SLAC–PUB–5771
UCSD/PTH 92-09
March 1992
T/E

The Residual Mass Term in the Heavy Quark Effective Theory*

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ABSTRACT

We reformulate the heavy quark effective theory in the presence of a residual mass term, which has been taken to vanish in previous analyses. While such a convention is permitted, the inclusion of a residual mass allows us to resolve a potential ambiguity in the choice of the expansion parameter which defines the effective theory. We show to subleading order in the mass expansion that physical quantities computed in the effective theory do not depend on the expansion parameter.

Submitted to *Nuclear Physics B*

⋆ Work supported by the Department of Energy, contracts DE–AC03–76SF00515 and DE–FG03–90ER40546.

1. 1. Introduction

There has been much recent interest in the limit of QCD in which the mass of a heavy quark is taken to be much larger than the characteristic scale $\Lambda_{\rm QCD}$ of the strong interactions [1-14]. In this limit it is natural to consider an expansion of the QCD lagrangian in inverse powers of the heavy quark mass, as well as kinematics in which the heavy quark, even though bound into a hadron, is almost on shell. Such an expansion has been used to lowest order in models of heavy hadrons [15,16], in QCD sum rule calculations [17-21], and in lattice gauge theory [22-25]. However, to put the predictions of this heavy quark effective theory (HQET) on firm footing it will be necessary to perform computations to subleading order in the expansion in inverse mass $1/m_Q$. But here arises a potential ambiguity, namely in what mass should one expand: the pole mass computed in perturbation theory, the $\overline{\rm MS}$ mass, the mass of the physical hadron, or some other parameter? Our intuition tells us that nothing physical can depend on this choice, and this is in fact the case. It is the purpose of this note to demonstrate in detail how physical matrix elements are insensitive to variations in the expansion parameter.

Throughout this paper we will use dimensional regularization to cut off the ultraviolet divergences which arise in the HQET. We will not address the important issue of whether power divergences, which appear when a dimensionful regulator is employed, require additional nonperturbative subtractions in the effective theory [26]. However, the arguments given here are sufficient to show that if such effects are important and introduce additional uncertainties into computations, they are unrelated to ambiguities in the choice of the expansion parameter which defines the effective theory. We feel that it is important to disentangle these two issues.

The potential ambiguity in the choice of expansion parameter arises because in a confining theory such as QCD there is no true "pole" mass which can be assigned to a heavy quark. In the absence of such a canonical choice, a variety of perturbative prescriptions (*e.g.*, the pole mass to a given order, or the mass renormalized at some scale) compete with "physical" prescriptions such as the mass of the lightest hadron containing the heavy quark, or this mass minus some fixed number.

In this paper we will show that these various choices correspond in the effective theory to various values of a new parameter, the "residual mass" δm. Section 2 is devoted to the construction of this more general form of the HQET. In Section 3 we discuss, to subleading order in $1/m_Q$, hadronic matrix elements of currents containing two heavy quarks. Currents with one heavy and one light quark are addressed in Section 4. Section 5 contains a brief summary of our results.

2. 2. HQET with a Residual Mass Term

Let us fix any reasonable choice for the heavy quark mass and denote it by m_Q. Each value of m_Q yields a different effective theory; it is the parameter which defines, once and for all, the relationship between the HQET and full QCD. Following refs. [9-11], we derive the effective theory as an expansion in $1/m_Q$. We rewrite the heavy quark momentum P_μ as

$$P_\mu = m_Q v_\mu + k_\mu \ , \tag{1}$$

where v_μ is the four-velocity of the hadron containing the heavy quark; in the $m_Q \to \infty$ limit it is also the velocity of the heavy quark itself. Note that k_μ does not scale with m_Q; in fact, it is this property of separating out the part of P_μ that scales with the heavy quark mass which characterizes a "reasonable" choice of m_Q.* We also define effective fields $h_Q(x)$ in terms of the original fields $Q(x)$ by

$$h_Q(x) = \exp(\mathrm{i} m_Q \not{v}\, v\cdot x)\, Q(x) \ . \tag{2}$$

These fields create and annihilate heavy quarks and antiquarks moving at velocity v_μ. We further specialize to heavy quarks by imposing the condition $\not{v} h_Q = h_Q$. To lowest order in the $1/m_Q$ expansion, the effective lagrangian is obtained by matching matrix elements in QCD with those in the effective theory. We find

$$\mathcal{L}_{\rm eff} = \overline{h}_Q(\mathrm{i} v\cdot D) h_Q - \delta m\, \overline{h}_Q h_Q = \overline{h}_Q v^\mu (\mathrm{i} D_\mu - \delta m v_\mu) h_Q \ , \tag{3}$$

where $D_\mu = \partial_\mu - \mathrm{i} g A_\mu$ is the gauge-covariant derivative.

