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Theoretical Aspects of Lepton-Hadron Scattering*

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FIRST LECTURE

I will emphasize two points in my lectures on Theoretical Aspects of Lepton-Hadron Scattering:

(1) The crucial importance of testing the "exact" sum rules as tests of the local current algebra. Discrepancies, if found, between experiment and theory cannot be "interpreted away" in terms of more complex parton wave functions for the hadronic ground state. The three sum rules of interest are those of Adler, Bjorken, and Gross and Llewellyn-Smith.

(2) An understanding of the corrections to scaling in QCD and what they teach us.

To begin with, I will review the parton model, its intuitive physical basis, its predictions, and its limitations.◇

The power and beauty of lepton-hadron scattering is that the electroweak field generated during the lepton scattering is as well understood as anything known in particle physics. This permits us to probe the unknown structure of the target hadron by means of a known current operator. Furthermore, at the same time, its strength is weak enough to allow a perturbative treatment in powers of the electroweak charge and strong enough to permit accurate measurements under physically interesting conditions of large energy and momentum transfers.

The original round of high energy measurements of elastic electron scattering by Hofstadter and collaborators demonstrated that protons and neutrons, similar to the nuclei of which they are the constituents, have extended charge distributions. For nuclei, in which the nucleons are highly nonrelativistic—being bound by less than 1% of their rest energies—the charge distributions are measured by (to leading order in $Z\alpha < 1$, and neglecting center-of-mass corrections $\sim 1/A$)

$$\left\langle \mathrm{P}_{Z,A} \right| \int d^3 r \, e^{i\vec{q}\cdot\vec{r}} \sum_P \frac{e^2}{|\vec{r} - \vec{R}_p|} \left| \mathrm{P}_{Z,A} \right\rangle = \frac{4\pi Z e^2}{|q|^2} \left\langle \mathrm{P}_{Z,A} \right| e^{i\vec{q}\cdot\vec{R}} \left| \mathrm{P}_{Z,A} \right\rangle , \quad (1)$$

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◇Rather than give individual references to the original literature during these lectures, I append at the end the titles and authors of relevant books and reviews which contain the references.

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where the electric form factor is defined by

$$
\begin{aligned}
F(|q|^2) &\equiv \left\langle \mathrm{P}_{Z,A} \right| e^{i\vec{q}\cdot\vec{R}} \left| \mathrm{P}_{Z,A} \right\rangle \\
&= \int d^3 R\, e^{i\vec{q}\cdot\vec{R}}\, \rho(R) \\
&= 1 \text{ for a point charge} \\
&\rightarrow 0 \text{ for a distributed charge, for } |q|^2 \gg \frac{1}{\langle R^2 \rangle} \\
&\approx 1 - \frac{1}{6}\, |q|^2 \left\langle R^2 \right\rangle + \ldots, \text{ for small } |q|^2 \ll \frac{1}{\langle R^2 \rangle}\ .
\end{aligned}
\tag{2}
$$

By standard steps we find the elastic cross section to be $F^2(q^2)$ times the point charge cross section:

$$
\frac{d\sigma_{e\ell}}{d\,|q|^2} = \frac{4\pi\,(Z\alpha)^2}{|q|^4}\, F^2(|q|^2)\ . \tag{3}
$$

For elastic scattering the energy and momentum transfer are related by the mass shell condition for the target nucleus [in the lab $q_\mu = (\nu, \vec{q})$]

$$
\begin{aligned}
\mathrm{P}^2 = (MA)^2 = (\mathrm{P}+q)^2 &= (MA)^2 + q^2 + 2\mathrm{P}\cdot q\ , \\
&= (MA)^2 + \nu^2 - |q|^2 + 2MA\nu\ , \\
\text{or}\quad \nu &\approx |q|^2/2MA\ .
\end{aligned}
\tag{4}
$$

For inelastic scattering, ν and q^2 are independent variables. For one-arm experiments that measure only the angle and energy of the scattered electron,

$$
(\mathrm{P}+q)^2 = \mathcal{M}_f^2 = (MA)^2 + q^2 + 2\mathrm{P}\cdot q
$$

and

$$
\nu = \frac{-q^2}{2MA} + \frac{\mathcal{M}_f^2 - (MA)^2}{2MA}\ . \tag{5}
$$

The hadronic structure is probed in such measurements by independently varying ν and q^2. The scattering cross section as derived by standard steps is

$$
\frac{d^2\sigma_{\mathrm{in}}}{d\,|\vec{q}|^2\, d\nu} = \frac{4\pi\,\alpha^2}{|q|^4} \sum_n \delta(E_P + \nu - E_n)\, \langle \mathrm{P}| \sum_i e^{-i\vec{q}\cdot\vec{r}_i} |n\rangle\, \langle n| \sum_i e^{i\vec{q}\cdot\vec{r}_j} |\mathrm{P}\rangle\ , \tag{6}
$$

which contains sums, $\sum_i$, over all protons in the nucleus and $\sum_n$ over all nuclear states, $|n\rangle$, satisfying energy conservation. To a good approximation for high energies and small scattering angles we can use closure when summing over all energy transfers ν for fixed momentum transfer $|q|^2$:

$$\int_{|q|^2 \text{ fixed}} d\nu \, \frac{d^2\sigma_{\text{in}}}{d|\vec{q}|^2 \, d\nu} = \frac{4\pi\,\alpha^2}{|q|^4} \, \langle \mathrm{P} | \sum_{i,j} e^{i\vec{q}\cdot(\vec{r}_j - \vec{r}_i)} | \mathrm{P} \rangle$$

$$= \frac{4\pi\,\alpha^2}{|q|^4} \, \{ Z + Z(Z-1) \, f_2 \, |q^2| \} \; , \tag{7}$$

where f_2 is the two-body correlation function.

Several observations of interest may be made about (6) and (7):

(1) For $|q| \gg$ [mean internucleon separation]$^{-1} \approx 150$ MeV, $f_2(q) \to 0$ and

$$\frac{d^2\sigma_{\text{in}}}{d|q|^2} = \frac{4\pi\,\alpha^2}{|q|^4} \, Z \; . \tag{8}$$

Equation (8) tells us that there is a finite area under the inelastic scattering curve summed over all final resonance plus continuum states of the nucleus at fixed large $|q|^2$. It corresponds to Coulomb scattering from Z-independent, incoherent point charges. This is the same result as applying the impulse approximation to each individual proton in the nucleus, treated as free, and neglecting correlations and binding forces.

(2) There is a peak in the continuum inelastic scattering curve at an energy loss corresponding to quasi-elastic scattering from a single nucleon; i.e., for

$$\nu_{QE} = \frac{|q|^2}{2M} \; . \tag{9}$$

As shown in Fig. 1, this peak remains, fulfilling the sum rule (8) as $|q|^2$ increases, and contributions from individual resonance states are suppressed by their form factors analogous to (3). By the uncertainty principle, we expect the impulse approximation to be valid when the energy transfer ν from the scattered electron is larger than the characteristic frequencies, or excitation energies, of the proton in a nucleus; i.e.,

$$\nu \gg \omega_{\text{Binding}}$$

or, by (9),

$$|q|^2 \gg 2M\omega_{\text{Binding}} \sim (150 \text{ MeV})^2 \; , \tag{10}$$

which is the same condition for (8) to be valid.

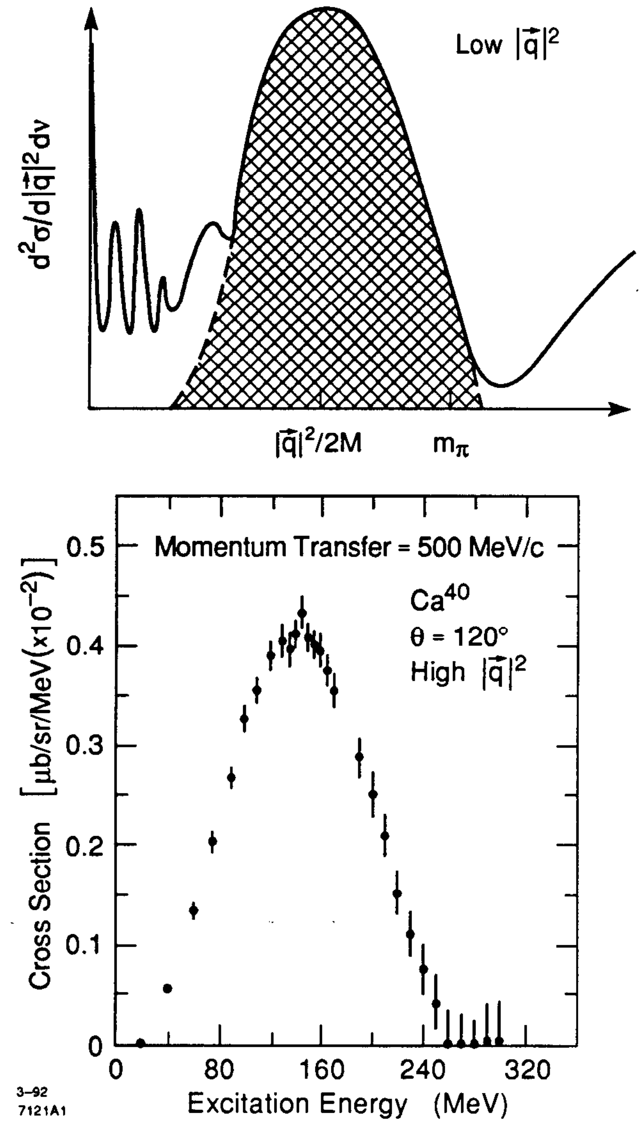

Fig. 1. Inelastic electron-nucleus scattering showing the quasi-elastic peak at both low and high $|\vec{q}|^2$.

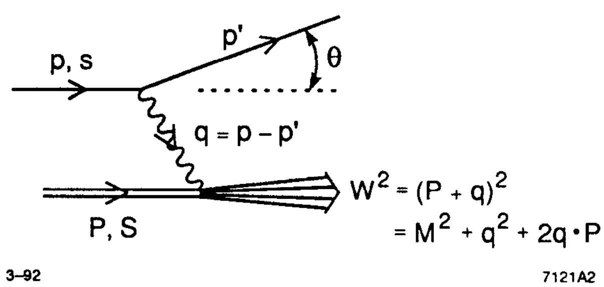

Fig. 2. Feynman diagram, with indicated kinematics, for inelastic electron-nucleon scattering.

(3) The quasi-elastic peak in Fig. 1 is broadened by the Fermi motion of the nucleons inside of the nucleus.

