ is barely possible kinematically and hence severely suppressed. As for decays to ground state D_s mesons, there is not enough energy to emit the isospin-0 η, while the decay to two pions in an isospin-0 state is induced by our effective lagrangian only at the one loop level. The strong decays of the D_{s1} are thus most likely mediated by operators which are subleading in the mass expansion. Although one might expect the current mass of the strange quark to induce larger $1/M$ corrections in the D_s system than in the D^+ and D^0, this suppression might help explain the relatively narrow width ($< 5\,\mathrm{MeV}$) observed for the D_{s1}.

This formalism could also be applied to semileptonic decays from a B meson to an excited D plus soft pions, as has been done for decays to ground state D mesons [15]. There may also be significant contributions to the decay $B \to D\pi\ell\overline{\nu}$, in which the B first decays semileptonically to an excited D, which then decays strongly to a D or D^* and a pion. We are currently studying these processes.

Finally, we point out that the same heavy-light chiral lagrangian could be used as well to describe the strong transitions of excited bottom mesons. In fact, we would expect $1/M$ corrections to be considerably smaller than in the case of charm. However these states have not yet been produced and studied, and their masses, to which the decay rates are so sensitive, are not known.

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References

[1] N. Isgur and M. B. Wise, Phys. Lett. B232 (1989) 113, Phys. Lett. B237 (1990) 527

[2] M. B. Wise, Caltech preprint CALT-68-1765 (1992)

[3] G. Burdman and J. Donoghue, U. Mass. preprint UMHEP-365 (1992)

[4] T.-M. Yan, H.-Y. Cheng, C.-Y. Cheung, G.-L. Lin, Y. C. Lin and H.-L. Yu, Cornell preprint CLNS 92/1138 (1992)

[5] P. Cho, Harvard preprint HUTP-92/A014 (1992)

[6] A. F. Falk, SLAC preprint SLAC-PUB-5689 (1991), to appear in Nucl. Phys. B

[7] Particle Data Group, Phys. Lett. B232 (1990) 1

[8] A. F. Falk, M. Luke and M. Neubert, SLAC and UCSD preprint SLAC-PUB-5771 and UCSD/PTH 92-09 (1992)

[9] M. Luke and A. V. Manohar, UCSD preprint UCSD/TH 92-15 (1992)

[10] N. Isgur and M. B. Wise, Phys. Rev. Lett. D66 (1991) 1130

[11] See, for example, H. Georgi, *Weak Interactions and Modern Particle Theory*, Benjamin/Cummings Publishing Co., Menlo Park, CA (1984)

[12] A. Manohar and H. Georgi, Nucl. Phys. B234 (1984) 189

[13] S. Godfrey and N. Isgur, Phys. Rev. D32 (1985) 189;
S. Godfrey and R. Kokoski, Phys. Rev. D43 (1991) 1679

[14] J. Rosner, Comm. Nucl. Part. Phys. 16 (1986) 109

[15] C. L. Y. Lee, M. Lu and M. B. Wise, Caltech preprint CALT-68-1771 (1992)

Figure Captions

Fig. 1. Full two pion widths for D_2^* and D_1 as functions of the ${D_1}'$ width, for $M_{D_0^*} = M_{{D_1}'} = 2300\,\mathrm{MeV}$ and $2400\,\mathrm{MeV}$. Note that for $M_{{D_1}'} = 2300\,\mathrm{MeV}$ the D_2^* partial width is nonzero as the ${D_1}'$ width goes to zero, since the ${D_1}'\pi$ intermediate state may be produced on shell. In this limit the D_2^* two pion width approaches the $D_2^* \to {D_1}'\pi$ partial width.

