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LOW ENERGY RING ISSUES*

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ABSTRACT

The Low Energy Ring (LER) in an Asymmetric B Factory is always too large in circumference. The requirements for the LER in a B Factory are developed, including, damping, emittance and power density, and some examples of technical solutions are given. The main emphasis is on ensuring flexibility of operation.

1. INTRODUCTION

The Low Energy Ring (LER) in an Asymmetric B Factory has special problems as the circumference is always too big. The "optimum" circumference for an electron storage ring scales as the square of the beam energy. This means that the LER is 4-6 times too large depending on the energy asymmetry. This paper will cover most of design constraints (and desires) of the LER and also some general ideas about the design.

1.1 General Considerations

a) The bending radius should be small to keep the damping times short.
b) The maximum synchrotron radiation power density should not exceed some engineering value (10 kW/linear meter for PEP-II). (The conditions for calculating the peak power (energy, current and orbit error) must be carefully examined.)
c) The natural emittance of the LER is usually too small and must be increased.
d) The natural damping time of the LER is usually too long and should be reduced.
e) The RF voltage required to obtain the short bunch length could be reduced if the momentum compaction factor could be reduced. However, a small momentum compaction factor is incompatible with large emittance in a simple FODO lattice.
f) The engineering of the vacuum chamber cooling drives the choice of the bending radius not the design of the magnet.

Nice features (not vital)
g) The bend should be placed close to the quadrupole for ease of installation.
h) The bend should be downbeam of the quadrupole to avoid high synchrotron radiation power levels in the quadrupole.

1.2 Beam Energies

PEP-II [1] was designed for nominal colliding beam energies of 9.0 GeV for the HER and 3.1 GeV for the LER. Most of the other studies have chosen a smaller asymmetry, usually 8.0 GeV for the HER and 3.5 GeV for the LER. Both values will be used in the examples but the reasons for preferring one or the other will not be addressed.

There is a further consideration - at what energy should the maximum beam power occur? When the machine is operating, the center-of-mass energy will be varied by changing the beam energies. The energy asymmetry is a free parameter and will be chosen in a way that is not fully predictable now. The assumption will be made that the beam power will not exceed the maximum power at the nominal energy. This implies that the beam current, and therefore the maximum attainable luminosity, will be reduced at energies above the design beam energy

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1.3 Beam Currents

PEP-II was based on the concept that both the vacuum chambers should be engineered for a maximum current of 3 Amps. This criteria should be maintained, so all calculations in this paper will be done for 3 Amps rather than the currents required to produce the nominal luminosity.

All of the arguments in this paper will be valid for other projects if equal maximum beam currents are assumed in the two rings. In some optimizations of the beam parameters, an equal "tune print" is assumed. In this case, the beam current is inversely proportional to the energy - E.I is a constant. This makes the design of the LER Vacuum chamber even harder.

In the calculations, I have used PEP-II parameters. This does not affect any of the conclusions significantly.

2 BEAM-BEAM TUNE SHIFT ξ

It is now common in most of the B Factory designs to adopt a nominal value of the beam-beam tune shift of 0.03 in both planes for both beams. This is about half the value achieved routinely in PEP and about that obtained in CESR. This value was chosen to be conservative but nevertheless we should examine the strategy to be followed if the nominal value cannot be reached and also for the case where higher values can be attained.

Initially, let us examine the case of both beams having equal tune shifts.

2.1 Lower ξ

Since the maximum current will be limited by other factors (installed RF power, multi-bunch instabilities etc.), the luminosity will be directly proportional to ξ if the emittances are scaled according to $1/\xi$. So if a worst case for x is taken as 0.015, the emittance required would be

ε_{x0} = *twice the nominal value.*

2.2 Higher ξ

If the beam-beam tune shift is higher than 0.03 in both planes (as it always was in PEP) then the emittance will need to be reduced or the number of bunches decreased with the same total current. In this case the luminosity will increase as $1/\xi$ If the beam-beam tune shift is 0.06 with the nominal bunch number, then the the emittance required would be

ε_{x0} = *half the nominal value.*

2.3 Higher ξ_v

Let us assume that the beam-beam tune shift is higher than 0.03 in the vertical dimension, a common occurrence in many storage rings. The appropriate response would be to reduce the vertical beam height. In this case the luminosity will increase proportional to $\xi_v^{1/2}$.

