

Superconductivity Solves the Monopole Problem for Alice Strings^{*}

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ABSTRACT

Alice strings are cosmic strings that turn matter into antimatter. Although they arise naturally in many GUT's, it has long been believed that because of the monopole problem they can have no cosmological effects. We show this conclusion to be false; by using the Langacker-Pi mechanism, monopoles can in fact be annihilated while Alice strings are left intact. This opens up the possibility that they can after all contribute to cosmology, and we mention some particularly important examples.

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1. Introduction.

Alice strings [1,2,3] are a class of cosmic string with the remarkable property that a particle travelling around one will come back as its own antiparticle. They may be formed in any Grand Unified Theory in which the charge conjugation operator is contained in the original gauge group. This occurs, for example, in the standard $SO(10)$ GUT. The existence of such objects could obviously have dramatic consequences for cosmology. It has long been appreciated [4] that a simple spontaneous symmetry breaking pattern that gives rise to Alice strings will also create magnetic monopoles. Since the monopole density of the present Universe is known to be very low, some mechanism must exist for getting rid of them. The most common solution is to claim that the pertinent phase transition occurred before some inflationary era. The monopoles would then have been swept outside our horizon, easily satisfying any observed bounds on their density. However, a feature of this solution is that inflation would have erased any other topological defects in exactly the same way. In particular, since Alice strings were produced at the same phase transition, none would survive to cosmologically interesting times.

There is, however, an alternative explanation for the scarcity of present day monopoles due to Langacker and Pi [5]. In this, the Universe enters a temporary superconducting phase after the original phase transition. This causes the magnetic flux to be confined to tubes that end at (anti) monopoles. The high tension in these tubes causes monopoles to annihilate extremely quickly. This rate may be increased even further by the tendency of the flux tubes to break, forming monopole-antimonopole pairs along their length [6,7]. Annihilation then need only occur between neighbouring particles. When the superconducting phase is exited, the monopole density has been reduced to acceptable values, which may be far less than one per horizon volume [7].

At first glance, it may seem that the Langacker-Pi mechanism is rather contrived; we might worry about how elaborate a theory must be in order to give an intermediate superconducting phase. Surprisingly, though, it turns out that even a simple extension of the Standard Model with one additional charged scalar is capable of producing this phase structure [9]. The Langacker-Pi mechanism must therefore be taken very seriously as a potential solution to the monopole problem.

In the present paper we examine the effects of this mechanism in a model containing

Alice strings. We find that breaking $U(1)_{\text{em}} \times \mathbf{Z}_2^\dagger$ to different discrete subgroups leads to different networks of flux tubes, which contain both Alice strings and magnetic flux lines. For our purposes, there are three categories into which such networks fall, and we analyze them using simple toy models. In the first case, $U(1)_{\text{em}} \times \mathbf{Z}_2$ is partially broken such that each magnetic monopole has at least two flux tubes attached to it. Because these tubes will generally be pulling the monopoles in different directions, the annihilation efficiency is fairly low [11], and it is not known whether it can occur fast enough to solve the monopole problem. In the second case, $U(1)_{\text{em}} \times \mathbf{Z}_2$ is completely broken, which means that the flux tubes all have trivial holonomy. If the hierarchy[‡] is large, then as before a network of strings will form. Although each monopole is attached to just one flux tube, there will also be loops of string that are multiply magnetically charged. These will not be neutralised quickly, and so the usual Langacker-Pi mechanism will be evaded. The efficacy of monopole annihilation in either of these cases is not yet known and a verdict must await more thorough studies of network evolution.

For the third case, we introduce a new model containing Alice strings. In this, it is possible to break $U(1)_{\text{em}}$ completely while leaving \mathbf{Z}_2 unbroken. If this is done at a low hierarchy, then we can show that monopoles annihilate much faster than the rest of the network. Thus the monopole bounds may be satisfied while leaving behind a high density of Alice strings. A variant of this mechanism, motivated by the work of Kibble and Weinberg [7], enables us to prevent monopoles from forming in the first place.

We conclude that it is possible for the pertinent phase transition to occur *after* any inflationary era. This means that Alice strings may contribute interesting effects to cosmology, particularly after the superconducting period. Of course we know that today C is *not* a symmetry of the vacuum. This is not a problem, since a necessary condition for a model to contain Alice strings is that C be a member of the original gauge group. It can later be spontaneously broken (together with CP), whereupon Alice strings become the boundaries of, possibly superconducting, domain walls. String-bounded walls eventually decay, and can have interesting consequences without dominating the energy density of the Universe [8]. In a future publication we discuss the cosmological implications of Alice strings further, and show in particular how these domain walls could account for the baryon asymmetry [12].

[†] Here \mathbf{Z}_2 is the discrete group $\{1, C\}$ generated by the charge conjugation operator C .

[‡] The term “hierarchy” refers to the energy-scale difference between the monopole-forming and the flux-tube-forming phase transitions. That is, it defines the relative scales of the monopole mass and the flux tube tension.

