Universal Properties of the Electromagnetic Interactions of Spin-One Systems[*]

STANLEY J. BRODSKY

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

and

JOHN R. HILLER

Department of Physics, University of
Minnesota–Duluth, Duluth, Minnesota 55812

ABSTRACT

The dominance of helicity–conserving amplitudes in gauge theory is shown to imply universal ratios for the charge, magnetic, and quadrupole form factors of spin–one bound states: $G_C(Q^2) : G_M(Q^2) : G_Q(Q^2) = (1 - \frac{2}{3}\eta) : 2 : -1$. These ratios hold at large space–like or time–like momentum transfer in the case of composite systems such as the ρ or deuteron in QCD. They are also the ratios predicted for the electromagnetic couplings of the $W^\pm$ for all Q^2 in the standard model at tree level. In the case of the deuteron, the leading–twist perturbative QCD predictions are valid at $Q^2 = |q^2| \gg \Lambda_{QCD}M_d$, but do not require the kinematical ratio $\eta = Q^2/4M_d^2$ to be large. These results provide new all–angle predictions for the leading power behavior of the tensor polarization $T_{20}(Q^2,\theta)$ and the invariant ratio $B(Q^2)/A(Q^2)$. We also use a generalization of the Drell-Hearn Gerasimov sum rule to show that the magnetic and quadrupole moment of any composite spin–one system take on the canonical values $\mu = e/M$ and $Q = -e/M^2$ in the strong binding limit of zero bound-state radius or infinite excitation energy. This allows new empirical constraints on the possible internal structure of the Z^0 and $W^\pm$ vector bosons. Simple gauge invariant and covariant models and null zone theory are used to illustrate these results. Complications that arise when the Breit frame is used for form factor analyses are also pointed out.

Submitted to *Physical Review D*

I. INTRODUCTION

The low-energy theorem [1] for the forward Compton amplitude at threshold, and the helicity selection rules [2] of perturbative quantum chromodynamics (QCD) for exclusive scattering amplitudes at high-momentum transfer, indicate that many properties of a system bound by a gauge theory are universal and are the same as those of a corresponding elementary particle of the same spin and charge. In this paper, we shall explore this universality for the case of spin-one bound states in QCD, including both the ρ meson and the deuteron. In particular, we shall focus on the behavior of the electromagnetic form factors of composite spin-one systems at large momentum transfer, and on the fundamental constraints on the magnetic and quadrupole moments of hadronic and nuclear states imposed by Compton-scattering sum rules.

In order to motivate the notion of universality, we first discuss the application of the Drell-Hearn-Gerasimov (DHG) sum rule [3–5] to the anomalous magnetic moment of a spin-one state. We then show how one can use Tung's [6] extension of this analysis to obtain a new sum rule for the anomalous quadrupole moment of a general spin-one system. Together these sum rules show that in the limit of zero radius or large excitation energies, the magnetic moment μ_1 and quadrupole moment Q_1 approach canonical values:

$$\mu_1 = \frac{e}{M}, \quad Q_1 = -\frac{e}{M^2}, \tag{1.1}$$

where e is the total charge and M is the mass of the spin-one system. These are the same values obtained [7] for the intermediate vector bosons $W^\pm$ in the tree approximation to the standard model. It should be emphasized that the sum rule constraints on Q_1 and μ_1 do not rely on perturbation theory, but only on the existence of unsubtracted dispersion relations for the relevant helicity-flip Compton amplitudes. The deviation of the observed magnetic and quadrupole moments from the canonical values thus define the "anomalous" moments of a general spin-one system: $\mu_a \equiv \mu_1 - \frac{e}{M}$ and $Q_a \equiv Q_1 + \frac{e}{M^2}$, dynamical contributions which must be strictly due to internal structure.

Various theoretical and experimental constraints have already been suggested for the magnetic and quadrupole moments of the W. The electromagnetic couplings of the intermediate vector boson are constrained by renormalizability and tree-level unitarity [8] to be those of the standard model. Experiments that have or will place bounds on nonstandard couplings include measurements of $g - 2$ for the muon [9], $p\bar{p} \to W\gamma X$ [10], the decay $\mu \to e\gamma$ [11], heavy-ion collisions [12] and e^+e^- annihilation processes [13].

The definition of the three parity-conserving and time-reversal invariant electromagnetic form factors of a spin-one object is well known [14]. We will discuss the implications of perturbative QCD and helicity selection rules for these form factors at high momentum transfer in terms of the ratio B/A of Rosenbluth form factors

[15] and the tensor polarization [16] T_{20}. A general derivation of the form factors of the deuteron in a light-front formulation has also been given by Chung et al. [17]. Our emphasis in this paper will be on the predictions of perturbative QCD for the large momentum transfer behavior and understanding the scale for their validity. Our analysis will be carried out using the standard light-cone frame (LCF) ($q^\pm = q^0 \pm q^3$):

$$q = (q^+, q^-, \vec{q}_\perp) = (0, \frac{Q^2}{p^+}, \vec{q}_\perp), \quad p = (p^+, p^-, \vec{p}_\perp) = (p^+, \frac{M^2}{P^+}, \vec{0}_\perp), \quad (1.2)$$

where, as in the Drell-Yan [18] frame, the photon momentum is transverse to the direction of the incident spin-one system, with $q_\perp^2 = Q^2 = -q^2$, and $q^+ = 0$ for space-like photons. Elastic kinematics requires $(p + q)^2 = M^2$, $2p \cdot q = Q^2$. Although this frame is often referred to as the infinite momentum frame, the light-cone kinematics are exact, and no limiting procedure has to be taken. In particular, the value of the frame-dependent momentum p^+ is irrelevant. In the transverse frame analysis, the dominance of the helicity-zero to zero matrix element of the electromagnetic current is sufficient as an assumption to determine the relationship between all three form factors. Predictions for time-like photons, such as in $e^+e^- \to \rho^+\rho^-$ can be obtained from crossing relations.

We repeat the analysis using the Breit frame with $\vec{p}$ and $\vec{q}$ parallel. Here we find that predictions for form factors require information about nonleading matrix elements; helicity-conserving matrix elements alone are not sufficient to determine the magnetic form factor. In addition, as recently emphasized by Sawicki [19], light-cone perturbation theory analyses in the Breit frame must take into account nondiagonal Z-graph contributions to the electromagnetic current.