The term proportional to δm corresponds to a residual mass for the heavy quark in the effective theory, of a size of order $\Lambda_{\rm QCD}$. In general, such a term arises due to an incomplete cancelation of the full theory mass by the field redefinition (2). Changes in m_Q will induce changes in this uncompensated mass: if $m_Q \to m_Q + \alpha$, then $\delta m \to \delta m - \alpha$. Similarly the residual momentum changes according to $k_\mu \to k_\mu - \alpha v_\mu$. We may write these relations in differential form as

$$\frac{\partial \delta m}{\partial m_Q} = -1 \ , \quad \frac{\partial k_\mu}{\partial m_Q} = -v_\mu \ . \tag{4}$$

The residual mass term has been neglected in all previous derivations based on the HQET. This is because m_Q always has been fixed implicitly, by imposing *post*

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* Note that the $\overline{\rm MS}$ mass $m_Q(m_Q)$ is not a "reasonable" choice, since in this case k_μ contains a piece proportional to $\alpha_s(m_Q) m_Q v_\mu$.

facto the condition $\delta m = 0$. Clearly there is some choice of m_Q which accomplishes this, even if we cannot compute it. In other words, these derivations have fixed the equations of motion of the heavy quark in the HQET to be $(\mathrm{i}v \cdot D)h_Q = 0$, rather than the more general condition $(\mathrm{i}v \cdot D - \delta m)h_Q = 0$. Of course setting $\delta m = 0$ is as valid a choice of m_Q as any other; in fact, for perturbative calculations it is by far the most convenient one. Yet nowhere it is forced upon us. Finally we note that although it is possible to formulate the HQET with any mass δm of the order of Λ_{QCD}, if there is more than one heavy quark in the theory it is convenient to choose δm to be the same for all of them, so as not to violate the heavy quark symmetry explicitly at leading order.

Essentially the only modification which we have introduced into the effective theory is the change in the equation of motion for the heavy quark, which we may write in momentum space as

$$v^\mu(k_\mu - \delta m v_\mu + gA_\mu)\, h_Q = 0 \ . \tag{5}$$

This equation is used in the matching conditions which determine the effective lagrangian to subleading order in $1/m_Q$, as well as in the matching of currents in the full theory onto currents in the effective theory. Recall that we may use the equations of motion here because the matching conditions are obtained by requiring the equality of *physical* matrix elements in the two theories.

We have recalculated, to leading logarithmic order in perturbation theory and to order $1/m_Q$, the operators and currents which arise in the HQET with two heavy quarks, which we shall call for concreteness b and c. This corresponds to QCD plus a contact term which describes the semileptonic decay of b to c quarks:

$$\mathcal{L}_{\mathrm{full}} = \overline{\mathrm{b}}(\mathrm{i}\,\not{D} - m_{\mathrm{b}})\mathrm{b} + \overline{\mathrm{c}}(\mathrm{i}\,\not{D} - m_{\mathrm{c}})\mathrm{c} + \overline{\mathrm{c}}\Gamma^\mu \mathrm{b} L_\mu + \mathcal{L}_{\mathrm{light}} \ , \tag{6}$$

where $\mathcal{L}_{\mathrm{light}}$ describes the interactions of the light quarks and gluons. Here L_μ is the charged lepton current (which includes constants such as G_F and V_{cb}), and we have written the left-handed heavy quark current in the general form $\overline{\mathrm{c}}\Gamma^\mu\mathrm{b}$. In the HQET, $\mathcal{L}_{\mathrm{light}}$ is unchanged, while the kinetic and interaction terms are expanded in powers of $1/m_Q$. The strong interaction lagrangian for the heavy quarks becomes

$$\begin{aligned}\mathcal{L}_{\mathrm{kin}} &= \overline{h_{\mathrm{b}}}(\mathrm{i}v_{\mathrm{b}} \cdot D - \delta m)h_{\mathrm{b}} + \frac{1}{2m_{\mathrm{b}}}\overline{h}_{\mathrm{b}}(\mathrm{i}D - \delta m v_{\mathrm{b}})^2 h_{\mathrm{b}} \\ &\quad + a_{\mathrm{b}}(\mu)\frac{g}{4m_{\mathrm{b}}}\overline{h}_{\mathrm{b}}\sigma_{\mu\nu}G^{\mu\nu}h_{\mathrm{b}} \ + \ (\mathrm{b} \to \mathrm{c}) \ ,\end{aligned} \tag{7}$$

where we have ignored an operator whose matrix elements vanish by the equations of motion. Such operators do not contribute to physical matrix elements. In leading logarithmic approximation, $a_Q(\mu) = [\alpha_s(m_Q)/\alpha_s(\mu)]^{9/\beta}$, where $\beta = 33 - 2N_f$, and N_f is the number of light quark flavors. The μ-dependence of the coefficients is required to cancel the subtraction-point dependence of the renormalized operators in the usual way. We emphasize that $m_{\rm b}$ and $m_{\rm c}$ are *fixed* (μ-independent) expansion parameters which define the effective theory. Finally, the effective lagrangian is written in terms of renormalized heavy quark fields, which are related to the bare fields by a factor $Z_Q^{1/2} = [\alpha_s(m_Q)/\alpha_s(\mu)]^{4/\beta}$.