2. Alice Strings and the Monopole Problem

In this section we introduce the simplest model containing Alice strings and show how it also leads to monopoles. This consists of a (3+1)-dimensional non-abelian theory with gauge group $G = \text{SO}(3)$ and a Higgs field Φ in the 5-dimensional irreducible representation. We can regard Φ as a real symmetric traceless 3×3 matrix transforming as

$$\Phi \rightarrow M\Phi M^{-1} \quad \text{for } M \in \text{SO}(3). \quad (2.1)$$

The Higgs potential is chosen so that $\langle \Phi \rangle$ acquires two degenerate eigenvalues. In the unitary gauge,

$$\langle \Phi \rangle = p \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & -2 \end{pmatrix}, \quad (2.2)$$

where p is some constant. The unbroken subgroup of $\text{SO}(3)$ is then $H = \text{U}(1) \times_{\text{SD}} \mathbf{Z}_2 \equiv \text{O}(2)$, and this is disconnected.* Since we wish to interpret the $\text{U}(1)$ factor as being electromagnetism, it is clear that H also contains the charge-conjugation operator C .

As our interest lies in the topological structures of the theory, it is more convenient to work with a simply connected unbroken gauge group. We can do this by considering the double cover of the above model. This has a gauge group $G = \text{SU}(2)$, which is broken by a Higgs field Φ in the spin-2 representation. As before, Φ may be regarded as a real traceless symmetric 3×3 matrix that transforms as in equation (2.1) and acquires the expectation value (2.2). The unbroken subgroup is now $H = \text{Pin}(2)$, where $\text{Pin}(2)$ is the double cover of $\text{O}(2)$ and may be parametrised as

$$\text{Pin}(2) = \{e^{i\frac{\theta}{2}\sigma_3}, i\sigma_2 e^{i\frac{\theta}{2}\sigma_3}\}, \quad \theta \in [0, 4\pi). \quad (2.3)$$

Because $G = \text{SU}(2)$ is simply-connected, we have $\Pi_1(G/H) = \Pi_0(H)$, and also $\Pi_2(G/H) = \Pi_1(H)$.

Since $\Pi_1(G/H) \neq 0$, the theory admits topologically stable cosmic strings. The result of parallel transport around such a string defines its ‘‘magnetic flux’’, Ω , an element of H :

$$\Omega = \text{Pexp} \left(i \oint \mathbf{A} \cdot d\mathbf{x} \right) \in H. \quad (2.4)$$

For a string whose flux lies in the component of H disconnected from the identity, Ω does

* The subscript SD denotes a semi-direct product.

not commute with the charge operator $Q \equiv \frac{1}{2}\sigma_3$ that generates $U(1) \subset \text{Pin}(2)$. Rather,

$$\Omega Q \Omega^{-1} = -Q. \quad (2.5)$$

Hence, a particle circumnavigating the string will have the sign of its charge flipped when it returns. This behaviour defines an Alice string. As explained in [2, 3], electric and magnetic field lines in the presence of an Alice string have their directions reversed as they cross some gauge-dependent branch cut.

Notice that since $\Pi_2(G/H) = \Pi_1(H) \neq 0$, the spontaneous symmetry breaking also generates magnetic monopoles. This behaviour is generic [4], and in particular persists even if G contains an explicit factor of $U(1)$. We may think that the situation can be saved in the usual way by simply asserting that an era of inflation is entered after the $G \rightarrow H$ phase transition. However, as the monopoles are swept outside the horizon, so too will be any other topological structures such as Alice strings. It is the purpose of this paper to provide a toy model in which the density of monopoles is reduced to satisfy experimental bounds, while the density of Alice strings remains high enough to contribute interesting effects to cosmology.

3. Alice Strings in a Superconducting Universe

In order to erase monopoles, we shall require the Universe to enter a temporary superconducting phase, following Langacker and Pi [5]. To understand the effects of this, it is very instructive to first consider the fate of monopoles and Alice strings when the group $\text{Pin}(2)$ is broken down to some discrete subgroup. In particular $O(2)$ contains a discrete subgroup D_2 . This is the dihedral group, and consists of rotations of π about the principal axes, so

$$D_2 = \{1, a, b, c\} \quad \text{where} \quad a^2 = b^2 = c^2 = 1, \quad ab = c, \quad ac = b, \quad bc = a. \quad (3.1)$$

Now, $O(2) \subset SO(3)$, and as $SO(3)$ is lifted to its double cover $SU(2)$, then $O(2)$ is lifted to $\text{Pin}(2)$. Similarly, D_2 is lifted to the quaternionic group Q , where

$$Q = \{\pm 1, \pm i, \pm j, \pm k\} \equiv \{\pm 1, \pm i\sigma_1, \pm i\sigma_2, \pm i\sigma_3\}. \quad (3.2)$$

If we start with the group $SU(2)$ and break it down to $\text{Pin}(2)$, then, as discussed in the previous section, we will generate both Alice strings and magnetic monopoles. Now we break the symmetry further to Q , and let k correspond to the generator of $U(1)_{\text{em}}$.

