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CP Violation in B Physics*

YOSEF NIR†

Physics Department, Weizmann Institute of Science, Rehovot 76100, Israel

and

HELEN R. QUINN

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

ABSTRACT

We discuss the physics of CP violation. The Standard Model predictions for CP violation in B physics are reviewed. A program of studies of neutral B decays that can test the Standard Model predictions is described. We briefly summarize a study of how these predictions are changed in various extensions of the Standard Model. The topic of direct CP violations arising from interference between the W-tree diagrams and the W-loop (the so-called "penguin") contributions in the Standard Model is then discussed; uncertainties in the calculation of penguin terms affect predictions for charged B decay asymmetries and also some neutral channels. We then summarize the experimental outlook, both for an e^+e^- B factory and for high energy hadron colliders.

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1. Introduction

1.1 CP is Not a Symmetry of Nature

It was long thought that CP symmetry was exact in nature and that only theories that had this property were viable descriptions of the observed world. The observation of the decay $K_L \rightarrow \pi\pi$ by Christenson, Cronin, Fitch and Turlay in 1964 (1) changed that view dramatically. The phenomenon of CP violation was unambiguously demonstrated by this decay. Since that time much effort has gone into studying the nature of CP violation. We have understood that CP violation is a crucial feature of any theory that attempts to explain the observed asymmetry between matter and anti-matter in the universe starting from initially symmetric conditions (2). We have also found that it is a natural feature of the three generation Standard Model (3). However, we have been unable to calculate and to measure enough about the Kaon system, which so far has been our only laboratory for CP violation measurements, to really test the Standard Model picture of CP violation.

The observation of baryon asymmetry in the universe can be explained if some CP-conjugate pairs of processes have different rates. If the initial conditions of the universe were baryon-symmetric, then the asymmetry should be generated by dynamical baryogenesis, which requires three ingredients (2): *i*) there exist baryon number violating processes, *ii*) these processes must go out of equilibrium sometime during the history of the universe, and *iii*) C and CP must be violated. While CPT requires that the *total* decay rates for a particle and its antiparticle are equal, CP symmetry requires that the *partial* rates of CP-conjugate processes are equal. If this were always the case, then for any process which violates baryon number there would be a CP-conjugate process of equal rate and no asymmetry could be generated. Thus, CP violation seems to be a necessary ingredient of any theory of elementary particles. Moreover, detailed analyses of baryogenesis imply that sources of CP violation beyond the Standard Model are required (for a recent review see Ref. (4)).

The possibility that other physics beyond the Standard Model plays an important role in CP violation is still an open one. This makes the prospect of investigating CP violation in the B meson system extremely interesting. At the very least, it will allow us to measure some of the remaining parameters of the Standard Model, parameters as fundamental as the quark masses themselves. If we are lucky it could do a lot more: if the results are inconsistent with Standard Model predictions then they may provide some clues about physics beyond the Standard Model. We have precious few ways to seek these clues, so a source of B mesons with luminosity high enough to study CP violation physics would be a truly exciting physics facility.

1.2 CP Violation in the Neutral Kaon System

To date three CP violating processes have been measured (5). The results are parameterized as follows:

$$\begin{aligned} |\eta_{+-}| &= \left[\frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)}\right]^{1/2} =(2.268 \pm 0.023) \times 10^{-3}, \\ |\eta_{00}| &= \left[\frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^0\pi^0)}\right]^{1/2} =(2.253 \pm 0.024) \times 10^{-3}, \end{aligned} \tag{1.1}$$

$$\delta = \frac{\Gamma(K_L \to \pi^-\ell^+\nu) - \Gamma(K_L \to \pi^+\ell^-\nu)}{\Gamma(K_L \to \pi^-\ell^+\nu) + \Gamma(K_L \to \pi^+\ell^-\nu)} = (3.27 \pm 0.12) \times 10^{-3}. \tag{1.2}$$

All three processes in Eqs. (1.1) and (1.2) are due to CP violation from *mixing*, that is CP is violated in $\Delta S = 2$ processes. The neutral Kaon mass eigenstates are not CP eigenstates but instead:

$$\begin{aligned} |K_L\rangle &= \frac{1+\epsilon}{\sqrt{2(1+|\epsilon|^2)}}|K^0\rangle + \frac{1-\epsilon}{\sqrt{2(1+|\epsilon|^2)}}\left|\overline{K}^0\right\rangle, \\ |K_S\rangle &= \frac{1+\epsilon}{\sqrt{2(1+|\epsilon|^2)}}|K^0\rangle - \frac{1-\epsilon}{\sqrt{2(1+|\epsilon|^2)}}\left|\overline{K}^0\right\rangle. \end{aligned} \tag{1.3}$$

Thus ϵ parameterizes the deviation from the CP-conserving limit. Were K_L a pure CP-odd state, it could not decay into two pions, which are here in a CP-even state ($J = 0$), and it would decay into leptons of opposite charges at equal rates. If $|\epsilon| = 0$ the observables in (1.1) and (1.2) would vanish. Instead, they are all compatible with the single value

$$|\epsilon| \approx 2.26 \times 10^{-3}. \tag{1.4}$$

There is yet another CP-violating parameter in the neutral Kaon system. The values of $|\eta_{00}|$ and $|\eta_{+-}|$ in (1.1) may slightly differ from each other. One parametrizes this difference by the parameter ϵ':

$$\eta_{+-} \approx \ \epsilon + \epsilon', \quad \eta_{00} \approx \ \epsilon - 2\epsilon', \tag{1.5}$$

where

$$\epsilon' \approx \frac{i}{\sqrt{2}}\,\mathrm{Im}(a_2/a_0)e^{i(\delta_2-\delta_0)}. \tag{1.6}$$

Here a_I is the amplitude for K^0 to decay into a final two pion state of isospin I, with the strong phase $e^{i\delta_I}$ factored out. (We explain the term strong phases in the next paragraph.) If CP violation could be attributed exclusively to the $\Delta S = 2$

mixing, then it would not depend on the final state and ϵ' would vanish. Non-zero ϵ' would signify *direct* CP violation, that is CP violation in $\Delta S = 1$ (decay) processes. The most recent measurements give (6)

$$\epsilon'/\epsilon = \begin{cases} (2.3 \pm 0.7) \times 10^{-3} & \text{NA31} \\ (0.6 \pm 0.7) \times 10^{-3} & \text{E731} \end{cases} \tag{1.7}$$

Thus, there is as yet no compelling evidence for direct CP violation: the results in (1.7) are consistent with Standard Model estimates but the weighted average for ϵ' is only two standard deviations from zero.

Before proceeding, let us discuss in brief the mechanism for CP violation: how does a complex phase in the Lagrangian translate into a prediction of a CP-violating observable? Physical amplitudes for any decay or scattering process are in general complex, even when the Lagrangian itself is real and CP conserving. Phases arise from the fact that there are coupled channels in most real physical processes. Amplitudes acquire phases from the absorptive parts associated with these coupled channels. These phases are here referred to as *strong phases*, since the rescatterings are dominated by strong interaction processes. The strong phases of a CP-conjugate pair of processes are always of the same magnitude and sign. The CP violating Lagrangian phases are generally referred to as *weak phases* because they appear in the weak interaction parts of the Lagrangian. The weak phases of a pair of CP-conjugate amplitudes are always of the same magnitude but of opposite sign.

If there is only a single term in the amplitude for a process then the CP-conjugate process would proceed at an identical rate, despite the fact that the amplitude has a different phase. To see a CP violating effect, one needs two different contributions to the amplitude. Then interference terms are sensitive to the difference between the phases and hence can give CP violating effects, that is rates which are different for a process and for its CP-conjugate.

In the case of the neutral K (or B) system, the particle can decay directly to a given final state or it can mix to its CP-conjugate and then decay to the same final state. Thus *mixing* provides the necessary second contribution. In addition, the Standard Model suggests that there are two different mechanisms at work in the *direct* decay, generically called tree and penguin decays (see Chapter 4). These can contribute to the amplitude with different weak and strong phases, thus giving direct CP violation. Note, however, in (1.6) that ϵ' depends not only on the weak phase difference, $\arg(a_2/a_0)$, but also on the strong phase difference, $\delta_2 - \delta_0$, and on the magnitude of the amplitudes, $|a_2/a_0|$. This dependence on hadronic physics is a common feature of direct CP violation effects, which explains why clean theoretical predictions for these effects are not available.

One beauty of the B system is the great variety of channels that can be studied. A second advantage is that, because the b-quark is more massive than the s-quark, we can more reliably calculate certain strong corrections by perturbative techniques, since they are more dominated by short-distance physics. Thus the relation between the measured asymmetries and Standard Model parameters has fewer uncertainties in the B case.

1.3 CP VIOLATION IN THE STANDARD MODEL

Under what conditions is a Lagrangian CP conserving? The general answer is: whenever all the coupling and mass terms in the Lagrangian can be made real by an appropriate set of field redefinitions. Within the Standard Model, the most general theory with only two quark generations and only a single Higgs multiplet is of this type. However, when we add a third quark generation then the most general quark mass matrix allows for CP violation. Similarly, when we extend the fermion sector in various other ways or extend the scalar sector beyond the single doublet of the minimal Standard Model, additional parameters appear that cannot all be made simultaneously real by field redefinitions and hence allow further CP violating effects.

The three generation Standard Model with a single Higgs multiplet has only a single non-zero phase. It appears in the matrix which relates weak eigenstates to mass eigenstates, commonly known as the CKM (Cabibbo–Kobayashi–Maskawa) matrix (7,3). That matrix must be unitary, a constraint that provides relationships between its elements. All of this translates with relatively few further assumptions into very specific predictions for the relationships between the parameters measured in different B decay processes. This makes the B decays an ideal laboratory to probe for physics beyond the Standard Model; theories with other types of CP violating parameters typically do not predict the same relationships (see Chapter 3).

We do not here consider CP violations that arise from the terms induced in the Lagrangian by instanton effects. For the weak $SU(2)$ gauge theory, such a term can always be removed by appropriate rephasings of lepton fields. For the strong $SU(3)$ gauge theory, such a term gives *strong* CP violation even in the two generation case. Experimentally, the bound on strong CP violation from the absence of an electric dipole moment of the neutron ($d_n \leq 12 \times 10^{-26}$ e-cm (5)) is $\theta \lesssim 10^{-9}$. Any effect of such a term in the processes discussed here is completely negligible.

Let us now discuss CP violation from quark mixing in more detail. In the Standard Model, the charged current interactions are given by

$$-\mathcal{L}_W = \frac{g}{\sqrt{2}} \left(\overline{u_L^I} \quad \overline{c_L^I} \quad \overline{t_L^I} \right) \gamma^\mu \begin{pmatrix} d_L^I \\ s_L^I \\ b_L^I \end{pmatrix} W_\mu^+ + \text{h.c.} \tag{1.8}$$

The superscript I denotes interaction eigenstates. The mass matrices M_D and M_U, are not simultaneously diagonal in this basis. However, any 3×3 hermitian matrix can be diagonalized via a bi-unitary transformation. Thus

$$V_{dL} M_D V_{dR}^\dagger = M_D^{\text{diag}}; \qquad V_{uL} M_U V_{uR}^\dagger = M_U^{\text{diag}}, \tag{1.9}$$

where the M_Q^{diag} matrices are real and diagonal. The matrices V_{qL}, V_{qR} define the transformation from the interaction eigenstates to the mass eigenstates. Thus the W-interactions of Eq. (1.8) can be rewritten in the mass eigenbasis:

$$-\mathcal{L}_W = \frac{g}{\sqrt{2}} \left(\overline{u_L} \quad \overline{c_L} \quad \overline{t_L} \right) \gamma^\mu (V_{uL} V_{dL}^\dagger) \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W_\mu^+ + \text{h.c.} \tag{1.10}$$

The matrix $(V_{uL} V_{dL}^\dagger)$ is the mixing matrix for three quark generations. It is a 3×3 unitary matrix. It contains 9 parameters, of which three can be chosen as real angles, θ_{12}, θ_{23} and θ_{13}, and six are phases. We can reduce the number of phases in the mixing matrix V by redefining the phases of the quark mass eigenstates. Then

$$(V_{uL} V_{dL}^\dagger) \rightarrow V = P_u (V_{uL} V_{dL}^\dagger) P_d^*, \tag{1.11}$$

with P_u and P_d unitary diagonal matrices. For three generations there are *five* independent phase differences between the elements of P_u and those of P_d, while there are *six* phases in $(V_{uL}V_{dL}^{\dagger})$. Consequently, the mixing matrix V contains one physically meaningful phase, which we denote by δ (3). The three generation Standard Model predicts CP violation unless $\delta = 0$.