The standard Rosenbluth cross section [15] for elastic electron scattering on a target of any spin in the lab frame,

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)} \frac{E'}{E} \left[A(Q^2) + B(Q^2)\tan^2(\theta/2) \right], \quad (1.3)$$

in terms of invariants, is:

$$\frac{d\sigma}{dt} = \frac{4\pi\alpha^2}{t^2} \left[\left(1 + \frac{ts}{(s - M^2)^2} \right) A(-t) - \frac{M^2 t}{(s - M^2)^2} B(-t) \right]. \quad (1.4)$$

The dimensional counting rules [20] of perturbative QCD for exclusive two-body scattering processes at large s, with t/s fixed, predict

$$\frac{d\sigma}{dt} \sim \frac{1}{t^{n-2}} f(t/s), \quad (1.5)$$

where n is the total number of incident and outgoing fields. In the case of electron-deuteron elastic scattering, $n = 14$. This implies that $A(-t)$ falls as t^{-10} and that B falls at least as fast as [21] tA/M^2 for the deuteron. Thus the ratio B/A could rise

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as fast as t/M^2. However, a more complete analysis [22] finds that the ratio becomes a constant. We elaborate on this in a later section.

A critical issue is the determination of the momentum-transfer scale at which perturbative QCD can make meaningful predictions for quantities such as B/A and the tensor polarization for spin-one targets T_{20}. Because of kinematic factors, this scale is different from the scale at which A can be predicted. We discuss an appropriate choice of scale and obtain predictions for B/A and T_{20}. In the latter case, our predictions for the deuteron differs significantly from what is usually quoted [23] for the experimentally accessible region [24].

With regard to perturbative calculations at large momentum transfer, we would like to draw attention to the work of Farrar, Huleihel, and Zhang [25] on the helicity-zero to zero deuteron form factor. They find that hidden-color degrees of freedom in the deuteron wavefunction may be important in obtaining the correct perturbative QCD predictions for normalization of the deuteron form factors at experimentally accessible momentum transfers. In addition, as shown in Ref. [26], the evolution of the deuteron's distribution amplitude leads to the dominance of hidden color state contributions in the asymptotic domain of very large momentum transfer. Our analysis will be independent of the existence of the relative normalization of hidden-color states.

To confirm the generality of the form factor analysis, we also consider a simple model in which the composite spin-one system is constructed in a Lorentz-invariant and gauge-invariant way from two spin-1/2 constituents in a zero-binding limit of one-boson exchange. The analysis of the electromagnetic interactions in this model gives a simple demonstration of the connection between radiation null zones [27] and the natural magnetic and quadrupole moments of spin-one systems.

An outline for the remainder of the paper is as follows. In Section II we discuss the sum rules and anomalous moments of spin-one systems. Form factors and their ratios are analyzed in Section III. The zero-binding model is presented in Section IV; this includes discussion of null zones in radiative processes. Finally, Section V contains a brief summary.

II. SUM RULES

The low momentum transfer properties of both elementary particles and composite systems can be related by general principles to integrals over scattering amplitudes. The best-known of these relationships is the Drell-Hearn-Gerasimov (DHG) sum rule [3, 4] for the anomalous magnetic moment of spin-1/2 systems. It can be obtained by using an unsubtracted dispersion relation and a low-energy theorem [1] for the helicity-flip Compton amplitude. The generalization to arbitrary spin has been made [4, 5]; the form for the spin-one case is

$$\mu_a^2 = \frac{1}{\pi} \int_{\omega_{\text{th}}}^{\infty} \frac{d\omega}{\omega} \left[\sigma_P(\omega) - \sigma_A(\omega) \right] , \qquad (2.1)$$

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where $\mu_a = \mu_1 - \frac{e}{M}$ is by definition the anomalous magnetic moment for the spin-one system, σ_P (σ_A) is the total cross section for absorption of a photon with spin parallel (antiparallel) to the spin of the target, and ω is the photon energy, with $\omega_{\rm th}$ the threshold energy. Although an experimental verification of the DHG sum rule for nucleons has been carried out [28], it would be interesting to verify this result for deuterons.

The extension of the DHG sum rule analysis to include the quadrupole moment of a spin-one system requires a low-energy theorem to second-order in the photon energy. At this order, the polarizability enters the forward Compton amplitude [29] in addition to the quadrupole moment. However, Tung [6] has shown that one can obtain the following sum rule for the non-forward Compton amplitude:

$$\mu_a^2 + \frac{2t}{M^2}\left(\mu_a + \frac{M}{2}Q_a\right)^2 = \frac{1}{4\pi}\int_{\nu_{\rm th}^2}^{\infty} \frac{d\nu^2}{(\nu - t/4)^3}\left(\mathcal{I}m\, f_P(s,t) - \mathcal{I}m\, f_A(s,t)\right),$$

(2.2)

where M is the mass, $Q_a = Q_1 + \frac{e}{M^2}$ defines the anomalous quadrupole moment, ν is $(s-u)/4$, and f_P (f_A) is the helicity amplitude for parallel (antiparallel) photon and target spins. The standard Mandelstam variables s, t and u are used. The normalization of the helicity amplitudes given by Bardeen and Tung [30] is used to derive (2.2). With this normalization, the optical theorem takes the form

$$\mathcal{I}m\, f_{P,A} = 2\nu\sigma_{P,A}.$$

(2.3)

In the forward direction, (2.2) reduces to (2.1), with use of $\omega = \nu/M$. A sum rule that relates Q_a to total cross sections does not exist [6].

It is interesting to apply (2.2) to composite spin-one systems which become point-like in some limit. In such a case the photoabsorption cross section and the integrals that appear in (2.1) and (2.2) vanish as the size $R \to 0$ or the excitation energy $\nu_{\rm th} \to \infty$. Thus in this limit, $Q_a \to 0$ and $\mu_a \to 0$. Therefore $\mu_1 = \frac{e}{M}$ and $Q_1 = -\frac{e}{M^2}$ are the canonical moments of a spin-one system. Note that this analysis is non-perturbative. In the case of the standard model, the integrals in (2.1) and (2.2) are higher order $\sim \mathcal{O}(\alpha^2)$; thus again $\mu_W = \frac{e}{M}$ and $Q_W = -\frac{e}{M^2}$, up to Schwinger-like radiative corrections of order α/π. Specific models for compositeness of leptons and intermediate vector bosons are discussed by Brodsky and Drell [31], Abbott and Farhi [32], and Claudson, Farhi, and Jaffe [33]. The DHG sum rule has also been used to place constraints on quark and lepton compositeness and excited states in the strong-coupling standard model [32] by Jaffe and Ryzak [34].

Note that any spin-one system is required to satisfy the extended DHG sum rule (2.2). This implies universal behavior for the properties of spin-one particles in the zero-radius limit. In the next section we explore a complimentary universality for the form factors of such particles at large momentum transfer in gauge theory.

