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CP Asymmetries in B^0 Decays in the Presence of Flavor-Changing Neutral Currents

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ABSTRACT

We study in detail models with vector-like quarks, with special emphasis on their implications for CP asymmetries in B^0 decays. In this class of models there are deviations from unitarity in the Cabibbo-Kobayashi-Maskawa matrix and flavor-changing neutral currents which, although naturally suppressed, may have important consequences. We show that even a relatively small contribution of Z mediated flavor changing neutral currents to $B^0 - \overline{B}^0$ mixing can lead to significant departures from the predictions of the standard model for CP asymmetries in B^0 decays.

I - Introduction

The measurement of CP asymmetries in B^0 decays [1] provides an opportunity to test various aspects of the standard model (S.M.), including the unitarity of the Cabibbo-Kobayashi-Maskawa matrix (CKM) and the standard K.M. mechanism of CP violation. The simplest extension of the standard model where deviations from unitarity of the CKM matrix naturally arise consists of introducing extra quarks which are isosinglets but which mix with the standard quarks. Isosinglet quarks have been suggested in a variety of models, including E_6 grand-unified theories and some of the superstring inspired models. The addition of isosinglet quarks to the S.M. provides [2] a possible connection between CP breaking at a high energy scale and the observed CP violation at low energies and furthermore it gives a simple solution to the strong CP problem [3], [4].

Some of the features of models with isosinglet quarks and their implications for CP violation have been analysed by Branco and Lavoura [5] and by Nir an Silverman [6]. The present work complements these two previous analyses.

The paper is organized as follows. In the next section we briefly describe the model, identifying the new CP violating phases which arise when both $Q = -\frac{1}{3}$ and $Q = \frac{2}{3}$ isosinglet quarks are present and also show how deviations from unitarity and flavor-changing neutral currents (F.C.N.C.) are closely related and both naturally suppressed in the model. In section III we advocate the use of rephasing invariant parametrizations which are specially convenient for models with isosinglet quarks. We give two examples, one with one down-type vector-like quark, and another with one down-type and one up-type vector-like quark. This section of the paper is logically independent of the other sections and therefore it may be skipped by the reader not interested in the question of how to parametrize the CKM matrix. In section IV we study B^0-$\bar{B}^0$ mixing and CP asymmetries in B^0 decays. For simplicity, we consider the case of one down-type vector-like quark (1dVLQ), but we will show that the analysis continues to be valid for an arbitrary number of down-type vector-like quarks. Nir and Silverman have analysed [6] in detail CP asymmetries in the 1dVLQ model under the assumption that B^0-$\bar{B}^0$ mixing is dominated by Z exchange tree diagrams. We will do the analysis so that it can be applied to the general case, including the one where the Z exchange and the S.M. box diagram contributions to B^0-$\bar{B}^0$ mixing are of comparable strength. We point out that if one takes into account the recent upper limit [7] on $B^0 \rightarrow \mu^+\mu^- X$ decays, then B_d-$\bar{B}_d$ mixing can still be dominated by the Z mediated F.C.N.C., while in the case of B_s-$\bar{B}_s$ mixing Z exchange can at most compete with the S.M. box diagram. We show that even a relatively small contribution by the Z exchange diagrams to B^0-$\bar{B}^0$ mixing can drastically modify the predictions of the S.M. for CP asymmetries in B^0 decays. Finally in section V we present our conclusions.

II - The Model

We will assume the standard SU(2)×U(1) gauge theory, with the addition of N_d charge $-\frac{1}{3}$ and N_u charge $\frac{2}{3}$ isosinglet quarks. The quark field content of the model will be denoted in the following way:

$$
\begin{array}{ll}
(\mathrm{u}^0\ \mathrm{d}^0)_{\mathrm{L}i} & i = 1, \ldots, \mathrm{n} \\
\mathrm{D}^0_{\mathrm{L}p} & p = 1, \ldots, \mathrm{N}_d \\
\mathrm{U}^0_{\mathrm{L}q} & q = 1, \ldots, \mathrm{N}_u \\
\mathrm{D}^0_{\mathrm{R}\alpha} & \alpha = 1, \ldots, \mathrm{n} + \mathrm{N}_d \\
\mathrm{U}^0_{\mathrm{R}\beta} & \beta = 1, \ldots, \mathrm{n} + \mathrm{N}_u
\end{array}
\tag{1}
$$

The quark mass terms are:

$$\mathcal{L}_{\mathrm{M}} = \bar{\mathrm{u}}^0_{\mathrm{L}i}(\mathrm{m}_u)_{i\beta}\mathrm{U}^0_{\mathrm{R}\beta} + \bar{\mathrm{U}}^0_{\mathrm{L}q}(\mathrm{M}_{\mathrm{U}})_{q\beta}\mathrm{U}^0_{\mathrm{R}\beta} + \bar{\mathrm{d}}^0_{\mathrm{L}i}(\mathrm{m}_d)_{i\alpha}\mathrm{D}^0_{\mathrm{R}\alpha} + \bar{\mathrm{D}}^0_{\mathrm{L}p}(\mathrm{M}_{\mathrm{D}})_{p\alpha}\mathrm{D}^0_{\mathrm{R}\alpha} \tag{2}$$

The dimensions of the four mass matrices are readily inferred from the index range convention of Eq. (1). Although most of our considerations apply to arbitrary n, N_d, N_u, we will, for simplicity, take $n = 3$, $N_d = N_u = 1$. The weak gauge currents can be written:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{g}} &= \mathcal{L}_{\mathrm{W}} + \mathcal{L}_{\mathrm{Z}} \\
\mathcal{L}_{\mathrm{W}} &= \frac{\mathrm{g}}{\sqrt{2}}\bar{\mathrm{u}}_{\mathrm{L}\alpha}\mathrm{V}^{\mathrm{CKM}}_{\alpha\beta}\gamma_\mu \mathrm{d}_{\mathrm{L}\beta}\mathrm{W}^\mu \qquad\qquad (\alpha, \beta = 1,\ldots,4) \\
\mathcal{L}_{\mathrm{Z}} &= \frac{\mathrm{g}}{2\cos\theta_{\mathrm{W}}}\left[z^u_{\alpha\beta}\bar{\mathrm{u}}_{\mathrm{L}\alpha}\gamma^\mu \mathrm{u}_{\mathrm{L}\beta} - z^d_{\alpha\beta}\bar{\mathrm{d}}_{\mathrm{L}\alpha}\gamma^\mu \mathrm{d}_{\mathrm{L}\beta} - \sin^2\theta_{\mathrm{W}}\mathrm{J}^\mu_{\mathrm{em}}\right]\mathrm{Z}_\mu
\end{aligned}
\tag{3}
$$

where u_α, d_β are mass eigenstates and:

$$\mathrm{V}^{\mathrm{CKM}}_{\alpha\beta} = \sum_{i=1}^{3} U^*_{i\alpha} W_{i\beta} \tag{4a}$$

$$z^u_{\alpha\beta} = \delta_{\alpha\beta} - U^*_{4\alpha} U_{4\beta} \tag{4b}$$

$$z^d_{\alpha\beta} = \delta_{\alpha\beta} - W^*_{4\alpha} W_{4\beta} \tag{4c}$$

where U, W denote the matrices which relate the weak and mass eigenstates:

$$\begin{bmatrix} u_i^0 \\ U^0 \end{bmatrix}_L = U \begin{bmatrix} u_i \\ T \end{bmatrix}_L , \qquad \begin{bmatrix} d_i^0 \\ D^0 \end{bmatrix}_L = W \begin{bmatrix} d_i \\ B \end{bmatrix}_L \tag{5}$$