When the SLAC-MIT deep inelastic scattering experiments were started twenty-five years ago, there was little reason to suspect that the nuclear-inspired ideas of quasi-elastic peaks and large continuum scattering from point-like constituents were applicable to individual nucleons. All we knew from the elastic scattering measurements was that the nucleon form factors—defined by a generalization of (3) known as the Rosenbluth formula, including relativistic corrections to the kinematics and a second form factor arising from the anomalous magnetic moments—decreased as $(1/q^2)^2$ for large q^2 corresponding to distributed average charge and moment distributions with mean square radii of ~ 0.8 fermis. The pivotal theoretical contributions of Bjorken and Feynman, and the experimental findings of Friedman, Kendall, Taylor, and collaborators were to show that we could, with appropriate care, transfer to the structure of hadrons the ideas illustrated above for nuclei. This spectacular progress and what we have learned from lepton scattering since that breakthrough are the subjects of these two lectures.

For high energy lepton-nucleon scattering, we must generalize the previous discussion to include hadronic recoil in the kinematics, the spin of the electron and the target hadron, and the transition current in addition to the charge of the scattered electron. To lowest order in $\alpha = 1/137$, the Feynman graph—with the defining notation for this process—is shown in Fig. 2.

In terms of laboratory variables, with lepton mass set to 0,

$$\begin{aligned} p_\mu &: (E, 0, 0, p) \\ p'_\mu &: (E', 0, p' \sin\theta, p' \cos\theta) \\ \mathrm{P}_\mu &: (M, 0, 0, 0) \\ Q^2 &\equiv -q^2 = 4EE' \sin^2(\theta/2) \\ q_0 &= E - E' \equiv \nu \\ W^2 - M^2 &= 2M\nu - Q^2 = 2M\nu(1-x) \ , \end{aligned} \tag{11}$$

and for longitudinally polarized initial leptons and hadrons we define the covariant spin pseudovectors ($s \cdot p = 0$; $s^2 = -1$)

$$\begin{aligned} s_\mu &: \frac{1}{m}\,(p, 0, 0, E) \\ \mathrm{S}_\mu &: (0, 0, 0, \pm 1) \ . \end{aligned}$$

The inclusive inelastic cross section—the generalization of (6)—is given for electromagnetic scattering by

$$\frac{d^2\sigma}{dQ^2\,d\nu} = \frac{d^2\sigma}{2EE'\,d\cos\theta\;dE'}\,, \tag{12}$$

$$= \frac{4\pi\alpha^2}{Q^4}\,\frac{E'}{E}\sum_n (2\pi)^3\,\delta^4\left(q+\mathrm{P}-\mathrm{P}_n\right)\left|\langle n|_{p'}\langle j_\mu\rangle_{s,p}\,\mathrm{J}_\mu\,|\mathrm{PS}\rangle\right|^2\,,$$

where j_μ and J_μ are, respectively, the lepton and hadron electromagnetic current operators and $_{p'}\langle j_\mu\rangle_{s,p}$ is the lepton current matrix element for momentum transfer q from an initial state with (p,s). For scattering by the weak interaction current,* we make the substitution in (12)

$$\frac{4\pi\alpha^2}{Q^4} \to \frac{G_F^2}{2\pi}\,, \tag{13}$$

where G_F is the Fermi coupling constant, and replace the electromagnetic current operators by the weak currents. For the electron currents the sum over final state spin gives the tensor (since $s_\mu \propto 1/m$, the lepton mass, must be retained in the spin term)

$$\begin{aligned} I_{\mu\nu} &= \frac{1}{4EE'}\,\mathrm{Tr}(\not p' + m)\gamma_\mu\,(\not p + m)\left(\frac{1+\gamma_5\,\not s}{2}\right)\gamma_\nu \\ &= \left[\sigma_\mu\sigma_\nu - q_\mu q_\nu + q^2 g_{\mu\nu} + 2i\epsilon_{\mu\nu\sigma\tau}\,q_\sigma s_\tau\,m\right]\frac{1}{4EE'} \end{aligned} \tag{14}$$

with $\sigma_\mu \equiv (p_\mu + p'_\mu)$.

Note $q_\mu I_{\mu\nu} = q_\nu I_{\mu\nu}$ as it must by current conservation. We rewrite (12) as

$$\frac{d^2\sigma}{dQ^2\,d\nu} = \frac{4\pi\alpha^2}{Q^4}\,\frac{E'}{E}\,I_{\mu\nu}\frac{1}{2\pi}\sum_n\int d^4x\,e^{i(q+\mathrm{P}-\mathrm{P}_n)\cdot x}\langle\mathrm{PS}\,|\mathrm{J}_\mu|\,n\rangle\langle n\,|\mathrm{J}_\nu|\,\mathrm{PS}\rangle\,, \tag{15}$$

$$= \frac{4\pi\alpha^2}{Q^4}\,\frac{E'}{E}\,I_{\mu\nu}\frac{1}{2\pi}\int d^4x\,e^{iq\cdot x}\langle\mathrm{PS}\,|\,[\mathrm{J}_\mu(x),\mathrm{J}_\nu(0)]\,|\,\mathrm{PS}\rangle\,, \tag{16}$$

$$\equiv \frac{4\pi\alpha^2}{Q^4}\,\frac{E'}{E}\,I_{\mu\nu}\,W_{\mu\nu}\,. \tag{17}$$

Equation (15) is the relativistic generalization of (6). In Eq. (16) we used closure and changed the product of currents to a commutator by adding zero (since $q_0 > 0$; exercise for the reader). Commutators are always attractive if they can be brought to equal-time limits as we shall see later.

$\star$ i.e., $\alpha \to \alpha_W$ and $1/Q^2 \to 1/(Q^2+M_W^2)$, which leads to $[4\pi(e^2/4\pi)^2]/Q^4 \to 1/4\pi[e_W^2/(Q^2+M_W^2)]^2 \approx 1/2\pi[(e_W^2/\sqrt{2})/M_W^2]^2 \equiv G_F^2/2\pi$.

The general form of the hadronic tensor introduced in (17) is, upon averaging over hadron spin and enforcing current conservation, $q_\mu W_{\mu\nu} = q_\nu W_{\mu\nu} = 0$,

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(q^2,\nu) + \frac{1}{M^2}\left(\mathrm{P}_\mu - \frac{\mathrm{P}\cdot q}{q^2}\, q_\mu\right)\left(\mathrm{P}_\nu - \frac{\mathrm{P}\cdot q}{q^2}\, q_\nu\right) W_2(q^2,\nu)\,. \tag{18}$$

W_1 and W_2 are scalar functions of q^2 and $\nu = q\cdot \mathrm{P}/M$.

No term of form $\epsilon_{\mu\nu\sigma\tau} q_\sigma \mathrm{P}_\tau$ appears because the electromagnetic current is a polar vector. For the weak current that is (V–A), such an odd parity term that changes sign under charge conjugation does arise, and with it a third form factor W_3. From (14), (17) and (18) we obtain for the unpolarized cross section

$$\frac{d^2\sigma}{dQ^2\, d\nu} = \frac{4\pi\alpha^2}{Q^4}\,\frac{E'}{E}\,\left[W_2(q^2,\nu)\,\cos^2(\theta/2) + 2W_1(q^2,\nu)\,\sin^2(\theta/2)\right]\,. \tag{19}$$

The interesting physics describing the electromagnetic structure of nucleons is wrapped up in the two-scalar structure function W_1 and W_2 for unpolarized processes. From the form of (18) and (19) we can anticipate that W_2, which appears together with the tensor structure of a product of Schrodinger currents, will be a charge structure term.

In general the structure functions depend on two variables Q^2 and ν and experiments probe the parameter space as illustrated in Fig. 3:

For completeness we write here the generalization of (18) for polarized hadrons.

$$\begin{aligned}(W_{\mu\nu})_{\text{spin}} &= W_{\mu\nu} \\ &- \frac{i}{\pi M}\left[\left(\mathrm{P}_\mu\,\epsilon_{\nu\alpha\beta\tau} - \mathrm{P}_\nu\,\epsilon_{\mu\alpha\beta\tau}\right)\,\mathrm{P}_\alpha\,\mathrm{S}_\beta\, q_\tau + \mathrm{P}\cdot q\,\epsilon_{\mu\nu\alpha\beta}\,\mathrm{P}_\alpha\,\mathrm{S}_\beta\right] g_1 \\ &- \frac{i}{\pi M}\left[\left(q_\mu\,\epsilon_{\nu\alpha\beta\tau} - q_\nu\,\epsilon_{\mu\alpha\beta\tau}\right)\,\mathrm{P}_\alpha\,\mathrm{S}_\beta\, q_\tau + q^2\,\epsilon_{\mu\nu\alpha\beta}\,\mathrm{P}_\alpha\,\mathrm{S}_\beta\right] g_2\,.\end{aligned}$$

The ratio of the spin dependent to the spin independent cross sections is

$$\frac{d\sigma_{\uparrow\uparrow} - d\sigma_{\uparrow\downarrow}}{d\sigma_{\uparrow\uparrow} + d\sigma_{\uparrow\downarrow}} = \frac{4}{\pi}\left[\frac{M(E+E'\,\cos\theta)\,g_1(q^2,\nu) + q^2\,g_2(q^2,\nu)}{2W_1(q^2,\nu) + W_2(q^2,\nu)\,\cot^2(\theta/2)}\right]\,. \tag{20}$$

