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The Axial Vector Coupling and Magnetic Moment of the Quark *

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Abstract

Details of an estimate of the corrections of order $1/N$ to the axial vector coupling of the constituent quark, where N is the number of colors, are presented. Analogous estimate is made for the magnetic moment of the constituent quark.

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Some time ago Weinberg [1] argued that in the leading order in $1/N$, where N is the number of colors, the axial vector coupling of the constituent quark is equal to one and its anomalous magnetic moment is zero. This justifies the usual treatments of the constituent quark and bag models, where the quark is treated as a bare Dirac particle, provided the corrections in $1/N$ are shown to be small, especially for the magnetic moments. More recently Weinberg [2] (see also ref. [3]) has given an estimate of the corrections of order $1/N$ to the axial vector coupling of the constituent quark. His calculation was done using the chiral quark model Lagrangian [4] in the chiral limit and the limit of large number of colors. The essential input was the analogue of the famous Adler-Weisberger sum rule for pion-quark scattering. First order corrections in $1/N$ to the leading result were shown to come from tree-level pion-quark scattering and quark-antiquark pair production diagrams. The latter contribution turned out to be logarithmically divergent but relatively small even for the values of a cutoff as large as 5 GeV.

It is our aim in this paper to elaborate on this last result, already quoted in [2]. Following the same pattern of reasoning we also analyze the magnetic moment of the constituent quark using the same chiral Lagrangian and the analogue of the Drell-Hearn-Gerasimov sum rule for photon-quark scattering. Our results seem to indicate that the chiral quark model with coefficients obtained by sum rules works well.

The lowest order terms in the chiral Lagrangian density in which the relevant degrees of freedom are the constituent u- and d-quarks, treated as massive particles subject to color force only at large separations, and pions treated as pseudo-Goldstone bosons, have the following form[1]:

$$\begin{aligned}\mathcal{L} &= -\frac{1}{2(1+\frac{\vec{\pi}^2}{F_\pi^2})^2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{1}{2(1+\frac{\vec{\pi}^2}{F_\pi^2})}m_\pi^2\vec{\pi}^2 \\ &\quad -\bar{\psi}[\not{\partial} + m + \frac{2i}{F_\pi^2}\frac{1}{1+\frac{\vec{\pi}^2}{F_\pi^2}}\vec{t}\cdot(\vec{\pi}\times\not{\partial}\vec{\pi}) \\ &\quad +\frac{2ig_A}{F_\pi}\frac{1}{1+\frac{\vec{\pi}^2}{F_\pi^2}}\gamma_5\vec{t}\cdot\not{\partial}\vec{\pi}]\psi \qquad (1)\end{aligned}$$

[1] Here we neglect isospin breaking due either to a quark mass difference in the QCD Lagrangian or to electromagnetic interactions. Accordingly, we do not include purely electromagnetic contributions to the sum rules.

where $\vec{\pi}$ denotes the pion field, ψ is the quark field, m_π (≈ 135 MeV) is the pion mass, m stands for the mass of the constituent quark (≈ 360 MeV), F_π is the pion decay constant (≈ 190 MeV), g_A is the axial vector coupling in the leading order in $1/N$, hence is set to one, and $\vec{t} = \frac{\vec{\sigma}}{2}$ where $\vec{\sigma}$ are the usual Pauli matrices in isospin space.

In order to estimate corrections to g_A to first order in $1/N$ we use the Adler-Weisberger [5] sum rule for pion-quark scattering in the form used by Weinberg [2] in the chiral limit ($m_\pi = 0$):

$$g_A^2 \;=\; 1 - \frac{F_\pi^2}{2\pi}\int_0^\infty \frac{d\omega}{\omega}[\sigma_-(\omega) - \sigma_+(\omega)] \tag{2}$$