Due to the presence of the vector-like quarks there are flavor changing neutral currents which are closely connected to the deviations from unitarity in V^{CKM}. Indeed, using the unitarity of U and W, one readily obtains:

$$(VV^+)_{\alpha\beta} = z^u_{\alpha\beta} \tag{6a}$$

$$(V \equiv V^{CKM})$$

$$(V^+V)_{\alpha\beta} = z^d_{\alpha\beta} \, . \tag{6b}$$

An attractive feature of models with vector-like quarks is the fact that although deviations from unitarity in V^{CKM} and FCNC arise, they are related through Eqs. (6) and are both suppressed in the standard quark sector by the ratio of standard quark masses to the vector-like quark masses. This can be readily seen by making an approximate diagonalization of the quark mass matrices. By choosing an appropriate weak basis one can put, without loss of generality, the quark mass matrices in the form:

$$\mathcal{M}_d = \begin{bmatrix} m_d \\ M_D \end{bmatrix} = \begin{bmatrix} G_d & J_d \\ 0 & \hat{M}_d \end{bmatrix} \tag{7a}$$

$$\mathcal{M}_u = \begin{bmatrix} m_u \\ M_u \end{bmatrix} = \begin{bmatrix} \hat{G}_u & J_u \\ 0 & \hat{M}_u \end{bmatrix} \tag{7b}$$

where $\hat{G}_u$, $\hat{M}_u$, $\hat{M}_d$ are diagonal real positive matrices of dimension n, N_u, N_d. The matrix G_d is n-dimensional and complex, while J_d, J_u are $(n \times N_d)$, $(n \times N_u)$ complex matrices. Through a phase redefinition, one can eliminate N_d, N_u phases from J_d, J_u, respectively. It is convenient to write in block form the unitary matrices W, U which diagonalize $\mathcal{M}_d\mathcal{M}_d^+$, $\mathcal{M}_u\mathcal{M}_u^+$, respectively:

$$W = \begin{bmatrix} A_d \\ B_d \end{bmatrix} = \begin{bmatrix} K_d & R_d \\ S_d & T_d \end{bmatrix} \tag{8}$$

with analogous expressions for U. Let m the mass scale of (G_d, J_d) and M be the mass scale of $\hat{M}_d$. Since G_d, J_d are $\Delta I = \frac{1}{2}$ mass terms while $\hat{M}_d$ is a $\Delta I = 0$ mass term, it is natural to assume $M \gg m$. One can then find an approximate solution for W:

$$T_d \approx \mathbb{1}_{N_d} \qquad S_d \approx -(\hat{M}_d^{-1} J_d) K_d$$

$$R_d \simeq J_d \hat{M}_d^{-1} \tag{9}$$

while K_d is, up to $O(m^2/M^2)$, the unitary matrix which diagonalizes $G_d G_d^+$. Analogous expressions obviously apply to U. The V^{CKM} matrix is then given by :

$$V^{CKM} = (A_u^+ A_d) = \begin{bmatrix} K_u^+ K_d & K_u^+ J_d \hat{M}_d^{-1} \\ \hat{M}_u^{-1} J_u^+ K_d & \hat{M}_u^{-1} J_u^+ J_d \hat{M}_d^{-1} \end{bmatrix} \tag{10}$$

Using unitarity of W, U one readily obtains:

$$(VV^+)_{ij} = \delta_{ij} - \left[J_u M_u^{-2} J_u^+\right]_{ij}$$

$$(V^+V)_{ij} = \delta_{ij} - \left[K_d^+ J_d M_d^{-2} J_d^+ K_d\right]_{ij} \tag{11}$$

where we have taken into account the fact that we have chosen to work in the weak basis where $\hat{G}_u$ is diagonal and therefore $K_u \simeq \mathbb{1}_3$. Since J_u, J_d are $O(m)$ it is clear from Eqs. (11), that deviations from unitarity and FCNC for the standard quarks will be suppressed by the ratio m^2/M^2.

III - Rephasing Invariant Parametrization of V^{CKM}

Since the CKM matrix is no longer unitary, it is less obvious to find the number of independent CP violating phases for arbitrary n, N_d, N_u. In Ref. [5] Branco and Lavoura have studied the restrictions that CP invariance imposes on the quark masses of Eq. (1). This was done by constructing the most general CP transformation which leaves invariant the charged and neutral current interactions. They obtained for the number of CP restrictions:

$$N_c = \frac{1}{2}(n\text{-}1)\left[(n\text{-}2)+2(N_d+N_u)\right] \tag{12}$$

This corresponds in general to the number of independent CP violating phases N_ϕ which appear in V^{CKM}. At this point it is worth noting that although the expression for V^{CKM} given by Eq. (10) is only approximate, it contains the correct number of physical phases, namely:

$$K_d \to \frac{1}{2}(n-1)(n-2)$$

$$J_d \to (n-1)N_d \tag{13}$$

$$J_u \to (n-1)N_u$$

We turn now to the question of finding an exact parametrization of V^{CKM} for models with vector-like quarks. There are two different approaches to the problem: one , more traditional, parametrizes V^{CKM} through Euler angles and phases; the other uses rephasing invariant quantities [8] to parametrize V^{CKM}. In the case where there are only isosinglet quarks of a given charge (e.g. $N_u=0$, N_d=arbitrary) the V^{CKM} matrix consists of the first n lines of a $(n+N_d)$ dimensional unitary matrix, and the problem amounts to finding a parametrization of this unitary matrix where $\frac{1}{2}(n-1)\left[(n-2)+2N_d\right]$ physical phases appear in the first n lines. This problem was solved in Ref. [5], where an explicit parametrization through Euler angles and phases was given. At this point, it is worth mentioning that, for more than one vector-like quark the "standard" parametrization [9] of V^{CKM} does not have the above property and therefore it is not adequate. Although the solution presented in Ref. [5] is mathematically correct, parametrizations through Euler angles and phases are not the most convenient, especially when isosinglet quarks are present. Therefore, we will propose here the use of rephasing invariant parametrizations and analyse the two simplest cases, namely ($N_d=1$, $N_u=0$) and ($N_d=N_u=1$).

The case $N_d=1$, $N_u=0$

In this case there are three phase variables and six angle variables. We propose the following choice:

Phase variables:

$$\begin{aligned} \varphi_1 &= \arg(V_{11}V_{23}V_{13}^*V_{21}^*) \\ \varphi_2 &= \arg(V_{11}V_{33}V_{13}^*V_{31}^*) \\ \varphi_3 &= \arg(V_{23}V_{32}V_{22}^*V_{33}^*) \end{aligned} \tag{14a}$$

Angle variables:

$$|V_{11}|,\ |V_{21}|,\ |V_{31}|,\ |V_{23}|,\ |V_{13}|,\ |V_{32}| \tag{14b}$$

We have chosen a complete set of variables containing quantities that are either already measured or likely to be directly measured in the future. Indeed it will be seen that ϕ_1, ϕ_2, ϕ_3

correspond to the angles γ, α, β_s respectively, which appear in the unitarity quadrangles of the 1dVLQ model (see Figs. 1, 2).