III. SPIN-ONE FORM FACTORS

A. General formulas

For a spin-one particle, the matrix elements of the electromagnetic current J^μ can be written in terms of three form factors, assuming parity and time-reversal invariance [14]. We define

$$G^\mu_{h'h} = \langle p'h'|J^\mu|ph\rangle\,, \tag{3.1}$$

where $|ph\rangle$ is an eigenstate of momentum p and helicity h. This matrix element can be written in the form [14]

$$G^\mu_{h'h} = -\{G_1(Q^2)\epsilon'^*\cdot\epsilon[p^\mu + p'^\mu] + G_2(Q^2)[\epsilon^\mu\epsilon'^*\cdot q - \epsilon'^{*\mu}\epsilon\cdot q]$$
$$-G_3(Q^2)\epsilon\cdot q\,\epsilon'^*\cdot q(p^\mu + p'^\mu)/(2M^2)\}\,, \tag{3.2}$$

with $Q^2 = -q^2$, $q = p' - p$, and $\epsilon \equiv \epsilon_h$ and $\epsilon' \equiv \epsilon_{h'}$ the initial and final polarization vectors. The Lorentz-invariant form factors $G_i(Q^2)$ are related to the charge, magnetic and quadrupole form factors [14]

$$G_C = G_1 + \frac{2}{3}\eta G_Q\,, \quad G_M = G_2\,, \quad G_Q = G_1 - G_2 + (1+\eta)G_3\,, \tag{3.3}$$

where $\eta = \frac{Q^2}{4M^2}$ is a kinematic factor. At zero momentum transfer, these form factors are proportional to the usual static quantities of charge e, magnetic moment μ_1 and quadrupole moment Q_1:

$$eG_C(0) = e\,, \quad eG_M(0) = 2M\mu_1\,, \quad eG_Q(0) = M^2Q_1\,. \tag{3.4}$$

The Rosenbluth cross section (1.3) for elastic electron scattering on a spin-one particle is determined by these form factors via the definitions

$$A = G_C^2 + \frac{2}{3}\eta G_M^2 + \frac{8}{9}\eta^2 G_Q^2\,,$$
$$B = \frac{4}{3}\eta(1+\eta)G_M^2\,. \tag{3.5}$$

The tensor polarization [16] T_{20} can also be written in terms of these form factors as

$$T_{20}(Q^2,\theta) = -\frac{\frac{8}{9}\eta^2 G_Q^2 + \frac{8}{3}\eta G_C G_Q + \frac{2}{3}\eta G_M^2\left[\frac{1}{2} + (1+\eta)\tan^2(\frac{\theta}{2})\right]}{\sqrt{2}\left[A + B\tan^2(\frac{\theta}{2})\right]}\,. \tag{3.6}$$

The relationship (3.2) between the covariant form factors and current matrix elements can be inverted [17, 35] for any choice of Lorentz frame. In the standard LCF, defined by [18] $q^+ = 0$, $q_y = 0$, $q_x = Q$, all the form factors can be obtained from the plus component of three matrix elements:

$$G_C = \frac{1}{2p^+(2\eta+1)}\left[\frac{16}{3}\eta\frac{G^+_{+0}}{\sqrt{2\eta}} - \frac{2\eta-3}{3}G^+_{00} + \frac{2}{3}(2\eta-1)G^+_{+-}\right]$$

$$G_M = \frac{2}{2p^+(2\eta+1)}\left[(2\eta-1)\frac{G^+_{+0}}{\sqrt{2\eta}} + G^+_{00} - G^+_{+-}\right] \tag{3.7}$$

$$G_Q = \frac{1}{2p^+(2\eta+1)}\left[2\frac{G^+_{+0}}{\sqrt{2\eta}} - G^+_{00} - \frac{\eta+1}{\eta}G^+_{+-}\right].$$

In contrast, in the Breit frame, where $\vec{q}_\perp = 0$, $\vec{p} = -\frac{1}{2}\vec{q} = -\vec{p}'$, an x component of a current matrix element is needed to extract G_M:

$$G_C = \frac{-1}{2M\sqrt{1+\eta}}\left[\frac{1}{3}G^+_{00} - \frac{2}{3}G^+_{+-}\right]$$

$$G_M = \frac{2}{2M\sqrt{1+\eta}}\frac{G^x_{+0}}{\sqrt{2\eta}} \tag{3.8}$$

$$G_Q = \frac{-1}{2M\sqrt{1+\eta}}\frac{G^+_{00} + G^+_{+-}}{2\eta}.$$

Predictions of the behavior of the matrix elements as functions of momentum transfer can then be used to extract the Q^2 dependence of form factors.

B. Asymptotic forms

Perturbative QCD predicts [36] that the helicity-zero to zero matrix element G^+_{00} will be the dominant helicity amplitude at large Q^2 for lepton scattering on a spin-one bound state. This follows since quark helicity is conserved in the hard-scattering quark-gluon amplitude, and the dominant wave function coefficient, or distribution amplitude, has $L_z = 0$. However, it is important to distinguish two scales in the form-factor analysis. The primary scale for the validity of perturbative QCD predictions is set by the requirement that the momentum transfer through the hard-scattering amplitude and the propagators be large compared to the QCD scale $\Lambda_{\rm QCD}$. Since the current value of $\Lambda_{\overline{\rm MS}}$ lies between 120 MeV and 200 MeV [37], we take $\Lambda_{\rm QCD}$ to be of order 200 MeV. From estimates by Carlson and Gross [35] we conclude that the LCF helicity-flip amplitudes G^+_{+0} and G^+_{+-} are suppressed by factors of $\frac{\Lambda_{\rm QCD}}{Q}$ and $\left(\frac{\Lambda_{\rm QCD}}{Q}\right)^2$, respectively.

The second scale is a purely kinematic one. In order to control the kinematic factors in (3.7), and thereby retain dominance of G^+_{00}, one needs

$$Q \gg \sqrt{2M\Lambda_{\rm QCD}}. \tag{3.9}$$

This follows from the assumptions $G^+_{+0} \sim \frac{\Lambda_{\rm QCD}}{Q}G^+_{00}$ and $G^+_{+-} \sim \left(\frac{\Lambda_{\rm QCD}}{Q}\right)^2 G^+_{00}$. For the dimensionless ratio $\eta = \frac{Q^2}{4M^2}$, this requirement translates to $\eta \gg \frac{\Lambda_{\rm QCD}}{2M}$. Thus for

$M > \Lambda_{\mathrm{QCD}}$, the validity of perturbative QCD predictions does not depend on taking $\eta \gg 1$.

The present data for the deuteron form factor [38] $\sqrt{A}$, and also for the photodisintegration of the deuteron at large momentum transfer [39], appear to be consistent with perturbative QCD dimensional counting rules [20] and reduced amplitude scaling [21, 40]. The scaling of the dominant form factor $\sqrt{A}$ can be seen in Fig. 1, where the data is plotted in terms of the reduced form factor [21] $f_d(Q^2) \equiv A(Q^2)/F_N^2(Q^2/4)$, with F_N the dominant nucleon form factor. Perturbative QCD predicts asymptotic scaling for $Q^2 f_d(Q^2)$ up to calculable logarithms [26]. Thus, optimistically, one could expect that the dominance of G_{00}^+ begins at $Q^2 \sim 1\,\mathrm{GeV}^2$. We emphasize that the kinematic quantity η can be small in the perturbative QCD regime: for example for the deuteron $Q^2 = 5\,\mathrm{GeV}^2$ corresponds to $\eta \sim 0.35$, whereas we only require $\eta \gg \frac{\Lambda_{\mathrm{QCD}}}{2M} \simeq 0.05$.