where σ_+ and σ_- stand for the total cross-sections for scattering of π^+ and π^- respectively on a constituent u-quark at rest. The incoming energy of the pion is denoted by ω. In the large N limit F_π goes as $\sqrt{N}$. Taking this fact into account it is easy to conclude that the only relevant processes making contributions of order $1/N^2$ in the total cross-sections, or equivalently of order $1/N$ in g_A are tree-level pion-quark elastic scattering (Figure (1)) and the quark-antiquark pair production (Figure (2)) [2]. (In the latter case the extra $1/\sqrt{N}$ in the amplitude, coming from the coupling to the produced pairs, is canceled in the expression for the total rate, by the sum over the colors of the produced pair). Using the Lagrangian density given by (1) one finds that the only relevant contribution for pion-quark scattering comes from $\pi^- u \rightarrow \pi^0 d$ (contributions coming from $\pi^+ u \rightarrow \pi^+ u$ and $\pi^- u \rightarrow \pi^- u$ cancel). The corresponding differential cross section is [2] :

$$\left(\frac{d\sigma}{d\Omega}\right)_{\pi^- u \rightarrow \pi^0 d} \;=\; \frac{m^2}{2\pi^2 F_\pi^4}\frac{u^2(1-\cos\theta)^2}{[1+u(1-\cos\theta)]^3} \tag{3}$$

where θ is the angle between $\vec{p'_\pi}$ and $\vec{p_\pi}$ (see Figure (1)) and $u = \frac{\omega}{m}$.

The situation is a bit more involved for quark-antiquark pair production. Three processes contribute: $\pi^- u \rightarrow dd\bar{d}$, $\pi^+ u \rightarrow uu\bar{d}$ and $\pi^- u \rightarrow du\bar{u}$. The last process can be broken into two non-interfering parts (to leading order in $1/N$). The first comes from diagrams of Figure (2) where the produced pair with momenta p_1 and p_2 consists of a d-quark and a $\bar{u}$-quark. The contribution from these diagrams to the sum rule (2) is exactly canceled by the corresponding contribution from the process $\pi^+ u \rightarrow uu\bar{d}$. The second part of the process $\pi^- u \rightarrow du\bar{u}$ comes from diagrams of Figure (2) where the produced pair is $u\bar{u}$. The corresponding contribution to g_A^2 is exactly equal to that of $\pi^- u \to dd\bar{d}$. Because of the complication coming from the three particle phase space we only quote the expression for the amplitude squared for $\pi^- u \to dd\bar{d}$:

$$|M|^2 = 2^9 \frac{m^6[(p-p\prime)\cdot p_\pi]^2}{F_\pi^6 (p_1+p_2)^2 (p\cdot p_\pi)(p\prime \cdot p_\pi)} + 2^9 \frac{m^4 (p_1+p_2)^2}{F_\pi^6 (p\prime - p)^2}. \tag{4}$$

Doing the three particle phase space integrals and making use of the Adler-Weisberger sum rule (2) we find a logarithmically divergent contribution to g_A^2. The final expression for g_A^2 up to first order in $1/N$ is:

$$g_A^2 = 1 - \frac{m^2}{2\pi^2 F_\pi^2} - 2N\left(\frac{m^2}{2\pi^2 F_\pi^2}\right)^2 \mathcal{I}. \tag{5}$$

The second term in (5) comes from pion-quark elastic scattering and the last term comes from quark-antiquark pair production. $\mathcal{I}$ denotes a numerical integral evaluated from $4m$ to a cutoff Λ.

In the following table we summarize the dependence of $\mathcal{I}$ on different values of the cutoff Λ:

Λ (MeV)	$\mathcal{I}$
2000	0.022
2800	0.135
5000	0.634
10000	1.76
20000	3.42

As pointed out by Weinberg [2], the validity of the described estimate of the axial vector coupling of the constituent quark is based on the assumption that the integral over ω in (2) is dominated by energies less than or of the order of a typical N-independent QCD energy scale, such as the mass of the ρ-meson ($m_\rho \approx 770$ MeV). This condition is satisfied for the tree-level pion-quark scattering. On the other hand the contribution for the pair production process is logarithmically divergent and it obviously does not meet the above condition[2]. In particular the threshold for the pair production process is at

$4m \approx 1400$ MeV which is not small compared to the above scale. Thus, although this process is formally of the same order in $1/N$ as the tree-level pion-quark elastic scattering, one could think that its inclusion would be problematic even if it did not diverge. In any case its contribution turns out to be small for any reasonable cutoff.