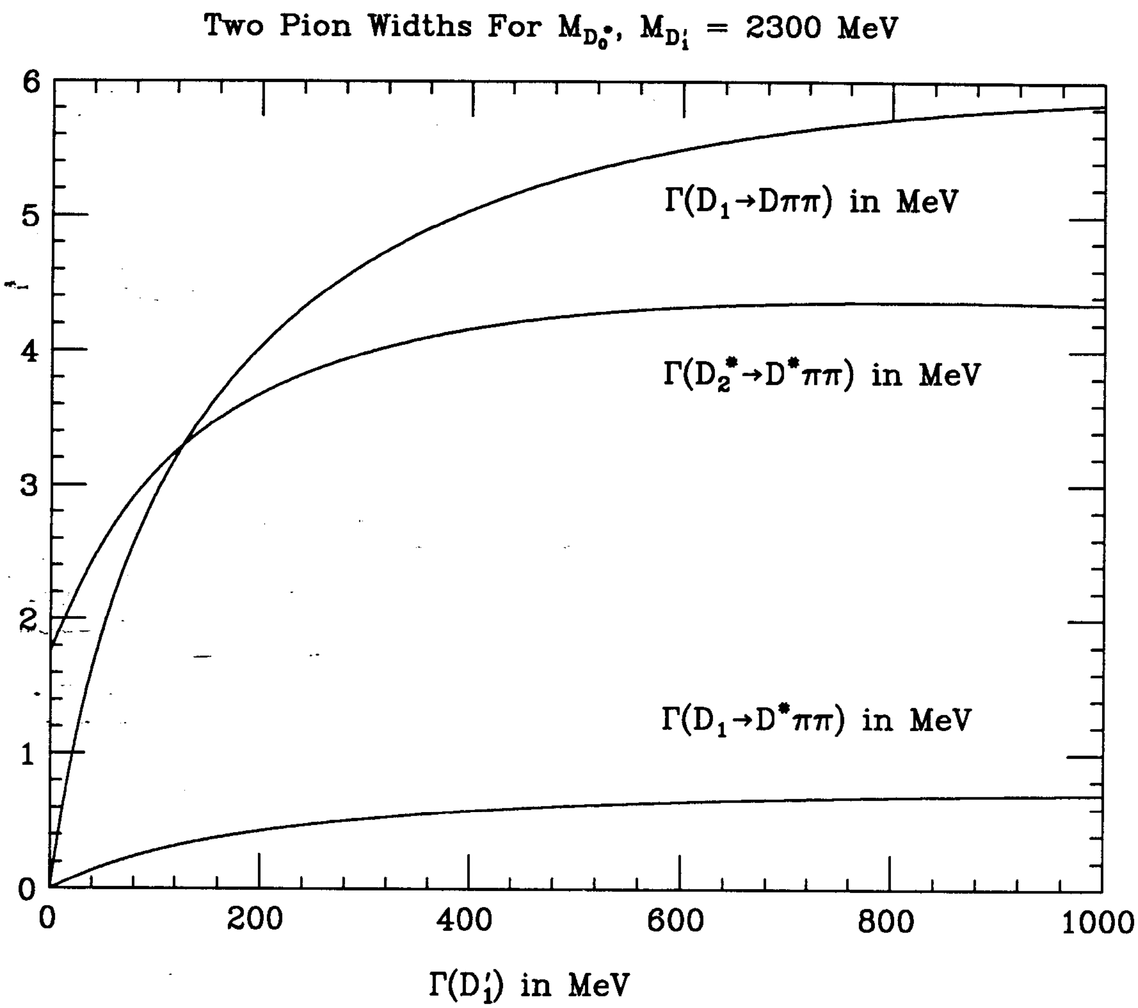
Two Pion Widths For $M_{D_0^*}$, $M_{D_1'}$ = 2300 MeV
6
5
4
3
2
1
0
0
200
400
600
800
1000
$\Gamma(D_1 \to D\pi\pi)$ in MeV
$\Gamma(D_2^* \to D^*\pi\pi)$ in MeV
$\Gamma(D_1 \to D^*\pi\pi)$ in MeV
$\Gamma(D_1')$ in MeV