Thus the domain for leading-power perturbative QCD predictions for the deuteron form factors is $Q^2 \gg 2M_d\Lambda_{\mathrm{QCD}} \sim 0.8\,\mathrm{GeV}^2$. In this domain, one obtains [22] from (3.7),

$$\frac{B}{A} \simeq \frac{4\eta(\eta+1)}{\eta^2 + \eta + 3/4}, \quad T_{20}(\theta) \simeq -\sqrt{2}\frac{\eta[\eta - \frac{1}{2} + (\eta+1)\tan^2\frac{\theta}{2}]}{\eta^2 + \eta + \frac{3}{4} + 4\eta(\eta+1)\tan^2\frac{\theta}{2}}. \quad (3.10)$$

In the extreme limit $\eta \gg 1$, these reduce to

$$\frac{B}{A} \simeq 4, \quad T_{20} \simeq -\sqrt{2}\frac{1 + \tan^2\frac{\theta}{2}}{1 + 4\tan^2\frac{\theta}{2}}. \quad (3.11)$$

The asymptotic value of $-\sqrt{2}$ usually quoted [23] for T_{20} only applies when θ is zero, and when Q^2 is much larger than $4M^2$, which is nearly 8 GeV^2 for the deuteron. For $\eta \ll 1$, which is relevant to experiment, we obtain

$$\frac{B}{A} \simeq \frac{16}{3}\eta, \quad T_{20} \simeq \frac{2\sqrt{2}}{3}\eta(1 - 2\tan^2\frac{\theta}{2}). \quad (3.12)$$

The essential assumption made in all of these results is that the G_{00}^+ amplitude is dominant.

The expressions derived for B/A and T_{20} are compared with experiment [38, 41, 24] in Figs. 2 and 3. Clearly, the presently available data do not come close to the prediction for B/A. However, for T_{20} the trend of the data is not inconsistent with the prediction. Data at larger momentum transfer is clearly needed. It would also be useful to compare elastic electron and positron deuteron scattering to check the size of the two-photon exchange interference contribution to $B(Q^2)$ in the dip region.

At lower Q^2, where perturbative QCD is inapplicable, the behavior of the ρ and deuteron form factors can have completely different properties. For example, the deuteron quadrupole moment is measured to be [42] $Q_d = eG_Q(0)/M^2 = (25.84 \pm$

$0.13)\frac{e}{M^2}$, whereas at large Q^2, $G_Q(Q^2)$ is predicted to be negative. The change in sign has led Carlson [43] to infer the existence of a zero in $G_Q(Q^2)$.

In the Breit frame, the assumption of G_{00}^+ dominance is insufficient for determination of G_M. In fact, in any frame where the momenta are collinear, G_{00}^+ does not contribute to G_M, and, therefore, not to B. Collinear momenta keep the spin quantization axis fixed; a magnetic interaction then requires a change in the spin state. We therefore retain the ratio

$$z \equiv \sqrt{2\eta}\frac{G_{+0}^x}{G_{00}^+},$$
(3.13)

which, using the scaling obtained by Carlson and Gross [35], can be estimated to be of order $z \sim \frac{\Lambda_{QCD}}{\sqrt{2}M} \sim 0.07$. For general η, the resulting perturbative estimates of B/A and T_{20} are

$$\frac{B}{A} \simeq \frac{4(\eta+1)z^2}{\eta+2z^2}, \quad T_{20} \simeq -\sqrt{2}\frac{\eta + \frac{1}{2}z^2 + z^2(\eta+1)\tan^2\frac{\theta}{2}}{\eta + 2z^2 + 4z^2(\eta+1)\tan^2\frac{\theta}{2}}.$$
(3.14)

For $\eta \gg 1$, they become

$$\frac{B}{A} \simeq 4z^2, \quad T_{20} \simeq -\sqrt{2}\frac{1 + z^2\tan^2\frac{\theta}{2}}{1 + 4z^2\tan^2\frac{\theta}{2}},$$
(3.15)

and, for $\eta \ll 1$, they reduce to

$$\frac{B}{A} \simeq \frac{4z^2}{\eta+2z^2}, \quad T_{20} \simeq -\sqrt{2}\frac{\eta + \frac{1}{2}z^2 + z^2\tan^2\frac{\theta}{2}}{\eta + 2z^2 + 4z^2\tan^2\frac{\theta}{2}}.$$
(3.16)

In the asymptotic limit, agreement with the LCF analysis is obtained only if the ratio z is unity. In the regime currently accessible to experiment, the two analyses produce completely different results. It should be noted that the evaluation of matrix elements of the transverse current J^x is treacherous in light-cone quantized theories, usually requiring Z-graph contributions [19] or surface terms [44].

The fewer assumptions required in the LCF analysis clearly make this method the preferred approach. To confirm that is the correct analysis, we compare the behavior of form factors for composites with the tree-level form factors of the W in the standard model.

C. Tree-level properties of the W^+

At tree level, the form factors of the W^+ are given by

$$G_C = 1 - \frac{2}{3}\eta, \quad G_M = 2, \quad G_Q = -1.$$
(3.17)

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These follow directly from the photon-absorption vertex in the standard model. At $Q^2 = 0$ this corresponds to [7] the canonical magnetic moment of e/M_W and quadrupole moment of $-e/M_W^2$. For comparison, the form factors of a composite spin-one object are, in the LCF, assuming helicity-zero to zero dominance,

$$G_C = (1 - \frac{2}{3}\eta)\frac{G_{00}^+}{2p^+(2\eta + 1)}, \quad G_M = 2\frac{G_{00}^+}{2p^+(2\eta + 1)}, \quad G_Q = -\frac{G_{00}^+}{2p^+(2\eta + 1)}.$$

(3.18)

Notice that the ratios of the three electromagnetic form factors $G_C : G_M : G_Q = (1 - \frac{2}{3}\eta) : 2 : -1$ are identical for elementary spin-one W's and for composite spin-one hadrons in QCD when G_{00}^+ is dominant. In particular, B/A and T_{20} for the W^+ are given by (3.10). Thus at large Q^2, perturbative QCD predicts that the ratio of form factors for deuterons, $\rho^\pm$, etc. become identical to those of the point-like spin-one fields of the standard model. We will see explicit realization of these results in the next section.