We have also checked how the second term in (5) changes if the mass of the pion is taken into account. Numerical calculation shows that the change in the final result is only three percent even if the mass of the pion is taken to be half of the constituent quark mass. Also, because the energy threshold is large the pion mass can be neglected in calculating $\mathcal{I}$.

Following the same logic we calculate the corrections of order $1/N$ to the magnetic moment of the constituent quark. Here we have to take into account the lowest order terms in the chiral Lagrangian density describing the electromagnetic interaction of quarks and pions. They are given by the following expression:

$$\begin{aligned} \Delta\mathcal{L} &= -\frac{1}{(1+\frac{\vec{\pi}^2}{F_\pi^2})^2}(eA^\mu(\vec{\pi}\times\partial_\mu\vec{\pi})_3 + \frac{e^2}{2}A_\mu A^\mu(\vec{\pi}^2-\pi_3^2)) \\ &\quad + ieA_\mu\bar{\psi}\gamma^\mu[(\frac{(z_u+z_d)}{2} + (z_u-z_d)t_3) + \frac{2}{F_\pi^2}\frac{1}{1+\frac{\vec{\pi}^2}{F_\pi^2}}(\vec{\pi}^2 t_3 - \vec{t}\cdot\vec{\pi}\pi_3) \\ &\quad - \frac{2g_A}{F_\pi}\frac{1}{1+\frac{\vec{\pi}^2}{F_\pi^2}}\gamma_5(\vec{t}\times\vec{\pi})_3]\psi \end{aligned} \tag{6}$$

where A_μ denotes the photon field, $-e$ stands for the charge of the electron and $z_u e$ and $z_d e$ stand for the charges of the up and down quark respectively. Again we set $g_A = 1$ in the leading order in $1/N$.

In order to estimate the anomalous magnetic moment of the constituent quark, we use the analogue of the Drell-Hearn-Gerasimov sum rule [6] for

[2]Because we work only to leading order in $1/N$ certain interference terms have been neglected in (5). An example is the diagrams of Figure (2) for $\pi^- u \to dd\bar{d}$ multiplied by those with the identical d-quarks interchanged. To check the numerical validity of the $1/N$ approximation, and to check that the neglect of these terms was not the cause of the divergence of the sum rule integral, we have calculated them for $\pi^- u \to dd\bar{d}$. They change the values of $\mathcal{I}$ given in the previous table by only about ten percent and do not improve the convergence of the integral.

photon-quark interaction, given by the following formula:

$$\kappa^2 = \frac{m^2}{2\pi^2\alpha z^2}\int_t^\infty \frac{d\omega}{\omega}[\sigma_P(\omega) - \sigma_A(\omega)] \quad (7)$$

where σ_P and σ_A represent the cross-sections for parallel and antiparallel photon and quark spins and ω denotes the incoming energy of the photon in the frame where the target quark is at rest. Also, ze denotes the charge of the target, $\alpha = \frac{e^2}{4\pi}$, t is the threshold for the relevant process and κ stands for the anomalous magnetic moment of the constituent quark.

Counting the powers of N is done as before. Only two types of processes make contributions of order $1/N$ to the cross-sections σ_P and σ_A and correspondingly of the same order to κ^2. These are: pion-photoproduction (Figure (3)) and quark-antiquark pair photoproduction (Figure (4)). The relevant calculations have to be performed while keeping the pion mass finite and then letting it to zero. Taking this fact into account,we obtain the following expression for the cross-sections coming from pion-photoproduction diagrams (with $\Delta\sigma = \sigma_P - \sigma_A$):