It is therefore important to design the insertion for a conservative value of the emittance ratio to be certain that we can profit from higher vertical tune shifts. (A ratio of 25:1 was adopted for PEP-II, comfortably below the 40:1 routinely attained in PEP.)

Flexibility for optimizing the emittance and the coupling is vital

2.4 Unequal Beam-Beam Tune Shifts ($\xi_{HER} > \xi_{LER}$)

Note that to date, all of the proposals have set the beam-beam tune shift for the two beams equal. Since the high energy beam is more rigid than the low energy beam, the tune shift is likely to be higher for the high energy beam. Let us assume that the ratio of the tune shifts in the two beams is R_ξ where:

$$R_\xi = \frac{\xi_x^-}{\xi_x^+} = \frac{\xi_y^-}{\xi_y^+} \qquad (1)$$

The standard tune shift formula for unequal energy beams is [2]:

$$\xi_i^{\pm} = \frac{r_e \beta_i^{\pm} N^{\mp}}{2\pi\gamma^{\pm}\sigma_i^{\mp}(\sigma_x^{\mp}+\sigma_y^{\mp})} \qquad (2)$$

where the subscript i means either x or y.

The optimum luminosity is obtained when the beam sizes are equal:

$$\sigma_i^{+} = \sigma_i^{-} \qquad (3)$$

(although this is not absolutely necessary and, in reality, may be very difficult to attain.)
The expression for the ratio of beam-beam limits is then:

$$R_{\xi} = \frac{\xi_i^{-}}{\xi_i^{+}} = \frac{\beta_i^{-} N^{-}}{\gamma^{-}} \frac{\gamma^{+}}{\beta_i^{+} N^{+}} \qquad (4)$$

and the horizontal emittance ratio R_{ε} is:

$$R_{\varepsilon} \doteq \frac{\varepsilon_x^{-}}{\varepsilon_x^{+}} = \frac{\beta_i^{+}}{\beta_i^{-}} = \frac{1}{R_{\xi}} \frac{N^{-}}{\gamma^{-}} \frac{\gamma^{+}}{N^{+}} \qquad (5)$$

So if it is possible to put more current in the low energy beam, the emittance can stay constant and the luminosity will increase.

It may not be possible to increase the current in the low energy beam (lack of RF power, single bunch instabilities, for example). In this case, the luminosity may only be increased by reducing the spot sizes.

The preferred way of reducing the spots is:
LER - reduce the beta function
HER - reduce the emittance

Since it is rather likely that the beam-beam tune shifts are unequal, this possibility should be included in the design of the Interaction Region right from the start.

> *Care should be taken in the LER design to keep open the option of lower IP beta functions.*

3. RADIATION DAMPING

The transverse damping time in an isomagnetic ring is given by [3]

$$\tau_x(\text{sec}) = \frac{7.533\text{x}10^{-5}\ C(\text{m})\ \rho(\text{m})}{J_x\ E(\text{GeV})^3} \qquad (6)$$

where C is the circumference of the ring (2200 m) and J_x is the horizontal damping partition coefficient (nominally approximately 1).

One of the indications from the beam-beam simulations is that the damping of the low energy beam should be increased over the natural value. Calculations will be made here for the extreme case where equal damping times are required in the two rings. Using the natural values of J_x, the bending radius of the LER is given by:

$$\rho_{LER} = \rho_{HER} \frac{E_{LER}^3}{E_{HER}^3} \qquad (7)$$

Putting in numerical values gives

HER Energy	LER Energy	HER Bend Radius	LER Bend Radius
9.0 GeV	3.1 GeV	165 m	6.7 m
8.0 GeV	3.5 GeV	165 m	13.8 m

With beam energies of 9.0 GeV and 3.1 GeV, it is almost impossible to produce equal damping times unless wigglers are used. For beam energies of 8.0 GeV and 3.5 GeV, the situation does not seem so hopeless.

3.1 Wiggler Ring

The CESR-B proposal [4] uses a "wiggler" ring. The simplest version of this would be to use magnets of the same bending field, but alternate sign. The exact geometry will not be calculated here as we are more interested in scaling laws here.