The standard parametrization of V is (8,5)

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{1.12}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The matrix V for the three generation mixing is called the Cabibbo–Kobayashi–Maskawa matrix or, in short, the CKM matrix.

The unitarity of the CKM matrix leads to relations such as

$$V_{ub}^{*}V_{ud} + V_{cb}^{*}V_{cd} + V_{tb}^{*}V_{td} = 0. \tag{1.13}$$

The *unitarity triangle* is a geometrical representation of this relation in the complex plane: the three complex quantities, $V_{ub}^{*}V_{ud}$, $V_{cb}^{*}V_{cd}$ and $V_{tb}^{*}V_{td}$, should form a triangle, as shown in Fig. 1.

Rescaling the sides of the triangle by $1/|V_{cb}^{*}V_{cd}|$ and choosing a phase convention where $V_{cb}^{*}V_{cd}$ is real (this holds to a very good approximation for the parametrization (1.12)), the coordinates of the three vertices A, B and C become

$$A\left[\frac{\text{Re }V_{ub}}{s_{12}|V_{cb}|}, -\frac{\text{Im }V_{ub}}{s_{12}|V_{cb}|}\right], \quad B(1,0), \quad C(0,0). \tag{1.14}$$

Another commonly used parametrization is that of Wolfenstein (9), in which the coordinates of the vertex A are denoted by (ρ,η). The unitarity triangle gives a relation between the two most poorly determined entries of the CKM matrix, V_{ub} and V_{td}. Thus it is convenient to present constraints on the values of the

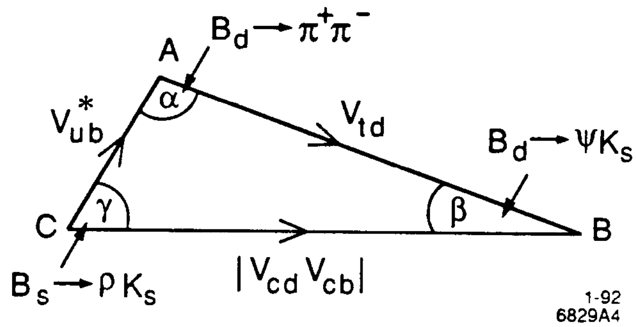

Figure 1. The unitarity triangle is a representation in the complex plane of the triangle formed by the CKM matrix elements $V_{ud}V_{ub}^*$, $V_{cd}V_{cb}^*$ and $V_{td}V_{tb}^*$.

CKM parameters as bounds on the coordinates of the vertex A. Furthermore, the Standard Model predictions for the CP asymmetries in neutral B decays into certain CP eigenstates are fully determined by the values of the three angles of the unitarity triangle, α, β and γ. Their measurement will test these Standard Model predictions and consequently provide a probe for physics beyond the Standard Model.

2. CP Violation in Neutral B Decays

2.1 General Formalism

We consider a neutral meson B^0 and its antiparticle $\overline{B}^0$. The two mass eigenstates are B_H and B_L (H and L stand for Heavy and Light, respectively):

$$\begin{aligned} |B_L\rangle =& p\,|B_0\rangle + q\left|\overline{B}^0\right\rangle, \\ |B_H\rangle =& p\,|B_0\rangle - q\left|\overline{B}^0\right\rangle. \end{aligned} \tag{2.1}$$

The eigenvalue equation is

$$\left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} p \\ \pm q \end{pmatrix} = (M_{L,H} - \frac{i}{2}\Gamma_{L,H})\begin{pmatrix} p \\ \pm q \end{pmatrix}. \tag{2.2}$$

Here $\mathbf{M}$ (the mass matrix) and $\mathbf{\Gamma}$ (which describes the exponential decay of the system) are 2×2 Hermitian matrices. Since $\Delta\Gamma \equiv \Gamma_H - \Gamma_L$ is produced by channels with branching ratios of $\mathcal{O}(10^{-3})$ which contribute with alternating signs (10), we have $\Delta\Gamma \ll \Gamma$ and therefore may safely set $\Gamma_H = \Gamma_L \equiv \Gamma$. We define $M \equiv (M_H + M_L)/2$ and $\Delta M \equiv M_H - M_L$. Furthermore, $\Gamma_{12} \ll M_{12}$ (see Chapter 3) gives $|q/p| = 1$. The amplitudes for the states B_H or B_L at time t can be written as

$$\begin{aligned} A_H(t) =& A_H(0)e^{-(\Gamma/2+iM_H)t}, \\ A_L(t) =& A_L(0)e^{-(\Gamma/2+iM_L)t}. \end{aligned} \tag{2.3}$$

The proper time evolution of states which at time $t = 0$ were either pure B^0 $[A_L(0) = A_H(0) = 1/(2p)]$ or pure $\overline{B}^0$ $[A_L(0) = -A_H(0) = 1/(2q)]$ is given by

$$\begin{aligned} \left|B^0_{\text{phys}}(t)\right\rangle =& g_+(t)\,|B_0\rangle + (q/p)g_-(t)\left|\overline{B}^0\right\rangle, \\ \left|\overline{B}^0_{\text{phys}}(t)\right\rangle =& (p/q)g_-(t)\,|B_0\rangle + g_+(t)\left|\overline{B}^0\right\rangle, \end{aligned} \tag{2.4}$$

where

$$\begin{aligned} g_+(t) =& \exp(-\Gamma t/2)\exp(-iMt)\cos(\Delta M t/2), \\ g_-(t) =& \exp(-\Gamma t/2)\exp(-iMt)i\sin(\Delta M t/2). \end{aligned} \tag{2.5}$$

We are interested in the decays of neutral B's into a CP eigenstate (11,12) which we denote by f_{CP}. We define the amplitudes for these processes as

$$A \equiv \langle f_{CP}|\mathcal{H}|B^0\rangle, \quad \overline{A} \equiv \left\langle f_{CP}|\mathcal{H}|\overline{B}^0\right\rangle. \tag{2.6}$$

We further define

$$\lambda \equiv \frac{q}{p}\frac{\overline{A}}{A}. \tag{2.7}$$

Then

$$\begin{aligned}\langle f_{CP}|\mathcal{H}|B^0_{\text{phys}}(t)\rangle =& A[g_+(t) + \lambda g_-(t)],\\ \left\langle f_{CP}|\mathcal{H}|\overline{B}^0_{\text{phys}}(t)\right\rangle =& A(p/q)[g_-(t) + \lambda g_+(t)].\end{aligned} \tag{2.8}$$

The time-dependent rates for initially pure B^0 or $\overline{B}^0$ states to decay into a final CP eigenstate at time t are given by:

$$\begin{aligned}\Gamma(B^0_{\text{phys}}(t) \to f_{CP}) =& |A|^2 e^{-\Gamma t}\left[\frac{1+|\lambda|^2}{2} + \frac{1-|\lambda|^2}{2}\cos(\Delta M t) - \text{Im}\lambda\sin(\Delta M t)\right],\\ \Gamma(\overline{B}^0_{\text{phys}}(t) \to f_{CP}) =& |A|^2 e^{-\Gamma t}\left[\frac{1+|\lambda|^2}{2} - \frac{1-|\lambda|^2}{2}\cos(\Delta M t) + \text{Im}\lambda\sin(\Delta M t)\right].\end{aligned} \tag{2.9}$$

We define the time dependent CP asymmetry as

$$a_{f_{CP}}(t) \equiv \frac{\Gamma(B^0_{\text{phys}}(t) \to f_{CP}) - \Gamma(\overline{B}^0_{\text{phys}}(t) \to f_{CP})}{\Gamma(B^0_{\text{phys}}(t) \to f_{CP}) + \Gamma(\overline{B}^0_{\text{phys}}(t) \to f_{CP})}. \tag{2.10}$$

Then

$$a_{f_{CP}}(t) = \frac{(1-|\lambda|^2)\cos(\Delta M t) - 2\text{Im}\lambda\sin(\Delta M t)}{1+|\lambda|^2}. \tag{2.11}$$

The quantity $\text{Im}\lambda$ which can be extracted from $a_{f_{CP}}(t)$ can be directly related to CKM matrix elements in the Standard Model.

2.2 Modes which Measure the Angles of the Unitarity Triangle

The measurement of the CP asymmetry (2.10) will determine $\text{Im}\lambda$ through (2.11). If $|A/\overline{A}| = 1$ (as well as $|q/p| = 1$), then (2.11) simplifies considerably:

$$a_{f_{CP}}(t) = -\text{Im}\lambda \sin(\Delta M t). \tag{2.12}$$

Moreover, in this case $\text{Im}\lambda$ depends on electroweak parameters only, without hadronic uncertainties. The condition which guarantees $|A/\overline{A}| = 1$ is easy to find (13). In the general case

$$A = \sum_i A_i e^{i\delta_i} e^{i\phi_i}, \quad \overline{A} = \sum_i A_i e^{i\delta_i} e^{-i\phi_i}, \tag{2.13}$$

where A_i are real, ϕ_i are CKM phases and δ_i are strong phases. Thus, $|A| = |\overline{A}|$ if all amplitudes that contribute to the direct decay have the same CKM phase, which we denote by ϕ_D: $\overline{A}/A = e^{-2i\phi_D}$. For $\Gamma_{12} \ll M_{12}$, we have $q/p = \sqrt{M_{12}^*/M_{12}} = e^{-2i\phi_M}$ where ϕ_M is the CKM phase in the $B - \overline{B}$ mixing. Thus

$$\lambda = e^{-2i(\phi_M + \phi_D)} \implies \text{Im}\lambda = -\sin 2(\phi_M + \phi_D). \tag{2.14}$$

Note that $\text{Im}\lambda$ is independent of phase convention and does not depend on any hadronic parameters. In what follows, we concentrate on those processes which, within the Standard Model, are dominated by amplitudes that have a single CKM phase. For some cases where there are two contributions with different weak phases, one can still cleanly extract the CKM parameters through isospin analysis, if sufficient data are available on the full set of isospin related channels. This will be discussed in Section 4.3.

For mixing in the B_d $[B_s]$ system $M_{12} \propto (V_{tb}V_{td}^*)^2$ $[(V_{tb}V_{ts}^*)^2]$. Consequently,

$$\left(\frac{q}{p}\right)_{B_d} = \frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}; \qquad \left(\frac{q}{p}\right)_{B_s} = \frac{V_{tb}^*V_{ts}}{V_{tb}V_{ts}^*}. \tag{2.15}$$

For decays via quark subprocesses $b \to \overline{u}_i u_i d_j$ which are dominated by tree diagrams,

$$\frac{\overline{A}}{A} = \frac{V_{ib}V_{ij}^*}{V_{ib}^*V_{ij}}. \tag{2.16}$$

Thus, for B_{d_j} decaying through $\overline{b} \to \overline{u}_i u_i \overline{d}_j$,

$$\mathrm{Im}\lambda = \sin\left[2\arg\left(\frac{V_{ib}V_{ij}^*}{V_{tb}V_{tj}^*}\right)\right]. \tag{2.17}$$

For decays with a single K_S (or K_L) in the final state, $K - \overline{K}$ mixing plays an essential role since $B^0 \to K^0$ and $\overline{B}^0 \to \overline{K}^0$. Interference is possible only due to $K - \overline{K}^0$ mixing. For these modes

$$\lambda = \left(\frac{q}{p}\right)\left(\frac{\overline{A}}{A}\right)\left(\frac{q}{p}\right)_K, \qquad \left(\frac{q}{p}\right)_K = \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}. \tag{2.18}$$

Decay processes $b \to \overline{s}sd_j$ are dominated by penguin diagrams. For these

$$\frac{\overline{A}}{A} = \frac{V_{tb}V_{tj}^*}{V_{tb}^*V_{tj}}. \tag{2.19}$$

Note that sign(Imλ) depends on the CP transformation properties of the final state. The analysis above corresponds to CP-even final states. For CP-odd states, Imλ has the opposite sign. In what follows, we specify Imλ of CP-even states, regardless of the CP assignments of specific hadronic modes discussed.