$$\begin{aligned}\Delta\sigma = & -\frac{m\alpha}{2F_\pi^2\omega}\{(z+z_d)[z(\frac{m_\pi^2}{m} - m - \omega) + z_d(\frac{m_\pi^2}{m} - 3m - \omega)]\frac{1}{\omega}\ln\frac{A_+}{A_-} \\ & + \frac{z^2 m_\pi^2}{m\omega}\ln\frac{A_+ - m^2 + m_\pi^2}{A_- - m^2 + m_\pi^2} + \frac{2}{m\omega}B[\frac{1}{m+2\omega}z_d(zm + z_d(2m - \frac{m_\pi^2}{m})) \\ & + (z+z_d)^2 + z_d(z+z_d) - z_d^2\frac{(m+\omega)(m^2 + m\omega - \frac{m_\pi^2}{2})}{m(m+2\omega)^2}]\}\end{aligned} \quad (8)$$

where $z = 1 (z = 0)$ for the case where the charged(neutral) pion is in the final state (the up quark charge has been eliminated by $z_u = z + z_d$). Also $A_\pm = m(m+\omega) - \frac{m_\pi^2}{2} \pm B$ with $B = [(m\omega - \frac{m_\pi^2}{2})^2 - m_\pi^2 m^2]^{1/2}$. The pion mass is important only in the second term which contributes to the sum rule as $\frac{2\alpha z^2 m_\pi^2}{F_\pi^2}\int_{m_\pi}^\infty \frac{d\omega}{\omega^3}\ln\frac{2\omega}{m_\pi}$ as m_π goes to zero. This term, which is lost if one naively sets m_π to zero before calculating $\Delta\sigma$, gives a large contribution to κ^2 but is exactly canceled by the other terms[3]. Numerical evaluation of the integral in (7) gives $\kappa^2 = 0$ for both the charged and neutral pion, independent of the value of z_d.

[3]We thank S. Drell for suggesting this to us.

Again, the contribution coming from quark-antiquark pair photoproduction is slightly more complicated, but in this case the sum rule integral is finite. The corresponding expressions for the magnetic moments of the constituent u and d quarks are:

$$\kappa_u^2 = \frac{2}{{z_u}^2} N \left(\frac{m^2}{2\pi^2 F_\pi^2}\right)^2 \mathcal{I}_1(z_u, z_d) \tag{9}$$

$$\kappa_d^2 = \frac{2}{{z_d}^2} N \left(\frac{m^2}{2\pi^2 F_\pi^2}\right)^2 \mathcal{I}_2(z_u, z_d) \tag{10}$$

where $\mathcal{I}_1, \mathcal{I}_2$ stand for numerical integrals whose values [4] are less than 0.001. Because the energy threshold is large the pion mass can be neglected in calculating $\mathcal{I}_1$ and $\mathcal{I}_2$. Of course the same objections as before can be raised to contributions from the pair production processes which obviously do not come from low energies but again, their contributions turn out to be numerically negligible[5].

Thus the $1/N$ corrections to κ^2 are essentially zero. Just by dimensional analysis we expect contributions to κ^2 of $O(1/N^2)$ to be of the following form:

$$\kappa_{u,d}{}^2 \sim \frac{1}{z_{u,d}^2} \left(\frac{m^2}{2\pi^2 F_\pi^2}\right)^2. \tag{11}$$

In order to discuss the meaning of the equations (5),(9) and (10) we argue that for consistency of our calculation the $N = 3$ limit should be taken after the axial vector coupling and the magnetic moments are obtained for the nucleons in the framework of the large N constituent quark model[6].(In the large N limit of the naive quark model[7] the proton is built from $k+1$ up and k down quarks and analogously the neutron consists of k up and $k+1$ down

[4] In fact as the number of sampling points in our Monte Carlo routine is increased the values of the above numerical integrals seem to converge to zero.

[5] The same comment would also apply to diagrams with possible four-quark vertices[8] which we have not considered.

[6] If this limit is taken first the values for the magnetic moments of the nucleons are identical to the ones that follow.The value of the axial vector coupling, on the other hand, decreases a bit.

quarks, where $N = 2k+1$). We also note that an extension of the usual power counting argument [8] indicates that the dominant contribution to both pion-nucleon and photon-nucleon interactions is the impulse approximation, in which the pion or photon interacts independently with each quark in the nucleon. All other processes are of higher order in the chiral expansion. On the other hand, all these processes are of the same order, N, in the $1/N$ expansion.