We will show next that one can obtain the remaining elements of V^{CKM} from the input data of Eqs. (14). In order to facilitate our task, we work in the weak basis where m_u is diagonal, real. Let us consider the unitary matrix W defined by Eq. (5), whose first three lines constitute V^{CKM}:

$$W = \begin{bmatrix} V^{CKM} \\ W_{4i} \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} & V_{uB} \\ V_{cd} & V_{cs} & V_{cb} & V_{cB} \\ V_{td} & V_{ts} & V_{tb} & V_{tB} \\ W_{41} & W_{42} & W_{43} & W_{44} \end{bmatrix} \tag{15}$$

Without loss of generality, one can choose the quark phases so that the second row and the third column are real. Then φ_1, φ_2, φ_3, fix the arguments of V_{11}, V_{31}, V_{32}, respectively. Normalization of the first column $\left(|W_{41}|^2 = 1 - \sum_{i=1}^{3}|V_{i1}|^2\right)$ gives us $|W_{41}|$. Then orthogonality of the first and third columns together with normalization of the third column give us $\arg(W_{41})$, $|V_{33}|$, $|W_{43}|$. At this stage, the first and the third columns are completely determined and in the second column $|V_{32}|$, $\arg V_{32}$, $\arg V_{23}$ are also known. The remaining elements in the second column can determined from orthogonality of the second column to the first and third, together with normalization of the second column. We have omitted the usual ambiguities [8], [10] which arise in reconstructing the CKM matrix from input data. Note that our parametrization is such that for angle variables we have only used the moduli of V^{CKM} connecting the standard quarks. Therefore V^{CKM} can be reconstructed without directly measuring the coupling of the isovector quark B to the standard quarks. We have considered the case N_d=1. The extension to N_d>1 is straightforward. However, there are some special features which only hold for N_d=1. For example, for N_d=1, it can easily be verified that one can choose the quark field phases in such a way that the couplings $z^d_{\alpha\beta}$ are all real. In general this is not possible for N_d>1.

We have presented a rephasing invariant parametrization of V^{CKM} where invariant phases and moduli were used. In the standard model, one can also parametrize V^{CKM} using only independent moduli [11]. One may ask whether that parametrization is also possible in the presence of isovector quarks. We will show that it is only possible for N_d=1. The number of independent moduli (N_m) is:

$$N_m = n(n+N_d-1) \qquad N_d \geq 1 \tag{16}$$

While the number of angles is:

$$N_a = \frac{1}{2}n\left[(n+N_d-1)+N_d\right] \tag{17}$$

Taking into account that the number of independent phases N_ϕ is given by Eq. (12), with $N_u=0$, one obtains:

$$N_p = N_\phi + N_a = N_m+(N_d-1)(n-1) \tag{18}$$

Therefore for $N_d>1$ the number of parameters exceeds the number of independent moduli, and a parametrization through moduli is no longer possible.

The case $N_d=N_u=1$

We consider now the case where there are both isovector quarks of charge $-\frac{1}{3}$ and of charge $\frac{2}{3}$. The parametrization of V^{CKM} is less obvious in this case, since there no longer exists a weak basis where either the up or down quark mass matrices are diagonal. It is convenient to introduce the auxiliary matrix X defined by:

$$X = \begin{bmatrix} V^{\mathrm{CKM}} & B_u^+ \\ B_d & 0 \end{bmatrix} \tag{19}$$

where $V^{\mathrm{CKM}} = A_d^+ A_d$, and A_u, A_d, B_u, B_d were defined in Eq. (8). For the moment n, N_d, N_u are arbitrary. X is a $(n+N_d+N_u)$ dimensional matrix and it can readily be verified that X is unitary. The fact that V^{CKM} is a submatrix of a unitary matrix, obviously facilitates the task of finding an appropriate parametrization. We will specialize now to the case $N_d=N_u=1$. There are five phase variables and nine angle variables. Our choice is:

Invariant phases:

$$\begin{aligned} \varphi_1 &= \arg(V_{11}V_{23}V_{13}^*V_{21}^*) \\ \varphi_2 &= \arg(V_{11}V_{33}V_{13}^*V_{31}^*) \\ \varphi_3 &= \arg(V_{32}V_{23}V_{33}^*V_{22}^*) \\ \varphi_4 &= \arg(V_{11}V_{43}V_{13}^*V_{41}^*) \\ \varphi_5 &= \arg(V_{12}V_{21}V_{11}^*V_{22}^*) \end{aligned} \tag{20}$$

Moduli:

$|V_{11}|$, $|V_{12}|$, $|V_{21}|$, $|V_{22}|$, $|V_{13}|$, $|V_{23}|$, $|V_{33}|$, $|V_{31}|$, $|V_{43}|$

It can be readily seen that these input parameters enable one to reconstruct the CKM matrix using unitarity of the auxiliary matrix X.

IV CP Asymmetries in B^0 decays

In this section we study B-$\bar{B}$ mixing and CP asymmetries in B^0 decays in models with vector-like quarks. In ref. [6], Nir and Silverman have studied these asymmetries under the assumption that Z mediated FCNC give the dominant contribution to B-$\bar{B}$ mixing. We will consider here the more general case where non-standard contributions compete with the standard box diagram at inducing B^0-$\bar{B}^0$ mixing. The relevance of this analysis stems from the fact that even a relatively small contribution from new physics can produce significant departures from the S.M. predictions for the CP asymmetries in B^0decays.

This section is organized as follows. First we present a general analysis of CP asymmetries in B^0 decays when new physics is added to the mixing matrix. Then we particularize to the model with charge -$\frac{1}{3}$ vector-like quarks. It turns out that it is sufficient to consider the case where there is only one such quark, since CP asymmetries cannot distinguish N_d=1 from N_d>1.

Let us assume that the off diagonal element of B_q-$\bar{B}_q$ is changed by a factor Δ_{qb} as a result of a new contribution from physics beyond the S.M.:

$$M_{12} = M_{12}^{(0)}\Delta_{qb} \qquad (q=d,\ s) \tag{21}$$

where $M_{12}^{(0)}$ is the box diagram contribution. We will assume that all amplitudes contributing to the decay have the same CKM phase and furthermore that $\Gamma_{12}^{(0)} \ll M_{12}^{(0)}$. In this case the CP asymmetry is given by:

$$a \equiv \frac{\Gamma(B^0\to f)-\Gamma(\bar{B}^0\to f)}{\Gamma(B^0\to f)+\Gamma(\bar{B}^0\to f)} \simeq -\sin(\Delta M t)\sin\phi \tag{22}$$

where:

$$\phi = \phi^{(0)} + \arg\Delta_{bq}, \quad \Delta_{bq} = \Delta_{qb}^{*}$$
$$\phi^{(0)} = \arg\left[\left(\frac{q}{p}\right)^{(0)} \frac{\bar{A}(f)}{A(f)}\right], \quad \left(\frac{q}{p}\right)^{(0)} = \left(\frac{M_{12}^{(0)*}}{M_{12}^{(0)}}\right)^{\frac{1}{2}} \tag{23}$$

The index (o) denotes the contributions arising within the three generation S.M., and A(f), $\bar{A}(f)$ stand for the decay amplitudes from the initial state $|B^0\rangle$, $|\bar{B}^0\rangle$ to a CP eigenstate $|f\rangle$. From Eq.(23) it follows that there are two possible sources which may change the S.M. prediction:

(i)- The presence of the phase of Δ_{bq}, which determines the deviation from the box diagram contribution $\phi^{(0)}$. It is possible to incorporate different new physics contributions for B_d and B_s, if $\arg\Delta_{bd} \neq \arg\Delta_{bs}$.

(ii)- Although the expression for $\phi^{(0)}$ is the one given by the S.M., the actual numerical value of $\phi^{(0)}$ may differ from the S.M. prediction. This is due to the fact that models beyond the S.M. allow in general for a different range of the CKM matrix elements.

In table I, we establish our notation by explicitly giving ϕ for various final states. For comparison, the standard model values are also shown.