CP asymmetries in $B^0 \to f_{CP}$ provide a way to measure the three angles of the unitarity triangle defined by (see Fig. 1)

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \quad \beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \quad \gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right). \tag{2.20}$$

The aim is to make enough independent measurements of the sides and angles that this triangle is overdetermined and thus check the validity of the Standard Model. We now give three explicit examples for asymmetries that measure the three angles α, β and γ:

(i) Measuring $\sin(2\beta)$ in $B \to \psi K_S$.

The mixing phase in the B_d system is given in Eq. (2.15). With a single final kaon, one has to take into account the mixing phase in the K system given in Eq. (2.18). The decay phase (2.16) in the quark subprocess $b \to c\bar{c}s$ is $\overline{A}/A = (V_{cb}V_{cs}^*)/(V_{cb}^*V_{cs})$. Thus

$$\lambda(B \to \psi K_S) = \left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)\left(\frac{V_{cs}^*V_{cb}}{V_{cs}V_{cb}^*}\right)\left(\frac{V_{cd}^*V_{cs}}{V_{cd}V_{cs}^*}\right) \Longrightarrow \mathrm{Im}\lambda = -\sin(2\beta). \qquad (2.21)$$

(As ψK_S is a $CP = -1$ state, there is an extra minus sign in the asymmetry which we suppress here.) There is a small penguin contribution to $b \to c\bar{c}s$. However, it depends on the CKM combination $V_{tb}V_{ts}^*$ which has, to a very good approximation, the same phase (mod π) as the tree diagram, which depends on $V_{cb}V_{cs}^*$. Hence only a single weak phase contributes in the decay. Other examples of final hadronic states in B_d decays which measure $\sin 2\beta$ are χK_S, ϕK_S, $\eta_c K_S$, ωK_S, D^+D^-, $D^0\overline{D}^0$ and similar modes with K_L instead of K_S. In addition, vector-vector modes such as ΨK^* and $D^{*+}D^{*-}$ can be used with angular analysis (see Section 2.4).

(ii) Measuring $\sin(2\alpha)$ in $B \to \pi^+\pi^-$.

Using (2.15) and, from (2.16), $(\overline{A}/A) = (V_{ub}V_{ud}^*)/(V_{ub}^*V_{ud})$, we get

$$\lambda(B \to \pi^+\pi^-) = \left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)\left(\frac{V_{ud}^*V_{ub}}{V_{ud}V_{ub}^*}\right) \Longrightarrow \mathrm{Im}\lambda = \sin(2\alpha). \qquad (2.22)$$

In this case, the penguin contribution is still expected to be small, but it depends on the CKM combination $V_{td}^*V_{tb}$ which has a phase different from that of the tree diagram. Uncertainties due to the penguin contribution can be eliminated using isospin analysis (14) (see Section 4.3). Other examples of final hadronic states in B_d decays which measure $\sin 2\alpha$ are $\rho\pi^0$, $\pi^0\pi^0$, $\omega\pi^0$ and, with angular analysis, $p\overline{p}$ and $\rho\rho$.

(*iii*) Measuring $\sin(2\gamma)$ in $B_s \rightarrow \rho K_S$.

The mixing phase in the B_s system is given in Eq. (2.15). Due to the final K_S, the mixing phase for the K system has to be taken into account. The quark subprocess is the same as in $B_d \rightarrow \pi\pi$, namely $b \rightarrow u\overline{u}d$. Thus we get

$$\lambda(B_s \rightarrow \rho K_S) = \left(\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}\right) \left(\frac{V_{ud}^* V_{ub}}{V_{ud} V_{ub}^*}\right) \left(\frac{V_{cs}^* V_{cd}}{V_{cs} V_{cd}^*}\right) \Longrightarrow \mathrm{Im}\lambda = -\sin(2\gamma). \qquad (2.23)$$

Other examples for final hadronic states in B_s decays which measure $\sin 2\gamma$ are ωK_S and similar modes with K_L instead of K_S.

These three examples demonstrate that the three angles of the unitarity triangle can, in principle, be measured independently of each other. In Tables 2.1 and 2.2 we list CP asymmetries for various channels in B_d and B_s decays, respectively. The channels are classified by the quark sub-process, denoted by i ($i = 1, \ldots, 6$) and by the type of decaying meson B_q ($q = d, s$). One possible hadronic final state for each class is listed as an example. Other states may be more favorable experimentally. We always quote the CP asymmetry for CP-even states, regardless of the specific hadronic state listed.

TABLE 2.1

CP Asymmetries in B_d Decays

Class (iq)	Quark sub-process	Final state (example)	SM prediction
1d	$\bar{b} \to \bar{c}c\bar{s}$	ψK_S	$-\sin 2\beta$
2d	$\bar{b} \to \bar{c}c\bar{d}$	D^+D^-	$-\sin 2\beta$
3d	$\bar{b} \to \bar{u}u\bar{d}$	$\pi^+\pi^-$	$\sin 2\alpha$
4d	$\bar{b} \to \bar{s}s\bar{s}$	ϕK_S	$-\sin 2\beta$
5d	$\bar{b} \to \bar{s}s\bar{d}$	$K_S K_S$	0
6d	$\bar{b} \to \bar{c}u\bar{s},\ \bar{u}c\bar{s}$	$D^0_{CP}K^{*0}$	$\sin\gamma$

TABLE 2.2

CP Asymmetries in B_s Decays

Class (iq)	Quark sub-process	Final state (example)	SM prediction
1s	$\bar{b} \to \bar{c}c\bar{s}$	$\psi\phi$	0
2s	$\bar{b} \to \bar{c}c\bar{d}$	ψK_S	0
3s	$\bar{b} \to \bar{u}u\bar{d}$	ρK_S	$-\sin 2\gamma$
4s	$\bar{b} \to \bar{s}s\bar{s}$	$\eta'\eta'$	0
5s	$\bar{b} \to \bar{s}s\bar{d}$	ϕK_S	$\sin 2\beta$

Perhaps the most difficult angle to measure will be γ, since in e^+e^- machines it is difficult to achieve a high production rate of B_s, and in hadron experiments a mode such as ρK_S is plagued with large backgrounds. An alternative way to measure γ (15,16), using B decays into non-CP eigenstates, is denoted as 6d in Table 2.1. One has to measure (16) the rates for B_d decays into $D^0_{CP}K^{*0}$, D^0K^{*0} and $\overline{D}^0K^{*0}$, and the three CP-conjugate $\overline{B}_d$ decays. Here D^0_{CP} denotes a CP eigenmode of a D^0 or $\overline{D}^0$ such as $\pi^+\pi^-$. The flavor of the initial B can be identified from a flavor-tagging decay of the K^{*0}. The six rates can be used to extract the value of $|\sin\gamma|$ up to some discrete ambiguity. The feasibility of this method depends on branching ratios and techniques to separate the modes of interest from backgrounds. A similar method, using charged B decays, was suggested in Ref. (15). (Previous studies of CP asymmetries in $B \to DK$ were presented in Refs. (17,18).)

Finally, we mention that the sign of the various asymmetries is predicted within the Standard Model (and not only the relative signs between various asymmetries). Measuring the signs of several asymmetries may serve to test whether the CKM phase δ is indeed the source of CP violation (19).

2.3 Current Constraints on Standard Model Parameters

This section presents the current status of our knowledge of the experimental constraints on the parameters of the Standard Model (20-21), updating and extending previous works (22-25). We use the unitarity triangle of the CKM matrix to show the constraints on the angles α, β and γ as a function of top quark mass. The consequent range of asymmetries allowed for a given type of B decay is evaluated. The luminosity of various colliders needed in order to guarantee a statistically significant measurement of CP violation in one or more types of B decay is then presented in Chapter 5.

We impose three constraints on the form of the unitarity triangle. (All the values of parameters quoted below are taken from Ref. (21) where the relevant references can be found.) First,

$$0.06 \leq |V_{ub}/V_{cb}| \leq 0.16, \tag{2.24}$$

is directly measured in semileptonic B decay and thus independent of m_t. The other two constraints depend on loop processes: CP violation in $K - \overline{K}^0$ mixing parametrized by ϵ, and $B_d - \overline{B}_d$ mixing parameterized by x_d. The resulting constraints depend strongly on the yet-unknown mass of the top quark, m_t. The analytical expressions are (26)

$$x_d \equiv \frac{\Delta M(B^0)}{\Gamma(B^0)} = \tau_b \frac{G_F^2}{6\pi^2} \eta_B M_B (B_B f_B^2) M_t^2 f_2(y_t) |V_{td}^* V_{tb}|^2, \tag{2.25}$$

$$|\epsilon| = \frac{G_F^2}{12\pi^2} \frac{M_K}{\sqrt{2}\Delta M_K} (B_K f_K^2) M_W^2$$
$$\times \left\{ \eta_1 y_c \mathrm{Im}\left[(V_{cd}^* V_{cs})^2\right] + \eta_2 y_t f_2(y_t) \mathrm{Im}\left[(V_{td}^* V_{ts})^2\right] + 2\eta_3 f_3(y_t) \mathrm{Im}\left[V_{cd}^* V_{cs} V_{td}^* V_{ts}\right]\right\}, \tag{2.26}$$

where $y_i \equiv m_i^2/M_W^2$ and

$$\begin{aligned} f_2(y_t) =& 1 - \frac{3}{4}\frac{y_t(1+y_t)}{(1-y_t)^2}\left[1 + \frac{2y_t}{1-y_t^2}\ln(y_t)\right], \\ f_3(y_t) =& \ln\left(\frac{y_t}{y_c}\right) - \frac{3}{4}\frac{y_t}{1-y_t}\left[1 + \frac{y_t}{1-y_t}\ln(y_t)\right]. \end{aligned} \tag{2.27}$$

The values of well-known quantities used here are:

$$\begin{aligned} f_K =& 0.165\ GeV;\ m_c = 1.4\ GeV;\ M_B = 5.28\ GeV;\ M_W = 80\ GeV; \\ G_F =& 1.166 \times 10^{-5}\ GeV^{-2};\ |V_{us}| = \sin\theta_C = 0.22;\ |\epsilon| = 2.26 \times 10^{-3}. \end{aligned} \tag{2.28}$$

The QCD correction factors for ϵ (27) and x_d (28) are

$$\eta_B = 0.85;\quad \eta_1 = 0.7,\quad \eta_2 = 0.6,\quad \eta_3 = 0.4. \tag{2.29}$$

We consider the range $0.038 \leq |V_{cb}| \leq 0.052$, and $90\ GeV \leq m_t \leq 200\ GeV$. As examples we choose $m_t = 90,\ 120,\ 160,$ and $200\ GeV$. The constraint on $|V_{ub}/V_{cb}|$

(see Eq. (2.24)) forces the vertex A to lie between two circles (dotted in Fig. 2) centered at the vertex $C(0,0)$. The constraint on x_d (see Eq. (2.25)) requires the vertex A to lie between two circles (dashed in Fig. 2) centered at $B(1,0)$. The width of this band arises mainly from theoretical uncertainties in $B_B f_B^2$ and, to a lesser extent, from lifetime and mixing measurements:

$$(0.1\ GeV)^2 \leq B_B f_B^2 \leq (0.2\ GeV)^2,$$

$$2.9 \times 10^9\ GeV^{-1} \leq \tau_b |V_{cb}|^2 \leq 4.1 \times 10^9\ GeV^{-1}, \tag{2.30}$$

$$0.55 \leq x_d \leq 0.77.$$

The constraint on ϵ (see Eq. (2.26)) demands that the vertex A lie between the two hyperbolas (solid curves in Fig. 2). The width of this band arises from the theoretical uncertainty in $|V_{cb}|$ and in the B_K parameter,

$$1/3 \leq B_K \leq 1. \tag{2.31}$$

The resulting allowed domain for the vertex A is given by the shaded region in Fig. 2.