As this breaking occurs, the flux emanating from a monopole will form four tubes,[†] each with holonomy k , as shown in figure 1. This is because the holonomy of any closed loop must now be an element of Q . Furthermore, it is energetically favourable for a flux tube to break into the smallest allowed flux fractions. Since $\pm i\sigma_1$ and $\pm i\sigma_2$ correspond to charge conjugation operators in the group $\text{Pin}(2)$, it should be clear that tubes carrying quaternionic flux i or j are the remains of Alice strings.

In addition to monopoles at which four k -tubes meet, there will be vertices at which four i -tubes meet; we call them i^4 vertices. These correspond to the joining of four Alice strings. An i^2k^2 vertex describes a half-charged monopole[‡] threaded onto a string, and an ijk vertex will correspond to a quarter-charged monopole on an Alice string, and will play a role in describing the topologically charged strings described below. It is in fact possible to describe all fourth-order vertices in terms of cubic vertices. For instance, the magnetic monopole of figure 1 is just the limit of the configuration shown in figure 3 as the loop shrinks to a point.

Note that in general a flux tube carries more information than just its holonomy. It is described fully by the type and direction of the flux that it carries. The holonomy may, of course, be computed from these. In our case, a tube carrying, say, j -flux in one direction is equivalent to a tube carrying $(-j)$ -flux in the opposite direction. It is clearly not equivalent to two k -tubes and three j -tubes bound together, though it could have the same holonomy.

In fact, the story is a little more complicated since loops of Alice strings can carry two types of magnetic charge. The first of these is so-called ‘‘Cheshire Charge’’. Before $U(1)_{\text{em}}$ is broken, imagine bringing a (magnetic) charge q from infinity, passing it through the string loop, and then returning it to infinity. Its charge has been changed to $-q$, and so by the conservation of magnetic charge the Alice string must have acquired a charge $+2q$. This is non-localised, hence the name ‘‘Cheshire Charge’’. Upon breaking $\text{Pin}(2)$ to Q , the associated magnetic flux will be confined to tubes, and so a Cheshire magnetically charged Alice loop will become, say, an i -loop with some k -tubes passing through its centre. As explained above, the direction of the k -flux will reverse as we cross some gauge-dependent branch cut. This is illustrated in figure 2.

The second type of magnetic charge is defined as follows [2]: consider some base point

[†] In the sequel we shall often use the terms ‘‘string’’ and ‘‘flux tube’’ interchangeably.

[‡] It is essential in any Grand Unified Theory that fractionally charged monopoles cannot exist by themselves. The objects that we are considering here, though, can never be separated from the Alice string, and so they are allowed.

x_0 . The holonomy of a closed path starting at x_0 and linking the Alice loop is an element of H . As this path is deformed around the Alice loop, its holonomy traces out a closed curve in H . Since H is not simply-connected, the curve may be topologically non-trivial, and so the Alice loop carries “twisting” magnetic charge. When $\text{Pin}(2)$ breaks to Q , there can no longer be smoothly varying paths in H . Twisting magnetic charge is then manifested by quarter-monopoles strung on the Alice string, as shown in figure 3. As the path based at x_0 is moved around the string, its holonomy jumps each time it crosses a k -tube, from i to j to $-i$ etc.[§] It is clear in this picture that twisting charge and Cheshire charge are essentially equivalent — at a finite cost in energy, the quarter-monopoles described above could be moved together and the k -tubes disentangled to leave an i -loop with several k -tubes passing through its centre. In fact, for a non-trivial hierarchy, this will be the energetically preferred configuration.

In this quaternionic superconducting phase, we have seen that monopoles and Alice strings become a network of tubes carrying i , j , and k flux, joined three-fold[¶] at vertices, such that the total holonomy at any vertex is trivial. A typically ugly section of such a configuration is shown in figure 4.

In order to actually implement this superconducting phase, we take the $SU(2)$ model of the previous section, and add a second Higgs field $\tilde{\Phi}$, also transforming in the spin-2 representation. When the first Higgs field acquired an expectation value

$$\langle \Phi \rangle = p \begin{pmatrix} 1 & 0 \\ & 1 \\ 0 & -2 \end{pmatrix}, \quad (3.3)$$

we saw that $SU(2)$ was broken to $\text{Pin}(2)$. If the second Higgs now gets expectation value

$$\langle \tilde{\Phi} \rangle = \tilde{p} \begin{pmatrix} 1 & 0 \\ & 2 \\ 0 & -3 \end{pmatrix}, \quad (3.4)$$

then the symmetry will be further broken to Q .

We thus require the effective potential of Φ and $\tilde{\Phi}$ to vary with temperature such that, as the Universe cools, it passes through the following phases:

[§] We thank J.Preskill for discussions on this point.

[¶] As mentioned before, a four-vertex can be constructed from three-vertices.