We turn now to the detailed analysis of models with charge $-\frac{1}{3}$ vector-like quarks. The new contribution to the $\Delta B=2$ effective hamiltonian arises from Z exchange tree graphs and one readily obtains:

$$\begin{aligned}
\Delta_{bq} &= 1 + r_q e^{2i\theta_{bq}} \\
r_q &= \frac{1}{\nu|\bar{E}(x_t)|}\left|\frac{z_{bq}}{V_{tb}V^*_{tq}}\right|^2 \qquad (24) \\
\theta_{bq} &= \arg\left[\frac{z_{bq}}{V_{tq}V^*_{tb}}\right]
\end{aligned}$$

where $\nu = \frac{\alpha}{4\pi\sin^2\theta_W}$ and $\bar{E}\left(x_t=\left(\frac{m_t}{m_W}\right)^2\right)$ is an Inami-Lim function for the top quark box diagram. Note that $\nu\bar{E}(x_t)=-0.0046$ for $m_t=140$ GeV. We assume the same QCD correction factor for both the box diagram and the Z exchange diagram. This should be a good approximation since QCD corrections above the scale of M_Z are negligible. From Eq.(24) one readily obtains:

$$\begin{aligned}
\arg\Delta_{bq} &= \tan^{-1}\left[\frac{r_q\sin 2\theta_{bq}}{1+r_q\cos 2\theta_{bq}}\right] \qquad (25) \\
|\Delta_{bq}| &= \left(1+r_q^2+2r_q\cos 2\theta_{bq}\right)^{1/2}
\end{aligned}$$

There are two distinct cases of interest in the study of CP asymmetries:

(a) - Z exchange and box diagrams give comparable contributions to B_q-$\bar{B}_q$ mixing.

(b) - Z exchange gives the dominant contribution to B_q-$\bar{B}_q$ mixing.

Thes two cases are distinguishable by the value of the parameter r_q:

(a) - $r_q \approx 1$

(b) - $r_q \gg 1$

Case (b) was studied in Ref.[6]. Therefore, our emphasis will be on case (a); we find that even a relatively small contribution from Z exchange to B_q-$\bar{B}_q$ mixing can imply very significant departures from the S.M. predictions for CP asymmetries.

In order to derive the numerical predictions for the CP asymmetries, one has to take into account the unitarity constraints. The relevant ones for our purposes are:

$$
\begin{aligned}
V^*_{tb}V_{td}+V^*_{cb}V_{cd}+V^*_{ub}V_{ud} &= z_{bd} \\
V^*_{tb}V_{ts}+V^*_{cb}V_{cs}+V^*_{ub}V_{us} &= z_{bs}
\end{aligned}
\tag{26}
$$

which lead to the unitarity quadrangles of figs. 1, 2. In order to determine the angle ϕ for the various final states (Table 1), one has to know the values of the angles α, β, β_S shown in figs. 1, 2. One readily obtains:

$$
\begin{aligned}
&\cos(\alpha\text{-}\delta) = \\
&\quad = \frac{|V_{ub}V_{ud}|^2+\left(|V_{tb}V_{td}|^2-2|V_{tb}V_{td}||z_{bd}|\cos\theta_{bd}+|z_{bd}|^2\right)-|V_{cb}V_{cd}|^2}{2|V_{ub}V_{ud}|\left(|V_{tb}V_{td}|^2-2|V_{tb}V_{td}||z_{bd}|\cos\theta_{bd}+|z_{bd}|^2\right)^{1/2}} \\
&\cos(\beta+\delta) = \\
&\quad = \frac{|V_{cb}V_{cd}|^2+\left(|V_{tb}V_{td}|^2-2|V_{tb}V_{td}||z_{bd}|\cos\theta_{bd}+|z_{bd}|^2\right)-|V_{ub}V_{ud}|^2}{2|V_{cb}V_{cd}|\left(|V_{tb}V_{td}|^2-2|V_{tb}V_{td}||z_{bd}|\cos\theta_{bd}+|z_{bd}|^2\right)^{1/2}}
\end{aligned}
\tag{27}
$$

$$
\delta = \arg\frac{V^*_{tb}V_{td}}{V^*_{tb}V_{td}-z_{bd}} = \tan^{-1}\frac{|z_{bd}|\sin\theta_{bd}}{|V_{tb}V_{td}|-|z_{bd}|\cos\theta_{bd}}
$$

$$
\cos\beta_s = \frac{|V_{tb}V_{ts}|^2+|V_{cb}V_{cs}|^2-|z_{bs}|^2}{2|V_{tb}V_{ts}||V_{cb}V_{cs}|^2}
$$

Where we have neglected $|V_{us}V_{ub}|$. When z_{bq}=0, δ=0, one recovers the S.M. expressions giving α, β, β_s in terms of the sides of the S.M. unitarity triangles. We turn now to the experimental constraints on z_{bq}. A recent experimental search [7] for the decays $B \rightarrow X\mu^+\mu^-$ by the UA1 collaboration [7] has led to the upper bound:

$$\mathrm{Br}(B \rightarrow X\mu^+\mu^-) \leq 5.0\times10^{-5} \tag{28}$$

At this point, it is worth noting that for $m_t \approx 150$ GeV and withtin the context of the S.M., the above branching ratio is predicted [13] to be $\mathrm{Br}(B^0 \rightarrow X\mu^+\mu^-)=(6\text{-}8)\times10^{-6}$, therefore one order of magnitude smaller than the UA1 bound. From Eq. (28) one derives the bounds:

$$\left|\frac{z_{bd}}{V_{cb}}\right| \leq .029 \qquad\qquad \left|\frac{z_{bs}}{V_{cb}}\right| \leq .029 \tag{29}$$

These bounds on z_{bq} are almost an order of magnitude stricter than the bounds considered in Ref.[6]. If one writes Eqs.(26) as:

$$\frac{V^*_{tb}V_{tq}}{V^*_{cb}} = \frac{z_{bq}}{V^*_{cb}} - V_{cq} - \frac{V^*_{ub}V_{uq}}{V^*_{cb}} \qquad\qquad q = d,\, s \tag{30}$$

and takes into account the experimental constraints:

$$\begin{array}{ll} .9734 \leq |V_{ud}| \leq .9754\ ; & .2173 \leq |V_{us}| \leq .2219\ ; \\ .187 \leq |V_{cd}| \leq .221\ ; & .07 \leq \left|\frac{V_{ub}}{V_{cb}}\right| \leq .13\ ; \\ |V_{cs}| \geq .8 & \end{array} \tag{31}$$

one readily obtains:

$$\left|\frac{V^*_{tb}V_{td}}{V^*_{cb}}\right| \geq .031\ ; \qquad\qquad \left|\frac{V^*_{tb}V_{ts}}{V^*_{cb}}\right| \geq .73 \tag{32}$$

Combining Eqs.(29, 32) one finally gets:

$$\left|\frac{z_{bd}}{V^*_{tb}V_{td}}\right| \leq .93\ ; \qquad\qquad \left|\frac{z_{bs}}{V^*_{tb}V_{ts}}\right| \leq .04\ . \tag{33}$$

Now the condition for Z exchange to give the dominant contribution to B_q-$\bar{B}_q$ mixing is that $\left|\frac{z_{bq}}{V^*_{tb}V_{tq}}\right| \geq .07$ for m_t=140 GeV. One therefore concludes that in the case of B_d the dominant contribution to the mixing may arise from Z exchange, while in the case of B_s, Z exchange can at most compete with the box diagram contribution. This completes our analysis of the unitarity constraints in the model.