The allowed value for the angles α, β and γ can be deduced from Fig. 2. Note that values of 45^o or 135^o correspond to a maximal CP asymmetry, while 90^o for an angle implies that there will be no CP asymmetry in the corresponding class of B decays. However, if one angle is 90^o, then CP violation will necessarily exhibit itself in the decays that measure the other two angles. Examining Fig. 2, we see that either α or γ may be 90^o for all top masses. Consequently, zero asymmetries may occur for class ($3d$), *e.g.* $B_d \to \pi^+\pi^-$, or class ($3s$), *e.g.* $B_s \to \rho K_S$ decays, respectively. In fact, for $\sin(2\alpha)$ and $\sin(2\gamma)$, all values are allowed. The possibilities

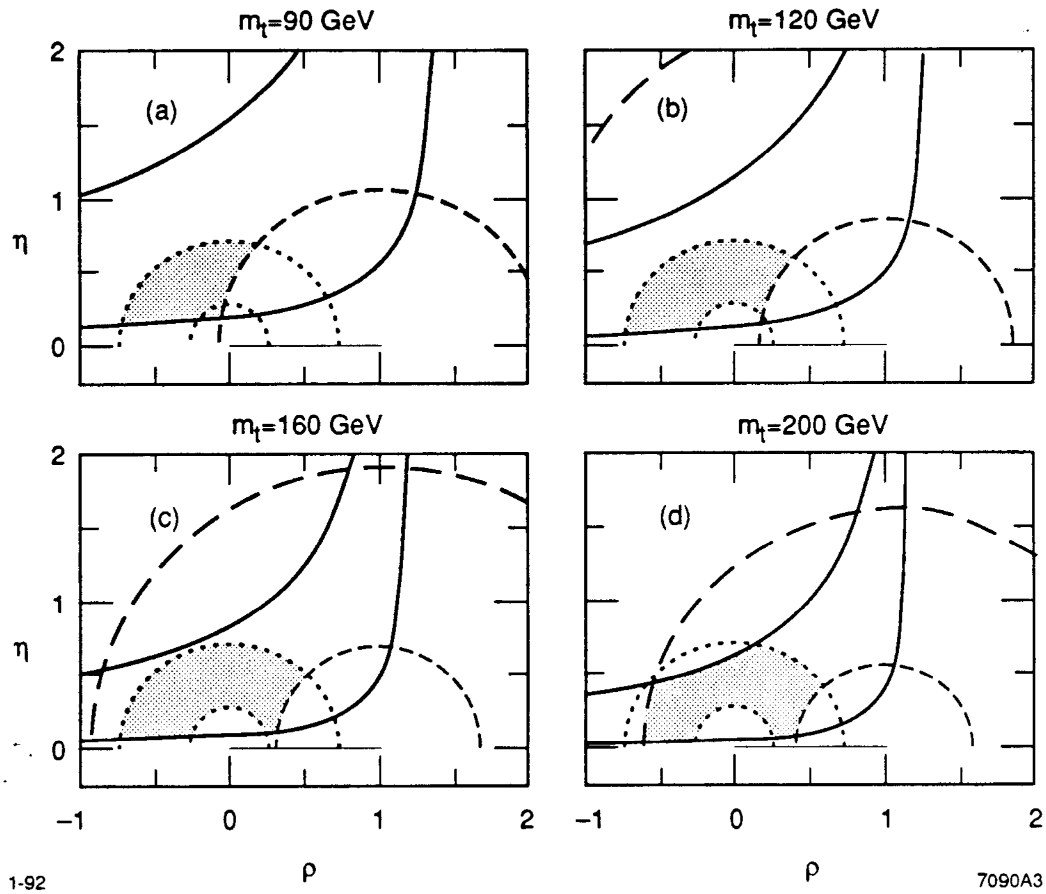

Figure 2. Constraints (21) from $|V_{ub}/V_{cb}|$ (dotted lines), x_d (dashed curves) and ϵ (solid curves) on the rescaled unitarity triangle for $m_t = 90,\ 120,\ 160$ and 200 GeV. The shaded region is that allowed for the vertex $A(\rho, \eta)$.

range from maximal ($|\mathrm{Im}\lambda| = 1$) to vanishing ($|\mathrm{Im}\lambda| = 0$) CP asymmetry. The fact that a particular interference term might vanish is disconcerting; however, failure to observe CP violation in just a class ($3d$) or just a class ($3s$) process would not be evidence against CP violation originating in the CKM matrix. Fortunately, a nonvanishing asymmetry is guaranteed in decays of classes ($1d$), ($2d$) and ($4d$) in the Standard Model, since the angle β satisfies (see Fig. 3(a))

$$2^o < \beta \leq \arcsin |V_{ub}/(V_{cd}V_{cb})| \lesssim 47^o, \tag{2.32}$$

giving (see Fig. 3(b)):

$$0.08 \leq \sin(2\beta) \leq 1. \tag{2.33}$$

Moreover, this is just the angle that can be most readily measured. As we discuss below, a leptonic B-factory with luminosity of 3×10^{33} cm^{-2} sec^{-1} is expected to achieve within one year of running an accuracy of ± 0.05 in the measurement of $\sin(2\beta)$, while a hadron collider such as the upgraded TeVatron at Fermilab could achieve an accuracy of about ± 0.15.

The three angles of the unitarity triangle are, of course, correlated. Thus, an experiment which measures asymmetries proportional to both $\sin(2\alpha)$ and $\sin(2\beta)$ is assured that $|\text{Im } \lambda| \geq 0.1 - 0.2$ (the exact value depends on the top mass) for at least one of the two processes (24). Similarly, an experiment searching simultaneously for CP asymmetries in processes sensitive to each of the three different angles is guaranteed to find that $|\text{Im } \lambda| \geq 0.2 - 0.3$ for at least one of the three classes of CP violating asymmetries.

The allowed range for the unitarity triangle is rather sensitive to the value of f_B. Recent lattice calculations (29–32) give, instead of the range in Eq. (2.30), $f_B \sim 0.2 - 0.4$ GeV. It is interesting to note that a recent QCD sum rule calculation, consistent with heavy quark symmetry constraints, yields (33) $f_B = 0.19 \pm 0.05$ GeV. We believe that it is premature to include these lattice results in our range for f_B since the calculations are still being tested and refined, and it appears that finite-mass corrections will lower the values (34, 35). However, we note that large values of f_B would be very favorable for the measurements of CP asymmetries. To demonstrate that, we give in Fig. 3(c) the lower bound on $\sin(2\beta)$ (*i.e.* $a_{\psi K_S}$) as a function of f_B for various values of m_t. Note that if $m_t \gtrsim 120$ GeV

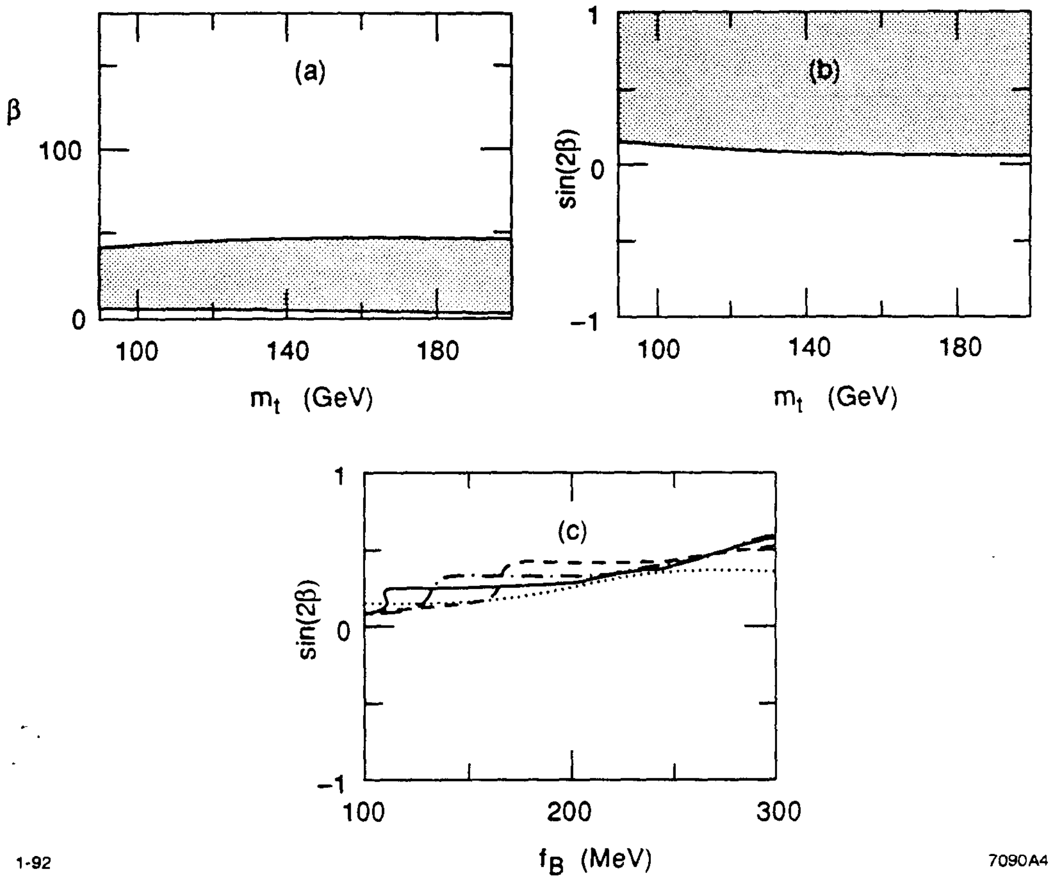

Figure 3. (a) The allowed range for the angle β of the unitarity triangle as a function of the top mass. (b) The allowed range for $\sin(2\beta)$ as a function of the top mass. (c) The lower bound on $\sin(2\beta)$ as a function of f_B for top masses of 90 GeV (dotted), 120 GeV (dashed), 160 GeV (dot-dashed) and 200 GeV (solid).

and $f_B \gtrsim 170\ MeV$, then $\sin(2\beta) \gtrsim 0.26$.

A further comment on the program of testing the unitarity triangle is in order. Suppose we cannot measure all three angles. (As mentioned above, $\sin(2\gamma)$ may be difficult to study.) The triangle can be overdetermined by measuring two angles and one additional side – for example, by determining $|V_{ub}|$. Measurements of the decay $B \to \rho e \nu$ and comparison to $D \to \rho e \nu$ can be done quite accurately in a B-factory. The question here is how well can theorists determine the model-dependent $1/m_c$ corrections to the heavy quark symmetry that relates the B- and D-decay form factors (see Ref. (21) for a review and references on this topic). A-priori these corrections can be large, but a variety of approaches may be used to try to achieve accurate estimates. These include lattice calculations as well as the more traditional models for wave-functions.

2.4 Angular Analysis and Other Ways to Treat Additional Modes.

The simplest modes to analyse for CP violating parameters are the decays to a pure CP eigenstate such as ψK_S or $\pi^+\pi^-$. There are many modes that contain both even and odd CP contributions, but which can still be used to extract the fundamental parameters of the CKM matrix by carefully selecting a definite CP contribution.

The simplest case is any collection of CP self-conjugate particles, where the mixture of CP states comes from the possibility of more than one orbital angular momentum state. Since $P \propto (-1)^L$, even- and odd-L states contribute with opposite CP. This apparently non-relativistic argument can be restated in a fully relativistic form using the helicity formalism. The problem of isolating a definite CP contribution can then be studied in terms of the possible reconstruction of helicities from an angular analysis of the decays of unstable spinning particles such as ρ or K^*. The use of such methods in the decays of a scalar to two vector particles was discussed in general in Refs. (36,37) and for B decays in Ref. (38). In Ref. (39), a special case of this analysis in the context of a modern B-factory is pointed out, namely that selecting zero helicity vector particles is one way to select a definite CP state. Subsequently, a study of various approaches to angular analysis for few body B decays was carried out in Ref. (40). We here summarize a few of the results.

When sufficient data is available on a set of isospin related channels, for example ψK^{*0} and ψK^{*+}, the best method is always to determine the various helicity and isospin amplitudes by a maximum likelihood fit to all the data. The CP asymmetry is one of the parameters that can be extracted from this fit. In such an analysis the possible small CP violations in charged B decay channels which can arise from penguin contributions are neglected.

In cases where the information from related channels is not available to help fix parameters, one can still perform an angular analysis to isolate definite CP contributions. A particularly attractive way to do this for final vector–vector states is to define a plane in the rest frame of the B that contains the decay products of one of the vector particles and the second vector particle and then to analyse the spin projection of the second vector particle on the direction perpendicular to this plane. This quantity, called transversity, is invariant under boosts in the plane and is directly related to the CP eigenvalue. One advantage of this method is that it does not require a true two body process; the three body state is all that is needed to define the plane and hence non-resonant production of the pair of scalars or of leptons can also be included in the statistical sample.