- I) Unbroken SU(2) with $\langle \Phi \rangle = \langle \tilde{\Phi} \rangle = 0$
- II) SU(2) broken to Pin(2) with $\langle \Phi \rangle = p \text{diag}(1, 1, -2)$ and $\langle \tilde{\Phi} \rangle = 0$
- III) SU(2) broken to Q with $\langle \Phi \rangle = p \text{diag}(1, 1, -2)$ and $\langle \tilde{\Phi} \rangle = \tilde{p} \text{diag}(1, 2, -3)$
- IV) Same as phase II.

At phase II, both Alice strings and monopoles are formed, and at phase III they become the quaternionic network described above. This network will evolve as the tension in the strings pulls the vertices around. In phase IV, the electromagnetic symmetry is restored, and the k -flux tubes become deconfined. The network will then dissolve into Alice string structures and monopoles. Notice that if we had broken SU(2) directly to Q, we would have obtained the same type of network before $U(1)_{\text{em}}$ was restored.

In the original Langacker-Pi scenario, the magnetic flux in the superconducting phase was completely confined. This meant that to each monopole was attached just one flux tube. The tension in the flux tubes then brought monopoles together very quickly, causing almost all of them to annihilate. In our model, the situation is more complicated. Each monopole is attached not to one, but to four tubes, which will generally be trying to pull it in different directions. Thus it is not clear whether the monopole-antimonopole pairs will be able to meet each other sufficiently quickly to satisfy the experimental density constraints.

In order to study this question, we must focus on the evolution of the network formed in phase III. The vertices will be pulled around by the tension in the strings. In addition, strings may cut through each other to form new vertices; for example, two k -tubes may “escape” from an i -loop to form a new magnetic monopole as shown in figure 5. The amplitudes for these processes will depend on the vertex masses, and in addition must satisfy various topological constraints; in this case it is not possible for just one k -tube to escape, for if it did it would have to form a free monopole of half the elementary charge, thus incurring Dirac’s wrath.

The qualitative development of string networks of this form is likely to depend crucially on the details of the particular model chosen. The effective Higgs potential will determine the string tensions and vertex masses, which in turn control the rates of interconnection. In addition, the introduction of friction will change the evolution dramatically. Not surprisingly, no thorough analysis of such networks has yet been presented. However, numerical simulations [11] have been undertaken for simplified network models. These indicate that, for some range of parameters, a network’s mass will not come to dominate the Universe.

For our applications we require far more than this — we need the network to have become sufficiently dilute that after restoration of $U(1)_{\text{em}}$ the surviving magnetic monopoles satisfy the experimental density bounds. Thus the question of whether the model so far described can solve the monopole problem will need a more thorough knowledge of network evolution. However, in section 5 we shall present a symmetry breaking scenario in which it is clear that the monopole density can easily be reduced to the desired level. It is this that we regard as definitively solving the monopole problem in an Alice model.

4. Breaking to Smaller Groups

In the previous section we considered a superconducting phase in which $\text{Pin}(2)$ was broken to $Q = \{\pm 1, \pm i, \pm j, \pm k\}$. As a natural extension of this, we ask what happens if instead we break to some subgroup of Q . Up to conjugacy, there are three such subgroups: $Z_4 = \{\pm 1, \pm i\}$, $Z_2 = \{\pm 1\}$, and the trivial group.

For ease of visualisation, suppose that we first break $\text{Pin}(2)$ to Q , and then break this further in stages. By adding a Higgs field \mathbf{v} in the spin-1 representation, we can break the symmetry to Z_4 . After $\langle \tilde{\Phi} \rangle$ has become

$$\langle \tilde{\Phi} \rangle = \tilde{p} \begin{pmatrix} 1 & 0 \\ 2 & \\ 0 & -3 \end{pmatrix}, \quad (4.1)$$

we let \mathbf{v} acquire the expectation value

$$\langle \mathbf{v} \rangle = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}. \quad (4.2)$$

This breaks Q to $Z_4 = \{\pm 1, \pm i\}$. At this point, strings of holonomy $\pm j, \pm k$ will no longer be allowed, and so they must pair up.* A magnetic monopole will now have two (-1) -holonomy flux tubes attached to it. Note that, as before, a 1-string will decompose into smaller flux

* The pairing-up mechanism can be understood in the following way. As the symmetry breaks, the j - and k -strings will become boundaries of domain walls on which $\langle \mathbf{v} \rangle$ remains zero. The tension in these walls, aided by string interconnections, causes them to collapse. As they do so they will bring pairs of j - and k -strings together. This will occur on a time-scale that is short compared to the subsequent evolution of the string network.

fractions. Hence the network will consist of i -strings and (-1) -strings, with $i^2(-1)$ vertices, and beads strung on (-1) strings. The beads will interpolate between the various kinds of flux that the (-1) -strings can carry. Like our previous quaternionic network, this one will evolve by the tension in the strings accelerating the vertices, and by strings interconnecting with each other to form new vertices. Again, the evolution will depend on the details of the model, and the question of whether experimental monopole bounds can be satisfied must await more refined network analyses.