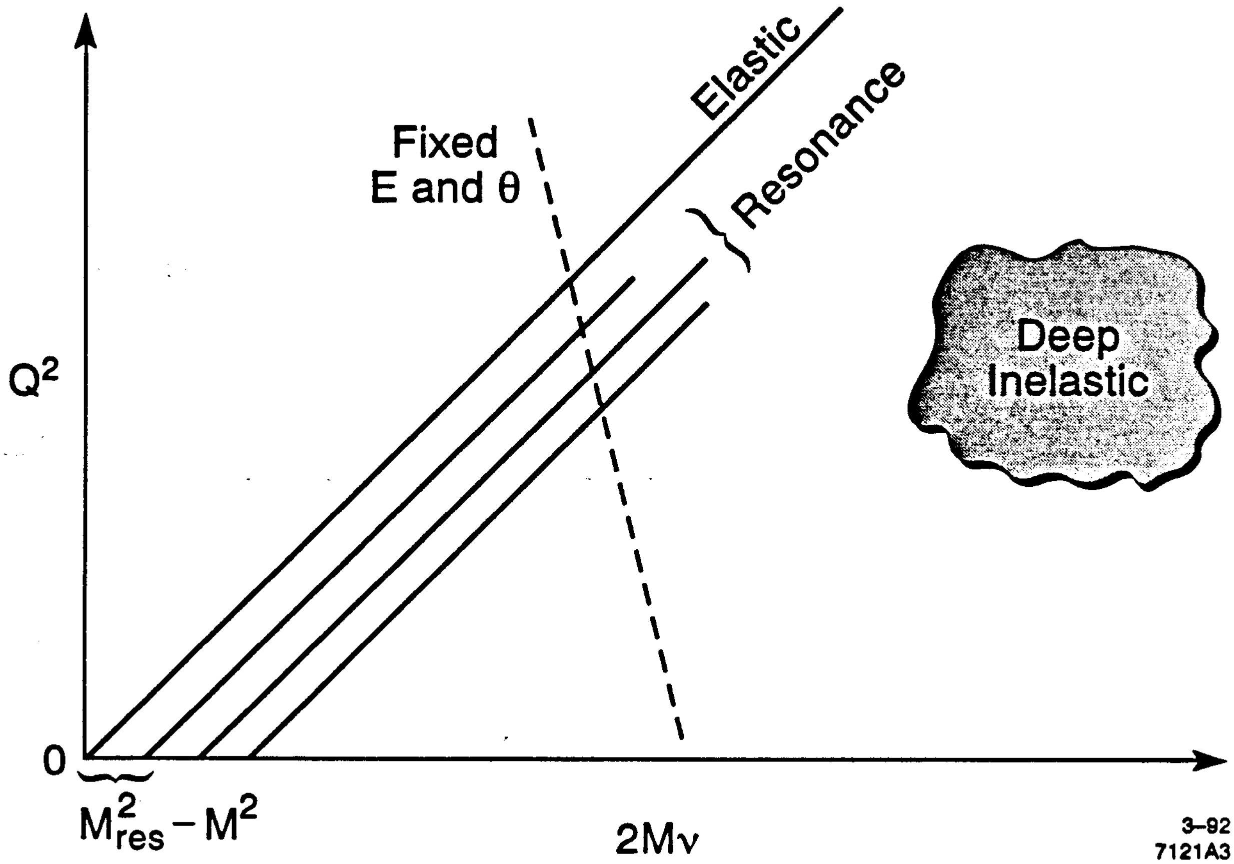

Fig. 3. Parameter space for inelastic electron-nucleon scattering.

To conclude the kinematics we write the inelastic cross section for weak scattering with the additional contribution arising from the parity violating contribution

$$\frac{d^2\sigma^{(\ell,\bar{\ell})}}{dQ^2 d\nu} = \frac{G_F^2}{2\pi}\frac{E'}{E} \tag{21}$$

$$\times \left[W_2^{\prime(\ell,\bar{\ell})} \cos^2(\theta/2) + 2W_1^{\prime(\ell,\bar{\ell})} \sin^2(\theta/2) \pm W_3^{\prime(\ell,\bar{\ell})} \sin^2(\theta/2) \left(\frac{E+E'}{M}\right) \right] ,$$

where $\ell(\bar{\ell})$ denotes lepton (anti-lepton) scattering. The weak interaction structure functions $W_{1,2}'$ can be related to the electromagnetic ones by the underlying conservation laws such as CVC or by specific models and isotopic rotations. The additional structure function corresponds to a parity violating contribution of form $-iW_3^{\prime(\ell,\bar{\ell})}\, \epsilon_{\mu\nu\sigma\tau}\, \mathrm{P}^\sigma\, q^\tau \,/\, M^2$, added to (18).

In order to probe the detailed structure of hadrons it is desirable to study the structure functions at large values of Q^2 and ν. For elastic scattering the structure functions reduce to squares of the familiar form factors (κ is the anomalous magnetic moment of the nucleon; $\kappa_p = 1.79$; $\kappa_n = -1.91$):

$$\begin{aligned} W_2 &\rightarrow \delta\left(\nu - \frac{Q^2}{2M}\right) \left[F_1^2(Q^2) + \frac{Q^2}{4M^2}\,\kappa^2\, F_2^2(Q^2)\right] \\ W_1 &\rightarrow \delta\left(\nu - \frac{Q^2}{2M}\right) \left[F_1(Q^2) + \kappa\, F_2(Q^2)\right]^2 \frac{Q^2}{4M^2} \end{aligned} \tag{22}$$

The form factors are experimentally found to fall off rapidly with increasing Q^2, $F(Q^2) \propto (1/Q^4)$ out to $\sqrt{Q^2} \sim 6$ GeV, indicating a diffuse and smooth structure with $\langle r^2 \rangle^{1/2} \simeq 0.8 \times 10^{-13}$ cm. This behavior is readily accommodated by a three-quark model of hadron structure, but is less natural to parameterize in dispersion theory models. A low-lying resonance leads more simply to $1/Q^2$ falloff. Cancellations between several resonances have to be arranged to account for a more rapid falloff.

Of primary interest is the study of the inelastic structure functions, the presence of quasi-elastic scattering and Bjorken scaling in the very inelastic region of large Q^2 and ν. In particular, note that in the high energy limit, with $E \rightarrow \infty$ and with fixed Q^2, so that $\theta \rightarrow 0$, comparison of (19) and (8) shows that

$$Z \rightarrow \int_{\nu_{\min}}^{\infty} W_2(\nu, Q^2)\, d\nu \, . \tag{23}$$

A finite value for (23) in some sense "measures" the charged constituents of the nucleon. But what are they? In contrast to nuclei as we discussed earlier, the nucleon's constituents had not been deciphered 25 years ago—and the debris emerging from hard collisions with a single nucleon would necessarily have been strongly bound within the nucleon if they were its actual constituents. To aid our intuitive understanding, we identify kinematic conditions that permit an impulse approximation analysis akin to the nuclear case. This is the parton model, valid in the Bjorken scaling region of deep inelastic scattering with large Q^2 and ν. Subsequently, to provide a solid theoretical underpinning, we will also appeal to quantum field theory and the current operator algebra to understand deviations from scaling and to establish sum rules of general validity which, if violated, would have profound implications for the validity of our basic theoretical understanding.

As first suggested by Feynman, we can gain an intuitive understanding of deep inelastic scattering by viewing the proton from an infinite momentum frame, a limiting idea for very high energy eP scattering. We understand that the constituents of a nucleon—gluons, quarks, or simply partons—are bound by strong forces, the color forces of QCD. In this $\mathrm{P} \to \infty$ frame, the partons will each share a finite fraction $0 < x_i < 1$ of $\mathrm{P} \to \infty$, and move closely parallel to P. They will behave as almost free on this energy scale, relative to which their binding is weak, and scattering from individual partons can be treated as incoherent for sudden perturbations. Stated more quantitatively, the lifetimes of the parton states are characteristically

$$\tau_{\rm life} \sim \frac{1}{\Delta E} \sim \frac{1}{\sqrt{\mathrm{P}^2 + M_g^2} - \sqrt{\mathrm{P}^2 + M_x^2}} \sim \frac{\mathrm{P}}{M_{eff}^2}, \tag{24}$$

where we expect $M_{eff} \sim 1$ GeV, a typical mass scale for the nucleon, and $\mathrm{P} \sim \sqrt{s}/2$ in the eP center-of-mass frame. Equation (24) exhibits the relativistic time dilation, which in effect "freezes" the proton in one of its virtual states of mass M_x.

The duration of the perturbing electromagnetic pulse from the scattered electron in this frame is (homework for you).

$$\tau_{\rm pulse} \sim \frac{4\mathrm{P}}{2M\nu - Q^2}. \tag{25}$$

Comparing (24) and (25) note that $\tau_{\rm pulse} \ll \tau_{\rm life}$ for

$$2M\nu - Q^2 \equiv 2M\nu(1-x) \gg M_{eff}^2. \tag{26}$$

Equation (26) defines the key condition for applying the impulse approximation.

To explore the meaning of (26) further, we make two observations:

(1) Magnitude of M^2_{eff}

Consider a proton with $\vec{\mathrm{P}}$ disassociating virtually into two constituents with momenta $x_0\,\vec{\mathrm{P}} + \vec{\kappa}_\perp$ and $(1-x_0)\vec{\mathrm{P}} - \vec{\kappa}_\perp$, respectively. The energy denominator (or reciprocal lifetime) of this virtual state is given by

$$\begin{aligned}\Delta E \; &= \; \sqrt{x_0^2\,\mathrm{P}^2 + \kappa_\perp^2 + m_1^2} + \sqrt{(1-x_0)^2\,\mathrm{P}^2 + \kappa_\perp^2 + m_2^2} - \sqrt{\mathrm{P}^2 + M^2} \\ &\approx \; \frac{1}{\mathrm{P}}\left[\frac{\kappa_\perp^2 + m_1^2(1-x_0) + m_2^2 x_0 - M^2 x_0(1-x_0)}{2x_0(1-x_0)}\right] \; \equiv \; \frac{M^2_{eff}}{\mathrm{P}} \,. \end{aligned} \tag{27}$$

For (26) to be satisfied in the lab, x_0 must approach neither 0 nor 1, and $\kappa_\perp$ must be bounded, as is observed experimentally from the small momentum width of secondaries and the rapid fall off of elastic form factors, which measure the probability of putting the proton back together in an elastic collision.

(2) Interpretation of x

The relation

$$x \; = \; \frac{Q^2}{2M\nu} \tag{28}$$

is just the condition for elastic scattering from a constituent with longitudinal momentum $x\vec{\mathrm{P}}$ in the $\mathrm{P} \to \infty$ frame.

To see this, ignore the "parton" mass and transverse momentum as relatively small $\lesssim 1$ GeV; then $\mathrm{P}^\mu_{\mathrm{constituent}} = x_0\mathrm{P}^\mu$ and the elastic scattering condition

$$(q+\mathrm{P}_c)^2 \; = \; \mathrm{P}_c^2\,, \quad \text{or} \quad (q + x_0\mathrm{P})^2 \; = \; (x_0\mathrm{P})^2$$

gives

$$Q^2 \; = \; 2x_0 q\cdot\mathrm{P} = \; 2x_0 M\nu\,, \quad \text{or} \quad x_0 \; = \; \frac{Q^2}{2M\nu} \; = \; x \quad \text{by (26)}\,.$$

Equation (26) shows that x cannot approach too close to 1; combined with (27), the condition that $x = x_0$ not approach too close to 0 is seen to be identical to $Q^2 \gg M^2$.

For scattering from point-like partons of spin 1/2 and charge $q_i\, e$, one finds by direct calculation, or from (22) with $M \to xM$,

$$W_1 \;=\; \frac{q_i^2}{2M}\,\delta\left(x - \frac{Q^2}{2M\nu}\right) \;, \tag{29}$$

$$\nu W_2 \;=\; xq_i^2\,\delta\left(x - \frac{Q^2}{2M\nu}\right) \;. \tag{30}$$

Thus for a proton built of N partons of charge q_i and momentum distribution $f_i(x_i)$,

$$\begin{aligned}\nu W_2 \;&=\; \int \sum_{i=1}^{N} f_i(x_i)\, q_i^2\, x_i\, \delta\left(x_i - \frac{Q^2}{2M\nu}\right)\, dx_i \\ &=\; \sum_{i=1}^{N} f_i\left(\frac{Q^2}{2M\nu}\right)\, q_i^2 \left(\frac{Q^2}{2M\nu}\right) \;=\; \sum_i x f_i(x)\, q_i^2 \;,\end{aligned} \tag{31}$$

and

$$\int_{\nu_{\rm th}}^{\infty} \frac{d\nu}{\nu}\, \nu W_2 \;=\; \sum_i q_i^2 \int_0^1 dx f_i(x) \;=\; \sum_i q_i^2 \;. \tag{32}$$

Equation (32) is for nucleons the analogue of (23), with which it coincides for Z constituents of unit charge. Most importantly, we observe that W_1 and νW_2 depend only on the ratio $x = Q^2/2M\nu$. This result is known as "bj" scaling, as first derived and predicted by Bjorken in 1966 from a formal study of the current matrix elements in the large ν and Q^2 limits and with x not too close to 0 or 1. Note also that

$$\nu W_2 \;=\; 2MxW_1 \;. \tag{33}$$

This prediction is valid for the charged partons having spin 1/2, and is expected to be true in QCD in the bj limit, since integer spin gluons are electrically neutral. This relation, known as the Callan-Gross Relation, is accurate to 10 to15% in current measurements.