As an example, consider a ring where the dipoles in each half period consist of three equal strength magnets of alternating sign. The bending radius required to obtain equal damping times in this three-bend wiggler lattice would be $\sqrt[3]{3}$ times bigger, a factor of 1.44. Putting in this factor gives:

HER Energy	LER Energy	HER Bend Radius	LER Bend Radius
9.0 GeV	3.1 GeV	165 m	9.7 m
8.0 GeV	3.5 GeV	165 m	19.8 m

Producing equal damping times with beam energies of 9.0 GeV and 3.1 GeV still seems hard even using wigglers.

4. ENERGY SPREAD AND RADIATION DAMPING IN A STORAGE RING WITH WIGGLERS

4.1 Energy Spread

The energy spread in a circular electron ring is given by the standard formula [3]:

$$\left(\frac{\sigma_E}{E}\right)^2 = \frac{1.476 \times 10^{-6}\, E(\mathrm{GeV})^2 \left\langle \frac{1}{\rho^3} \right\rangle}{J_\varepsilon \left\langle \frac{1}{\rho^2} \right\rangle} \qquad (8)$$

where J_e is approximately 2. Separating the effects of the wiggler and the arcs gives:

$$\left(\frac{\sigma_E}{E}\right)^2 = \frac{1.476 \times 10^{-6}\, E(\mathrm{GeV})^2 \left\{ \frac{L_W}{\rho_W^3} + \frac{2\pi}{\rho_A^2} \right\}}{J_\varepsilon \left\{ \frac{L_W}{\rho_W^2} + \frac{2\pi}{\rho_A} \right\}} \qquad (9)$$

There are two extremes: the damping is all done by the arc magnets; the damping is primarily due to the wigglers.

4.2 Synchrotron Radiation Loss

The formula for the synchrotron radiation energy loss in a storage ring is given by [3]:

$$U_o = \frac{8.85 \times 10^{-5}\, E(\mathrm{GeV})^4}{2\pi} \int \frac{1}{\rho^2}\, ds \qquad (10)$$

The total synchrotron radiation produced U_{LER} is the sum of the radiation in the wigglers (U_W) of bending radius r_W and total length L_W and the radiation produced by the arc magnets of bending radius r_A and length $2\pi r_A$ (the effect of the other bends used for the IR are usually much smaller)

$$U_{LER} = \frac{8.85 \times 10^{-5}\, E(\mathrm{GeV})^4}{2\pi} \left\{ \frac{L_W}{\rho_W^2} + \frac{2\pi}{\rho_A} \right\} \qquad (11)$$

If equal damping times are required, the additional damping produced by the wigglers is such that the total energy loss in the two rings is proportional to the beam energy:

$$\frac{U_{LER}}{E_{LER}} = \frac{U_{HER}}{E_{HER}} \qquad (12)$$

4.3 Uniform Ring - No Wigglers

The above formulae reduce to the well known expressions when there are no wigglers and all the damping is done by the arc magnets. In this case, the bending radius of the arcs is the minimum value ρ_{min} and the strength of the arc bends tends to be similar to the wigglers discussed below.

$$U_{LER} = 8.85 \times 10^{-5} \frac{E(\mathrm{GeV})^4}{\rho_{min}} \qquad (13)$$

and

$$\left(\frac{\sigma_E}{E}\right)^2 = \frac{1.476 \times 10^{-6}\, E(\mathrm{GeV})^2}{J_\varepsilon\, \rho_{min}} \qquad (14)$$

4.4 Approximation for Strong Wigglers

In the case where the wigglers are very strong compared to the bending magnets ($r_W \ll r_A$) the energy lost in the wigglers is approximately equal to the total energy lost in the ring. So in this case:

$$\frac{L_W}{\rho_W^2} \approx \frac{U_{LER}}{1.409 \times 10^{-5}\, E(GeV)^4} \qquad (15)$$

and

$$\left(\frac{\sigma_E}{E}\right)^2 \approx \frac{1.476 \times 10^{-6}\, E(GeV)^2}{J_\varepsilon\ \rho_W} \qquad (16)$$