The interaction term in the effective theory is fixed by the matching of the currents $J = \overline{\rm c}\Gamma{\rm b}$ in the full theory onto linear combinations of effective currents. Decomposing to order $1/m_Q$ the full theory on-shell spinor u in terms of the two-component spinor u_h which describes the heavy quark in the effective theory,

$$u = \left[1 + \frac{1}{2m_Q}(\not{k} - \delta m)\right] u_h \ , \tag{8}$$

we obtain the matching conditions at tree level. Additional operators are then induced by the renormalization group. To leading logarithmic order, the result is

$$J_{\rm eff} = c_0(\mu)Q_0 + c_1(\mu)Q_1 + c_2(\mu)Q_2 + c_3(\mu)Q_3 + c_4(\mu)Q_4 \ , \tag{9}$$

where

$$\begin{aligned}
Q_0 &= {\rm e}^{{\rm i}\phi}\,\overline{h}_{\rm c}\Gamma h_{\rm b} \ , \\
Q_1 &= {\rm e}^{{\rm i}\phi}\,\frac{1}{2m_{\rm c}}\overline{h}_{\rm c}(-{\rm i}\overleftarrow{\not{D}} - \delta m)\Gamma h_{\rm b} \ , \\
Q_2 &= {\rm e}^{{\rm i}\phi}\,\frac{1}{2m_{\rm c}}\overline{h}_{\rm c}(-{\rm i}v_{\rm b}\cdot\overleftarrow{D} - \delta m v_{\rm c}\cdot v_{\rm b})\Gamma h_{\rm b} \ , \\
Q_3 &= {\rm e}^{{\rm i}\phi}\,\frac{1}{2m_{\rm b}}\overline{h}_{\rm c}\Gamma({\rm i}\,\not{D} - \delta m)h_{\rm b} \ , \\
Q_4 &= {\rm e}^{{\rm i}\phi}\,\frac{1}{2m_{\rm b}}\overline{h}_{\rm c}\Gamma({\rm i}v_{\rm c}\cdot D - \delta m v_{\rm c}\cdot v_{\rm b})h_{\rm b} \ .
\end{aligned} \tag{10}$$

We have again neglected operators whose matrix elements vanish by the equations of motion. The phase $\phi = -(m_{\rm b}v_{\rm b} - m_{\rm c}v_{\rm c})\cdot x$ compensates for the field redefinition (2). At tree level, $c_0 = c_1 = c_3 = 1$, $c_2 = c_4 = 0$, while summing the leading logarithms yields

$$\begin{aligned}
c_0(\mu) = c_1(\mu) = c_3(\mu) &= \left(\frac{\alpha_s(\widetilde{m})}{\alpha_s(\mu)}\right)^{a_L} \ , \\
c_2(\mu) = c_4(\mu) &= -\frac{16}{\beta}\left(\frac{r(w)-w}{w^2-1}\right)\left(\frac{\alpha_s(\widetilde{m})}{\alpha_s(\mu)}\right)^{a_L}\ln\left(\frac{\alpha_s(\widetilde{m})}{\alpha_s(\mu)}\right) \ ,
\end{aligned} \tag{11}$$

where

$$
\begin{aligned}
w = v_{\rm c}\cdot v_{\rm b}, \qquad a_L = \frac{8}{\beta}\big[w r(w) - 1\big] \ , \\
r(w) = \frac{1}{\sqrt{w^2-1}}\ln\left(w+\sqrt{w^2-1}\right) \ .
\end{aligned}
\tag{12}
$$

These expressions arise when the transition from QCD to the effective theory is implemented at a single scale $\widetilde{m}$, which in this case is some mass between $m_{\rm c}$ and $m_{\rm b}$. Improvements such as including the full one loop matching conditions, summing the logarithms between $m_{\rm c}$ and $m_{\rm b}$ to resolve the ambiguity in $\widetilde{m}$, or working to higher order in perturbation theory, are discussed in the literature [27-32].

From (7) and (10) it is apparent that in the effective theory with a nonvanishing δm the covariant derivative acting on a heavy quark field $h_Q(v)$ always appears in the combination

$$
{\rm i}\mathcal{D}_\mu(v) \equiv {\rm i}D_\mu - \delta m v_\mu \ , \quad {\rm i}\overleftarrow{\mathcal{D}}_\mu(v) \equiv {\rm i}\overleftarrow{D}_\mu + \delta m v_\mu \ . \tag{13}
$$

Note that the "magnetic interaction" operator in (7) is also of this type, since $G_{\mu\nu}$ is proportional to $[\mathcal{D}_\mu, \mathcal{D}_\nu]$. From (4) we may conjecture that matrix elements of the operator $\mathcal{D}$ do not depend on the choice of m_Q. We shall now demonstrate that this is indeed the case.