The simple form of (31) suggests simple sum rules for deep inelastic scattering in terms of quark models of the hadron. Most directly, from (32) we expect in a pure three quark model of the hadron that

$$\int \frac{d\nu}{\nu}\, \nu W_2 \;=\; \int_0^1 \frac{dx}{x}\, (\nu W_2)_x \;=\; \begin{cases} 1 & \text{for a proton } (uud)\;, \\ 2/3 & \text{for a neutron } (udd)\;. \end{cases} \tag{34}$$

However, the measured shape of νW_2, as illustrated in Fig. 4 shows that a simple model of three-valence quarks does not accurately represent the nucleon. In particular if there were just three quarks in a proton that shared its momentum, we would expect to see a quasi-elastic peak as found earlier for nuclei and illustrated in Fig. 1. In contrast, Fig. 4 suggests that the integral (32) diverges logarithmically due to the presence of an infinite "sea" of $q\bar{q}$ pairs in the nucleon at low ("wee") values of x resulting from the quark-gluon interaction. This is the QCD analogue of soft bremsstrahlung and pair production in QED.

In a three-quark model we also expect

$$\int_0^1 dx\ (\nu W_2)_x = \int_0^1 x f_{(3)}(x)\ dx = \begin{cases} 1/3 & \text{for a proton ,} \\ 2/9 & \text{for a neutron .} \end{cases} \tag{35}$$

This is a more convergent relation than (34) as $x \to 0$. However, for many years it has been known that about 50% of the momentum of the proton is not on the three-valence quarks and must be shared with the sea quarks and gluon content—so the picture is not so simple as all that.

Evidently rigorous results will play a most crucial role in understanding inelastic scattering, beyond simple parton models. However, there are a number of simple and useful sum rules that can be derived from the quark-parton model of a nucleon built of spin-1/2 quarks plus neutral gluons. Several examples are as follows.

Reverting to the notation in (31) and defining the number density of u, d, s quarks in a proton by

$$\frac{f_{u,d,s}(x)}{x} \equiv \frac{1}{x}\ [u(x),\ d(x),\ s(x)]\ , \tag{36}$$

we can write

$$\begin{aligned} \frac{1}{x}\ F_2^{ep}(x) &\equiv \frac{1}{x}\ (\nu W_2)^{ep} \\ &= \frac{4}{9}\ [u(x)+\bar{u}(x)] + \frac{1}{9}\ [d(x)+\bar{d}(x)] + \frac{1}{9}\ [s(x)+\bar{s}(x)] + \dots\ . \end{aligned} \tag{37}$$

By isospin rotation $u \leftrightarrow d$ for a neutron, and

$$\frac{1}{x}\ F_2^{en}(x) = \frac{4}{9}\ [d(x)+\bar{d}(x)] + \frac{1}{9}\ [u(x)+\bar{u}(x)] + \dots\ . \tag{38}$$

The ratio $\dfrac{F_2^{en}(x)}{F_2^{ep}(x)}$ clearly is bounded between $\left(\dfrac{1}{4}, 4\right)$.

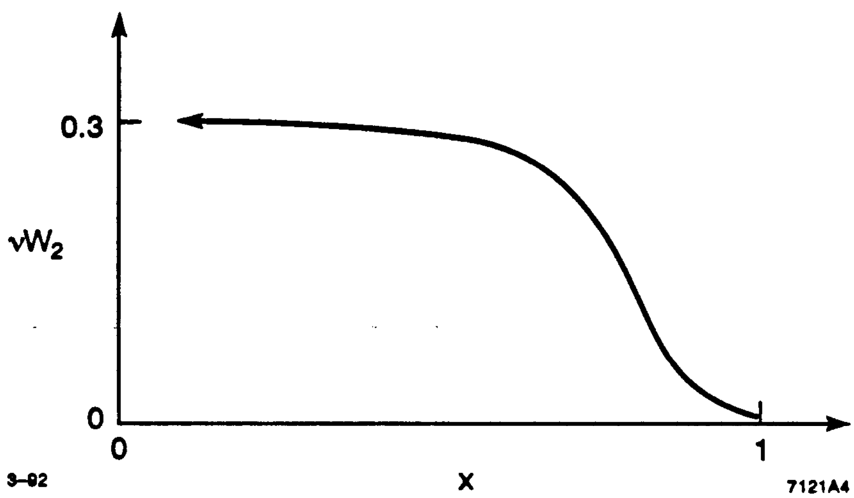

Fig. 4. Schematic representation of data observed for deep inelastic electron-proton scattering structure function, $\nu W_2(x)$, in the scaling region.

For small x where many pairs of sea $q\bar{q}$ may be dominant, we thus expect the ratio $(\sigma^{en}/\sigma^{ep}) \to 1$, consistent with experiment.

For large x on the other hand, the so-called valence quarks should dominate leading to $u(x) = 2d(x)$ in a proton with two up quarks and one down quark. Were that the case, the ratio $(\sigma^{en}/\sigma^{ep})$ should $\to 2/3$. However, experimentally the ratio falls to $1/4$, corresponding to $d/u \to 0$, and again cautioning that simple three-quark models can be dangerous.

Another example is the sum rule (32) applied to the difference between the proton and neutron for which the "sea" contributions are expected to cancel largely, leading to a finite result. This is called the Gottfried sum rule, and in terms of (37) and (38) reads

$$\int_0^1 \frac{dx}{x} \left[F_2^{ep}(x) - F_2^{en}(x)\right] = \int_0^1 dx \times \left\{\frac{1}{3}\left[u(x) + \bar{u}(x)\right] - \frac{1}{3}\left[d(x) + \bar{d}(x)\right] + \text{heavier "sea quarks"}\right\} . \tag{39}$$

Since the proton has two more up-quarks than anti-up, and one more down than anti-down, we can write exact integral relations

$$\begin{aligned} \int_0^1 dx \left[u(x) - \bar{u}(x)\right] &= 2 \\ \int_0^1 dx \left[d(x) - \bar{d}(x)\right] &= 1 . \end{aligned} \tag{40}$$

Inserting (40) into (39) gives

$$\int_0^1 \frac{dx}{x} \left[F_2^{ep}(x) - F_2^{en}(x)\right] = \frac{1}{3} + \int_0^1 dx \left\{\frac{2}{3}\left[\bar{u}(x) - \bar{d}(x)\right] + \text{heavier "sea quarks"}\right\} . \tag{41}$$

Recently published data by the NMC collaboration at CERN indicate that, for $Q^2 \simeq 4$ GeV^2, the simple three-valence quark prediction of $1/3$ is approximately 25% larger than the observed value of 0.240 ± 0.016.

Similar predictions can be constructed for deep inelastic neutrino scattering from Eq. 21, plus three additional findings:

(1) νW_3, like νW_2 and W_1, scales.

(2) The weak currents appear in (16) and the commutator becomes

$$\langle \mathrm{PS}|\,[\mathrm{J}^+_\mu(x),\mathrm{J}^-_\nu(0)]\,|\mathrm{PS}\rangle$$

where, neglecting Cabibbo mixing

$$\mathrm{J}^+_\mu \;=\bar{u}\,\gamma_\mu(1-\gamma_5)\,d \;=\; (\mathrm{J}^-_\mu)^\dagger\;. \tag{42}$$

Evidently under an isospin rotation, which turns a neutron into a proton and vice versa,

$$W_i^{\nu p} \;=\; W_i^{\bar{\nu} n}\;, \qquad \text{and} \qquad W_i^{\nu n} \;=\; W_i^{\bar{\nu} p}\;. \tag{43}$$

(3) Introducing $F_1^{(\nu)} \;=\; MW_1^{(\nu)}$, $F_2^{(\nu)} \;=\; \nu W_2^{(\nu)}$, and $F_3^{(\nu)} \;=\; \nu W_3^{(\nu)}$, Eq. (21) becomes in the scaling region ($x \equiv Q^2/2M\nu$; $y \equiv \nu/E$) and for $Q^2/E^2 \to 0$,

$$\frac{d^2\sigma}{dxdy} \;=\; \frac{G_F^2}{\pi}\,(ME)\left\{(1-y)\;F_2 + xy^2\,F_1 \pm \frac{1}{2}\;xy(2-y)\;F_3\right\} \tag{44}$$

showing a total cross section that rises linearly with energy (for $E \ll M_W!$).

Again various sum rules can be derived based on quark-parton wave functions. One can relate the structure functions probed by the weak currents with the electromagnetic ones on the basis of known symmetries.

In particular, recognizing that when $\nu \to e^-$, $d \to u$ or $\bar{u} \to \bar{d}$, whereas when $\bar{\nu} \to e^+$, $u \to d$ or $\bar{d} \to \bar{u}$, and that $(1-\gamma_5)^2 = 2(1-\gamma_5)$ for weak currents, it follows that (neglecting the Cabibbo angle as small)

$$\left.\begin{aligned} F_2^\nu(x) \;&=\; 2x\,[d(x)+\bar{u}(x)] \\ F_2^{\bar{\nu}}(x) \;&=\; 2x\,[\bar{d}(x)+u(x)] \end{aligned}\right\} \;+\; \text{heavier “sea” quarks}\;, \tag{45}$$

and therefore for isoscalar targets,

$$\frac{F_2^\nu(x)+F_2^{\bar{\nu}}(x)}{F_2^{ep}(x)+F_2^{en}(x)} \;=\; \frac{18}{5} + \text{strange quark corrections.} \tag{46}$$

The data agree well with this result. Deviations are a measure of the contribution of strange quarks (anti-quarks).