In the case of the axial vector coupling of the nucleon, we find in the nonrelativistic limit:

$$\begin{aligned}(g_A)_{nucleon} &= \frac{N+2}{3} g_A \\ &= \frac{1}{3}(N+2+N(g_A-1)) + O(1/N). \end{aligned} \tag{12}$$

Taking $\Lambda = 3500$ MeV for definiteness, which corresponds to the momenta of the incoming particles taken as m_ρ in the center of mass, $g_A = 0.87$ and $(g_A)_{nucleon} = 1.54$, which should be compared to the experimental value 1.25.

Analogously, we find the following expressions for the magnetic moments of the proton and neutron:

$$\mu_P = \frac{1}{6}(\mu_u(N+5) - \mu_d(N-1)) \tag{13}$$

$$\mu_N = \frac{1}{6}(\mu_d(N+5) - \mu_u(N-1)) \tag{14}$$

where $\mu_{u,d} = (1+\kappa_{u,d})\frac{z_{u,d}e}{2m}$ are the magnetic moments of the constituent up and down quarks. Using the usual quark charges in (13) and (14) we obtain:

$$\mu_P = \frac{e}{12m}(N+3+\frac{N}{3}(2\kappa_u+\kappa_d)) + O(1/\sqrt{N}) \tag{15}$$

$$\mu_N = -\frac{e}{12m}(N+1+\frac{N}{3}(2\kappa_u+\kappa_d)) + O(1/\sqrt{N}). \tag{16}$$

Since κ_u and κ_d are consistent with being zero the values for the magnetic moments are $\mu_P = 2.6\frac{e}{2m_N}$ and $\mu_N = -1.7\frac{e}{2m_N}$, where m_N is the nucleon mass. These values should be compared with the experimental results, $\mu_P = 2.79\frac{e}{2m_N}$ and $\mu_N = -1.91\frac{e}{2m_N}$. Alternatively one could use the values of the quark charges obtained from the quark model under the requirement that the nucleons have their usual charges for any N. Explicitly, $z_u = \frac{N+1}{2N}$ and $z_d = \frac{1-N}{2N}$ which in the large N limit goes to $z_u = \frac{1}{2}$ and $z_d = -\frac{1}{2}$. Then the corresponding values would be $\mu_P = 2.2\frac{e}{2m_N}$ and $\mu_N = -\mu_P$.

The value obtained for the axial vector coupling agrees within 20% with the experimental value. Perhaps the agreement could be improved by taking into account relativistic corrections [9]. Concerning the anomalous magnetic moment, we have shown explicitly that the potentially dangerous $O(1/\sqrt{N})$ corrections are negligible. We consider these results to be evidence that the constituent quark model can be understood in the large N limit in terms of chiral symmetry (in the form of the chiral quark model) and reasonable assumptions on the high-energy behavior of amplitudes (embodied in sum rules).

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Figure Captions

Figure (1) : Feynman diagrams for pion-quark scattering that contribute to the Adler-Weisberger sum rule.

Figure (2) : Quark-antiquark pair production diagrams that contribute to the Adler-Weisberger sum rule to the same order in $1/N$ as those of Figure (1).

Figure (3) : Feynman diagrams for pion photoproduction that contribute to the Drell-Hearn-Gerasimov sum rule.

Figure (4) : Quark-antiquark pair photoproduction diagrams that contribute to the Drell-Hearn-Gerasimov sum rule to the same order in $1/N$ as those of Figure (3).