4.5 Comparison of the Two Cases

Since the damping times are to be the same in the two cases:

$$\frac{L_W}{\rho_W^2} \approx \frac{2\pi}{\rho_{min}} \qquad (17)$$

and

$$\frac{\left[\left(\frac{\sigma_E}{E}\right)^2\right]_W}{\left[\left(\frac{\sigma_E}{E}\right)^2\right]_{min}} \approx \frac{\rho_{min}}{\rho_W} \qquad (18)$$

4.6 Scaling Laws

Of interest are the scaling laws. The relative energy spread is

$\left(\frac{\sigma_E}{E}\right)$ is proportional to $\rho_W^{-1/2}$ or $L_W^{-1/4}$

This is an extremely weak dependence.

The energy spread in the beam will be strongly dependent on the damping rate required but insensitive to the magnetic configuration used to obtain it.

5. EMITTANCE

The emittance of an electron storage ring is given by [3]:

$$\varepsilon_{x0} = \frac{C_q \gamma^2 \left\langle \frac{1}{\rho^3} \mathcal{H} \right\rangle}{J_x \left\langle \frac{1}{\rho^2} \right\rangle} \qquad (19)$$

where ε_{x0} is the uncoupled horizontal emittance, J_x is approximately 1,

$$C_q = \frac{55}{32\sqrt{3}} \frac{h}{2\pi} \frac{c}{(m_0 c^2)^3} = 1.47 \times 10^{-6}\, m(GeV)^{-2}$$

and

$$\mathcal{H} = \frac{\alpha_p^2 + \left[\alpha_p \beta_x + \alpha_x \alpha_p'\right]^2}{\beta_x}$$

The emittance of a ring with uniform bends then reduces to the well known formula

$$\varepsilon_{x0} = \frac{C_q \gamma^2 \langle \mathcal{H} \rangle_{Mag}}{J_x\, \rho_A}$$

where $\langle \mathcal{H} \rangle_{Mag}$ is the average over the arc bending magnets.

5.1 Emittance with Wigglers

It is useful to write the expression for the emittance with wigglers in a dimensionless form by comparing the emittance with wigglers ε_W to the emittance in the same lattice without wigglers ε_{x0}.

$$\frac{\varepsilon_W}{\varepsilon_{x0}} = \frac{1 + \frac{L_W}{2\pi r}\left(\frac{\rho_A}{\rho_W}\right)^3 \frac{\langle \mathcal{H}_W \rangle}{\langle \mathcal{H} \rangle}}{1 + \frac{L_W}{2\pi r}\left(\frac{\rho_A}{\rho_W}\right)^2} \qquad (20)$$

The emittance is therefore affected by the arc bending radius, the lattice functions in the arc, the wiggler length, the wiggler bending radius and the lattice functions at the wiggler. For maximum flexibility,

The wigglers should be placed in a region where the dispersion can be modified to tune the emittance to the desired value.

6 SYNCHROTRON RADIATION POWER

There are simple scaling laws for synchrotron radiation power production. The laws for energy deposition are somewhat more complicated and allow some degree of freedom for optimization. (For the HER, the bends are sufficiently long that the distinction is not important.) The peak power production is given by

$$P_{prod}(kW/m) = \frac{88.5\, E(GeV)^4\, I(A)}{2\,\pi\, \rho(m)^2} \qquad (21)$$

P_{prod}	the linear power production density
E	the beam energy in GeV
I	the current in Amps (calculate for 3 A)
ρ	the bending radius in metres

6.1 HER

The power production is the same as the peak power dissipation because the bends are long. For PEP-II, the power densities are:

Energy	Bend Radius	Linear Power Density
9.0 GeV	165 m	10.2 kW/m
8.5 GeV	165 m	8.1 kW/m

6.2 LER

The maximum power dissipation can be considerably less than the power production because of geometrical effects. The power production for PEP-II is:

Energy	Bend Radius	Linear Power Production	
3.1 GeV	6.7 m	87.6 kW/m	
3.1 GeV	9.7 m	41.8 kW/m	(Wiggler)
3.5 GeV	13.75 m	33.6 kW/m	
3.5 GeV	19.8 m	16.2 kW/m	(Wiggler)

6.3 GEOMETRICAL EFFECTS

There is a difference between the peak power production in the short bends and the peak power deposition on the chamber wall due to geometry. Let us take the bending radius of the LER to be 13.75 m, corresponding to a bending length of 90 cm per period, and examine the options available to reduce the peak synchrotron radiation power deposition on the chamber walls.