3. 3. Hadronic Matrix Elements

Let us evaluate the hadronic matrix elements of the weak current in the effective theory with a residual mass term, repeating the arguments of ref. [13] in this more general context. For simplicity, we shall restrict ourselves to the discussion of $1/m_{\rm c}$ corrections only. The inclusion of terms proportional to $1/m_{\rm b}$ is straightforward [16]. We will also specialize, for purpose of illustration, to transitions between $B^{(*)}$ and $D^{(*)}$ mesons. At next-to-leading order in the heavy quark expansion, one has to include the $1/m_{\rm c}$ corrections to the current and to the hadronic wave functions. Matrix elements of the effective current (9) can be evaluated most concisely by using a covariant trace formalism [12,13,33,34],

$$
\begin{aligned}
\langle\, D^{(*)}(v_{\rm c})\,|\,\overline{h}_{\rm c}\Gamma h_{\rm b}\,|\,B^{(*)}(v_{\rm b})\,\rangle &= -\xi_0(w,\mu)\,{\rm Tr}\left[\,\overline{M}_D\Gamma M_B\,\right] \ , \\
\langle\, D^{(*)}(v_{\rm c})\,|\,\overline{h}_{\rm c}\big[-{\rm i}\overleftarrow{\mathcal{D}}_\mu(v_{\rm c})\big]\Gamma h_{\rm b}\,|\,B^{(*)}(v_{\rm b})\,\rangle &= -\,{\rm Tr}\left[\,\xi_\mu(v_{\rm c},v_{\rm b},\mu)\,\overline{M}_D\Gamma M_B\,\right] \ ,
\end{aligned}
\tag{14}
$$

where pseudoscalar and vector meson states are represented respectively as

$$
M(v) = -\sqrt{m_M}\,\frac{1+\not{v}}{2}\gamma^5 \ , \qquad M^*(v,\varepsilon) = \sqrt{m_{M^*}}\,\frac{1+\not{v}}{2}\,\not{\varepsilon} \ . \tag{15}
$$

The universal Isgur-Wise function ξ_0 is the only form factor that appears in leading order of the $1/m_Q$ expansion [4]. At subleading order, there are additional functions. The most general Lorentz-invariant decomposition of ξ_μ is

$$\xi^\mu(v_\mathrm{c}, v_\mathrm{b}, \mu) = \xi_+(w,\mu)\,(v_\mathrm{c}^\mu + v_\mathrm{b}^\mu) + \xi_-(w,\mu)\,(v_\mathrm{c}^\mu - v_\mathrm{b}^\mu) - \xi_3(w,\mu)\,\gamma^\mu\ . \tag{16}$$

The form factors ξ_i, which parametrize matrix elements in the effective theory, are independent of the heavy quark masses. They depend, however, on the renormalization point μ and on the kinematic invariant $w = v_\mathrm{c}\cdot v_\mathrm{b}$. From here on we will suppress these arguments.

Performing an integration by parts on the interaction lagrangian and using the equations of motion (see ref. [13] for details), we find that in matrix elements

$$\overline{h}_\mathrm{c}\big[-\mathrm{i}v_\mathrm{b}\cdot \overleftarrow{\mathcal{D}}(v_\mathrm{c})\big]\Gamma h_\mathrm{b} \to (\overline{\Lambda} - \delta m)(w-1)\,\overline{h}_\mathrm{c}\Gamma h_\mathrm{b}\ . \tag{17}$$

Here $\overline{\Lambda}$ denotes the difference between the mass of the hadron containing the heavy quark and m_Q, *i.e.* in this case $\overline{\Lambda} = m_B - m_\mathrm{b} = m_D - m_\mathrm{c}$. The relation (17) implies that, in matrix elements, we may replace

$$Q_2 \to \frac{1}{2m_\mathrm{c}}(\overline{\Lambda} - \delta m)(w-1)\,\overline{h}_\mathrm{c}\Gamma h_\mathrm{b}\ . \tag{18}$$

The matrix elements of Q_2 (and Q_4, which obeys an analogous relation) are thus proportional to the Isgur-Wise function ξ_0. Furthermore, we see that they involve the combination $(\overline{\Lambda} - \delta m)$, which is invariant under shifts of m_c and m_b.

As in ref. [13], the equations of motion and relation (17) may be used to reduce the number of independent form factors. We find

$$\begin{aligned} \xi_- &= \frac{1}{2}(\overline{\Lambda} - \delta m)\,\xi_0\ , \\ (w+1)\,\xi_+ &= \frac{1}{2}(w-1)(\overline{\Lambda} - \delta m)\,\xi_0 - \xi_3\ . \end{aligned} \tag{19}$$

The corrections proportional to $1/m_\mathrm{b}$, although we do not present them explicitly, involve the same set of universal functions since the second of eqs. (14) can be related by an integration by parts to a formula for the matrix elements of $\overline{h}_\mathrm{c}\Gamma\mathrm{i}\mathcal{D}_\mu(v_\mathrm{b})h_\mathrm{b}$.