Figure 1

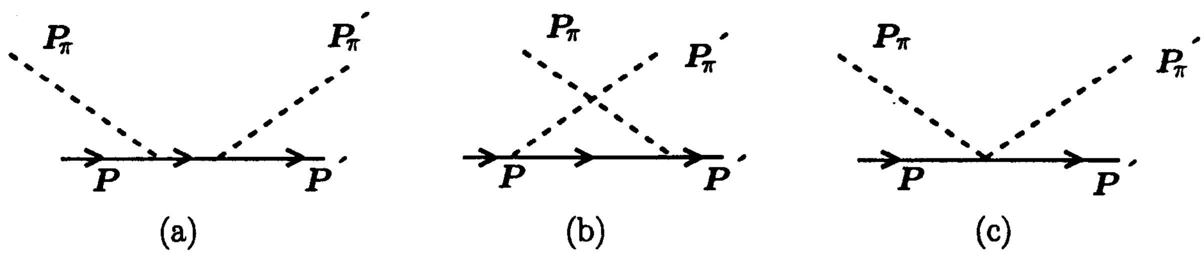

Figure 2

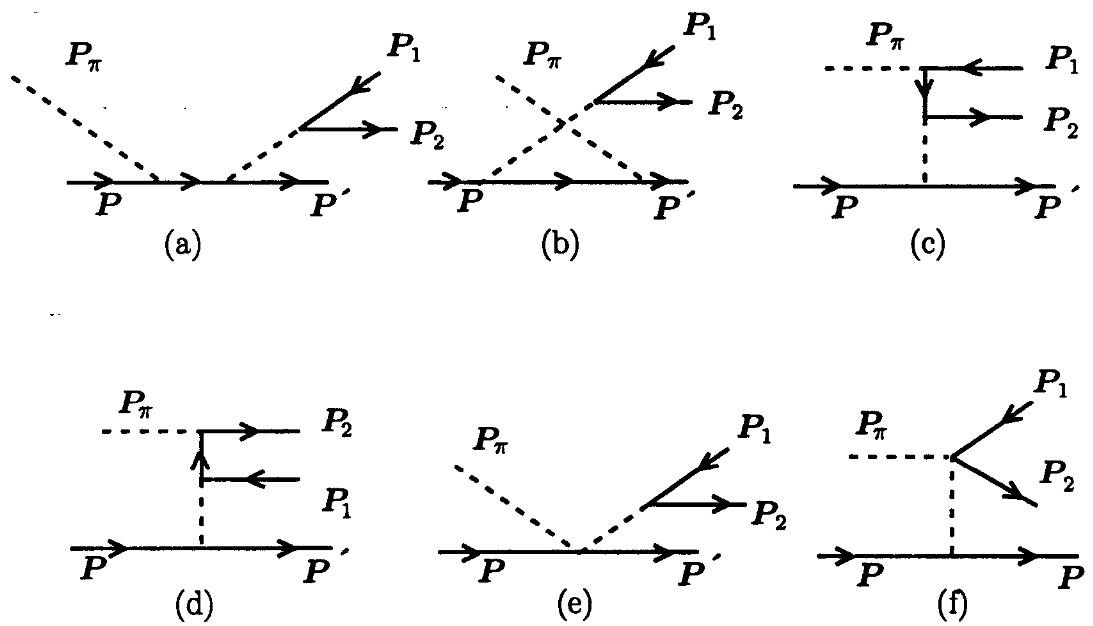

Figure 3

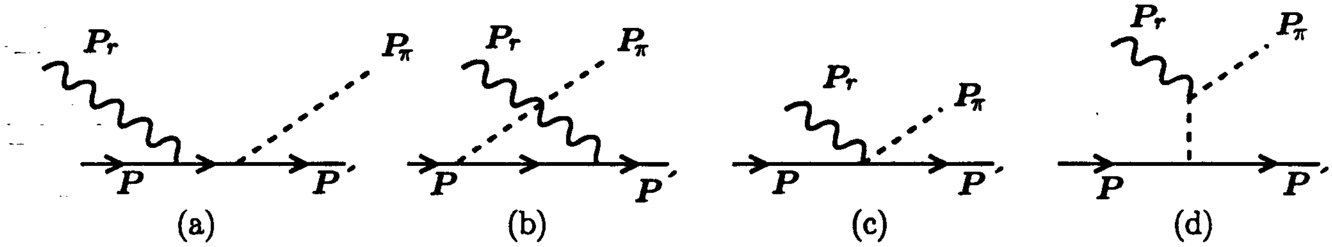

Figure 4