We consider now B_d-$\bar{B}_d$ mixing which is given by:

$$x_d = \frac{\Delta M_d}{\Gamma} = \frac{G_F^2}{6\pi^2}\tau_B \eta_{QCD} M_B B_B F_B^2 M_W^2 |\bar{E}(x_t)||V_{td}V_{tb}|^2|\Delta_{bd}| \tag{34}$$

Where we have followed standard notation. For $\sqrt{B_B F_B^2}$ we will use the range suggested in a recent review by Buras and Harlander [12]:

$$160 \text{ Mev} \leq \sqrt{B_B F_B^2} \leq 240 \text{ MeV} \tag{35}$$

following recent lattice calculations in the static limit. For η_{QCD} we wil use here the value η_{QCD}= 0.55, which is consistent with the renormalization u ed in obtaining Eq.(35), and for τ_B we will take τ_B= 1.28±0.06 ps which is the recent world average for τ_{B^0}, including the LEP results [13]. Eq.(34) fixes for us the experimentally allowed range for the product $|V_{td}V_{tb}||\Delta_{bd}|^{1/2}$, given by:

$$|V_{td}V_{tb}||\Delta_{bd}|^{1/2} = \left[\frac{6\pi^2}{G_F^2 \eta_{QCD} M_W^2 M_B}\right]^{1/2}\left[\frac{x_d^{1/2}}{\tau_B^{1/2} B_B^{1/2} F_B}\right]\frac{1}{\sqrt{|\bar{E}(x_t)|}} \tag{36}$$

The terms in the first bracket are taken as exact. In the second bracket, τ_B, $B_B^{1/2}F_B$ are constained to be within the indicated ranges, while x_d is within the range implied by the experimental results of ARGUS, CLEO, and LEP [14]:

$$x_d = 0.67 \pm 0.10 \tag{37}$$

We are now in a position to evaluate CP asymmetries in the model, taking into account all the experimental and theoretical constraints. At this point it should be obvious that the number of dVLQs is irrelevant to the discussion. None of the input in this section is dependent on the value of $N_d \geq 1$.

For given values of r_q and θ_{bq}, one obtains argΔ_{bq}, $|\Delta_{bq}|$ from Eqs. (25) and then Eq.(36) fixes the allowed range of $|V_{td}V_{tb}|$.

In fig.3 we present our main result. Recall that one of the most important predictions of the S.M. is the sign of some of the CP asymmetries in the B^0 decays. In particular, sen($\overline{\phi_{\psi K_s}}$) = $-$sen(ϕ_{1d}) is predicted to be positive and in fact [15] for $m_t \geq 120$ GeV and $F_B > 170$ MeV, $a(\psi K_s) \geq 0.26$. This is no longer true in the presence of dVLQs. In fig.3 we indicate the region in r_d, θ_{bd} space where the asymmetry $a(\psi K_s)$ has a sign opposite to the one predicted by the S.M.. It is seen from fig.3 that a positive sign for sen(ϕ_{1d}) can be obtained even for a relatively small value of r_q.

In Figs. 4-9 we give the values of various CP asymmetries as a funcion θ_{bd}, for various values of r_d. For comparison, we also give the prediction of the S.M., for the same choice of m_t, $B_B F_B^2$, τ_B, x_d, $|V_{qd}|$, $|V_{qb}|$ (q=c,t). We have used m_t= 140 MeV, and the central values of the ranges indicated above for the other parameters. We choose values of r_d ranging from 0.2 to 2.5. It is clear from the Figs. that even a relatively small contribution of the Z exchange diagrams (i.e. $r_q<1$) to B_q-$\bar{B}_q$ mixing can lead to substantial deviations from the S.M. predictions. For example in Fig. 5, sen(ϕ_{1d}) is plotted in terms of θ_{bd} for r_d=0.6, and one sees that for some regions of θ_{bd}, a_{1d} can have a sign opposite to the one predicted by the standard model. For large values of r_d, which corresponds to dominance of the Z exchange diagrams, one recovers the results presented in Ref. [6], that is that $\phi_{1d}\rightarrow 2\beta'$, $\phi_{3d}\rightarrow -2\alpha'$ (see Figs. 6 and 9).

Finally we consider briefly the influence of the Z mediated FCNC on the CP asymmetries of the B_s^0 decays. The S.M. predicts $\beta_s\simeq 0$, and due to the experimental restriction Eq. (29), this result holds in the dVLQ model also. However a significant value for the angle ϕ_{1s} is not ruled out in the dVLQ model. From Eqs. (24, 33) one deduces the maximum value of r_s possible:

$$r_s \leq 0.35, \qquad \text{for } m_t=140 \text{ GeV}. \tag{37}$$

One readily verifies that for $r_q<1$, the maximum value of $\arg\Delta_{bq}$ is given by:

$$(\arg\Delta_{bq})_{max} = \tan^{-1}\left[\frac{r_q}{\sqrt{1-r_q^2}}\right] \tag{39}$$

Therefore $\arg\Delta_{bs} \leq 21^\circ$. As the measured angle is $\phi_{1s}= -2\beta_s+\arg\Delta_{bs}$, strong deviations from the S.M. prediction are again possible.

V Conclusions

We have analyzed some of the main features of a minimal extension of the S.M. where vector-like quarks are introduced. We show that in these models deviations from unitarity and F.C.N.C. are related and both are naturally suppressed. We advocate the use of rephasing invariant parametrizations of the CKM matrix and give two examples for N_d=1, N_u=0 and for N_d=1, N_u=1.

Special emphasis was given to the consequences of the model for CP asymmetries in B^0 decays. It was shown that even a small contribution of the Z exchange diagrams to B^0-$\bar{B}^0$ mixing can lead to drastic deviations from the S.M. predictions for CP asymmetries in B^0 decays.

After this work was essentially completed we have received a preprint by Soares and Wolfenstein [17] where the authors examine the implications of new physics on CP asymmetries in B^0 decays. We thank João Soares for having given us a copy of the paper. We have also noticed a recent paper by D. Silverman [18] whose content partially overlaps with our section IV. However D. Silverman only considers the case of FCNC Z exchange dominating B_d^0-$\bar{B}_d^0$ mixing. As previously emphasized, the main point in section IV is that even a small contribution of Z-exchange to B^0-$\bar{B}^0$ mixing can lead to predictions for CP asymmetries in B^0 decays drastically different from those of the standard model.

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Table 1

initial state	quark subprocess	final state	ϕ	standard model	beyond standard model
B_d	$\bar{b} \to \bar{c} c \bar{s}$	ψK_s	ϕ_{1d}	-2β	$-2\beta + \arg\Delta_{bd}$
	$\bar{b} \to \bar{c} c \bar{d}$	$D^+ D^-$	ϕ_{2d}	-2β	$-2\beta + \arg\Delta_{bd}$
	$\bar{b} \to \bar{u} u \bar{d}$	$\pi^+ \pi^-$	ϕ_{3d}	2α	$2\alpha + \arg\Delta_{bd}$
B_s	$\bar{b} \to \bar{c} c \bar{s}$	$D_s^+ D_s^-$	ϕ_{1s}	$-2\beta_s$	$-2\beta_s + \arg\Delta_{bs}$
	$\bar{b} \to \bar{c} d \bar{d}$	ψK_s	ϕ_{2s}	$-2\beta_s$	$-2\beta_s + \arg\Delta_{bs}$
	$\bar{b} \to \bar{u} u \bar{d}$	ρK_s	ϕ_{3s}	$-2\gamma - 2\beta_s$	$-2\gamma - 2\beta_s + \arg\Delta_{bs}$

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Figure Captions

Figure 1: The unitarity quadrangle in the B_d sector, corresponding to the first of Eqs. (26).

Figure 2: Unitarity in the B_s sector. We have neglected $|V_{us}V_{ub}|$ and exaggerated $|z_{bd}|$ to show the angles.

Figure 3: The regions of r_d, θ_{bd} space where:

a) The model predicts sen(ϕ_{1d}) to be positive, thus contradicting the standard model result.

b) The model predicts sen(ϕ_{1d}) to be negative.

c) The unitarity quadrangle does not close. For these values of r_d, θ_{bd} Eq. (26) is not consistent with Eqs. (25, 36).