Reference (40) lists additional modes that could possibly be studied using some version of angular analysis. The usefulness will depend on branching ratios, a quantity that is not yet known for many of these modes. In a few cases more definite statements can be made. For example, $BR(B \to \psi K^*)$ is larger than $BR(B \to \psi K_S)$ by a factor of 3-6. Angular analysis applied to the decay of the K^*

can yield a measure of the angle β of comparable accuracy to that obtainable from the ψK_S channel, where no such analysis is needed. If both CP states contribute equally to the decay, it will require about 5000 reconstructed events, or about four times as many as for the pure CP channel ψK_S, to achieve $\delta(\sin 2\beta) = \pm 0.05$ (41). Results from Argus (42) indicate that the channel may be dominated by a single CP eigenmode, in which case it may even prove superior to ψK_S in accuracy for a given luminosity. For the comparison of $\pi\pi$ with $\rho\rho$, the branching ratios are not yet known, but it is likely that the latter channel has a significantly larger branching ratio and hence again may be as important as the simpler case in extracting an accurate value for the angle α.

Clearly one would like to use as many modes as possible for extracting the underlying CP violating parameters of the Standard Model. Another potentially useful class of modes (43) includes two body modes where the particles are not CP eigenstates but the underlying set of four quarks is unchanged in a CP transformation, *e.g.* $\rho^+\pi^-$. Again, the data will allow extraction of the underlying CKM parameters, provided a sufficient set of related channels is measured. Yet another possibility is to extract CP asymmetries from Dalitz plot distributions (44–46).

The usefulness of any of these methods will depend on the ability of the detectors to give good angular resolution of the particle decays and good particle identification. However, there is by now quite a bag of tricks awaiting any data with sufficient statistics. At present it appears that none of the more complicated analyses will yield better results for the underlying CKM parameters than the simplest modes first studied, but they could be competitive with them in accuracy.

3. Physics beyond the Standard Model

CP asymmetries in B decays are a sensitive probe of new physics in the quark sector, because they are likely to differ from the Standard Model predictions if there are sources of CP violation beyond the KM phase of the Standard Model. This can contribute in two ways:

1. there are significant contributions to $B - \overline{B}$ mixing (or $B_s - \overline{B}_s$ mixing) beyond the box diagram with intermediate top quarks; or if

2. the unitarity of the three generation CKM matrix does not hold, namely there are additional quarks.

Other ingredients in the analysis of CP asymmetries in neutral B decays are likely to hold in any model that satisfies current experimental constraints:

3. $\Gamma_{12} \ll M_{12}$. In order for this relation to be violated, one needs a new dominant contribution to tree-decays of B-mesons or strong suppression of the mixing compared to the Standard Model box diagram. Neither possibility is likely. The argument is particularly solid for the B_d system, as it is supported by experimental evidence: $\Delta M/\Gamma \sim 0.7$, while branching ratios into states which contribute to Γ_{12} are $\lesssim 10^{-3}$.

4. The relevant decay processes (in classes $i = 1, 2, 3$) are dominated by the Standard Model tree diagrams. Again, it is unlikely that new physics which typically takes place at a high energy scale, would compete with weak tree decays.

5. The phase of the mixing amplitude for neutral Kaons is approximately $\arg(V_{cd}^* V_{cs})$. This is practically guaranteed by the measurement of the ϵ

parameter (47).

Within the Standard Model, both b-quark decays and $B_q - \overline{B}_q$ mixing are determined by combinations of CKM matrix elements. The asymmetries then measure the relative phases between these combinations. Unitarity of the CKM matrix directly relates these phases (and consequently the measured asymmetries) to angles of the unitarity triangle. In models with new physics, unitarity of the three generation charged current mixing matrix may be lost and consequently the relation between the CKM phases and the angles of the unitarity triangle violated. But this is not the main reason that the predictions for the asymmetries are modified. The reason is rather that when $B_q - \overline{B}_q$ mixing has significant contributions from new physics, the asymmetries measure different quantities: the relative phases between the elements of mixing matrices in sectors of new physics (squarks, multi-Higgs, etc.) which contribute to $B_q - \overline{B}_q$ mixing and the CKM elements which determine b decays now enter the calculation.

Thus, when studying CP asymmetries in models of new physics, we look not only for violation of the unitarity constraints:

$$\begin{aligned} V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} &= 0, \\ V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} &= 0, \end{aligned} \tag{3.1}$$

but also for contributions to $B_q - \overline{B}_q$ mixing which are different in phase and at least comparable in magnitude to the Standard Model contribution:

$$M_{12}^{SM}(B_q) = \frac{G_F^2}{12\pi^2} \eta M_B (B_B f_B^2) M_W^2 y_t f_2(y_t) (V_{tb}^* V_{tq})^2. \tag{3.2}$$

The results of a survey of models beyond the Standard Model (48) are summarized in Fig. 4. Models beyond the Standard Model (SM) that have been studied include four quark generations (49-52), multi-scalar model with natural flavor conservation (NFC), with explicit or spontaneous CP violation (SCPV) (53), Z-mediated flavor changing neutral currents (FCNC) (54,55), Left-Right symmetry (LRS) (56,57), Supersymmetry (SUSY) (58) and "real superweak" models (59). Many of these models allow substantial deviations from the Standard Model predictions.

Some relations among the asymmetries do not depend on certain assumptions and thus may hold beyond the Standard Model or, conversely, if they are violated can help pinpoint which ingredients must be added to the Standard Model (47,19,48). The predictions

$$\text{Im }\lambda_{1d} = \text{Im }\lambda_{2d}, \quad \text{Im }\lambda_{1s} = \text{Im }\lambda_{2s}, \tag{3.3}$$

depend only on the mechanism for tree-level decays and the $K - \overline{K}$ mixing phase. Existing constraints already ensure that these will hold in any viable models. Violation of

$$\text{Im }\lambda_{1d} = \text{Im }\lambda_{4d}, \quad \text{Im }\lambda_{1s} = \text{Im }\lambda_{4s}, \tag{3.4}$$

would indicate that the second unitarity relation in (3.1) is violated. Similarly, there are clean tests of the first relation in (3.1). Violation of

$$\text{Im }\lambda_{1s} = 0 \tag{3.5}$$

would indicate new mechanism for $B_s - \overline{B}_s$ mixing. Violation of

$$\text{Im }\lambda_{2d} = -\sin(2\beta), \quad \text{Im }\lambda_{3d} = \sin(2\alpha), \tag{3.6}$$

would indicate new mechanism for $B_d - \overline{B}_d$ mixing. Finally, we note that the three angles deduced from measurements of Im λ_{1d}, Im λ_{3d} and Im λ_{3s} will sum to 180^o

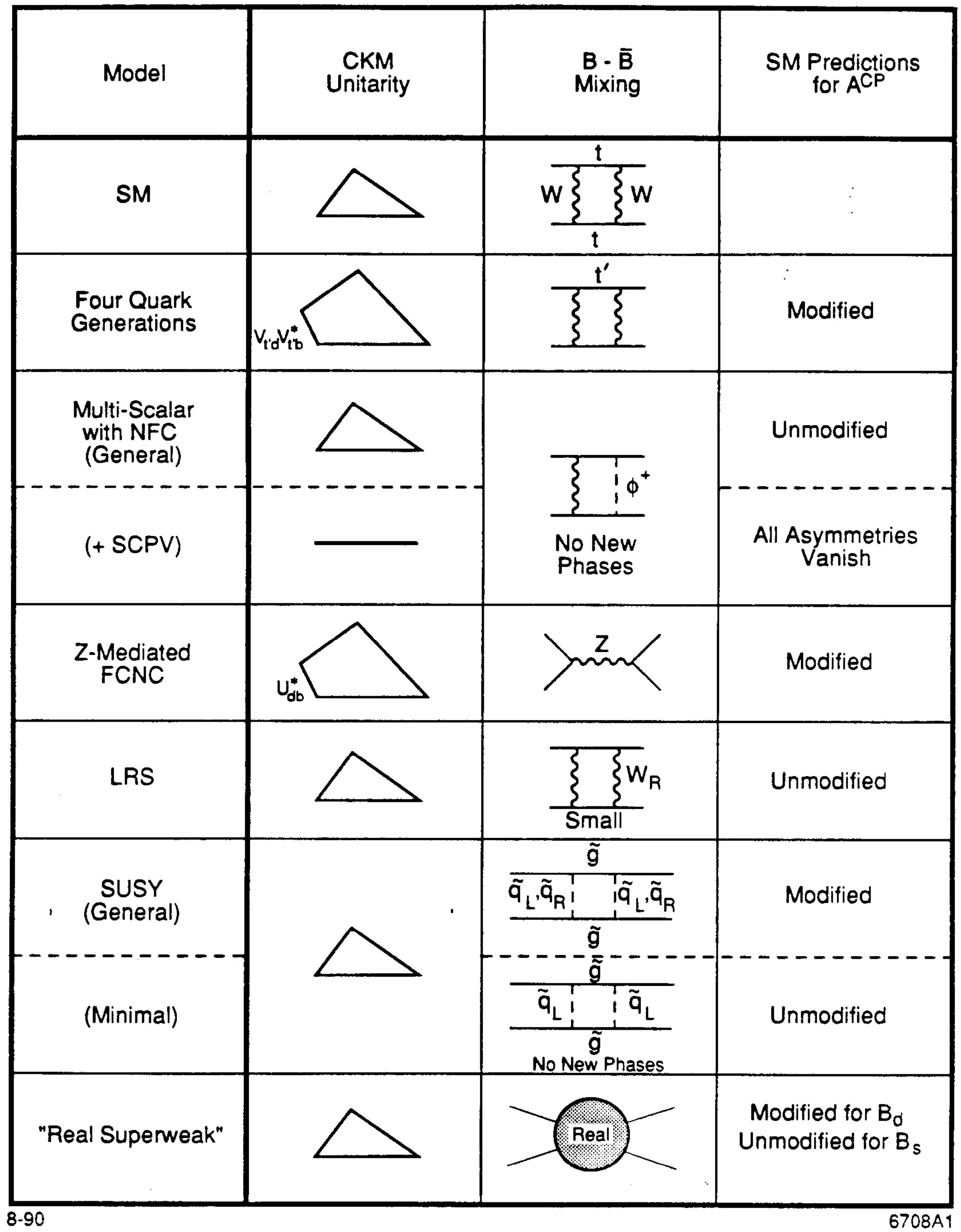

Figure 4. New physics effects on CP asymmetries in neutral B decays (48). The second column describes whether unitarity of the three generation CKM matrix is maintained (a triangle) or violated (a quadrangle). The third column gives an example of a new contribution to the mixing. Unless otherwise mentioned, the contribution could be large and carry new phases.

whenever the amplitude for $B_s - \overline{B}_s$ mixing is real (47). This is independent of whether they correspond to the angles of the unitarity triangle or not.

4. Tree and Penguin Processes

4.1 General Discussion

In the Standard Model the decays of any meson containing a heavy quark necessarily involves a W-boson. The amplitudes are generally divided into two classes, called tree or penguin type. If all the complications of strong interactions are ignored, this classification is readily explained in terms of the quark diagrams for the underlying weak transition, see Fig. 5. The penguin process is one where the W-boson is reabsorbed on the same quark line from which it was emitted, and all other diagrams are tree processes, that is they have no closed loop in the weak diagram. Tree diagrams can be further subdivided into spectator (where the light quark in the initial meson is disconnected in the weak decay diagram), exchange (where the W is exchanged between the two quarks of the initial meson) and annihilation (where the quark and antiquark of the initial meson annihilate to form the W). These subdivisions are unimportant in a discussion of CP violation because whenever two types of tree diagrams contribute to the same decay amplitude they do so with the same CKM matrix element and hence the same weak phase.