We find an intriguing result if, using (43) and (45), we form the difference

$$F_2^{\nu p}(x) - F_2^{\nu n}(x) = 2x\left\{\left[d(x) - \bar{d}(x)\right] - \left[u(x) - \bar{u}(x)\right] \right.$$
$$\left. + \text{ differences } [s(x) - \bar{s}(x)] \text{ of the heavier quarks}\right\} . \tag{47}$$

Using (40) and recalling that protons and neutrons have zero charm, strangeness, quantum numbers, etc., we find

$$\int_0^1 \frac{dx}{x}\left[F_2^{\nu p}(x) - F_2^{\nu n}(x)\right] = 2(1-2) = -2 . \tag{48}$$

Equation (48) is the Adler Sum Rule first derived in 1966 from current algebra. Its generality—i.e., independence of model-dependent statements about the quark"sea"—suggests that it may be an exact result, derivable from the algebra of the operator currents themselves rather than from models of the nucleon state. This suggests the importance of deriving exact sum rules and of testing them quantitatively. In 1984 the validity of (48) was established to ±20% at CERN.

There is an additional sum rule for the odd parity term, F_3, in (44) that suggests a more general validity than the parton model. One finds (homework!) that

$$xF_3^{\nu} = \pm F_2^{\nu} ,$$

where "+" applies for a fermion target and "−" for an anti-fermion. Correspondingly, for fermion targets

$$xF_3^{\nu} = +F_2^{\nu} ,$$
$$xF_3^{\bar{\nu}} = -F_2^{\bar{\nu}} . \tag{49}$$

This gives from (45)

$$F_3^{\nu}(x) = 2\left[d(x) - \bar{u}(x)\right] + \ldots ,$$
$$F_3^{\bar{\nu}}(x) = 2\left[u(x) - \bar{d}(x)\right] + \ldots . \tag{50}$$

Using $F_3^{\nu n} = F_3^{\bar{\nu} p}$, we obtain

$$\int_0^1 dx\left[F_3^{\nu p}(x) + F_3^{\nu n}(x)\right] = 2\int_0^1 dx\left[u(x) - \bar{u}(x) + d(x) - \bar{d}(x)\right] = 2(2+1) = 6 . \tag{51}$$

Equation (51) is the 1969 Gross-Llewellyn-Smith sum rule. Like the Adler sum rule (48) and the spin flip sum rule for electromagnetic currents derived by Bjorken in 1966, it too can be derived directly from current algebra—to which we now turn attention.

SECOND LECTURE

In the first lecture I emphasized the importance of sum rules. I suggested that the Adler Sum Rule might have a deeper basis than the parton model result, which expressed it as the difference between the up and down quark numbers of the proton. In this lecture I want to show how the Adler Sum Rule can be derived very generally from the Equal Time Commutator (ETC) algebra. It is one of the sum rules that I think deserve great attention in the experimental program in the years ahead.

We start with a quick run through of the simple algebra to derive the Adler Sum Rule. Remember that the structure functions are defined by $W_{\mu\nu}$, which can be expressed as a commutator formed with the electromagnetic currents, Eqs. (16) and (17). The weak currents carry isotopic spin, corresponding to W^+ and W^- vector bosons that change the down to an up quark, and the up to a down quark, respectively. Thus in a weak interaction scattering a neutrino into an electron, the current is carried by a W^+, which changes a down to an up quark in the target hadron. Calling the hadron current in this case J_μ, we write for neutrino scattering from a proton*

$$W^{\nu}_{\alpha\beta}(\mathrm{P},q) = \frac{1}{2\pi}\frac{E_P}{M}\int d^4y\; e^{iq\cdot y}\left\langle \mathrm{P}\left|\mathrm{J}^{\dagger}_{\beta}(y)\mathrm{J}_{\alpha}(0)\right|\mathrm{P}\right\rangle\,, \qquad q_0>0\,. \tag{52}$$

This is nonvanishing for $q_0 > 0$ as is clear by inserting a complete set of states in the matrix element and doing the time integral. Let us next write the corresponding expression for anti-neutrino scattering. The anti-neutrino now turns into a positron and transfers one unit of negative charge, which changes an up to a down quark. This is called $\mathrm{J}^{\dagger}_{\mu}$, the charge-lowering operator, and the corresponding scattering expression for the anti-neutrino is

$$\begin{aligned} W^{\bar\nu}_{\alpha\beta}(\mathrm{P},q) &= \frac{1}{2\pi}\frac{E_P}{M}\int d^4y\; e^{iq\cdot y}\left\langle \mathrm{P}\left|\mathrm{J}_{\beta}(y)\mathrm{J}^{\dagger}_{\alpha}(0)\right|\mathrm{P}\right\rangle \\ &= \frac{1}{2\pi}\frac{E_P}{M}\int d^4y\; e^{-iq\cdot y}\left\langle \mathrm{P}\left|\mathrm{J}_{\beta}(0)\mathrm{J}^{\dagger}_{\alpha}(y)\right|\mathrm{P}\right\rangle\,, \qquad q_0>0\,. \end{aligned} \tag{53}$$

In writing the second form of (53) we use the invariance of the diagonal matrix element under the displacement of y_μ to the origin, and then relabel the variable $y_\mu \to -y_\mu$. Equation (53) is, like Eq. (52), nonvanishing when $q_0 > 0$.

* The factor E_P/M, where E_P is the proton energy in the scattering frame, is included to give $W_{\alpha\beta}$ the correct Lorentz transformation property. This factor was replaced by unity in Eqs. (16) and (17) since we were working in the proton rest system.

Following Steve Adler's original derivation in 1966 of the neutrino scattering sum rule named for him, we will now take several simple steps of algebra to show that we can write the experimental quantity of interest as a commutator of currents. We start by constructing the commutator (54) which is related to the two expressions (52) and (53) as indicated.

$$\begin{aligned} \text{Define} \quad W_{\alpha\beta}(\mathrm{P},q) &= \frac{1}{2\pi}\,\frac{E_P}{M}\int d^4y\; e^{iq\cdot y}\;\left\langle \mathrm{P}\left|\left[\mathrm{J}_\beta^\dagger(y),\,\mathrm{J}_\alpha(0)\right]\right|\mathrm{P}\right\rangle \\ &= W^{\nu}_{\alpha\beta}(\mathrm{P},q) \qquad q_0>0 \\ &= -W^{\bar\nu}_{\beta\alpha}(\mathrm{P},-q) \qquad q_0<0\;. \end{aligned} \tag{54}$$

For $q_0 > 0$ this corresponds to ν scattering, and for $q_0 < 0$, to $\bar\nu$ scattering, with $\alpha \leftrightarrow \beta$ and multiplied by -1. Next we follow a kinematic trick introduced by Fubini and Furlan in 1965 by going to an infinite momentum reference frame. Remember we are interested in experimental quantities such as $W_1(q^2,\nu)$ and $W_2(q^2,\nu)$ in deep inelastic electron scattering which are scalars, and depend on q^2 and $\mathrm{P}\cdot q = M\nu$, which are Lorentz invariant scalars. The kinematics is simplified greatly if we choose a frame in which

$$\mathrm{P}\to\infty \qquad \text{and} \qquad \vec{\mathrm{P}}\cdot\vec{q} = 0\;, \tag{55}$$

so that

$$\mathrm{P}\cdot q = E_P\, q_0 = M\nu\;. \tag{56}$$

In the $\mathrm{P}\to\infty$ frame, it is the zero-zero component of $W_{\alpha\beta}$ that is of interest, as we see from (18):

$$W_{00} \to \left(\frac{E_P}{M}\right)^2\; W_2\left(Q^2,\nu\right)\;. \tag{57}$$

This is a simple result. There is nothing delicate about this limit. No limiting behavior is implied for W_1 or W_2. Let us next form the integral

$$\int_{-\infty}^{\infty} dq_0\; W_{00} = \int_0^{\infty} dq_0\;\left[W^{\nu}_{00}\left(\mathrm{P},q\right) - W^{\bar\nu}_{00}\left(\mathrm{P},q_0,-\vec{q}\right)\right]\;, \tag{58}$$

where in writing the right hand side we have replaced q_0 by $-q_0$ in the second term of the commutator in (54). From (55), (56), and (57) it is apparent that

W_{00} depends only on the magnitude, and not the sign of $\vec{q}$. Making the substitution $\vec{q} \to -\vec{q}$ we can carry through the manipulation shown in Eq. (59):

$$\int_0^\infty dq_0 \; [W_{00}^{\nu}(\mathrm{P},q) - W_{00}^{\bar\nu}(\mathrm{P},q)]$$

$$= \frac{1}{2\pi}\frac{E_P}{M}\int_{-\infty}^{\infty} dq_0 \int d^4y \; e^{iq\cdot y} \left\langle \mathrm{P}\left|\left[\mathrm{J}_0^\dagger(y),\, \mathrm{J}_0(0)\right]\right|\mathrm{P}\right\rangle$$

$$= \frac{E_P}{M}\int d^3y \; e^{i\vec{q}\cdot\vec{y}} \left\langle \mathrm{P}\left|\left[\mathrm{J}_0^\dagger(0,\vec{y}),\, \mathrm{J}_0(0)\right]\right|\mathrm{P}\right\rangle \tag{59}$$

$$= \left(\frac{E_P}{M}\right)^2 \frac{M}{E_P}\int_0^\infty d\nu \; [W_2^{\nu}(Q^2,\nu) - W_2^{\bar\nu}(Q^2,\nu)] \; .$$

The equal time commutator on the right hand side of (59) can be evaluated in terms of the local current algebra applied to the charge densities. This algebra reads in the SU(2) limit

$$\left[\mathrm{J}_0(\vec{y}),\, \mathrm{J}_0^\dagger(0)\right] = 2\mathrm{J}_0^3(0)\;\delta^3(\vec{y}) \; , \tag{60}$$

leading directly to the Adler Sum Rule

$$\int_0^\infty d\nu \left[W_2^{\nu}(Q^2,\nu) - W_2^{\bar\nu}(Q^2,\nu)\right] = -2 \; . \tag{61}$$

The quark currents in this limit are given by

$$\left.\begin{aligned} J_0 &= u^\dagger(1-\gamma_5)d \\ J_0^\dagger &= d^\dagger(1-\gamma_5)u \end{aligned}\right\} \; \left[\mathrm{J}_0(\vec{y}),\, \mathrm{J}_0^\dagger(0)\right] = 2\delta^3(\vec{y}) \; \left\{u^\dagger(1-\gamma_5)u - d^\dagger(1-\gamma_5)d\right\} \; . \tag{62}$$

Recalling (43), this result can also be expressed as

$$\int_0^\infty d\nu \left[W_2^{\nu n}(Q^2,\nu) - W_2^{\nu p}(Q^2,\nu)\right] = 2 \; . \tag{63}$$

A refinement of this result to include Cabibbo mixing of the down and strange currents (quarks) gives

$$\begin{aligned} \int_0^\infty d\nu \left[W_2^{\nu} - W_2^{\bar\nu}\right] &= -2\cos^2\theta_c - 4\sin^2\theta_c \qquad \text{proton target} \; , \\ &= +2\cos^2\theta_c - 2\sin^2\theta_c \qquad \text{neutron target} \; . \end{aligned} \tag{64}$$

The importance of the Adler Sum Rule is that it is derived on the basis of current algebra and general symmetry principles, and is independent of approximations on the dynamics or the hadron ground states as implied in parton model calculations. Experimental data from CERN in 1984, by Allasia et al. gives a 20% measurement with a 18% systemmatic error in good agreement with theory. This is a fundamental prediction of theory and more precise experimental confirmation is highly desirable.