The $1/m_\mathrm{c}$ corrections to the hadronic wave functions come from insertions of the subleading operators

$$\frac{1}{2m_\mathrm{c}}\,\overline{h}_\mathrm{c}(\mathrm{i}\mathcal{D})^2 h_\mathrm{c} + a_\mathrm{c}(\mu)\frac{g}{4m_\mathrm{c}}\,\overline{h}_\mathrm{c}\sigma^{\mu\nu}G_{\mu\nu}h_\mathrm{c} \equiv \mathcal{O}_1 + a_\mathrm{c}(\mu)\mathcal{O}_2 \tag{20}$$

into matrix elements of the leading order currents. This gives rise to additional unknown functions. In particular, insertions of $\mathcal{O}_2$ induce violations of the heavy quark spin symmetry. Extending the trace formalism to these contributions [13], we defined Lorentz and CP invariant form factors χ_i via the time-ordered products †

$$
\begin{aligned}
&\langle\, D^{(*)}(v_\mathrm{c})\,|\,\mathrm{i}\int d^4y\, T\left[\,\overline{h}_\mathrm{c}\Gamma h_\mathrm{b}(0), \mathcal{O}_1(y)\,\right]\,|\,B^{(*)}(v_\mathrm{b})\,\rangle \\
&\qquad\qquad = -\frac{\chi_1(w,\mu)}{m_\mathrm{c}}\ \mathrm{Tr}\left[\,\overline{M}_D\Gamma M_B\,\right]\ , \\
&\langle\, D^{(*)}(v_\mathrm{c})\,|\,\mathrm{i}\int d^4y\, T\left[\,\overline{h}_\mathrm{c}\Gamma h_\mathrm{b}(0), \mathcal{O}_2(y)\,\right]\,|\,B^{(*)}(v_\mathrm{b})\,\rangle \\
&\qquad\qquad = -\frac{1}{m_\mathrm{c}}\ \mathrm{Tr}\left[\left(\mathrm{i}\chi_2(w,\mu)\,\gamma^\mu v_\mathrm{b}^\nu + \chi_3(w,\mu)\,\sigma^{\mu\nu}\right)\overline{M}_D\sigma_{\mu\nu}\frac{(1+\not{v}_\mathrm{c})}{2}\Gamma M_B\right]\ .
\end{aligned}
\tag{21}
$$

Hence to order $1/m_\mathrm{c}$ the hadronic matrix elements of the weak current may be expressed in terms of five universal (real) functions of w and the parameter $(\overline{\Lambda}-\delta m)$. As mentioned above, the corrections proportional to $1/m_\mathrm{b}$ involve the same functions [16].

Evaluating the traces in (14) and (21), we obtain the vector and axial vector current matrix elements relevant for semileptonic $B\to D^{(*)}$ decays. To order $1/m_\mathrm{c}$, they are ‡

$$
\begin{aligned}
\langle\, D\,|\,V^\mu\,|\,B\,\rangle = \sqrt{m_D m_B}\,\Big\{ & c_0\,\xi_0\,(v_\mathrm{c}^\mu + v_\mathrm{b}^\mu) \\
& + \frac{c_1}{2m_\mathrm{c}}\left[\xi_0\,(\overline{\Lambda}-\delta m) - 2\xi_3\right](v_\mathrm{c}^\mu - v_\mathrm{b}^\mu) \\
& + \frac{c_2}{2m_\mathrm{c}}\,\xi_0\,(\overline{\Lambda}-\delta m)(w-1)\,(v_\mathrm{c}^\mu + v_\mathrm{b}^\mu) \\
& + \frac{c_0}{2m_\mathrm{c}}\left[2\chi_1 - 4(w-1)a_\mathrm{c}\,\chi_2 + 12a_\mathrm{c}\,\chi_3\right](v_\mathrm{c}^\mu + v_\mathrm{b}^\mu)\Big\}\ ,
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
\langle\, D^*(\varepsilon)\,|\,V^\mu\,|\,B\,\rangle = \mathrm{i}\sqrt{m_{D^*}m_B}\,\varepsilon^{\mu\nu\alpha\beta}\,\varepsilon_\nu^* v_{\mathrm{c}\alpha}v_{\mathrm{b}\beta}\Big\{ & c_0\,\xi_0 + \frac{c_1}{2m_\mathrm{c}}\,\xi_0\,(\overline{\Lambda}-\delta m) \\
& + \frac{c_2}{2m_\mathrm{c}}\,\xi_0\,(\overline{\Lambda}-\delta m)(w-1) + \frac{c_0}{2m_\mathrm{c}}\left[2\chi_1 - 4a_\mathrm{c}\,\chi_3\right]\Big\}\ ,
\end{aligned}
\tag{23}
$$

† This definition of χ_2 and χ_3 differs from that of ref. [13] by the prefactor $a_\mathrm{c}(\mu)$.
‡ We correct an error in expressions (3.11) and (3.14) of ref. [13].