If we add a further Higgs field \mathbf{v}' in the spin-1 representation, and let it acquire expectation value

$$\langle \mathbf{v}' \rangle = \begin{pmatrix} 0 \\ v' \\ 0 \end{pmatrix}, \quad (4.3)$$

then we see that \mathbf{Z}_4 is further broken to $\mathbf{Z}_2 = \{\pm 1\}$. Since strings of holonomy i are not allowed, they too must pair up. In contrast to the previous models, holonomy-1 strings can now be stable, since an ijk -string is unable to break up into anything smaller. Hence the network will consist of (-1) - and 1-strings carrying various types of flux, and with beads strung on them. There will be 1^3 and $(-1)^2 1$ vertices, and 1-tubes will be allowed to end (at generalised monopoles). No matter how low the hierarchy, a string of non-trivial holonomy can never break. However, a 1-string can break, forming a generalised monopole-antimonopole pair. Thus, in this example, if the $\text{Pin}(2) \rightarrow \mathbf{Z}_2$ phase transition occurs soon after the original $\text{SU}(2) \rightarrow \text{Pin}(2)$ transition, then the 1-strings will dissolve, leaving just a (-1) -string network. Magnetic monopoles, though, have two (-1) -strings attached to them, and so as before it is not clear from existing analyses that the subsequent network evolution will be fast enough to get the monopole density down to experimentally allowed levels.

In order to break $\text{SU}(2)$ completely, we add a doublet Higgs field, ψ . This is allowed to acquire the expectation value

$$\langle \psi \rangle = \begin{pmatrix} a \\ 0 \end{pmatrix}, \quad (4.4)$$

which is obviously not left invariant by any non-trivial subgroup of $\text{SU}(2)$. Now strings of holonomy (-1) are not allowed, and so they must pair up. This will leave just 1-tubes, carrying various types of flux, strung with beads, and joined together at vertices. The tubes will be able to end, either on magnetic monopoles or on other generalised monopoles whose nature will depend on the flux that they carry.

If the hierarchy is sufficiently low, then the entire network will dissolve almost immediately, leaving behind no topological structures. This case is as destructive to Alice strings as inflation, and hence is of no interest to us. We will assume, then, that the network does not polarise too quickly. In this case it will still exhibit a major difference from the partial symmetry breaking networks so far described; each monopole will now be connected to exactly one flux tube. This was the essential property of the usual Langacker-Pi mechanism, and so it may seem that the monopole problem would be solved. However, there will also be loops of string that are multiply magnetically charged. These will have many flux tubes attaching them to the rest of the network, and so they will move only slowly. At the termination of the superconducting era, we will be left with magnetically charged loops of Alice string which may subsequently break up and contract away to points, giving birth once again to the unwanted magnetic monopoles. The actual magnitudes of these effects are unknown and await a more detailed analysis.

We have seen that breaking the symmetry to smaller subgroups leads to the same sort of network that we found in the last section. When 1-strings are topologically stable, some qualitatively new features emerge, namely the breaking of strings and the fast motion of monopoles. Though it may possibly be found from more complete analyses that some of these networks *can* solve the monopole problem, we do not claim to have reached any such conclusion here. However, in the next section we shall present a new model containing Alice strings. Even without detailed knowledge of its evolution, it can clearly be seen to eliminate magnetic monopoles in the desired way.

5. Annihilating Monopoles Faster than Alice Strings

With the deliberations of the previous section in mind, we shall now develop a model in which monopoles are quickly annihilated to leave an Alice string network. We have seen that the only flux tubes that disappear quickly are those with trivial holonomy; if the hierarchy is low, they will dissolve into monopole-antimonopole pairs. Thus it would seem desirable that the magnetic flux be completely confined into 1-tubes soon after the monopoles are formed. However, if we *completely* break the symmetry then the Alice strings will also be dissolved. To avoid this we need magnetic flux to be completely confined, and Alice flux to be only partially confined. This could never have occurred for the $SU(2) \rightarrow Pin(2)$ model, since any

non-trivial subgroup of Q contains the element (-1) , and so any symmetry breaking that does not confine all flux would leave at least two flux tubes attached to each monopole.

To achieve our goal, then, we shall have to construct a new model containing Alice strings. For this we consider an $SU(3)$ gauge theory, with a Higgs field Λ in the $\mathbf{6} = (2, 0)$ representation. We can regard Λ as being a symmetric 3×3 matrix with the transformation law

$$\Lambda \rightarrow M \Lambda M^T \quad M \in SU(3). \quad (5.1)$$

Now let Λ acquire the unitary gauge expectation value

$$\langle \Lambda \rangle = h \begin{pmatrix} 2 & & 0 \\ & 2 & \\ 0 & & 1 \end{pmatrix}. \quad (5.2)$$

It is easy to see that this breaks $SU(3)$ down to $O(2)$, and so we have a model containing Alice strings. Note that since $SU(3)$ is simply-connected, we don't need to consider its covering group. This is the crucial difference from the $SO(3) \rightarrow O(2)$ model.