In the Breit frame analysis, the ratios of form factors do not match those for an elementary W. We therefore conclude that the LCF analysis is the correct approach.

IV. ZERO-BINDING MODEL

As a test of the correctness of the LCF analysis we shall study the form factors of a spin-one system in very simple gauge-invariant, Lorentz-invariant model, in which the composite system corresponds to two lightly-bound spin-$\frac{1}{2}$ constituents interacting via boson exchange. Results will be extracted in the zero-binding limit only. The analysis is similar to that required for constructing the hard scattering amplitude T_H in perturbative QCD analyses of mesonic form factors [36]. This model is thus directly applicable to the form factors of the ρ meson at large momentum transfer and, in the context of the reduced amplitude approach [40], is also applicable to the deuteron.

The wave functions used are generalizations of the vertex functions employed by Bagger and Gunion [45]. The functions factor into a spin-dependent part

$$\chi_{Jh} = \sum_{s_1 s_2} N_{s_1 s_2}^{Jh} \frac{1}{\sqrt{x_1 x_2}} u(p_1, s_1)\bar{v}(p_2, s_2),$$

(4.1)

where J is the total spin and h the helicity, and a spin-independent part $\phi(x)$. In the zero-binding limit, the constituents are collinear, $p_i = m_i p$, and the distribution amplitude $\phi(x)$ becomes $\delta(x_i - m_i/M)$. The spin wave functions then reduce to

$$\chi_{1h} = \frac{-1}{\sqrt{2}} \not{\epsilon}_h(\not{p} - M), \quad \chi_{00} = \frac{1}{\sqrt{2}}\gamma_5(\not{p} - M),$$

(4.2)

with ϵ_h a polarization vector given by

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$$\epsilon_\pm = \mp \frac{1}{\sqrt{2}}(0,\, 1,\, \pm i,\, 0)\,, \quad \epsilon_0 = \frac{1}{M}(|\vec{p}|, 0, 0, \sqrt{|\vec{p}|^2 + m^2}) \tag{4.3}$$

in any frame where $\vec{p}_\perp = 0$. The reduction can be done in a variety of ways, which differ by the choice of basis for the spinors. One can use the light-cone helicity basis of Brodsky and Lepage [36], a standard helicity basis [46], or the Weyl basis discussed by Hagiwara and Zeppenfeld [47]; except for different phases, they yield the same results.

To obtain form factors, we compute the matrix elements for the transition from p, h to p', h' in a one-boson exchange approximation. The corresponding diagrams are presented in Fig. 4. For a spin-one boson, with mass λ, the usual Feynman rules yield

$$G^\mu_{h'h} \propto \frac{e_1}{x_2 y_2 Q^2 + \lambda^2 - (x_2 - y_2)^2 M^2}\left[\frac{A^\mu_{h'h}}{x_2 Q^2 + m_1^2 - x_1^2 M^2} + \frac{B^\mu_{h'h}}{y_2 Q^2 + m_1^2 - y_1^2 M^2}\right] + (1 \leftrightarrow 2)\,, \tag{4.4}$$

where the proportionality constant is determined by the boson-fermion coupling, among other things, and where the numerators inside the brackets are Dirac traces. These traces are

$$A^\mu_{h'h} = \mathrm{Tr}\{\gamma_\nu \bar{\chi}_{Jh'}\gamma^\nu[\not{p}' - x_2 \not{p} + m_1]\gamma^\mu \chi_{Jh}\} \tag{4.5}$$

and

$$B^\mu_{h'h} = \mathrm{Tr}\{\bar{\chi}_{Jh'}\gamma^\mu[y_1 \not{p}' - \not{q} + m_1]\gamma^\nu \chi_{Jh}\gamma_\nu\}\,. \tag{4.6}$$

They can be reduced to

$$A^\mu_{h'h} = -4M^2\{\epsilon\cdot\epsilon'^*[p'^\mu + (\frac{m_1}{M} - x_2)p^\mu] + q\cdot\epsilon'^*(\frac{m_1}{M} + x_2)\epsilon^\mu - q\cdot\epsilon\,\epsilon'^{*\mu}\}\,. \tag{4.7}$$

and

$$B^\mu_{h'h} = -4M^2\{\epsilon\cdot\epsilon'^*[p^\mu + (\frac{m_1}{M} - y_2)p'^\mu] - q\cdot\epsilon(\frac{m_1}{M} + y_2)\epsilon'^{*\mu} + q\cdot\epsilon'^*\epsilon^\mu\}\,. \tag{4.8}$$

With the restriction to $x_i = y_i = m_i/M$, the matrix element is found to be

$$G^\mu_{h'h} \propto \frac{-e_1}{x_2^2 Q^2 + \lambda^2}\frac{M^2}{Q^2}\left[\epsilon\cdot\epsilon'^*\frac{x_1}{x_2}(p^\mu + p'^\mu) + \frac{1}{x_2}(q\cdot\epsilon'^*\epsilon^\mu - q\cdot\epsilon\,\epsilon'^{*\mu})\right] + (1 \leftrightarrow 2)\,. \tag{4.9}$$

Form factors can then be extracted by comparison with (3.2). We obtain

$$G_1 \propto \frac{e_1}{x_2^2 Q^2 + \lambda^2}\frac{M^2}{Q^2}\frac{x_1}{x_2} + \frac{e_2}{x_1^2 Q^2 + \lambda^2}\frac{M^2}{Q^2}\frac{x_2}{x_1}\,,$$

$$G_2 \propto \frac{e_1}{x_2^2 Q^2 + \lambda^2}\frac{M^2}{Q^2}\frac{1}{x_2} + \frac{e_2}{x_1^2 Q^2 + \lambda^2}\frac{M^2}{Q^2}\frac{1}{x_1}\,, \tag{4.10}$$

$$G_3 = 0\,.$$

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The magnetic form factor G_M is the same as G_2. The charge and quadrupole form factors are

$$G_C \propto \frac{e_1}{x_2^2 Q^2 + \lambda^2}\left[\frac{M^2}{Q^2}\frac{x_1}{x_2} - \frac{1}{6}\right] + \frac{e_2}{x_1^2 Q^2 + \lambda^2}\left[\frac{M^2}{Q^2}\frac{x_2}{x_1} - \frac{1}{6}\right] \qquad (4.11)$$

and

$$G_Q \propto -\frac{e_1}{x_2^2 Q^2 + \lambda^2}\frac{M^2}{Q^2} - \frac{e_2}{x_1^2 Q^2 + \lambda^2}\frac{M^2}{Q^2}. \qquad (4.12)$$

When the constituent masses are equal, these results become [48]

$$G_C \propto \frac{e_1 + e_2}{(Q/2)^2 + \lambda^2}\frac{M^2}{Q^2}\left(1 - \frac{2}{3}\eta\right),$$

$$G_M \propto 2\frac{e_1 + e_2}{(Q/2)^2 + \lambda^2}\frac{M^2}{Q^2}, \qquad (4.13)$$

$$G_Q \propto -\frac{e_1 + e_2}{(Q/2)^2 + \lambda^2}\frac{M^2}{Q^2}.$$

For the Rosenbluth form factor $\sqrt{A}$, given in (3.5), these results are consistent with those of Brodsky and Lepage [36]. The ratios of these form factors are identical to ratios of the tree-level form factors of the $W^{\pm}$, given in (3.17). They are then also identical to ratios obtained in the LCF, when helicity-zero to zero dominance is assumed.