Now we ask if there is another general prediction from current algebra which can be tested and is independent of detailed models. For this we look again to the electromagnetic current. It won't be as straightforward as for the Adler Sum Rule because there is no charge raising or lowering as in (59) and (62), and two electromagnetic charge densities separated by a space-like interval commute. However as first shown by Bjorken, in the deep inelastic scattering limit of very high energy electrons, the difference of spin-dependent parts of the cross section for electron scattering from hadrons can be expressed in terms of an equal time commutator of current densities. The isovector part of this commutator—i.e., the proton-neutron difference—can be expressed in terms of the axial vector β-decay coupling constant. Therefore in the deep inelastic limit—a limit not required for the Adler Sum Rule which is an identity for all Q^2—one has a sum rule independent of dynamical models that is derived directly from the equal time current algebra. This is the Bjorken, or bj, Sum Rule and presents a basic test of the current algebra. Verification of the bj Sum Rule presents an important challenge for the future.

I won't reproduce all the algebraic steps in Bjorken's derivation in this lecture, but will set up the derivation—details of which can be found in the original papers. It is useful to introduce the cross section for photoproduction by a virtual photon of invariant mass $q^2 \equiv -Q^2 < 0$ and energy $q \cdot \mathrm{P} = M\nu$ incident on a hadron (proton or neutron) of mass M. This is the part of the process below the dashed line in Fig. 5. Its cross section is expressed by

$$\sigma_i = \frac{4\pi^2\alpha}{\nu - (Q^2/2M)} \sum_n (2\pi)^3 \, \delta^4 (\mathrm{P}_n - \mathrm{P} - q) \left| \langle n \left| \epsilon_i \cdot \mathrm{J} \right| \mathrm{PS} \rangle \right|^2 , \qquad (65)$$

where we have used the Hand-Berkelman convention of defining the initial photon flux in terms of the equivalent energy of a real photon that would produce the same final hadron mass; i.e.,

$$\begin{aligned} (\mathrm{P}+q)^2 &= M^2 + 2M\nu - Q^2 \\ &= M^2 + 2M \left\{ \nu - \frac{Q^2}{2M} \right\} . \end{aligned} \qquad (66)$$

The final result is, of course, independent of this convention. The polarization vector appearing in (65) is defined by

$$\epsilon_i \cdot q \;=\; 0 \;, \qquad \epsilon_L^* \cdot \epsilon_L \;=\; \epsilon_R^* \cdot \epsilon_R \;=\; -1 \;; \qquad \epsilon_s^2 \;=\; +1 \;, \tag{67}$$

$$\text{For} \quad q \;:\; \left(\nu,\, 0,\, 0,\, \sqrt{\nu^2 + Q^2}\right) \;,$$

$$\epsilon_{R,L} \;:\; (0,\, \mp i,\, 1,\, 0)\; \frac{1}{\sqrt{2}} \;,$$

$$\epsilon_s \;:\; 1/\sqrt{Q^2}\; \left(\sqrt{\nu^2 + Q^2},\, 0,\, 0,\, \nu\right) \;,$$

for the virtual photon incident along the z-axis; $\epsilon_{R,L}$ denote right- and left-circularly polarized transverse photons and ϵ_s, the longitudinally polarized one present when $Q^2 \neq 0$.

To relate the cross section as defined for virtual photon scattering to deep inelastic electron scattering, Eq. (65) must be tied onto the electron current that describes the process above the dashed line in Fig. 5. Specifically, we are interested in the scattering of left-handed (or right-handed) electrons. The appropriate current in this case is

$$j_L^\mu \;=\; \bar{u}(p')\; \gamma_\mu (1 - \gamma_5)\; u(p) \;. \tag{68}$$

In the high energy deep-inelastic limit $\nu^2 \gg Q^2 \gg M^2$, (68) becomes (class homework)

$$j_L^\mu \;\simeq\; \frac{\sqrt{Q^2}}{\nu} \left[\epsilon_s^\mu + \sqrt{\frac{E}{2E'}}\, \epsilon_L^\mu + \sqrt{\frac{E'}{2E}}\, \epsilon_R^\mu\right] \;. \tag{69}$$

As a result of current conservation, $j_\mu Q_\mu = 0$, and it is possible to project j_L^μ as shown onto the three independent polarization vectors defined in (67). In the deep inelastic limit, it will be a good approximation to treat the direction of the incident electron spin as coincident with that of the momentum transfer $\vec{q}$. Note from (11) for the incident electron along the z-axis that

$$\frac{q_\perp}{q_\parallel} \;=\; \frac{p' \sin\theta}{p - p' \cos\theta} \;\approx\; \frac{E'\theta}{\nu} \;=\; \sqrt{\frac{E'}{E}}\; \frac{\sqrt{Q^2}}{\nu} \;\ll\; 1 \;, \tag{70}$$

and $\theta \approx \sqrt{Q^2/EE'}$. Henceforth, we neglect the angle between $\vec{q}$ and $\vec{p}$ in computing the cross section in the high energy, deep inelastic region for electrons with their spin polarized parallel or anti-parallel to that of the target protons. (Small correction terms can be included in comparing theory and data). With this simplifying assumption, which is also a good approximation, that electron and proton spins are aligned along the incident direction, we can perform the

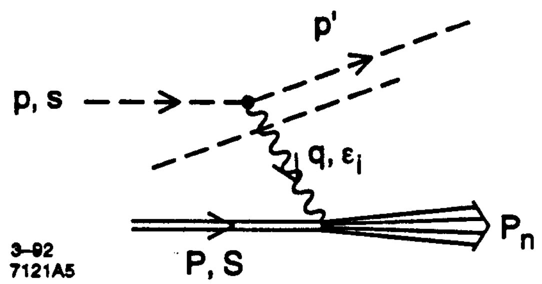

Fig. 5. Feynman diagram for inelastic scattering indicating the division into two parts corresponding to virtual photoproduction [Eq. (65)] multiplied by the electron current [Eq. (68)].

azimuthal average when we tie (65) and (69) together as in (12) to form the polarized electron cross section

$$\int \frac{d\phi}{2\pi} \frac{d^2\sigma_L^e}{dQ^2 d\nu} = \frac{\alpha}{\pi Q^2 \nu} \frac{E'}{E} (1-x) \left[\sigma_s(Q^2,\nu) + \frac{E}{2E'} \sigma_L + \frac{E'}{2E} \sigma_R\right] . \quad (71)$$

Interference between different polarizations is removed by the azimuthal averaging since such terms are proportional to $\cos\phi$ and $\cos 2\phi$. [Homework]. Comparing (71) with (19) in the $E \to \infty$, $\theta \to 0$ limit allows us to identify for deep inelastic scattering

$$W_2(Q^2,\nu) = \frac{Q^2}{4\pi^2\alpha\nu} (1-x) \; [\sigma_s + \sigma_T] \; , \quad \text{with} \quad \sigma_T \equiv \frac{1}{2} \; (\sigma_L + \sigma_R) \; . \quad (72)$$

The general identification is

$$\begin{aligned} \sigma_T &= \frac{4\pi^2\alpha}{\nu - (Q^2/2M)} W_1 \; , \\ \sigma_s &= \frac{4\pi^2\alpha}{\nu - (Q^2/2M)} \left[W_2 \left(\frac{\nu^2}{Q^2} + 1\right) - W_1\right] \; , \end{aligned} \quad (73)$$

which by (33) shows that $\sigma_s \to 0$ in the scaling limit for light spin-1/2 target quarks. The reason for this vanishing limit is easy to see in the Breit frame for the target quark (i.e., the quantum must bring a unit of z-axis spin because the quark helicity cannot flip). We can now rewrite (71), assuming henceforth that we work exclusively with the azimuthally averaged scattering cross sections

$$\frac{d^2\bar{\sigma}_L^e}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4} \frac{E'}{E} W_2(Q^2,\nu) \left\{1 + \frac{\nu}{2E'} \left[\frac{\sigma_L}{\sigma_s + \sigma_T}\right] - \frac{\nu}{2E} \left[\frac{\sigma_R}{\sigma_s + \sigma_T}\right]\right\}$$

$$\text{for} \quad \nu^2 \gg Q^2 \gg M^2 \; . \quad (74)$$

Finally, we have for the spin-dependent deep inelastic scattering to leading order in ν/E and for $\nu^2 \gg Q^2 \gg M^2$

$$\frac{d^2\sigma_A^e}{dQ^2 d\nu} - \frac{d^2\sigma_P^e}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4} \frac{2\nu}{E} \left(\frac{\sigma_A - \sigma_P}{\sigma_A + \sigma_P + 2\sigma_s}\right) W_2 \; , \quad (75)$$

where, following Bjorken, we identify

$$\begin{aligned} \sigma_R &\to \sigma_P \; , \\ \sigma_L &\to \sigma_A \; , \end{aligned} \quad (76)$$

for the target nuclear spin aligned parallel or anti-parallel to the z-axis direction of $\vec{q}$. The experimentalists will accurately determine the ratio of unpolarized-to-polarized cross sections (weighted of course by the degree of beam and target polarizations that are actually achieved):

$$\frac{\frac{d^2\sigma_A^e}{dQ^2d\nu} - \frac{d^2\sigma_P^e}{dQ^2d\nu}}{\frac{d^2\sigma_A^e}{dQ^2d\nu} + \frac{d^2\sigma_P^e}{dQ^2d\nu}} = \left(\frac{\nu}{E}\right) \frac{\sigma_A - \sigma_P}{\sigma_A + \sigma_P} \; \frac{1}{1 + \frac{2\sigma_s}{\sigma_A + \sigma_P}} \tag{77}$$

$$\equiv \left(\frac{\nu}{E}\right) A_1 \left(\frac{1}{1+R}\right) .$$

The factor ν/E is measured directly and the small ratio

$$R \equiv \frac{2\sigma_s}{\sigma_A + \sigma_P}$$

is deduced from measurement of the $\tan^2(\theta/2)$ slope of (19) and the definition (73). Typically $R \sim 0.1$. What one learns from (77) directly then is

$$A_1 \equiv \frac{\sigma_A - \sigma_P}{\sigma_A + \sigma_P} . \tag{78}$$

The crucial quantity computed by Bjorken in the deep inelastic, high energy limit is the sum rule

$$\int_{\nu_{th}=\frac{Q^2}{2M}}^{\infty} \frac{d\nu}{\nu} \; \nu W_2 \;\; A_1 \left(\frac{1}{1+R}\right) \equiv Z . \tag{79}$$

In the scaling limit, and using $F_2(x) \equiv \nu W_2$ as introduced earlier in discussing the parton model, (79) can be rewritten.

$$Z = \int_0^1 \frac{dx}{x} \, F_2(x) \, A_1 \left(\frac{1}{1+R}\right) \equiv 2 \int_0^1 g_1(x) \, dx . \tag{80}$$

What Bjorken showed is that Z can be written in the high energy limit as an equal-time commutator of two transverse components of the electromagnetic current densities:

$$Z = \lim_{P_z \to \infty} \frac{i}{2} \int d^3x \, \langle \mathrm{P} \, | \, [\mathrm{J}_x(\vec{x}, 0), \, \mathrm{J}_y(0)] \, | \, \mathrm{P} \rangle . \tag{81}$$

In contrast to the Adler Sum Rule, the Bjorken Sum Rule can be derived only in the large Q^2 high-energy limit, and on the basis of an assumption of "asymptotic freedom" in this limit. However it is a fundamental prediction of the theory relying in this limit only on the local current algebra, and on no further dynamical assumptions or models. The appearance of a commutator of the x-component with the y-component of the current represents the difference between scattering of left- and right-handed electrons. The formal manipulations leading to (81) are slightly more complicated than Eqs. (52) to (59) for the Adler Sum Rule, and I don't reproduce them here.