As before, the symmetry breaking will produce monopoles, since

$$\Pi_2(G/H) = \Pi_2(SU(3)/O(2)) = \Pi_1(O(2)) \neq 0. \quad (5.3)$$

In order to enter the superconducting phase, $O(2)$ must be broken down to some discrete subgroup. As before, for ease of visualisation, we shall imagine doing this in stages. First we break the symmetry to the dihedral group $D_2 = \{1, a, b, c\}$, consisting of rotations by π about each of the principal axes of Λ . This may be achieved by introducing a second Higgs field $\tilde{\Lambda}$, also in the $\mathbf{6}$ representation, and letting it acquire expectation value

$$\langle \tilde{\Lambda} \rangle = \tilde{h} \begin{pmatrix} 3 & & 0 \\ & 2 & \\ 0 & & 1 \end{pmatrix}, \quad (5.4)$$

in the unitary gauge. At this point, Alice strings have become a - and b - flux tubes, and magnetic flux is confined into c -flux tubes. Each monopole will have two c -tubes attached to it.

Now we break D_2 down to $Z_2 = \{1, a\}$. This can be done by introducing a Higgs field \mathbf{v} in the $\mathbf{3} = (1, 0)$ representation of $SU(3)$ and letting it acquire the (unitary gauge) expectation

value

$$\langle \mathbf{v} \rangle = \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}. \quad (5.5)$$

No longer will b - and c - flux tubes be allowed to exist in isolation, and so they will pair up. In particular, a magnetic monopole will now have a single 1-tube tied to it, carrying flux c^2 . We will thus have a network of strings of holonomy 1 and a , as shown in figure 6. Now suppose that the hierarchy is low. Then the 1-strings will dissolve into monopole-antimonopole pairs which immediately annihilate. This will leave only strings of holonomy a . There are two types of these — ones carrying a unit of a -flux, and ones carrying both b - and c - flux. The latter simply consist of a b -string and a c -string bound together. Because of the symmetry between a , b and c , it is clear that a -strings have approximately half the tension of bc -strings. Thus, since the hierarchy is low, the latter will decay into the former, via the process shown in figure 7. This will leave just a -strings sewn with beads at which the direction of the a -flux changes. There will be no stable vertices joining more than two a -strings. At the end of the superconducting era, then, we see that a relative abundance of Alice strings will remain. Moreover, these strings will be devoid of magnetic charge, and so there is no danger of them later decaying to give re-birth to monopoles.

Kibble and Weinberg [7] have pointed out a variant of the Langacker-Pi mechanism in which, rather than forming monopoles and then annihilating them, their initial production is prevented.* We can implement this scenario by breaking the symmetry directly to \mathbf{Z}_2 in the following way. Let Λ and \mathbf{v} , as before, lie in the $\mathbf{6}$ and $\mathbf{3}$ representations of $SU(3)$, and let the Higgs pair (Λ, \mathbf{v}) acquire the expectation value

$$\langle (\Lambda, \mathbf{v}) \rangle = \left(\begin{pmatrix} 2h & 0 \\ & 2h \\ 0 & h \end{pmatrix}, \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} \right) \quad (5.6)$$

in the unitary gauge. This breaks $SU(3)$ down to \mathbf{Z}_2 . In a non-singular gauge, then, we have

$$\langle (\Lambda, \mathbf{v}) \rangle = \left(M \begin{pmatrix} 2h & 0 \\ & 2h \\ 0 & h \end{pmatrix} M^T, M \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} \right), \quad M \in SU(3)/\mathbf{Z}_2. \quad (5.7)$$

Since $\Pi_2(SU(3)/\mathbf{Z}_2) = \Pi_1(\mathbf{Z}_2) = 0$, we can see that M , viewed as a 3×3 matrix, is

* This argument is valid when the symmetry breaking does not give rise to a network of stable cosmic strings with non-trivial vertices. In this case topological structures may be surrounded by spheres on which there are no singularities.

topologically trivial on a sphere S^2 , though not on a circle.

Next we let $\langle \mathbf{v} \rangle \rightarrow 0$, while leaving $\langle \Lambda \rangle$ unaltered. Then we have

$$\langle \Lambda \rangle = M \begin{pmatrix} 2h & & 0 \\ & 2h & \\ 0 & & h \end{pmatrix} M^T, \quad M \in \text{SU}(3)/\text{O}(2). \quad (5.8)$$

but still M is topologically trivial on S^2 , and so there are no monopoles. Summarising, the phases of symmetry breaking that we have described are:

I) $\text{SU}(3)$ unbroken.