The model (4.13) also provides a simple representation of the form factors of the W if it is a composite of two spin-1/2 fermions bound by a gauge interaction. In this case, λ^2 sets the scale of compositeness, and evidently $\lambda^2 \gg M_W^2$. In this model, which is constrained by gauge invariance and Lorentz invariance, we again obtain the usual ratios $G_C : G_M : G_Q = (1 - \frac{2}{3}\eta) : 2 : -1$, independent of the particulars of the theory.

There is an additional way to construct spin-one systems with natural magnetic and quadrupole moments. If one considers a spin-one state that consists of two collinear spin-1/2 constituents with charges e_i and masses m_i, such that $\frac{e_1}{m_1} = \frac{e_2}{m_2}$, and Dirac moments, then the composite will have $\mu_1 = \frac{e}{M}$, $Q_1 = -\frac{e}{M^2}$. The proof of this follows from the analysis of null zones [27] for radiative processes involving these particles. When $\frac{e_1}{m_1} = \frac{e_2}{m_2}$, then the null-zone condition [27] for the simultaneous vanishing of all helicity amplitudes,

$$\frac{e_1}{p_1 \cdot q} = \frac{e_2}{p_2 \cdot q}, \quad \frac{e_1}{p_1' \cdot q} = \frac{e_2}{p_2' \cdot q}, \qquad (4.14)$$

is satisfied at both the composite and constituent levels. However, null zones only arise if the spin currents cancel among themselves, that is when the charged particles have natural magnetic and quadrupole moments. Thus the moments defined by the

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sum rules discussed in Section II also are the moments that preserve null zones. In particular, all helicity amplitudes for the subprocess $u\bar{d} \to W^+\gamma$ vanish at $\cos\theta = e_d/e_W$ provided that the W^+ and the quarks have natural moments. A discussion of bounds on the W anomalous moment that can be obtained from present $p\bar{p} \to W\gamma X$ data has been recently given by Samuel *et al.* [10].

In the crossed reaction $e^+e^- \to \gamma^* \to V\bar{V}$ [49], where V is any massive vector particle with charge or charged constituents, one again predicts dominance of the $(0,0)$ helicity amplitude, so that the cross section $\frac{d\sigma}{d\Omega}$ is proportional to $\sin^2\theta$. This agrees with the perturbative QCD prediction of Ref. [2] for $e^+e^- \to \rho^+\rho^-$ and the standard-model prediction for $e^+e^- \to W^+W^-$ by Alles *et al.* [50]. Notice, however, that $e^+e^- \to W^+W^-$ receives contributions from ν exchange which do not appear in the above analysis. That the vector particle need not itself be charged implies that the $\sin^2\theta$ behavior of the cross section should hold at large $s = q^2$ for processes such as $e^+e^- \to K^{0*}\bar{K}^{0*}$. In each case, the time-like form factor ratios should satisfy $G_C : G_M : G_Q = (1 - \frac{2}{3}\eta) : 2 : -1$, where now $\eta = s/4M^2$.

V. SUMMARY

We have provided two new non-perturbative arguments for the selection of e/M and $-e/M^2$ as the natural magnetic and quadrupole moments of a spin-one particle. These are the canonical values that emerge in the strong binding limit of zero bound-state radius or infinite excitation energy of a composite spin-one system. The first argument, presented in Section II, is based on an extension [6] (2.2) of the DHG sum rule (2.1). The second uses the requirement that radiation null zones of composite particles must be the same as those of point-like particles; this is discussed near the end of Section IV. Arguments that have been given previously in the literature are perturbative in nature. They include the requirement of renormalizability and tree-level unitarity [8] which limits terms allowed in the interaction Lagrangian, and a perturbative analysis of the DHG sum rule [7]. In the case of the of the Z^0 and $W^\pm$ vector bosons, any deviation from these canonical values beyond that predicted from radiative corrections in the Standard model would provide empirical constraints on the possible internal structure of the gauge particles [10].

We have also established natural ratios for the electromagnetic form factors of spin-one systems in gauge theory:

$$G_C(Q^2) : G_M(Q^2) : G_Q(Q^2) = (1 - \frac{2}{3}\eta) : 2 : -1, \qquad (5.1)$$

where $\eta = |q^2|/4M^2$. These ratios hold at tree level for the W^+ in the standard model, and at large momentum transfer for hadrons in perturbative QCD. These results are most easily derived in the light-cone frame assuming the dominance of the helicity-zero to zero amplitude. In the Breit frame one has the complication of evaluating non-leading transverse current matrix elements.

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The ratios of the fundamental form factors (5.1) also determine the ratio B/A of Rosenbluth form factors [15] and T_{20}, the tensor polarization [16]. Both have been measured for the deuteron [38, 41, 24] out to momentum transfers where one might have thought perturbative QCD would apply. However, the expressions for these quantities contain kinematic factors that depend on $\eta = Q^2/4M^2$, which introduces M as a dimensional parameter in addition to the intrinsic QCD mass scale Λ_{QCD}. We have argued that the perturbative QCD predictions for B/A and T_{20} become valid for momentum transfers large compared to $\sqrt{2M\Lambda_{QCD}}$, not Λ_{QCD}. For the deuteron, this difference is significant and does postpone applicability of perturbative QCD.

The predictions made for B/A and T_{20} are given in (3.10). Comparisons of (3.10) with the deuteron data are shown in Figs. 2 and 3. The often-quoted prediction of $-\sqrt{2}$ for T_{20} applies only for momentum transfers so large that η is much larger than one. Such a large momentum transfer is not actually necessary for a prediction to be made. The general perturbative QCD prediction (3.10) should start to be valid at moderate momentum transfer $Q^2 >> 2M_d\Lambda_{QCD}$.

As a result of the universality of electromagnetic form factors, one can conclude that any seemingly point-like spin-one particle could actually be composite. This possibility has received considerable attention with respect to the W [32, 33]. In Section IV we explored a simple model for a composite spin-one particle and found that the expected form factor ratios can be obtained.

It should also be emphasized that the analysis presented here also applies to any spin-one bound state in gauge theory, for both space-like and time-like electromagnetic processes. In particular, the helicity zero amplitude should dominate and the form factor ratios (5.1) should hold for crossed reactions at large $s = q^2$ such as $e^+e^- \rightarrow \rho^+\rho^-$. It is clearly very important to verify the perturbative QCD predictions for this type of exclusive annihilation process.