For this figure the values m_t=140 GeV, $|V_{ud}|$=0.9744, $|V_{ub}|/|V_{cb}|$=0.1, $|V_{cd}|$=0.204, and $\sqrt{B_B F_B^2}$=0.2 GeV, were used.

Figure 4: sen(ϕ_{1d}) as a function of θ_{bd}, for r_d=0.2. The values of the other parameters are as in Fig. 3. Even for this low value of r_d, the dVLQ model value can differ significantly from the S.M. prediction, which is sen($\phi_{1d}^{S.M.}$)=-0.68 for the parameter values indicated.

Figure 5: The same as in Fig. 4, but with r_d=0.6. For this value of r_d, sen(ϕ_{1d}) can take on large positive values. Compare with Fig. 3.

Figure 6: sen(ϕ_{1d}) and senβ' for r_d=2.5. For this larger value of r_d these functions are almost the same.

Figure 7: sen(ϕ_{3d}) as a function of θ_{bd}, for r_d=0.2. Again all other parameter values are as in Fig. 3. The S.M. prediction is sen($\phi_{3d}^{S.M.}$)=0.99.

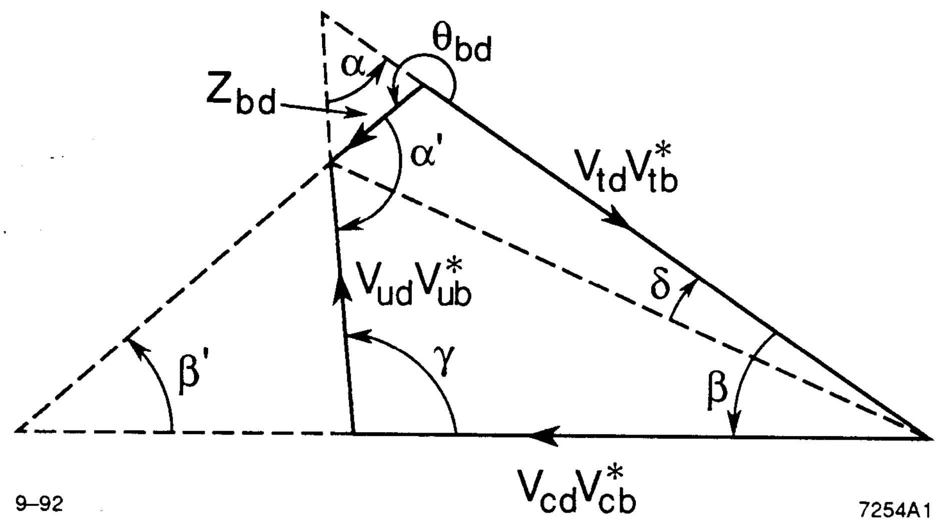
Figure 8: The same as Fig. 7 but with r_d=0.6.

Figure 9: sen(ϕ_{3d}) and sen($-2\alpha'$) for r_d=2.5.

Caption for table I

The predicted values for the angles ϕ_{iq}. The values shown are for CP even final states. Thus $\phi_{1d}=-2\beta=-\phi_{\psi K_s}$. By defintion $\alpha=\arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$; $\beta=\arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$; $\gamma=\arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$; $\alpha'=\arg\left(\frac{V_{ud}V_{ub}^*}{z_{bd}}\right)$; $\beta'=\arg\left(\frac{z_{bd}}{V_{cd}V_{cb}^*}\right)$; $\beta_s=\arg\left(-\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*}\right)$;

See Figs. 1 and 2.



θ_{bd}
α
Z_{bd}
α'
$V_{td}V_{tb}^*$
$V_{ud}V_{ub}^*$
δ
β'
γ
β
$V_{cd}V_{cb}^*$
9–92
7254A1