$$
\begin{aligned}
\langle\, D^*(\varepsilon)\,|\, A^\mu \,|\, B\,\rangle =& \sqrt{m_{D^*} m_B}\,\Big\{ c_0\,\xi_0\,\big[\varepsilon^{*\mu}(w+1) - \varepsilon^*\!\cdot v_{\rm b}\, v_{\rm c}^\mu\big] \\
&+ \frac{c_1}{2m_{\rm c}}\,\bigg[\frac{2}{w+1}\big[\xi_0\,(\overline{\Lambda}-\delta m) + \xi_3\big]\,\varepsilon^*\!\cdot v_{\rm b}\,(v_{\rm c}^\mu + v_{\rm b}^\mu) \\
&\qquad + \xi_0\,(\overline{\Lambda}-\delta m)\,\big[(w-1)\varepsilon^{*\mu} - \varepsilon^*\!\cdot v_{\rm b}\, v_{\rm c}^\mu\big]\bigg] \\
&+ \frac{c_2}{2m_{\rm c}}\,\xi_0\,(\overline{\Lambda}-\delta m)(w-1)\,\big[\varepsilon^{*\mu}(w+1) - \varepsilon^*\!\cdot v_{\rm b}\, v_{\rm c}^\mu\big] \\
&+ \frac{c_0}{2m_{\rm c}}\,\Big[4a_{\rm c}\,\chi_2\,\varepsilon^*\!\cdot v_{\rm b}\,(v_{\rm c}^\mu - v_{\rm b}^\mu) \\
&\qquad + \big[2\chi_1 - 4a_{\rm c}\,\chi_3\big]\,\big[\varepsilon^{*\mu}(w+1) - \varepsilon^*\!\cdot v_{\rm b}\, v_{\rm c}^\mu)\big]\Big]\Big\}\ .
\end{aligned}
\tag{24}
$$

It is clear that the requirement that these expressions be invariant under redefinitions of the expansion parameters $m_{\rm c}$ and $m_{\rm b}$ is equivalent to the requirement that the form factors be invariant, *i.e.*

$$
\frac{\partial \xi_i}{\partial m_Q} = \frac{\partial \chi_i}{\partial m_Q} = 0\ , \tag{25}
$$

where m_Q is any of the heavy quark masses. This is not unexpected, since we have defined the form factors in terms of matrix elements of operators built from the "generalized" covariant derivative $\mathcal{D}$. It is these form factors, together with the invariant combination $(\overline{\Lambda}-\delta m)$, which are observables of the effective theory.

We recall that by evaluating matrix elements of the charge operator V_0 at zero recoil one can show that the Isgur-Wise function is normalized at zero recoil, $\xi_0(1,\mu) = 1$, and that two of the subleading form factors vanish at this point, $\chi_1(1,\mu) = \chi_3(1,\mu) = 0$, if one uses the hadron masses in the prefactors in (22)–(24). These conditions are preserved in the effective theory with a residual mass term.

The physical matrix elements must be invariant not only under shifts in the expansion parameter m_Q, but also under changes in the renormalization point μ. We can use this fact to deduce the μ-dependence of the universal functions ξ_0, ξ_3 and χ_i from that of the short distance coefficients c_i. To first order in α_s we find that

$$
\begin{aligned}
\mu\frac{\partial \xi_i}{\partial \mu} &= -\frac{4\alpha_s}{3\pi}\,[w r(w) - 1]\,\xi_i\ ; \quad i = 0,3\ , \\
\mu\frac{\partial \chi_1}{\partial \mu} &= -\frac{4\alpha_s}{3\pi}\left\{[w r(w) - 1]\,\chi_1 - \xi_0\,(\overline{\Lambda}-\delta m)\,\frac{r(w) - w}{w+1}\right\}\ , \\
\mu\frac{\partial \chi_i}{\partial \mu} &= -\frac{4\alpha_s}{3\pi}\left[w r(w) + \tfrac{1}{8}\right]\chi_i\ ; \quad i = 2,3\ .
\end{aligned}
\tag{26}
$$

These conditions must be obeyed by any model calculation which is sensitive to the μ-dependence, such as leading-log improved QCD sum rules.

4. 4. Heavy-Light Currents

The arguments given above apply equally to currents with one heavy and one light quark. We have verified that again the only change with respect to the case $\delta m = 0$ consists in the replacement of the covariant derivative D_μ acting on a heavy quark field by $\mathcal{D}_\mu$ from (13). Derivatives acting on the light quark field remain unchanged. The study of heavy-light currents is interesting, however, since it will allow us to give an operational definition of the invariant combination $(\overline{\Lambda} - \delta m)$ which does not contain the residual mass explicitly.