Although we have considered only spin-one particles, one could imagine considering any class of composite systems with any fixed total spin. In particular, the general analysis of the spin-1/2 case can be applied to the form factors of the helium-3 nucleus in order to determine the scale where the underlying quark-gluon structure of more general nuclei becomes important.

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FIGURES

FIG. 1. Scaling of the reduced deuteron form factor f_d. The data are given in Ref. [38].

FIG. 2. Perturbative QCD predictions for B/A for the deuteron. Expressions in (3.10) and (3.12) of the text, which are valid in the regimes $Q^2 \gg 0.8\,\mathrm{GeV}^2$ and $16\,\mathrm{GeV}^2 \gg Q^2 \gg 0.8\,\mathrm{GeV}^2$, respectively, are plotted for comparison with values computed from data given in Ref. [38, 41]. We plot $\log(B/A)$ in order to show the full range of data.

FIG. 3. Perturbative QCD predictions for T_{20}. The expression in (3.10) of the text is plotted for various angles for comparison with data given in Ref. [24]. The prediction of the model suggested by Carlson in Ref. [43], which differs significantly, is also plotted. The horizontal line at $-\sqrt{2}$ is only relevant for large η and $\theta = 0$.

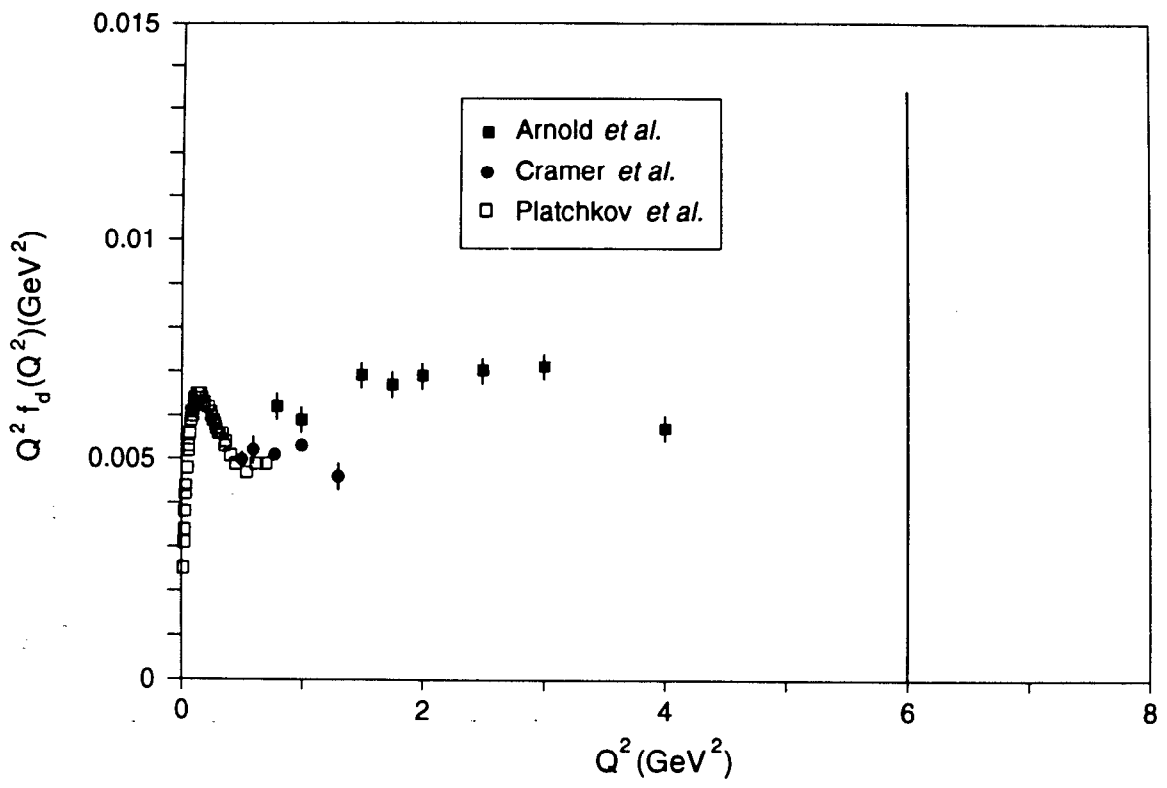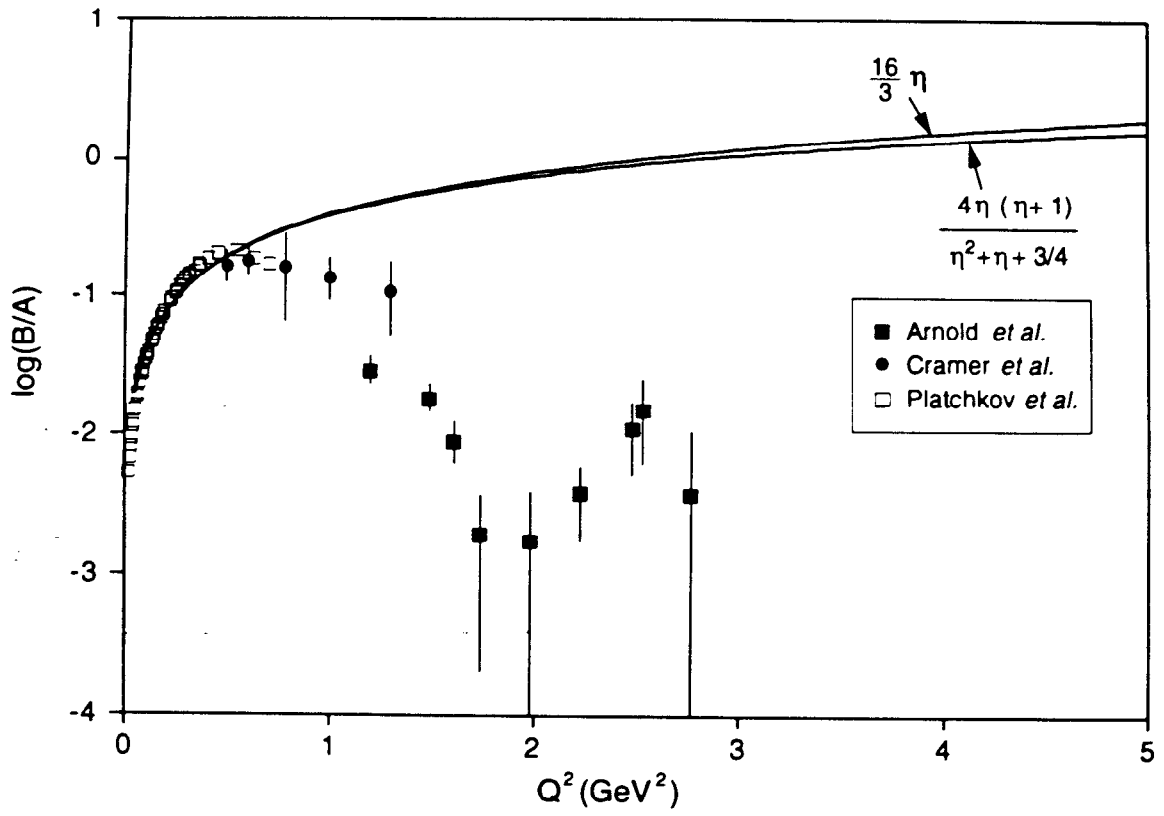FIG. 4. Feynman diagrams for the current matrix element in the one-boson exchange approximation. The model becomes gauge invariant in the collinear approximation.
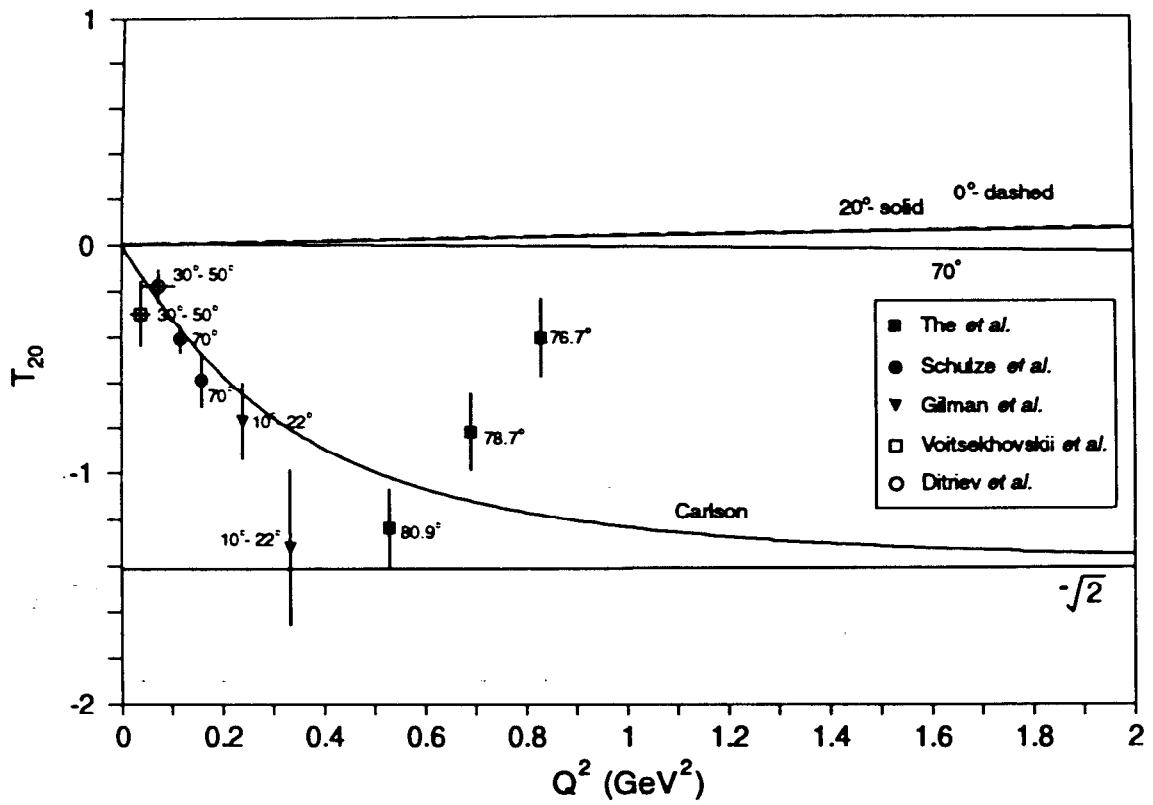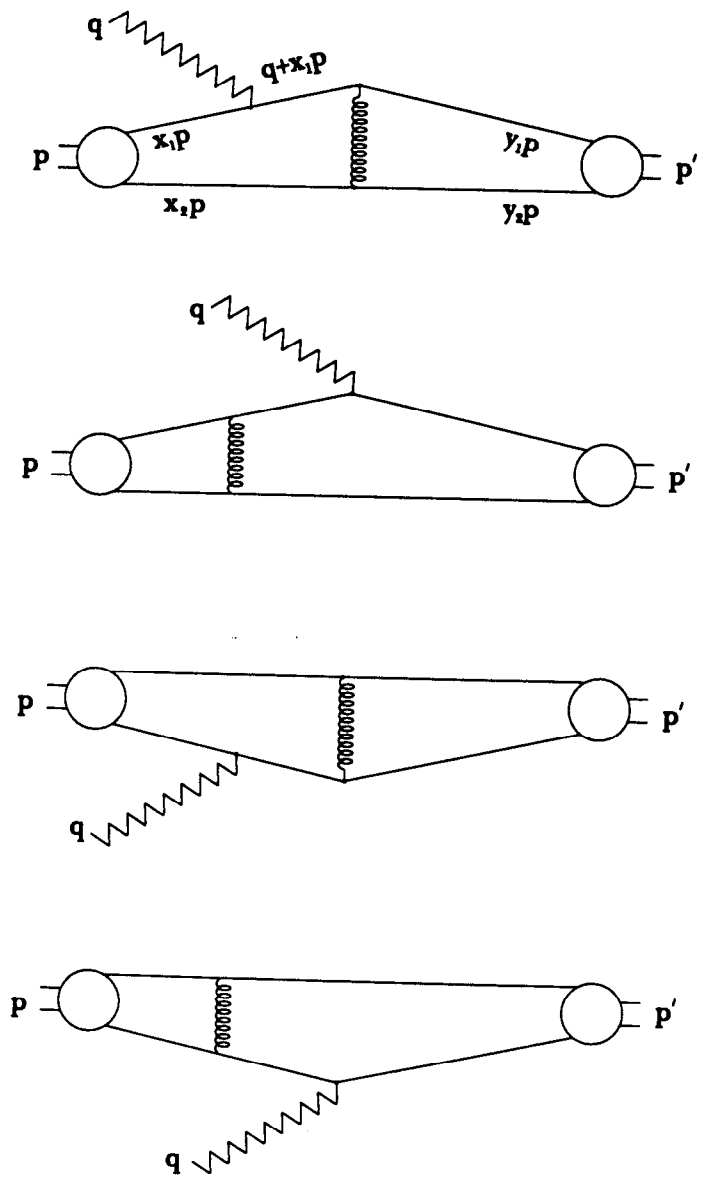
Figure 1

Figure 2

Figure 3

Figure 4