Proceeding from (81) to an experimental prediction, we introduce the quark currents as in (62), the only difference being that the vector currents alone contribute to electromagnetic scattering. We write the vector quark current

$$J_\mu = \bar{\psi}\, \gamma_\mu\, Q \psi \,, \text{ where } Q = \begin{bmatrix} \frac{2}{3} & & \\ & -\frac{1}{3} & \\ & & -\frac{1}{3} \end{bmatrix} \;;\quad Q^2 = \frac{2}{9} + \frac{Q}{3}\,, \tag{82}$$

including the strange quark in the SU(3) limit, although it is immediately clear that the algebra here is the same as for SU(2). Taking the equal time commutator gives

$$[\mathrm{J}_x(\vec{x},0),\, \mathrm{J}_y(0)] = 2i\delta^3(\vec{x})\, \psi^\dagger \sigma_z \left(\frac{2}{9} + \frac{1}{3}\, Q\right)\, \psi + \,\dots\,, \tag{83}$$

where the added so-called gradient, or Schwinger, terms are required to insure locality and lorentz invariance, but do not contribute to the integral (80). As noted by Bjorken, the isovector part of (83) is just the ratio of the axial to vector β-decay coupling strengths known from low-energy processes. Therefore the proton-neutron difference is determined

$$Z_{\mathrm{P}} - Z_N = \frac{1}{3} \left|\frac{g_A}{g_V}\right| \approx 0.41\,, \tag{84}$$

and by (80) we can write finally

$$\int_0^1 \left[g_1^P(x) - g_1^n(x)\right]\, dx = \frac{1}{6}\left|\frac{g_A}{g_V}\right| \approx 0.2\,. \tag{85}$$

Since it is an asymptotic sum rule, there are finite-energy corrections to the Bjorken Sum Rule that were not present for Adler's relations for neutrino-scattering. These have been studied in some detail by J. Kodaira and collaborators since 1979. To leading order in the strong coupling QCD corrections involving gluons, they find

$$\left|\frac{g_A}{g_V}\right| \to \left|\frac{g_A}{g_V}\right| \left[1 - \frac{\alpha_s(Q^2)}{\pi} + \cdots\right]\,. \tag{86}$$

This correction vanishes at infinite energy as it must by asymptotic freedom. At $Q^2 \approx 4$ GeV^2, it leads to a 9% reduction of (85).

An experimental program to test the Bjorken Sum Rule is now being undertaken at SLAC under the leadership of Emlyn Hughes and Charles Prescott. Plans include running at electron energies up to 50 GeV incident on hydrogen and He^3 gas targets to measure A_1 for both protons and neutrons. Figure 6 from the experimental proposals gives an idea of the anticipated accuracies in comparison with what is already known for proton targets from earlier SLAC and CERN observations. Nothing is known at present about neutron scattering. Theoretical calculations of $g_1^P(x)$ have led to a so-called "spin crisis". This arises from the Ellis-Jaffe calculation of $\int_0^1 g_1^P\, dx \approx 0.19$ for the proton based on the quark light-cone algebra that is generally accepted as valid, plus an assumption that strange sea quarks do not contribute in evaluating the contribution to Z_P from the isoscalar term in (83). However, this calculation is about 50% larger than the smooth extrapolation to a value

$$\int_0^1 g_1^P\, dx \;=\; 0.126 \pm (\sim .02)$$

from data shown in Fig. 7. The next lectures by Frank Sciulli will cover the experimental situation much more fully and accurately. Here I comment only to emphasize the importance of measuring the proton-neutron difference in order to avoid such model dependence and to rely on presumably accurate sum rule predictions.

There is by now a very extensive literature on the spin crisis. One recent notable observation by Jaffe and Lipkin** based on earlier ideas of Lipkin shows that one way to remove the spin crisis is to form the proton of the three-valence quarks plus a $Q\bar{Q}$ pair of quarks with $\underset{\sim}{L} = \underset{\sim}{S} = 1$ coupled to a $\underset{\sim}{J} = O^{++}$ or 1^{++} state. Another interesting analysis by Anselmino, Ioffe, and Leader** emphasizes the transition from the Bjorken Sum Rule for $Q^2 \to \infty$ with the analogous sum rule for real photons with $Q^2 = 0$.

The original dispersion relations for forward Compton scattering used causality, analyticity, and unitarity (the optical theorem) to give for $f_1(\nu)$ and $f_2(\nu)$, as defined by

$$f(\nu) \;=\; f_1(\nu^2)\; \vec{e}\,'^{*} \cdot \vec{e} + \nu f_2(\nu^2)\; i\vec{\sigma} \cdot \vec{e}\,'^{*} \times \vec{e}\,, \tag{87}$$

$$\mathrm{Re}\; f_1(\nu^2) \;=\; -\frac{\alpha}{M_P} + \frac{\nu^2}{8\pi^2}\, P\!\!\int_0^\infty \frac{d\nu'^2}{\nu'(\nu'^2 - \nu^2)})\left[\sigma_A(\nu') + \sigma_P(\nu')\right] \quad . \tag{88}$$

** R. L. Jaffe and H. J. Lipkin, Physics Lett. **B200**, 458 (1991).
** M. Anselmino, B. L. Ioffe, and E. Leader, Soviet Journal of Nuclear Physics **49**, 136 (1989).

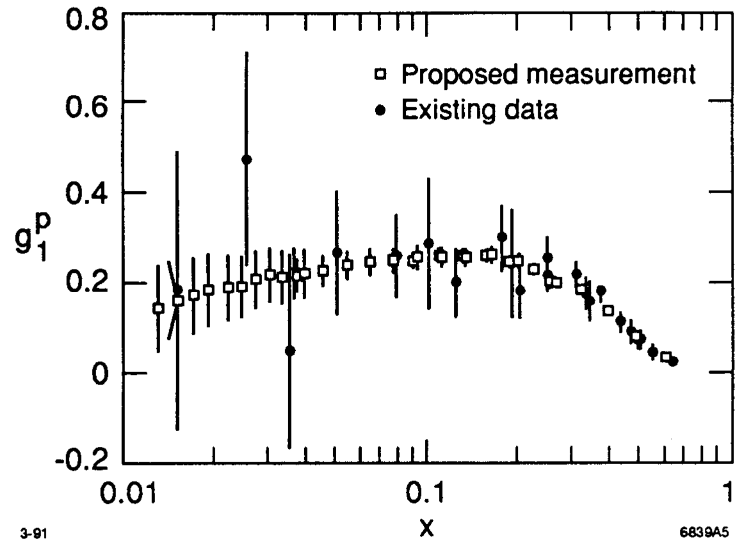

Fig. 6. Existing SLAC and CERN data for the proton spin structure function, together with data anticipated from the proposed E142 experiment at SLAC.

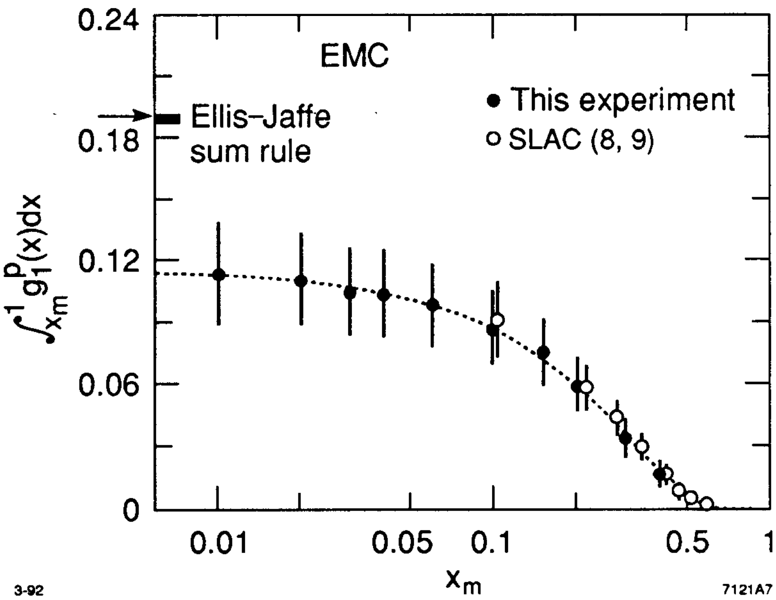

Fig. 7. Data for the sum rule for inelastic spin-dependent lepton-proton scattering. The black dots are the results from the EMC at CERN. $x_m \equiv Q^2/2M\nu_{\max} = \nu_{\text{th}}/\nu_{\max}$ denotes the minimum value of x measured, corresponding to the maximum energy loss by the muon in the CERN measurements and by the electron in the SLAC data.

$$\text{Re } f_2(\nu^2) = \frac{1}{8\pi^2} \oint_0^\infty [\sigma_A(\nu') - \sigma_P(\nu')] \frac{d\nu'^2}{\nu'^2 - \nu^2} . \tag{89}$$

Equation (88) for the spin independent amplitude is a once-subtracted dispersion relation with the zero energy limit given by the classical Thomson amplitude.