In leading logarithmic approximation, the $1/m_Q$ expansion of the current $J = \overline{\mathrm{q}}\Gamma Q$ in the effective theory reads

$$J_{\mathrm{eff}} = b_0(\mu)\, Q_0' + \frac{1}{2m_Q} \sum_{i=1}^{6} b_i(\mu)\, Q_i' \ , \tag{27}$$

where

$$Q_0' = \mathrm{e}^{\mathrm{i}\phi'}\, \overline{\mathrm{q}}\, \Gamma h_Q \ , \quad Q_1' = \mathrm{e}^{\mathrm{i}\phi'}\, \overline{\mathrm{q}}\, \Gamma\, \mathrm{i}\not{\mathcal{D}} h_Q, \tag{28}$$

and $\phi' = -m_Q v\cdot x$. The remaining operators in (27) involve the light quark mass m_{q} or derivatives acting on the light quark fields. They cannot easily be written in terms of an arbitrary Γ. For $\Gamma = \gamma_\mu$, it is convenient to choose

$$\begin{aligned} Q_2' &= \mathrm{e}^{\mathrm{i}\phi'}\, m_{\mathrm{q}}\, \overline{\mathrm{q}}\, \gamma_\mu h_Q \ , & Q_5' &= \mathrm{e}^{\mathrm{i}\phi'}\, \overline{\mathrm{q}}\, (-\mathrm{i}v\cdot \overleftarrow{D})\gamma_\mu h_Q \ , \\ Q_3' &= \mathrm{e}^{\mathrm{i}\phi'}\, m_{\mathrm{q}}\, \overline{\mathrm{q}}\, v_\mu h_Q \ , & Q_6' &= \mathrm{e}^{\mathrm{i}\phi'}\, \overline{\mathrm{q}}\, (-\mathrm{i}v\cdot \overleftarrow{D}) v_\mu h_Q \ . \\ Q_4' &= \mathrm{e}^{\mathrm{i}\phi'}\, \overline{\mathrm{q}}\, (-\mathrm{i}\overleftarrow{D}_\mu) h_Q \ , & & \end{aligned} \tag{29}$$

We neglect again operators whose matrix elements vanish by the equations of motion. A set of axial vector operators with the same coefficients $b_i(\mu)$ is obtained from (28) and (29) by replacing $\overline{\mathrm{q}} \to -\overline{\mathrm{q}}\gamma^5$ and $m_{\mathrm{q}} \to -m_{\mathrm{q}}$. From tree level matching one computes $b_0 = b_1 = 1$ and $b_i = 0$ for $i \geq 2$, while at leading logarithmic order (this may be extracted from ref. [27])

$$\begin{aligned} b_0(\mu) &= b_1(\mu) = z^{-6/\beta} \ , \\ b_2(\mu) &= -\tfrac{10}{9} - \tfrac{1}{9} z^{6/\beta} + \tfrac{8}{9} z^{3/\beta} + \tfrac{1}{3} z^{-6/\beta} \ , \\ b_3(\mu) &= \tfrac{10}{9} - \tfrac{26}{9} z^{6/\beta} + \tfrac{4}{9} z^{3/\beta} + \tfrac{4}{3} z^{-6/\beta} \ , \\ b_4(\mu) &= \tfrac{10}{3} - \tfrac{4}{3} z^{3/\beta} - 2 z^{-6/\beta} \ , \\ b_5(\mu) &= -\tfrac{10}{9} - \tfrac{4}{27} z^{3/\beta} + \tfrac{34}{27} z^{-6/\beta} - \tfrac{16}{\beta} z^{-6/\beta} \ln z \ , \\ b_6(\mu) &= -\tfrac{20}{9} + \tfrac{88}{27} z^{3/\beta} - \tfrac{28}{27} z^{-6/\beta} \ , \end{aligned} \tag{30}$$

where $z = \alpha_s(m_Q)/\alpha_s(\mu)$ and $\beta = 33 - 2N_f$.

Let us now consider matrix elements of the effective current between a heavy meson and the vacuum. The application of the trace formalism to this particular case allows us to write

$$\begin{aligned}
\langle\, 0\,|\, \overline{\mathrm{q}}\,\Gamma h_Q\,|\, M(v)\,\rangle &= \mathrm{i}\,\frac{F(\mu)}{2}\,\mathrm{Tr}\left[\,\Gamma\, M\,\right] , \\
\langle\, 0\,|\, \overline{\mathrm{q}}\,\Gamma\, \mathrm{i}\mathcal{D}_\mu\, h_Q\,|\, M(v)\,\rangle &= \frac{\mathrm{i}}{2}\,\mathrm{Tr}\left[\,\big(F_1(\mu) v_\mu + F_2(\mu)\gamma_\mu\big)\,\Gamma\, M\,\right] , \\
\langle\, 0\,|\, \overline{\mathrm{q}}\,(-\mathrm{i}\overleftarrow{D}_\mu)\Gamma h_Q\,|\, M(v)\,\rangle &= \frac{\mathrm{i}}{2}\,\mathrm{Tr}\left[\,\big(F_3(\mu) v_\mu + F_4(\mu)\gamma_\mu\big)\,\Gamma\, M\,\right] .
\end{aligned} \tag{31}$$