For the penguin contributions we have drawn the diagram in Fig. 5 without identifying the gluon which is emitted from the W–quark loop with that which produces the additional quark-antiquark pair to stress the fact that the term penguin in principle includes all possible such contributions. Note that the many gluons

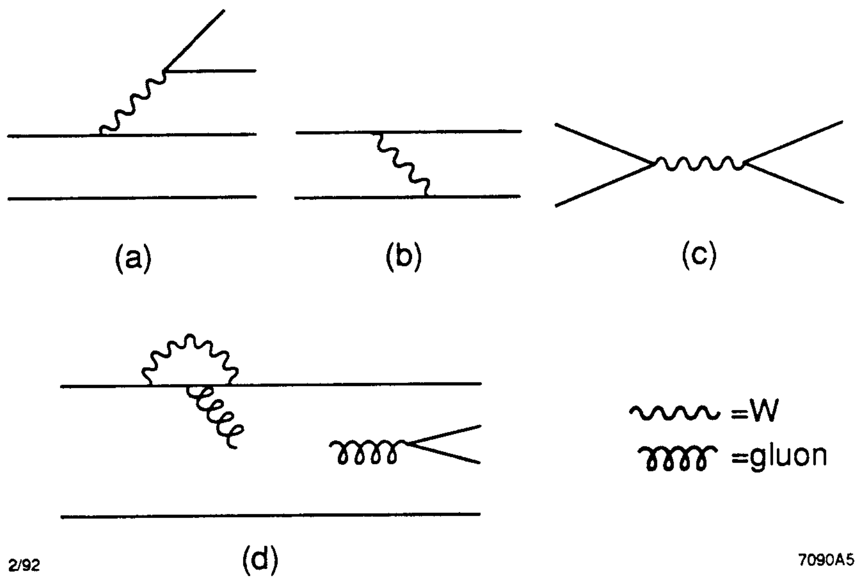

Figure 5. Tree and Penguin Contributions. Diagrams (a) Spectator, (b) W-exchange and (c) Annihilation are all tree contributions. Diagram (d) represents the penguin contribution. The gluons associated with binding are not shown.

involved in the binding are not drawn here, so the disconnected line simply means a gluon absorbed in and another produced from the general glue.

In contrast to the various tree diagrams, the penguin contributions must be treated separately in analysing CP violations because, in general, they will depend on different CKM matrix elements. In fact, in the penguin transition $b \rightarrow q$ ($q = d, s$), there are three penguin terms with different quarks in the loop ($i = u, c, t$), each contributing with a different CKM combination:

$$v_{iq} = V_{ib}^{*} V_{iq}. \tag{4.1}$$

(With three quark generations, unitarity requires

$$v_{uq} + v_{cq} + v_{tq} = 0 \tag{4.2}$$

which allows one of these three phases to be eliminated in terms of the remaining two.) Hence it becomes important to be able to calculate the relative strengths and strong phases of the tree and penguin terms.

The penguin involves strong interaction processes, as shown in Fig. 5. When the penguin contribution is evaluated perturbatively one identifies the two gluon lines in Fig. 5 for the leading contribution and then adds additional gluon corrections for a higher order calculation. The justification for this perturbative treatment is that the gluon emitted from the quark loop is quite hard because of the large mass difference between the initial b quark and the final s or d quark. We are doubtful that this approach is completely correct. For example, a contribution in which the hard gluon is absorbed by the other quark of the original meson and then hadronization occurs non-perturbatively could be comparable to the one usually calculated, particularly in inclusive rates, but is less readily evaluated. The standard approach is to calculate the penguin contribution perturbatively.

Such a calculation gives an estimate of the inclusive asymmetries summed over all states with a particular quark content. It is difficult to convert these numbers into reliable estimates for rates and asymmetries in particular exclusive (few body) channels (see *e.g.* Ref. (60)). Each configuration of final hadrons corresponds to some weighted integral over quark kinematics, but unfortunately we have no way to reliably determine that integral. Since the calculated quark-level asymmetries depend on the momentum transfer to the $q\overline{q}$ pair and even change sign as a function of this variable in some cases, it is very difficult to convert the quark estimates into estimates for exclusive hadron processes.

Furthermore, because of the dependence of the asymmetry on the difference of strong phases as well as that of the weak phases, calculations are sensitive to other aspects of hadronization. In the quark diagram calculation, the long-range final state hadron-hadron interaction phase shifts are ignored. The only strong phases included are those that arise from the absorptive parts of penguin loop diagrams. This assumption that no additional phase shifts are caused by final state rescattering is known as factorization. It is built into the calculations but has not yet been well tested. For a discussion of the justification of this assumption see Ref. (45). On the other hand it has been argued (61) that hadronization can result in final-state phase shifts which could decrease the resulting asymmetries compared to the quark-diagram perturbative calculations. This question remains an open one.

In the following section we review CP asymmetries in charged B decays. With the exception of the special case $B \to D^0_{CP} K(+n\pi)$, the penguin processes are central to the existence of any CP asymmetries in these decays. Hence it is much more difficult to reliably predict these asymmetries than those of the neutral B decays. If such asymmetries are observed they will verify the existence of penguin type processes and direct CP violation, but it will be difficult to extract from these measurements any of the fundamental CKM parameters.

4.2 CP Violation in Charged B Decays

In charged B decays, one can search for CP violating differences of the form

$$a_f = \frac{\Gamma(B^+ \to f) - \Gamma(B^- \to \overline{f})}{\Gamma(B^+ \to f) + \Gamma(B^- \to \overline{f})} \tag{4.3}$$

where f is any final state and $\overline{f}$ is its CP conjugate. Although CPT symmetry requires that the total B^+ and B^- decay widths are the same, specific channels or sums over channels can contribute to asymmetries of the form (4.3). For this to occur, there must be interference between two separate amplitudes that contribute to the decay $B^+ \to f$ with different weak phases, $\phi_1 \neq \phi_2$, *and* with different strong phase shifts, $\delta_1 \neq \delta_2$. Then

$$a_f \propto \sin(\delta_1 - \delta_2)\sin(\phi_1 - \phi_2). \tag{4.4}$$

It was recognized by Bander, Silverman and Soni (62) that, within the Standard Model, these conditions can readily be met in three types of B decays:

(i) CKM suppressed B decays. The tree amplitudes for $b \to u\bar{u}s$ can interfere with penguin type processes.

(ii) CKM forbidden B decays. In the channels $b \to sd\bar{d}$, $b \to dd\bar{d}$, $b \to ss\bar{s}$, and $b \to ds\bar{s}$, which have no tree contributions, there can still be asymmetries due to the interference of penguin contributions with different charge 2/3 quarks in the loop. For example, in $b \to ds\bar{s}$

$$A_{b\to ds\bar{s}} = v_{ud}A_{u,ds\bar{s}} + v_{cd}A_{c,ds\bar{s}} + v_{td}A_{t,ds\bar{s}} \tag{4.5}$$

where v_{iq} is defined in (4.1), and $A_{i,ds\bar{s}}$ is the corresponding penguin amplitude (including the strong phase shift). We can use the unitarity constraint (4.2) to eliminate v_{tq}:

$$A_{b\to ds\bar{s}} = v_{ud}\Delta_{ut}(ds\bar{s}) + v_{cd}\Delta_{ct}(ds\bar{s}) \ . \tag{4.6}$$

This then leads to a nonvanishing asymmetry,

$$a_{ds\bar{s}} \propto \mathrm{Im}\,(v_{ud}^{*}v_{cd})\,\mathrm{Im}\,(\Delta_{ut}^{*}\Delta_{ct}), \tag{4.7}$$

provided the quantities Δ_{ct} and Δ_{ut} have different strong final state phases. The phases of the penguin amplitudes can be evaluated by examining the various possible cuts of the diagrams. If the u and c quarks were degenerate, the two contributions Δ_{ut} and Δ_{ct} would be identical and the asymmetry would vanish.

Since the penguin amplitudes are each of order α_s, the penguin-penguin interference term is of order α_s^2. Thus a consistent perturbative calculation must take into account all other order α_s^2 contributions to the rate (63).

(iii) Radiative B decays. The mechanism for CP asymmetries is similar to that of the pure penguin cases discussed above, except that the leading contribution to the decay is an electromagnetic penguin, that is one where the gluons shown in Fig. 5 are replaced by a single photon line.

A special case (15) is the channel $B^{\pm} \to D^0_{CP} K^{\pm}$ where D^0_{CP} represents the decay of a D^0 or $\overline{D}^0$ to a CP eigenstate such as $\pi^+\pi^-$. Here interference between the D^0 and $\overline{D}^0$ tree contributions can give rise to a CP violation. This is an exception to the general statement that observation of CP violation in charged B decays requires non-vanishing penguin contributions. CP violation in this mode occurs because the D^0 and $\overline{D}^0$ have common decay channels, and the weak amplitudes for $B^0_d \to D^0, \overline{D}^0$ have different phases.

Detailed studies of expected asymmetries in charged B decays for classes (i) and (ii) above have been recently carried out by two groups (64,65) with similar conclusions; namely, $a_{su\bar{u}}$, $a_{sd\bar{d}}$ and $a_{ss\bar{s}}$ are a few tenths of a percent, while for the rarer processes $a_{dd\bar{d}}$ and $a_{ds\bar{s}}$ could be as large as a few percent. These estimates are for the inclusive sum of all channels with a given quark content, which is not readily measured. The cases $b \to q\bar{d}d$ would be particularly difficult to extract since many modes with a contribution from this quark content will also have $b \to q\bar{u}u$ contributions. (For the $q = d$ case such terms dominate all channels.) Even to extract the $q\bar{d}d$ contribution requires careful subtraction of different isospin combinations. It is unlikely that one could measure the small asymmetry in the resulting quantity. Earlier estimates based on model calculations (66) give larger asymmetries for some exclusive channels but the results are highly model dependent. Estimates of asymmetries in baryonic modes are given in Ref. (67). Asymmetries in radiative B decays have been studied in Ref. (68), finding $a_{s\gamma} \sim (1 - 10) \times 10^{-3}$ and $a_{d\gamma} \sim (1 - 30) \times 10^{-2}$.

The calculations of Refs. (64, 65) suggest that the CP violations in charged B decays will be extremely difficult to observe, requiring $\mathcal{O}(10^{10})$ produced B's for exclusive $b \to s$ modes and $\mathcal{O}(10^9)$ B's for exclusive $b \to d$ modes. In Ref. (64) it is suggested that this can be improved to perhaps as low as 10^7 produced B's if one can sum all two-body or quasi two-body $b \to ds\bar{s}$ modes, but the experimental difficulties of such a semi-inclusive measurement may defeat this theoretical improvement. These estimates include neither any factors for the inefficiencies introduced by triggering and detection requirements, nor suppression due to final-state rescattering, and thus are quite optimistic. The situation is even more difficult for the radiative decays (68), which give comparable asymmetries but have lower branching ratios.

Although the uncertainties inherent in the calculations described above leave some small possibility of larger effects (see for example the model predictions of Ref. (66)), it appears that the calculations are sufficiently reliable that asymmetries an order of magnitude *larger* than predicted in Refs. (64, 65, 68) would have to be interpreted as evidence for some CP violating mechanism that arises from sources beyond the Standard Model. Various "beyond standard" models contain novel CP violating decay mechanisms which could be comparable to the Standard Model penguin contributions. For example, with four quark generations there is a penguin diagram with an intermediate t' which depends on additional phases of the 4×4 mixing matrix; in models with Z-mediated flavor changing neutral currents there is a tree diagram which depends on new phases in the neutral current mixing matrix.

The net conclusion of this section is unfortunately that with regard to the charged B decays the situation is not unlike that for ϵ' of the K system. The Standard Model predicts a small effect that will be experimentally very difficult to measure. However, any program of B physics should certainly attempt to measure as many different asymmetries of the form (4.3) as possible. Any large non-zero result could provide a clue to physics beyond the Standard Model.