II) $\text{SU}(3)$ broken to \mathbf{Z}_2 . $\langle (\Lambda, \mathbf{v}) \rangle = \left(\begin{pmatrix} 2h & & 0 \\ & 2h & \\ 0 & & h \end{pmatrix}, \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix} \right)$

III) $\text{SU}(3)$ broken to $\text{O}(2)$. $\langle \Lambda \rangle \neq 0$, $\langle \mathbf{v} \rangle = 0$.

And in phase III we are left with Alice strings but no monopoles.

This mechanism for preventing the formation of monopoles is really just the limit of the previous model as the hierarchy is taken to zero. In any case, either of the scenarios of this section can easily account for an abundance of Alice strings with a paucity of monopoles.

6. Conclusion

In this paper we have studied the Langacker-Pi mechanism as a means of solving the monopole problem in models containing Alice strings. We found that the superconducting phase transition gives rise to a network of flux tubes which evolves in a complex way. It is difficult to analyse this evolution in detail, and we have not done so here. However, for the particular symmetry breaking pattern $\text{U}(1)_{\text{em}} \times \mathbf{Z}_2 \rightarrow \mathbf{Z}_2$ with a low hierarchy, we have shown that magnetic monopoles are quickly annihilated, leaving behind a network of Alice strings.

This observation opens up the possibility that Alice strings were formed after any inflationary eras, and hence could have important cosmological effects. Since our Universe today has a vacuum which is not symmetric under C , we must postulate that this is the result of some spontaneous symmetry breaking of a GUT gauge group containing C . When this

occured, each Alice string became the boundary of a domain wall. These domain walls are possibly superconducting, and have the property that they can act as “filters” that convert antimatter into matter. This could account for the baryon asymmetry of the Universe. We shall investigate these matters further in a separate publication [12].

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FIGURE CAPTIONS

- 1) The flux tubes associated with a magnetic monopole after $Pin(2)$ has been broken down to Q .
- 2) After the superconducting phase is entered, Cheshire magnetic charge is expressed in terms of flux tubes whose direction reverses as they pass through the centre of an Alice loop.
- 3) Twisting magnetic charge is described, in the superconducting phase, by quarter-monopoles threaded on the Alice string. As the path based at x_0 is moved around the Alice string, its holonomy jumps each time it crosses a k -tube, from i to j to $-i$ etc.
- 4) Breaking $SU(2) \rightarrow Q$ leads to a network of i -, j -, and k -strings joined at vertices such that the total holonomy at any vertex is trivial.
- 5) Two k -tubes passing through an i -loop can escape, forming a magnetic monopole and an uncharged Alice loop.
- 6) Breaking $SU(3) \rightarrow \mathbf{Z}_2 = \{1, a\}$ leads to a network of 1-strings and a -strings, with each magnetic monopole attached to the end of a 1-string.
- 7) A bc -string has approximately twice the tension of an a -string, and so, if the hierarchy is low, sections of it will be replaced by sections of a -string. These will then expand until the entire bc -string has been transformed into an a -string.

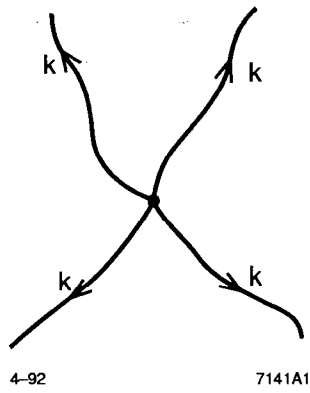
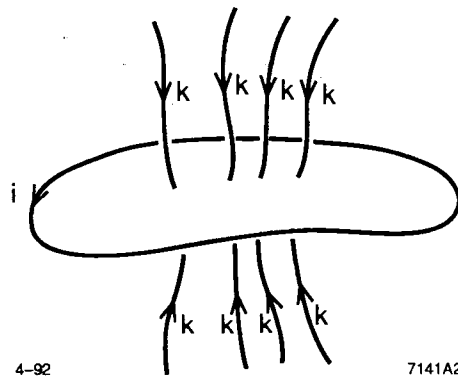