Fig. 1a

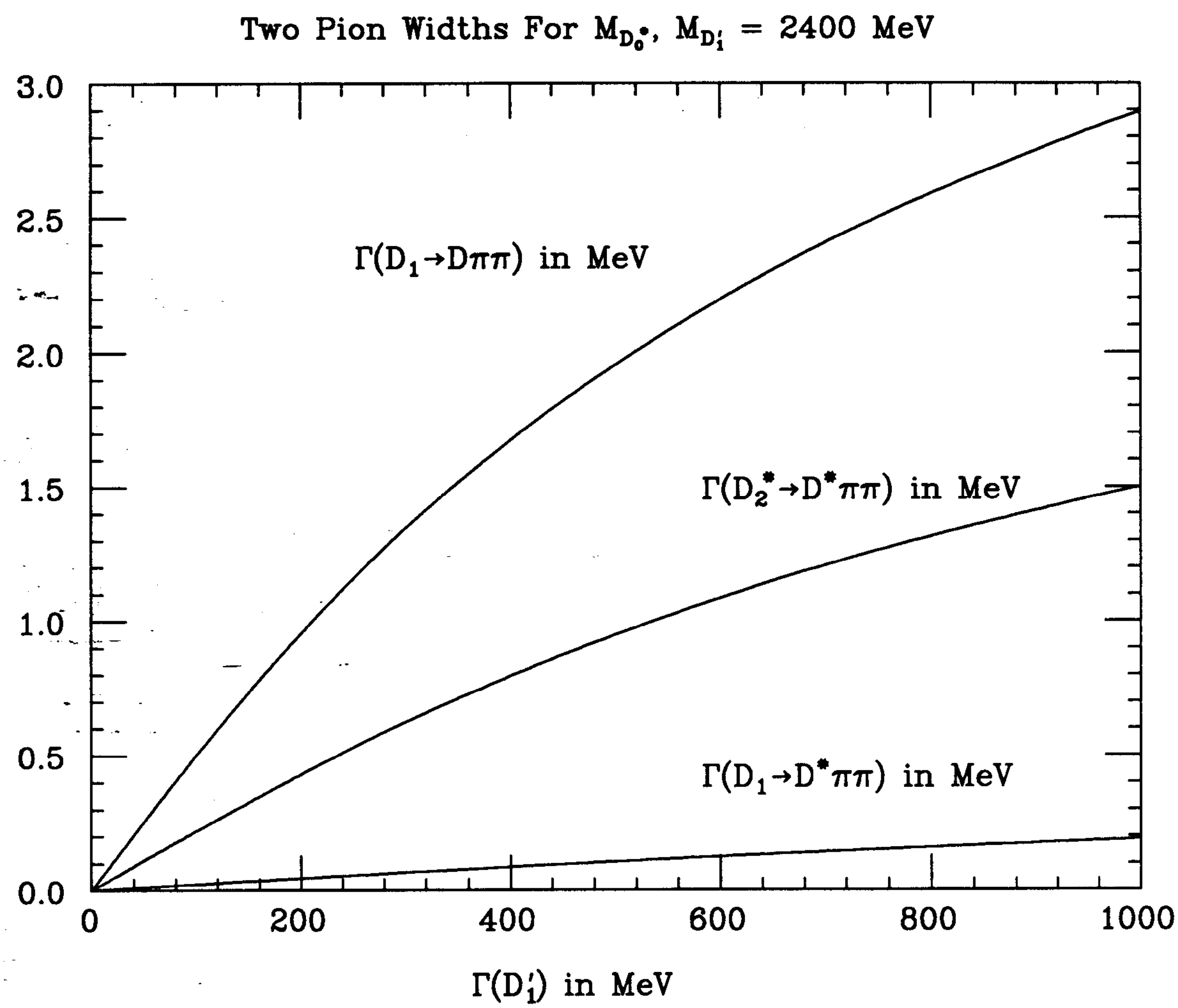
Two Pion Widths For $M_{D_0^*}$, $M_{D_1'}$ = 2400 MeV
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
200
400
600
800
1000
$\Gamma(D_1 \to D\pi\pi)$ in MeV
$\Gamma(D_2^* \to D^*\pi\pi)$ in MeV
$\Gamma(D_1 \to D^*\pi\pi)$ in MeV
$\Gamma(D_1')$ in MeV

Fig. 1b