4.3 Eliminating Penguin Uncertainties with Isospin Analysis

In neutral B decays, the penguins are not an essential part of the CP violation mechanism because the mixing mechanism provides two paths to many final states even in the absence of any penguin contributions. The theoretical uncertainties in estimating relative strength of penguin and tree contributions can, however, lead to uncertainties in the relationship between measured asymmetries and fundamental CKM matrix parameters (69-71). In $b \to c\bar{c}s$ processes (*e.g.* $B \to \psi K_S$) both amplitudes carry the same CKM phase; extracting $\sin 2\beta$ from this asymmetry is thus independent of the relative strength of tree and penguin contributions and hence free of such uncertainties. In $b \to u\bar{u}d$ processes (*e.g.* $B \to \pi\pi$) the two amplitudes carry different CKM phases. The perturbative estimates indicate that the contribution from the penguin amplitude is small (a few percent), but it could be larger than the naive expectation if the matrix element for the penguin operator is enhanced; thus the value of $\sin 2\alpha$ deduced from this asymmetry suffers from hadronic uncertainties. In this section, we describe a method of isospin analysis which can test for the existence of a significant penguin contribution and, with sufficient data, would provide a clean extraction of CKM parameters even if such a contribution were large (14). For $b \to u\overline{u}s$ processes (*e.g.* $B \to K\pi$) the situation is even worse: not only do the tree and penguin amplitudes carry different CKM phases, but also perturbative estimates suggest that they are comparable in magnitude (the tree process is strongly CKM-suppressed). Extraction of CKM parameters from data on this channel would require isospin analysis. Since the expected rates are low, it is unlikely that there will be enough data accumulated, even at a high luminosity B-factory, to carry out such an analysis for this channel (72–74).

We here briefly review the analysis of the $\pi\pi$ channel (14). There are three amplitudes for B^+ and B^0 decays into final $\pi\pi$ states,

$$A^{ij} \equiv \left\langle \pi^i \pi^j | \mathcal{H} | B^{i+j} \right\rangle . \tag{4.8}$$

Similarly, there are three amplitudes, $\overline{A}^{ij}$, for $\overline{B}^0$ and B^- decays to two pions. We define $\overline{A}^{ij}$ to be the amplitude for the CP-conjugated process of A^{ij}, *e.g.* $\overline{A}^{+0}$ corresponds to $B^- \to \pi^-\pi^0$. The $\overline{A}^{ij}$ amplitudes carry weak phases opposite to those of A^{ij}, but unchanged strong phases.

In the general case, there is one independent amplitude A_{I_t,I_f} for each possible combination of $\{I_t, I_f\}$, where I_t is the transition isospin and I_f is the final state isospin (including the spectator quark). However, there is no $I_f = 1$ state because it is forbidden by Bose symmetry for an angular momentum zero system of two pions. Consequently, $I_t = 3/2$ transitions lead to $I_f = 2$ states only, while $I_t = 1/2$ transitions lead to $I_f = 0$ states only. Therefore, we have two independent amplitudes only. This leads to a triangle relation within each set of decay amplitudes:

$$\begin{aligned} \sqrt{\tfrac{1}{2}}A^{+-} &= A^{+0} - A^{00}, \\ \sqrt{\tfrac{1}{2}}\overline{A}^{+-} &= \overline{A}^{+0} - \overline{A}^{00}. \end{aligned} \tag{4.9}$$

For the neutral modes, the decay rates are given in (2.9), with possibly $|\lambda| \neq 1$. Measuring the total rates for the charged and neutral B decays gives all six magnitudes, $|A^{ij}|$ and $|\overline{A}^{ij}|$, and consequently the shapes of the two triangles can be determined. In addition, from the time-dependent decay rates into $\pi^+\pi^-$, one can deduce $\mathrm{Im}\lambda^{+-} = \mathrm{Im}\left[e^{-2i\phi_M}(\overline{A}^{+-}/A^{+-})\right]$. Let us replace the barred amplitudes by rotated amplitudes, $\widetilde{A}^{ij} = e^{2i\phi_T}\overline{A}^{ij}$, where the phase ϕ_T is the CKM phase in the tree diagram. Since the penguin diagram is purely $I_t = 1/2$, only tree diagrams contribute A^{+0} and $\overline{A}^{+0}$ which are pure $I_t = 3/2$ transitions. Thus, the triangle formed by the $\widetilde{A}$'s shares a common side with that formed by the A's, $A^{+0} = \widetilde{A}^{+0}$. Any difference between the two triangles is now due to the penguin contributions. The figure thus formed allows us to measure the angle between A^{+-} and $\widetilde{A}^{+-}$ (up to an overall ambiguity which arises from the four possible orientations of the two triangles relative to their common side).

The CP asymmetry is

$$\mathrm{Im}\lambda^{+-} = \mathrm{Im}\left[e^{-2i(\phi_M+\phi_T)}\frac{\widetilde{A}^{+-}}{A^{+-}}\right]. \tag{4.10}$$

Were $A_{1/2,0}$ dominated by the tree-level diagram, we would have $(\widetilde{A}^{+-}/A^{+-}) = 1$, and Eq. (4.10) would reduce to the usual $\sin 2(\phi_M + \phi_T)$ expression. However, from the triangles we know both the magnitude and the phase of $(\widetilde{A}^{+-}/A^{+-})$; we need not make the assumption of a small penguin amplitude anymore. We are able to disentangle the value of $(\phi_M + \phi_T)$ without any uncertainty from the unknown penguin contribution to $A_{1/2,0}$. If we assume the Standard Model, we can in principle also use the construction described above to extract a measure of the penguin contribution to $A_{1/2,0}$ (72). If, as expected, the penguin contribution is small then the two triangles will be the same within errors. In that case this isospin analysis will simply place an experimental bound on the errors in $\sin(2\alpha)$ due to penguin contributions.

Similar isospin analysis can in principle be applied to many other modes (73), *e.g.* $\rho\rho$, $\rho\pi$, $K\pi$, $K\rho$ and $K\pi\pi$. It is doubtful that sufficient data will be available to allow such analyses. The number of discrete ambiguities further reduces the usefulness of this approach in all but the simplest case (74, 75).

5. Experimental Prospects

The many interesting features of CP violation in B decays can only be studied with a copious source of B mesons. We review here several different experimental approaches.

5.1 B-FACTORIES

The term B-factory is used for a high-luminosity e^+e^- collider running at the energy of the $\Upsilon(4S)$ resonance. The $\Upsilon(4S)$ decays almost exclusively into $B^0\overline{B}^0$ pairs (50%) and B^+B^- pairs (50%). The consensus of various design groups (most noticeably at SLAC and Cornell in the US and at KEK in Japan) has now settled on a design based on an 'asymmetric' collider, with two rings at different energies (76). The purpose of this asymmetry in energies is to produce a $B^0\overline{B}^0$ system which is moving with a significant relativistic γ-factor in the laboratory. This will cause the two B mesons to decay typically far enough apart in space that they can be separately identified by the detector and that the separation between them can be measured. (In contrast, an $\Upsilon(4S)$ state at rest would produce B mesons almost at rest, and their decay vertices could not be resolved experimentally.) This then allows a reconstruction of the time difference between the two decays. When one of the decays, say at time t_{tag}, is to a tagging mode, that is a mode which identifies the parent particle as either a B^0 or a $\overline{B}^0$, and the other decay, at time t_{CP}, is to a mode that can be used for reconstruction of a well defined CP asymmetry, then knowledge of the time difference $t = t_{CP} - t_{tag}$ between the two decays gives a time-dependent measurement of that asymmetry. The $B^0\overline{B}^0$ pair are produced in a well-defined coherent CP eigenstate which does not evolve until one of the two B's decays. This decay then effectively starts the clock for the remaining particle (B or $\overline{B}$) to evolve due to mixing with its CP conjugate. Notice that t can be either positive or negative. Since the CP asymmetry is an odd function of t, $a_{f_{CP}} \propto \sin(\Delta M t)$, a time-dependent measurement is essential to any study of CP violation for this system. A time-integrated result from an e^+e^- collider of this type will have no CP violating contribution.

The challenge is thus to design an asymmetric collider with sufficient luminosity at the $\Upsilon(4S)$ to allow these interesting measurements to be made. A number of studies have been made and the common conclusion is that a luminosity of 3×10^{33}

cm^{-2} sec^{-1} is sufficient for the task and can be achieved. (With this luminosity, a 3σ measurement of $\sin 2\beta$ is possible within three years of running even if it is at the lower bound (2.33).) Detailed machine designs in SLAC, Cornell and KEK have been developed and agree in their basic ideas. To study B_s decay modes the machines can be run at the $\Upsilon(5S)$ resonance, but the cross-section, $\sigma(e^+e^- \to \Upsilon(5S)) = 0.16\ \mu b$, and the branching ratio, $BR(\Upsilon(5S) \to B_s\overline{B}_s) \leq 0.1$, make it difficult to achieve a sufficient statistical sample for CP asymmetry measurements.

The ability to reconstruct both the tagging modes and the CP modes is of course as important as the luminosity in determining what can be achieved in a given running time. Extensive studies have been made by the proponents of these machines and preliminary detector designs exist. In Table 5.1 we display estimates from Refs. (77,78) of the accuracy with which the asymmetries of various modes can be measured.

TABLE 5.1

$\delta(\sin 2\phi)$ at $\Upsilon(4S)$ with $30 fb^{-1}$

ϕ	Mode	$10^4 \times BR$	$\delta(\sin 2\phi)$
β	ψK_S	4.0 ± 1.4 ($E^\dagger$)	0.05
	ψK^*	37 ± 13 (E)	
	$\psi' K_S$	11 ± 8 ($E^\dagger$)	
	D^+D^-	$1 - 10$ (T)	
	$D^{*+}D^{*-}$	$1 - 10$ (T)	
α	$\pi\pi$	0.2 (T); ≤ 0.9 (E)	0.18
α	$\rho\pi$	0.6 (T); ≤ 60 (E)	0.12
α	$a_1\pi$	0.6 (T); ≤ 5.7 (E)	0.18

The estimates are made using the general formula

$$\delta(\sin 2\phi) = [(1-2w)d]^{-1}[2\epsilon_f\epsilon_t B_f f_0 \sigma(b\bar{b}) \int \mathcal{L}dt]^{-1/2} \tag{5.1}$$

where $\epsilon_{f(t)}$ are the detection efficiencies for the CP mode (the tagging mode), B_f is the branching fraction for the CP mode, w is the fraction of wrong tags, f_0 is the fraction of b quarks that appear as $\overline{B}^0$ mesons and d is a dilution factor introduced by any time integration, by background contributions, by fitting procedures and by mixing of the tagged decay. The integrated luminosity $\int \mathcal{L}dt$ is given in nb^{-1} for running at the $\Upsilon(4S)$. The estimates given here are from the SLAC detector workshop studies; similar estimates have been made by other groups. These numbers are meant as a 'ball-park' figure only, since the precise numbers depend on details of detector design and on branching ratios that are not yet known. Note that (5.1) can be used also to provide similar estimates for hadron machines when the appropriate values for the cross-section and $\overline{B}^0$ fraction (here assumed equal to that for B^0) are used. Table 5.1 shows the assumed branching ratio and labels it by E for direct experimental determination and $E^\dagger$ when deduced from a charged channel (based on Ref. (5)), or T for theoretical estimates (based on Refs. (79,80)). We list only the most commonly discussed modes. The estimates suggest that the CP violating angles α and β can be well measured in such a B-factory. A 3σ measurement of $\sin 2\beta \gtrsim 0.15$ can be made within one year of running (see Eq. (2.33) for the Standard Model predictions). Additional channels that require angular analysis or other techniques to isolate a particular CP asymmetry can give confirming measurements of comparable accuracy.

It will be more difficult to constrain the γ-angle of the unitarity triangle in this way. Not only is $\sigma(e^+e^- \to \Upsilon(5S)) \ll \sigma(e^+e^- \to \Upsilon(4S))$, but also $BR(\Upsilon(5S) \to B_s\overline{B}_s) < BR(\Upsilon(4S) \to B_d\overline{B}_d)$. In addition, as $x_s \gg x_d$, $B_s - \overline{B}_s$ oscillations are very fast and hence the dilution factor d is small for this channel. Estimates show that an integrated luminosity of $300 fb^{-1}$ will be needed to achieve $\delta(\sin 2\gamma) = \pm 0.05$ for $x_s = 4\pi$ (78). If x_s is larger, the measurement would become even more difficult. For present machine designs, this means many years of running and is not a feasible goal.

We note that a B factory will allow a measurement of form factors in $B \rightarrow \rho e \nu$ and $D \rightarrow \rho e \nu$. Consequently, the determination of $|V_{ub}|$ will be limited in accuracy mainly due to theoretical uncertainties. As discussed in Chapter 2, such a measurement is useful to overdetermine the unitarity triangle even in the case that $\sin 2\gamma$ cannot be measured.