Using an integration by parts and the equations of motion of the effective theory, one can show that [20]

$$\begin{aligned}
F_1(\mu) = F_2(\mu) = F_4(\mu) &= -\frac{F(\mu)}{3}\left[\overline{\Lambda} - \delta m - m_{\mathrm{q}}\right] . \\
F_3(\mu) &= -\frac{F(\mu)}{3}\left[4(\overline{\Lambda} - \delta m) - m_{\mathrm{q}}\right] .
\end{aligned} \tag{32}$$

Additional parameters $G_1(\mu)$ and $G_2(\mu)$ are induced by insertions of subleading operators from the effective lagrangian (7), in analogy to the functions $\chi_i(w,\mu)$.

It is clear from (32) that physical matrix elements depend only on the combination $(\overline{\Lambda} - \delta m)$ and the form factors F, G_1, and G_2, which must therefore be invariant under redefinitions of m_Q. As an example, we present the result for the ratio of the decay constants of a heavy vector and pseudoscalar meson [20]

$$\begin{aligned}
\frac{f_{M^*}\sqrt{m_{M^*}}}{f_M\sqrt{m_M}} = 1 + \frac{1}{2m_Q}\Big\{ &(\overline{\Lambda} - \delta m - m_{\mathrm{q}})\, c_0^{-1}(\mu)\left[\tfrac{4}{3} b_1(\mu) + \tfrac{2}{3} b_4(\mu) + b_6(\mu)\right] \\
&+ m_{\mathrm{q}}\, c_0^{-1}(\mu)\left[2b_2(\mu) + b_3(\mu) + b_4(\mu) + b_6(\mu)\right] - 16 G_2(\mu)\Big\} .
\end{aligned} \tag{33}$$

From the fact that this ratio is μ-independent we conclude that the scale dependence of G_2 involves m_{q} and $(\overline{\Lambda} - \delta m)$, in analogy to (26).

The relations (32) allow us to give an operational definition of the combination $(\overline{\Lambda} - \delta m)$, which is invariant under redefinitions of m_Q, without reference to the residual mass term. The reason is that this combination is induced when the *usual* covariant derivative acts on the light quark. In particular, it follows that

$$\frac{\langle\, 0\,|\, \overline{\mathrm{q}}\,(\mathrm{i}v\cdot\overleftarrow{D})\Gamma h_Q\,|\, M(v)\,\rangle}{\langle\, 0\,|\, \overline{\mathrm{q}}\,\Gamma h_Q\,|\, M(v)\,\rangle} = \overline{\Lambda} - \delta m . \tag{34}$$

Note that this relation expresses $(\overline{\Lambda} - \delta m)$ directly in terms of properties of the light degrees of freedom, instead of in terms of a difference of heavy masses. We may even propose (34) as a definition of the energy of the light degrees of freedom in the background of a static color source. This equation might also be useful in extracting the value of $(\overline{\Lambda} - \delta m)$ from lattice gauge theory.

5. 5. Summary

We have demonstrated in detail that there is no important ambiguity in the choice of the HQET expansion parameter $1/m_Q$, in the sense that all physical quantities are independent of this choice. This should hardly be surprising. The ambiguity is resolved by the introduction of a new quantity δm, which is a non-trivial dynamical parameter of the effective theory. In order to be insensitive to the choice of the expansion parameter m_Q, hadronic form factors in the HQET must be defined in terms of matrix elements containing the operator $\mathrm{i}\mathcal{D}_\mu = \mathrm{i}D_\mu - \delta m v_\mu$. Physical quantities will then depend on these invariant form factors as well as on the combination $(\overline{\Lambda} - \delta m)$, which replaces $\overline{\Lambda}$ in previous analyses. This combination is invariant under redefinitions of m_Q.

While calculable in continuum perturbation theory, the residual mass term contains power divergences when the theory is regulated with a dimensionful cutoff, divergences which may be associated with incalculable nonperturbative processes [26]. The issue of whether such effects make the HQET useless beyond leading order in $1/m_Q$ is beyond the scope of this letter. It is important, however, to disentangle this larger question from that of whether the theory is plagued by a fundamental ambiguity in the choice of expansion parameter. We have shown that it is not, in the presence of the residual mass δm. We believe that this is the correct framework within which to investigate further the possible importance of nonperturbative effects.

ACKNOWLEDGEMENTS

It is a pleasure to thank Nathan Isgur, Chris Sachrajda and Guido Martinelli for provocative questions and interesting discussions. Portions of this work were initiated at the 1992 Santa Barbara Workshop on Heavy Quark Symmetry, and we are most grateful to the Institute for Theoretical Physics for their kind hospitality. One of us (M.N.) gratefully acknowledges financial support from the Research Fellowship Program of the BASF Aktiengesellschaft and the German National Scholarship Foundation.

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