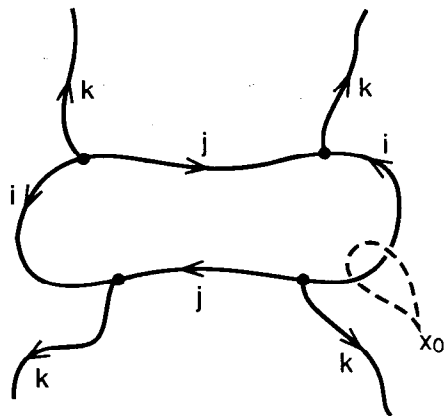
Fig. 1



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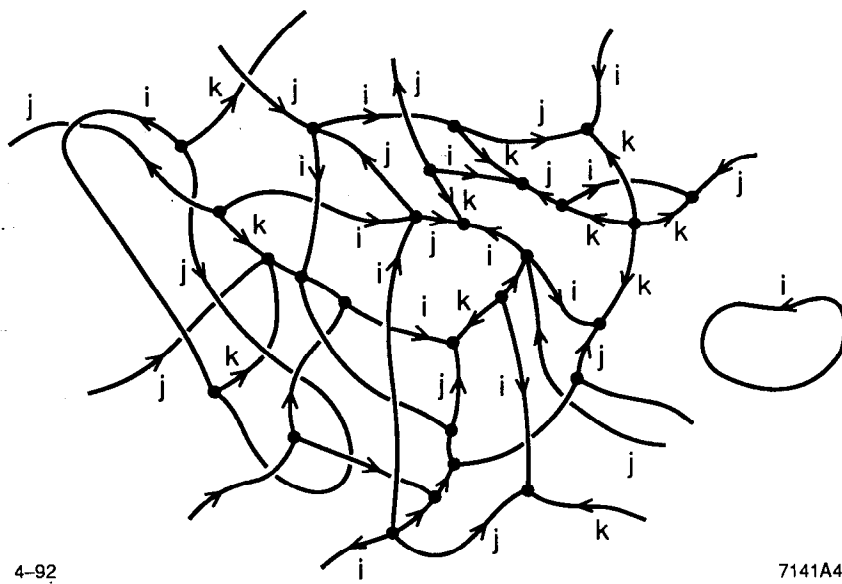
Fig. 2



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Fig. 3



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Fig. 4

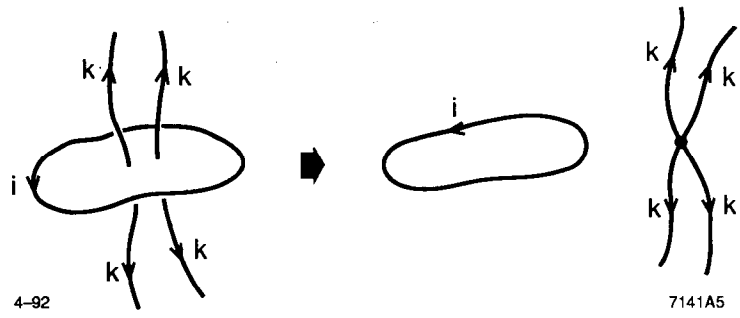
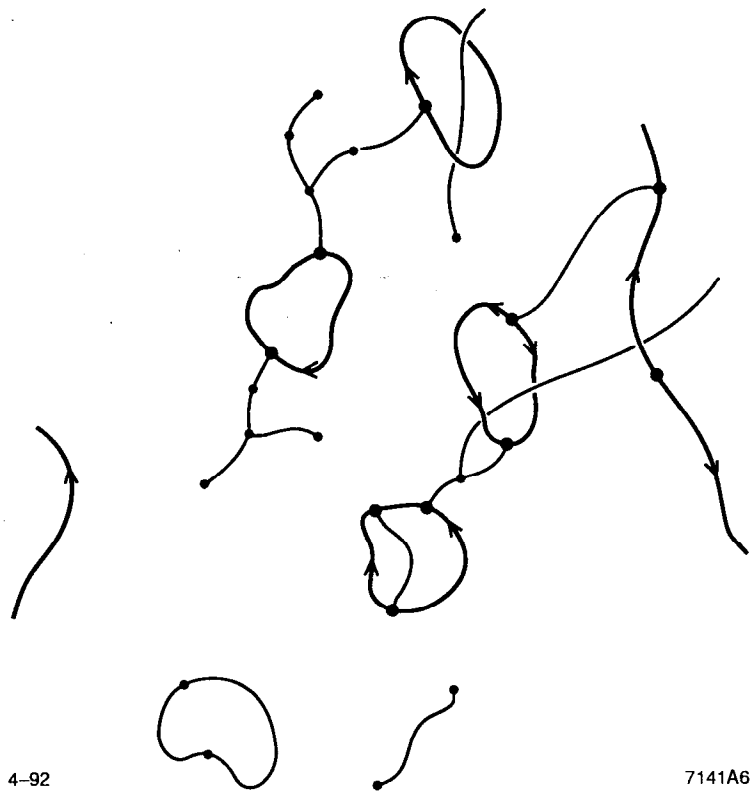


Fig. 5



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Fig. 6

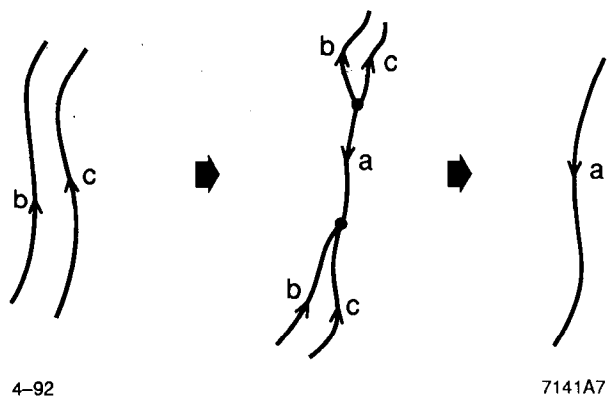


Fig. 7