Fig. 1

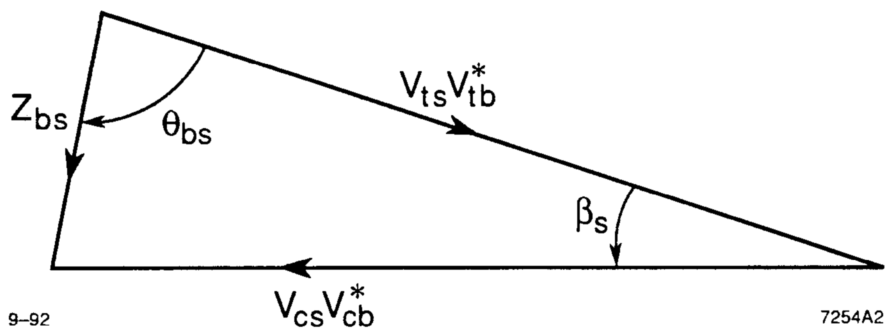


Fig. 2

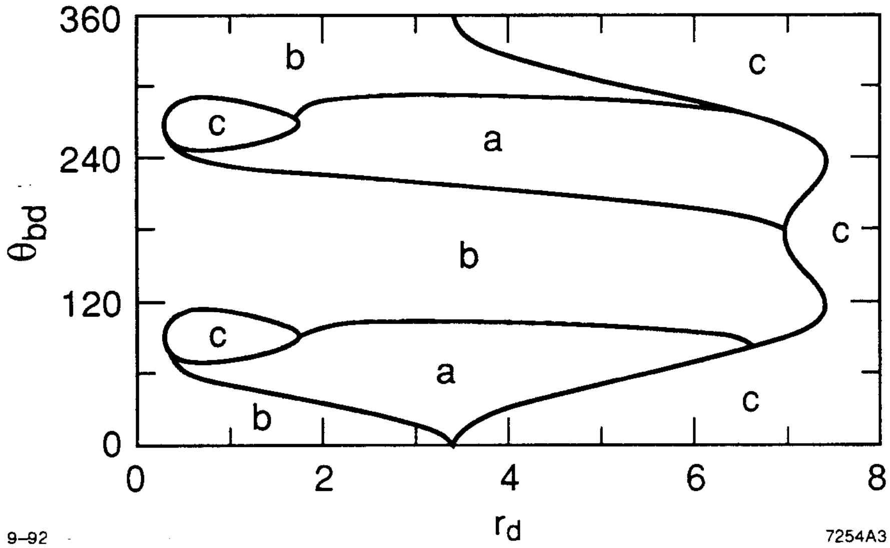


Fig. 3

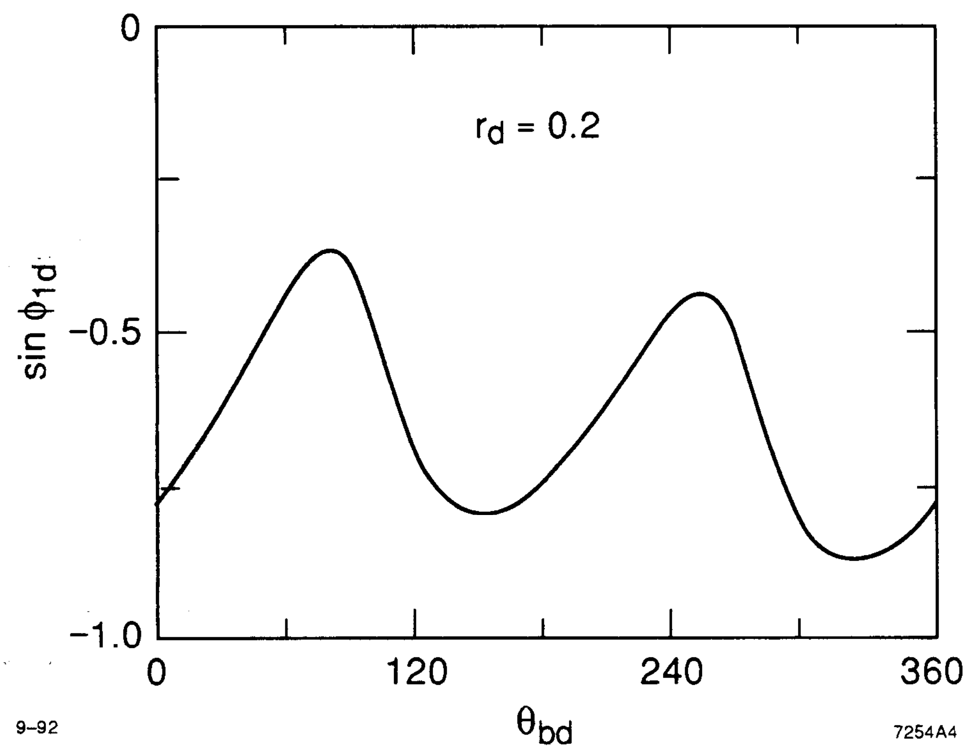


Fig. 4

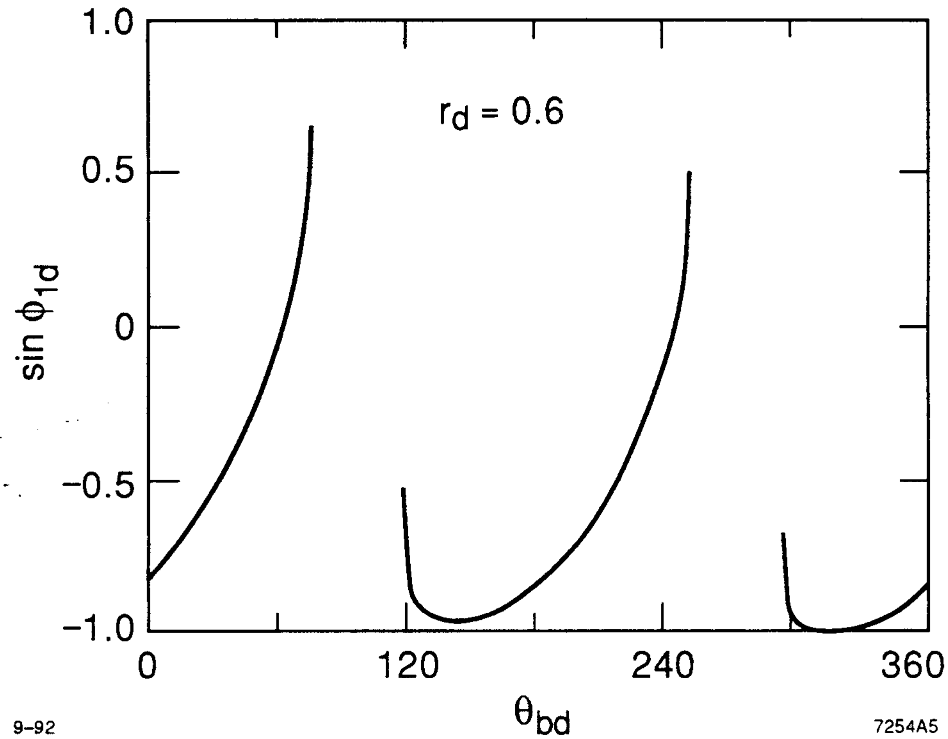


Fig. 5

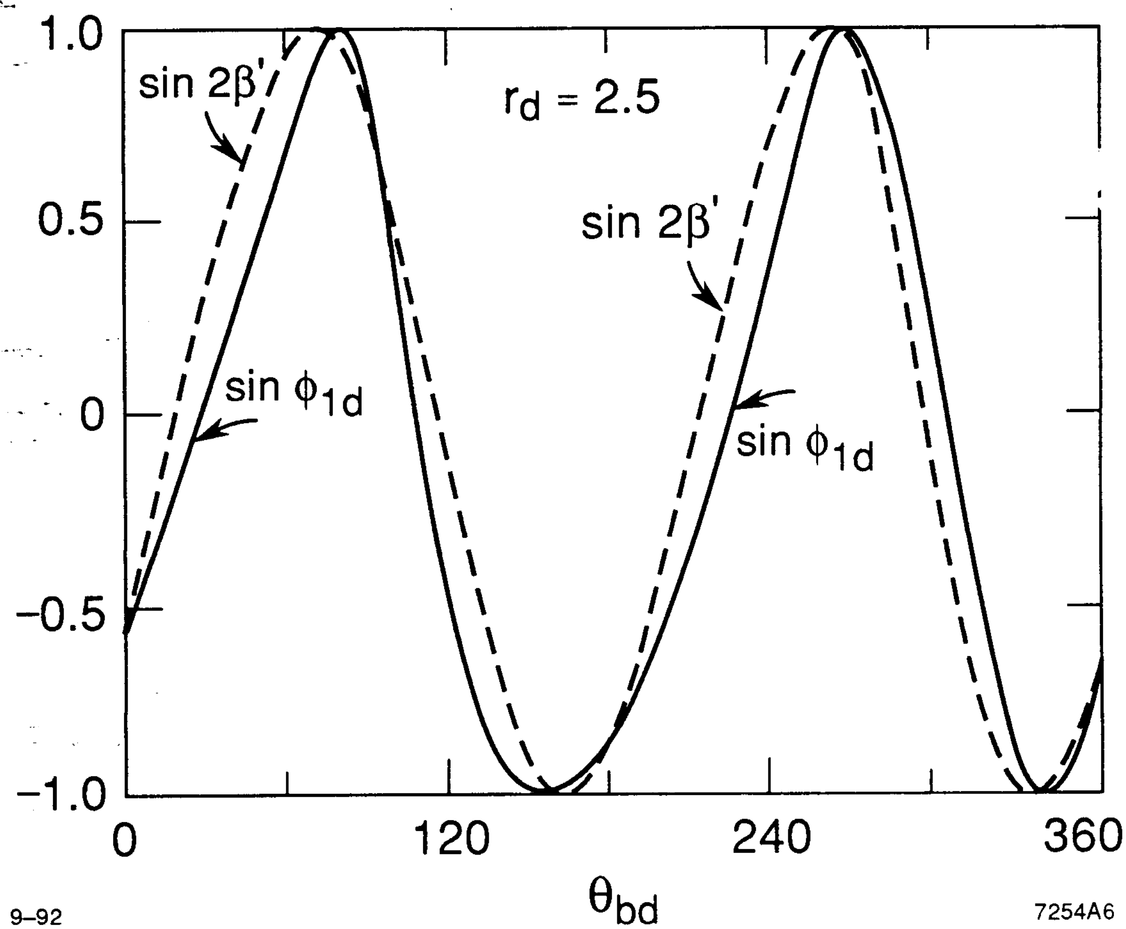

1.0
0.5
0
−0.5
−1.0
0
120
240
360
θ_{bd}
sin 2β'
sin 2β'
r_d = 2.5
sin ϕ_{1d}
sin ϕ_{1d}
9–92
7254A6