A second suggestion for a study of CP asymmetries in B decays in an e^+e^- machine is based on the idea of a polarized Z factory which would then preferentially produce B^0 or $\overline{B}^0$ in certain directions (81,82). While the idea is intriguing, it does not now seem likely that a sufficiently high luminosity in polarized Z particles will be available at either LEP (CERN) or the SLC (SLAC), nor is there any current proposal for a facility of this type to be built at any other location.

5.2 Hadron Experiments

The prospects for B physics studies with hadron colliders are not as clear, simply because the studies of what can be done with such machines are less advanced than for the B-factory proposals. This section is based on preliminary studies and estimates for various hadron colliders, most noticeably the upgraded TeVatron, the CERN LHC and the SSC. The processes that occur in hadron colliders are quite different from those of a lepton B factory. Instead of production of a coherent

$B^0\overline{B}^0$ state, we have the production of a pair of $b\bar{b}$ quarks which then hadronize independently, either as B mesons, charged or neutral, or as baryons. Thus, the experimental challenges in measuring CP asymmetries are very different in the two types of machines.

TABLE 5.2

Comparison of $b\bar{b}$ Production for various facilities.

Quantity	Units	TeVatron	SSC	Fixed Target at SSC	$\Upsilon(4s)$
Collision		$\bar{p}p$	pp	pNucleus	e^+e^-
Energy	TeV	2	40	.2	10.63×10^{-3}
Luminosity	$10^{33}\text{cm}^{-2}\text{s}^{-1}$	.01	.1	.1	3
$\sigma_{b\bar{b}}$	μb	20	200	2	1.2×10^{-3}
σ_{tot}	mb	50	100	10	4.8×10^{-3}
$N_{b\bar{b}}^{prod}$	$(\text{year})^{-1}$	2×10^9	2×10^{11}	2×10^9	3.6×10^7
$fraction_{b\bar{b}}$		.0004	.002	.0002	.25

The most significant advantage of hadron colliders in conducting B-physics researches is that they will produce copious B mesons, via either $b\bar{b}$ pair production or t-quark decays. The estimated number of $b\bar{b}$ pairs to be produced in the LHC or the SSC is $\mathcal{O}(10^{11})$ per year, about a thousand times larger than in a lepton B factory (83,84). A fixed target experiment could produce $\mathcal{O}(10^9)$ pairs. Estimates for the upgraded Fermilab TeVatron are similar (85,86). A comparison of various machines is given in Table 5.2. The principal experimental problems for hadron colliders are the methods for triggering on B events at such high rates, and the signal to noise ratio caused by the additional hadrons in the underlying events. What remains to be seen is whether these problems can be overcome in a way that retains enough of the produced B's to compete in accuracy with a B-factory on

CP violation measurements.

The fact that, unlike the $\Upsilon(4S)$ machines, there is no coherent $B - \overline{B}$ production, provides the option of making time-integrated measurements of CP asymmetries. We note, however, that time-integrated asymmetries are diluted by a factor $d_q = \frac{x_q}{1+x_q^2}$ $(q = d, s)$. In particular, d_s is expected to be very small. With sufficiently good position resolution on the decay vertex, time-dependent measurements will also be possible.

On the other hand, the lack of coherence means that the fraction f of wrong tags here includes the mixing of a tagging B^0 or $\overline{B}^0$ to its conjugate particle. Moreover, (particularly in time integrated-measurements) the observed asymmetry is affected by possible confusion of B_s decays with B_d decays since the mass resolution in these experiments will probably not be good enough to separate these contributions. The problem is most severe if the ratio of the contribution to a specific channel from B_s to that from B_d, denoted x in the expression below, is $\mathcal{O}(1)$. Another source of uncertainty is that B^0 and $\overline{B}^0$ may be produced in different numbers. In a pp machine, $r_B \equiv \frac{N_B - N_{\overline{B}}}{N_B + N_{\overline{B}}} \neq 0$ is expected because the probability for a b or $\overline{b}$ to hadronize as a baryon is affected by the population of quarks and antiquarks in its vicinity. Even in a $p\overline{p}$ machine there could be a forward-backward asymmetry. All these sources of uncertainties modify the observed asymmetry:

$$a_{\text{observed}} = \frac{(1-2f)[d_d a(B_d) + x d_s a(B_s)] + \delta}{1 + x + \delta'}. \tag{5.2}$$

Note that the dilution factors d_q can be avoided if a time-dependent measurement is made. The quantity δ vanishes if r_B vanishes but for non-zero r_B it does not vanish even when all CP violating asymmetries are zero. However, we expect r_B and hence δ to introduce only small corrections, of order a percent or so. The most severe problem will be background contributions. The quantity δ' depends on the ratio of background to B_d events and on r_B. These latter corrections depend on a number of factors such as the flavor-tagging efficiencies for baryons and for mesons and the fraction of wrong sign tags in each case as well as r_B. These factors are presumably not even universal but vary across phase space. Thus Eq. (5.2) is also not universal; the corrections must be calculated for each kinematic situation. The general form of Eq. (5.2) is given here mainly to stress that an accurate measurement of the asymmetry requires good control of backgrounds including those from B_s decays (or B_d decays when a B_s channel is studied). Note also that the background problems eliminate many of the tagging modes that can be used in the e^+e^- environment.

Let us now examine the feasibility of measuring CP asymmetries in various specific modes. A mode such as $B \to \psi K_S$ which has a μ-pair signature for the ψ can probably be studied and CP violating parameters extracted. For this mode, B_s contamination will be small since $B_s \to \psi K_S$ is CKM suppressed. For the upgraded TeVatron with an upgraded D0 detector, Roe (85) has estimated that two years running would allow an accuracy of about $\delta(\sin 2\beta) = \pm 0.15$. She included only the $\mu^+\mu^-$ decay mode of the ψ and only muon tags so this number can perhaps be improved by adding electron channels. A similar estimate for the CDF detector was given in (86). Further upgrades to either detector to improve particle identification may make additional tagging channels viable and thus improve the data collection rate.

Purely hadronic final states such as $\pi\pi$ or ρK_S will be much more difficult to separate from backgrounds. For LHC or SSC, widely different estimates of possible efficiencies for B reconstruction and signal to noise ratio exist and further study is clearly needed. (See *e.g.* Ref. (84) compared to Ref. (83).) Typical TeVatron events are estimated to have about one K and more than one π in the underlying event; the problem becomes worse at higher energies where the multiplicities are higher. Selection of B events will require p_T-cuts and possibly other cuts. The CDF group has already shown that one can pursue B physics in this hadronic environment but the requirements of CP violation physics are challenging. Methods for rapid triggers which select B events and cuts which reduce backgrounds need to be developed.

Since measuring the angle γ seems difficult for e^+e^- colliders, it is interesting to consider whether a hadron collider can do better. The production rate of B_s is comparable to that of B_d (within an order of magnitude), but as $x_s \geq 5$ the mixing is much more rapid. This means that the dilution of the asymmetry due to the time integrated measurement, $d_s = \frac{x_s}{1+x_s^2}$, will be a significant loss. Furthermore, for $B_s \to \rho K_S$, the hadronic backgrounds as well as background from B_d events present a severe problem. The CKM suppression of $B_d \to \rho K_S$ is compensated by a higher B_d production rate. As the two channels have different predicted asymmetries, the B_d initiated events do provide a serious background problem. Measurement of γ by this approach does not appear feasible.

Another mode of interest in B_s decays is $\psi\phi$. The Standard Model prediction is zero asymmetry, but with new physics in B_s mixing, even maximal asymmetry is possible (54). The hadronic background problems can be avoided by observing the $\psi \to \mu^+\mu^-$ decay, but the branching ratio makes the measurement difficult (86). Further studies of these modes and of strategies to improve signal to noise and/or detection efficiencies are needed. At present we cannot give numerical estimates.

For asymmetries in charged B decay, the problems of mixing and tagging are not relevant but the problem of hadronic backgrounds is severe. Given the small asymmetries predicted for charged B decays, it appears that they will be very difficult to observe in a hadronic environment. Studies of CP asymmetry measurements with baryonic modes are not yet available, but need to be pursued to evaluate the full range of physics options for a hadron collider (87).

Before concluding this section, let us mention a few more intriguing ideas concerning hadron colliders. It has been suggested (88) that one can turn the asymmetry in production of B^0 and $\overline{B}^0$ into a useful tool, as it allows a (time-dependent) measurement of CP asymmetries without tagging. The time-dependent rate in the $|\lambda| = 1$ case, summing over both B^0 and $\overline{B}^0$ contributions, is

$$\Gamma^{(\text{no tagging})} = |A|^2 e^{-\Gamma t} \left[1 - r_B \text{Im}\lambda \sin(\Delta M t)\right], \tag{5.3}$$

where r_B is defined at $t = 0$. Of course, if $r_B = 0$ the untagged rate gives no asymmetry measurement. To extract useful information, r_B has to be accurately known. Present estimates vary greatly, but probably $r_B \lesssim 0.01$ even in the forward direction (88). In principle r_B can be measured by looking at flavor-tagging decays, though the effect is diluted by mixing and other sources of wrong-sign tags. An experiment with no flavor-tagging requires accurate vertex reconstruction to isolate the $\sin(\Delta M t)$ contribution which is suppressed by r_B even when $\text{Im}\lambda$ is large. For example, $\text{Im}\lambda \sim 0.3$ would give a fraction of a percent deviation from a purely exponential decay. Hence it does not appear to us that this method provides a feasible measurement of any CP violating asymmetries.

It was recently suggested that microvertex detectors placed very close to the beam, in roman pots, can view a much larger fraction of the B mesons produced close to the forward direction and provide better vertex reconstruction (89). Unfortunately, not only the signal events but also the backgrounds are strongly forward-backward peaked and so this method does not alleviate the signal to noise and triggering problems. The problems of data rate and of possible radiation damage to the detector in such a configuration are severe. However, the idea certainly merits further study.

Another possible way to study B physics at high-energy proton accelerators is in a fixed target mode, using either extracted beams or a gas jet target (90). Preliminary studies indicate that such an experiment may result in a sample of B events of comparable size and purity to that expected from to colliding-beam experiments. This requires very efficient beam extraction and vertex triggers. Again this is a subject where more study is needed to reach a conclusion, but the approach appears at present to offer a possible alternative.

To summarize, it appears to us that the only CP violating asymmetry that can be readily measured in a hadron collider or a high energy hadron fixed-target experiment is $\sin(2\beta)$ via $B_d \to \psi K_S$. For this mode, a measurement of comparable accuracy to that achievable at an e^+e^- collider is feasible at the upgraded TeVatron or the SSC. For any other mode, significant improvements in event selection methods are required before this interesting physics can be tackled.

6. Summary

The physics of CP violation is an area where the Standard Model makes definite predictions that have yet to be fully explored experimentally. B mesons provide an excellent system with which to test these predictions and thus to search for any clues to physics beyond the Standard Model. Current efforts to understand baryogenesis in the early universe suggest that there must be CP violating physics beyond the Standard Model, which makes this search even more attractive. We have here reviewed the predictions of the Standard Model and have shown how the relationships between various measurements can be used to test the Kobayashi-Maskawa picture of CP violation, to measure some remaining unknown Standard Model parameters and to seek for clues to physics beyond the Standard Model.

Considerable effort by several groups has now been devoted to the design of asymmetric e^+e^- B factories and to studies of the physics opportunities offered by such facilities. The progress made in machine and detector design show that such a facility would be a very exciting laboratory for the program of CP violation studies and other aspects of B physics.

The study of B physics at hadron colliders or in very high-energy fixed target hadron experiments also offer some interesting possibilities. Early results show that the identification of B mesons in high energy hadron-hadron collisions is feasible. Much more work is needed on detectors particularly designed for the task and the necessary triggering and event selection procedures must be further studied before definitive conclusions can be reached about the capability of hadron machines to carry out a competitive and complementary program to that possible at lepton machines for B Physics and CP violation studies.

The prospects for these experiments at either type of facility are as yet quite uncertain. Although several groups around the world have been working on designs for B factories as yet none of these projects is funded for construction. On the hadron side the TeVatron upgrade at Fermilab, the European LHC and the SSC are all proceeding, but designs for detectors especially suited to study B physics are still in the preliminary study stage. Much interesting and fundamental physics waits for the experiment that can reconstruct a sufficient number of B decays in a variety of modes.

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