For LER at 3.5 GeV, full current and a bending radius of 13.75 m, a geometrical factor of 3.4 is required for the maximum radiation density to be less than 10 kW/m. This means that the synchrotron radiation produced in a 90 cm bend should be spread out over 3.1 m.

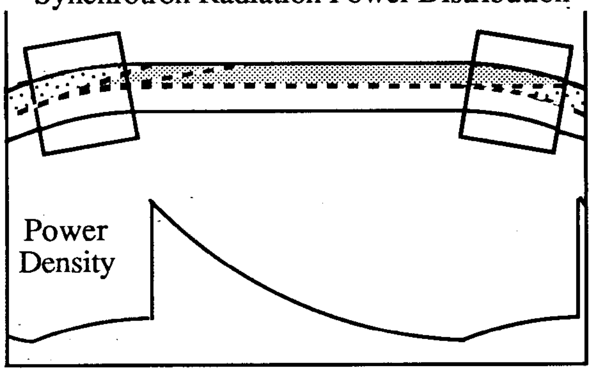
6.4 Magnet Layout

Let us consider the synchrotron radiation distribution from a short bend. In this case, the synchrotron radiation is projected out of the magnet and strikes the vacuum chamber wall downbeam.

The maximum value of the linear synchrotron radiation deposition occurs at the nearest point of contact where the angle of incidence is greatest. This maximum is always less than the linear synchrotron radiation production by an amount that is larger if the synchrotron radiation first strikes the chamber further away. This means:

Short bending magnet
Wide vacuum chamber

Synchrotron Radiation Power Distribution



The formula for the power dissipation is [5]:

$$P_{diss} = \frac{14.1\,E^4\,I}{\frac{\rho^2}{2} + \frac{r\rho^3}{2\left(L^2 + r\rho - L\sqrt{L^2 + 2\,r\rho}\right)}} \qquad (22)$$

where r is the distance of the trajectory from the chamber wall and L is the distance along the straight section (L= 0 at the exit face of the bend).

$$\frac{P_{prod}}{P_{diss}} = \frac{1}{2} + \frac{r\rho}{2\left(L^2 + r\rho - L\sqrt{L^2 + 2r\rho}\right)} \qquad (23)$$

The expression on the right hand side of this equation is the geometrical reduction factor. If L = 0, the factor reduces to unity as expected.

If all the synchrotron radiation strikes the chamber wall outside of the bending magnet, the peak value of the dissipation occurs at a value of L given by:

$$L_{min} = \frac{r}{\varphi_b} - \frac{\rho\,\varphi_b}{2} \qquad (24)$$

where ϕ_b is the bending angle of the dipole.

6.4 Magnet End Effects

In the model discussed up till now, the peak power density occurs at the nearest point of contact of the synchrotron radiation with the chamber wall. In fact, the magnet is not "hard edged", the field falls off from the core end over a distance roughly equal to the gap width. The radiation striking the wall at the point of closest approach actually starts from the fringe field and does not correspond to the maximum power density.

The field extends beyond the core for a distance roughly equal to the gap width. So a core of 38 cm with a gap of 7 cm will have a magnetic length of 45 cm. The maximum synchrotron radiation density on the chamber wall will actually be produced near the front face of the core which results in a smoothing of the peak over more of the chamber wall.

The equations given for the square magnetic field can be generalized to the case of the trapezoidal field. In practice, it is necessary to do an exact calculation to include all of the details of the vacuum chamber profile.

7. SUMMARY

In a new kind of colliding beam machine such as an Asymmetric B Factory, the most important design goal should be flexibility.

The LER lattice should provide:

An emittance that can be varied by a factor of two above and below nominal.

Variable coupling down to half of nominal.

Variable IR Beta functions down to half of nominal.

The synchrotron power distribution must be carefully evaluated since geometrical effects have an important influence.

8. REFERENCES

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