Fig. 6

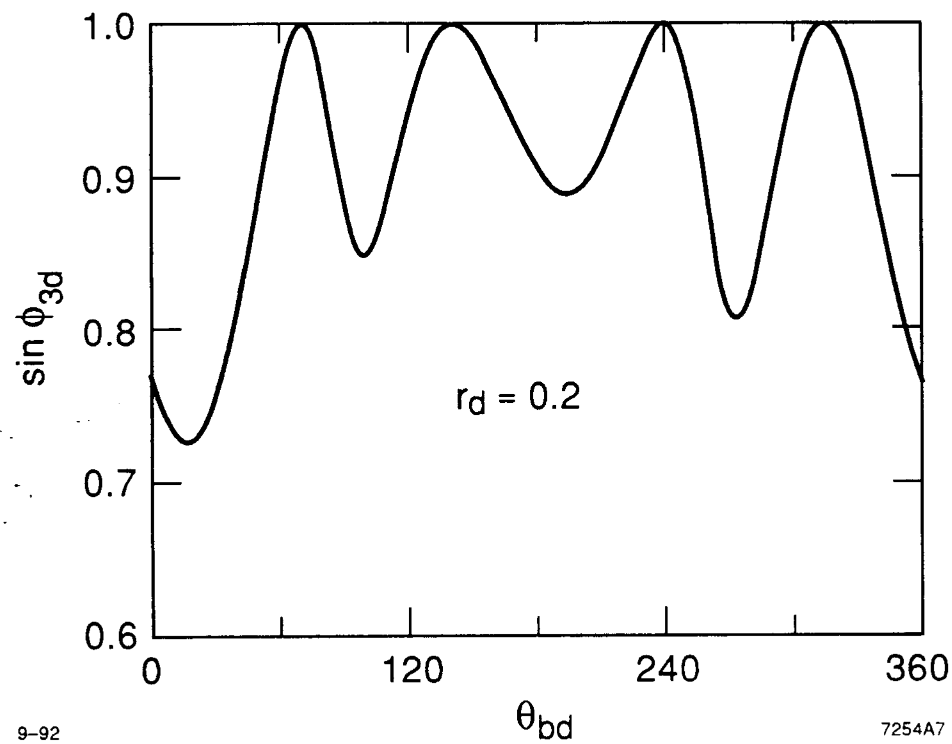


Fig. 7

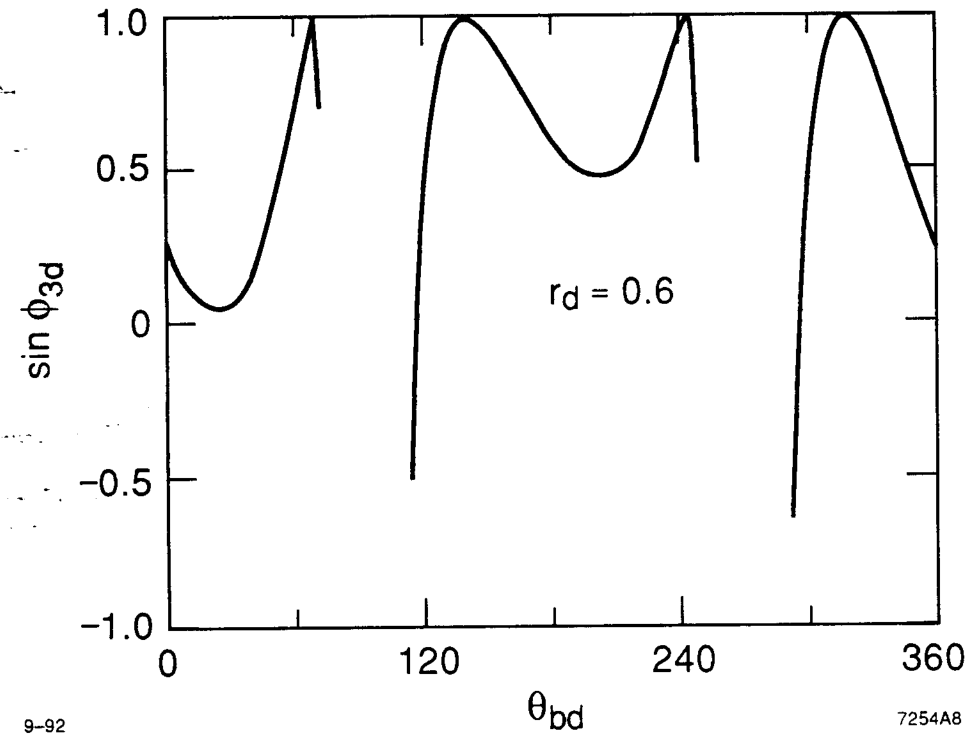


Fig. 8

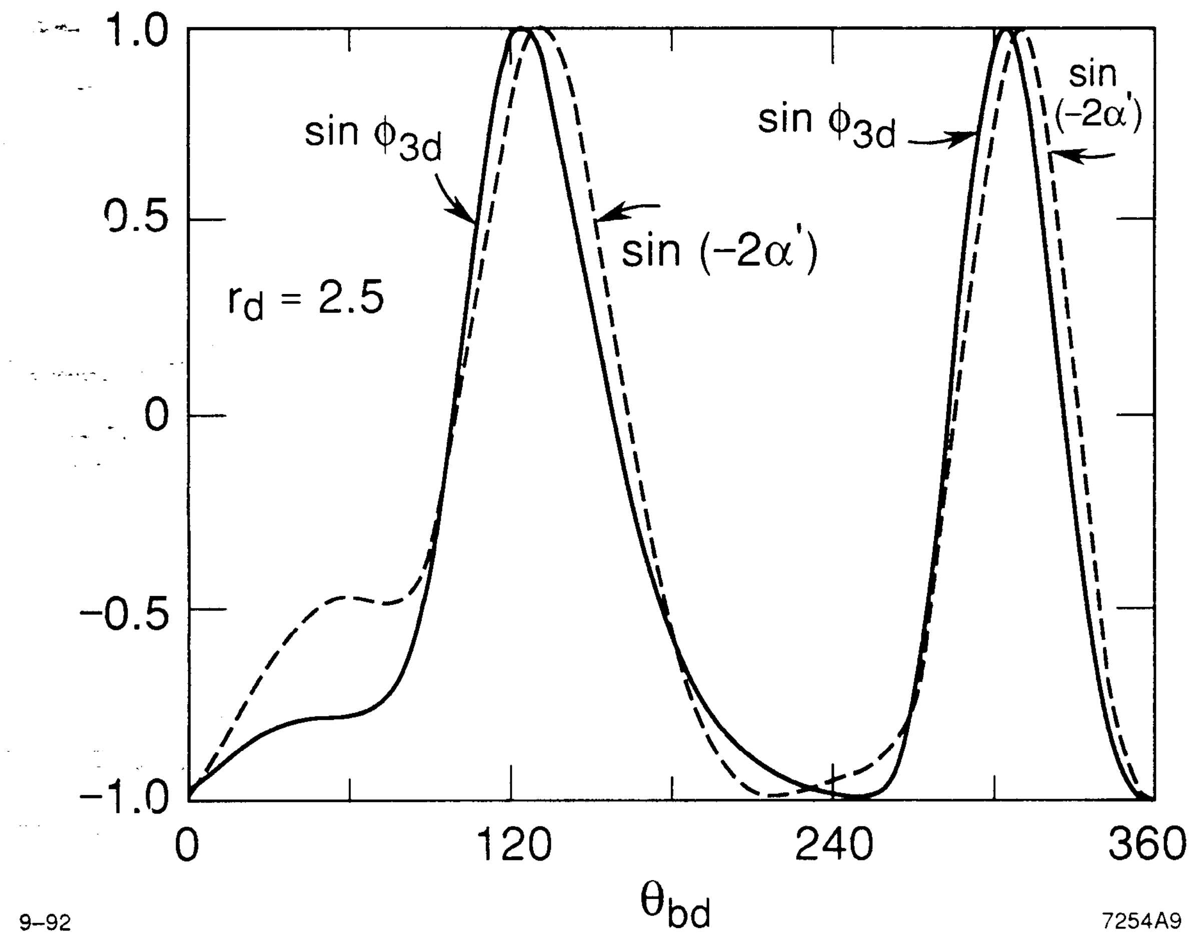

1.0
0.5
0
-0.5
-1.0
0
120
240
360
θbd
sin φ3d
sin (-2α')
sin φ3d
sin (-2α')
rd = 2.5
9-92
7254A9

Fig. 9