If $\lim_{\nu \to \infty} (\sigma_P(\nu) - \sigma_A(\nu)) \to 0$ faster than $1/(\ell n \nu)$, as indicated by low-order perturbation theory, Eq. (89) for the spin dependent amplitude requires no subtraction and converges as written. For pure QED this means no arbitrary parameter other than the electron charge α enters the theory. With this assumption the low-energy limit of $f_2(\nu)$ is given by

$$f_2(0) = \frac{1}{4\pi^2} \int_0^\infty \frac{d\nu'}{\nu'} [\sigma_A(\nu') - \sigma_P(\nu')] . \tag{90}$$

A low energy theorem for $f_2(0)$ derived in 1954 by F. Low, and independently by M. Gell-Mann and M. Goldberger, from the general structure of the scattering amplitude, including in particular current conservation, gives the exact result

$$f_2^{(P)}(0) = -\frac{1}{2} \frac{\alpha}{M_P^2} \kappa_P^2 , \tag{91}$$

where κ is the anomalous nucleon magnetic moment in units of the nuclear Bohr magneton; i.e., $\kappa_P = 1.79$ and $\kappa_N = -1.91$ for the proton and neutron, respectively. Together, Eqs. (90) and (91) give an exact sum rule relation for the anomalous moment

$$\begin{aligned} \int_0^\infty \frac{d\nu}{\nu} [\sigma_P^p - \sigma_A^p] &= \frac{2\pi^2\alpha}{M_P^2} \kappa_P^2 = 205 \ \mu b \\ \int_0^\infty \frac{d\nu}{\nu} [\sigma_P^n - \sigma_A^n] &= \frac{2\pi^2\alpha}{M_N^2} \kappa_N^2 = 233 \ \mu b . \end{aligned} \tag{92}$$

Of particular importance to us here is that the difference of cross sections in the left side of (92) is precisely the $Q^2 \to 0$ limit of the integrand in the Bjorken sum rule expression (79), multiplied by $(-Q^2/4\pi^2\alpha)$. [This is your homework; use Eq. (73)]. Thus defining

$$\int_{\nu_{\text{th}}}^\infty \frac{d\nu}{\nu} [\nu W_2] \left. \frac{\sigma_A - \sigma_P}{\sigma_A + \sigma_P + 2\sigma_s} \right|_{\text{proton}} \equiv Q^2 \ I_P(Q^2) , \tag{93}$$

we can write generally

$$I_P(Q^2) \begin{array}{l} \nearrow \quad \dfrac{Z_p}{Q^2} \qquad\qquad Q^2 \to \infty \\ \searrow \quad -\dfrac{1}{4}\,\dfrac{\kappa_P^2}{M_P^2} \qquad Q^2 \to 0 \end{array} , \tag{94}$$

and for the proton-neutron difference determined by the Bjorken Sum Rule

$$I_P(Q^2) - I_N(Q^2) \begin{array}{l} \nearrow \quad \dfrac{1}{3}\left|\dfrac{g_A}{g_V}\right| \dfrac{1}{Q^2} = \dfrac{.41}{Q^2} \qquad Q^2 \to \infty \\ \searrow \quad +.11\,\dfrac{1}{M_P^2} \qquad\qquad\qquad Q^2 \to 0 \end{array} . \tag{95}$$

Thus both limits are determined exactly. Anselmino, Ioffe and Leader have studied the transition region between $Q^2 \to \infty$ and $Q^2 \to 0$, and presented models to help interpolate. Figure 8 shows that a smooth extrapolation for the proton-neutron difference becomes a very different one for the proton and neutron individually. The important point to emphasize is that in (93) we have a relation that is fixed by general predictions of local, relativistic quantum field theory in both the deep inelastic and the real photon limits. Confirmation of this result and study of the behavior between these two limits is of fundamental interest and importance. In both limits the predictions are in terms of zero-energy behavior. There is a world of interesting and clean physics to be studied here. It is a rich challenge to the experimenters.

Finally, there is one additional sum rule that I mentioned at the end of the first lecture—the Gross-Llewellyn-Smith Sum Rule (51) for neutrino scattering. As noted there, the parton result suggested a more general basis for it. It too can be deduced similarly to the Bjorken Sum Rule as an asymptotic result, and with the same QCD correction factor as in (86). It has been tested with modest accuracy, as Sciulli will discuss in his lectures. That is the end of the Sum Rule story.

We turn next to deviations from scaling. The underlying physical picture of the proton leading to scaling is that there are point-like constituents in the nucleon. It would be startling if this were strictly true. It would mean the end of our search for nature's building blocks if these were truly points. However, we know from all previous advances in physics that even if a particle looks point-like on one resolution scale, at higher resolution you find some structure on a smaller scale. The electron was point-like in the Dirac hydrogen atom for a long time until we looked with such precision that quantum radiative corrections had to be included. To understand the electron's anomalous magnetic moment, $g-2$, and the Lamb Shift we had to include the coupling of the electron to the radiation field, which gave it structure, characteristically $\sim \sqrt{\alpha}\;\lambda_c$ where $\alpha = 1/137$ and λ_c is the electron compton wavelength, $\lambda_c \equiv \hbar/mc \simeq 3.8 \times 10^{-11}$ cm.

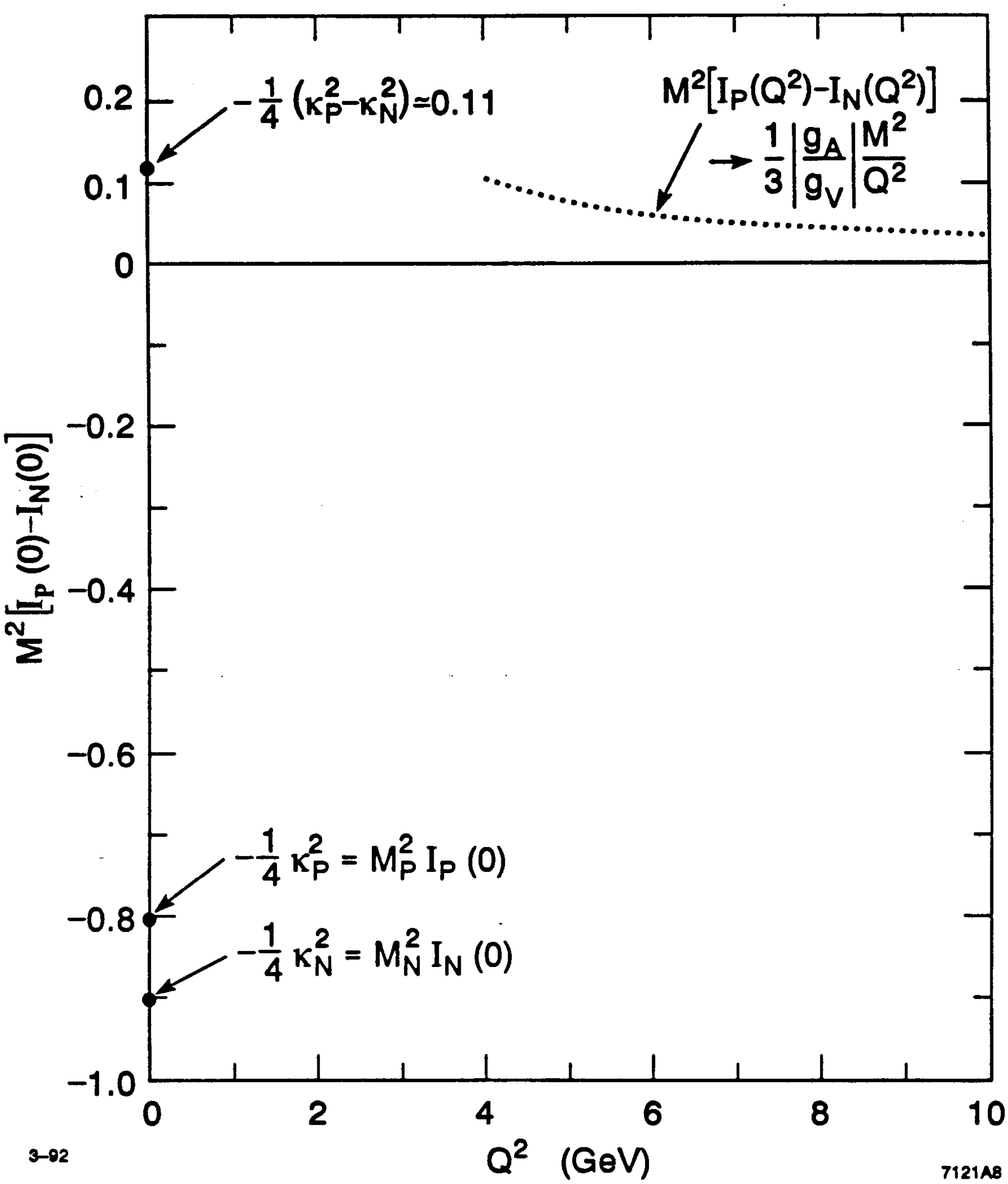

Fig. 8. The dotted line indicates the prediction of the Bjorken Sum Rule for large Q^2. The existing data for the proton alone lie near this prediction. The intercepts at $Q^2 = 0$ for real photons are the dispersion relation predictions, Eqs. (94) and (95).

It would indeed be amazing if quarks didn't have a structure because of their color interaction in QCD to gluons. Analogously to the electron in the atom, we expect to see deviations from scaling due to quark-gluon coupling in the proton. Scaling deviations should be seen—and in fact, they are observed experimentally.

In contrast to quantum electrodynamics, QCD is asymptotically free. Therefore, when we go to asymptotically high momentum transfer, we should observe scattering from free, point-like objects. But in QCD there is no length scale, and the approach to asymptopia is only logarithmic. In contrast, for a super-renormalizable theory with a length scale, interactions look really point-like asymptotically. However, QCD is a renormalizable theory with dimensionless interactions, and the approach to asymptopia is very slow, or logarithmic. This leads to the expectation of logarithmic deviations from scaling. The rest of this lecture is devoted to developing an understanding of the logarithmic corrections to scaling and to showing how they measure properties of theory. For large Q^2 there will be "higher twist" corrections of order M^2/Q^2, where M represents a proton or quark mass, or a threshold for creating massive quarks. These are important, and must be included as part of the technology of the field in making accurate analyses. I am not talking about these. I am talking about an in-principle deviation from scaling, even if Q^2 is very large and one can forget about the masses of quarks.

This deviation is to be expected if you think about it as follows. When you (theoretically) "look at a quark with a limited-power microscope"—which is what deep inelastic scattering does—you will see a quark carrying a certain momentum. But if you look with a very high resolving power with your microscope, you may not see that quark, but a quark with a fraction of that momentum because it has radiated a gluon that is moving with it. Or you may also see a quark–anti-quark pair. You will see more structure when you look with finer resolution, just like the atom became a structure and wasn't a point, and also the nucleus revealed its structure, and so forth, as the resolution increased. And as one goes from Q_1 to a higher $Q_2 > Q_1$, you may ask what is the quark momentum distribution in the proton that will be seen. In general one might expect that, for increasing Q, the quark momentum distribution—i.e., its distribution as a function of the Bjorken scaling variable x, which is the fraction of the $P \to \infty$ momentum of the proton that it carries—would be more highly concentrated at smaller x than for small Q values. This expresses the fact that at higher $Q_2 > Q_1$, one is more likely to be seeing a quark that has radiated a fraction of its momentum to a gluon, which it interacts with in the usual way of field theory. This effect is illustrated in Fig. 9.

The formal field theoretic analysis of the Q^2 dependence of the structure functions relies on the fact that deep inelastic scattering processes are dominated

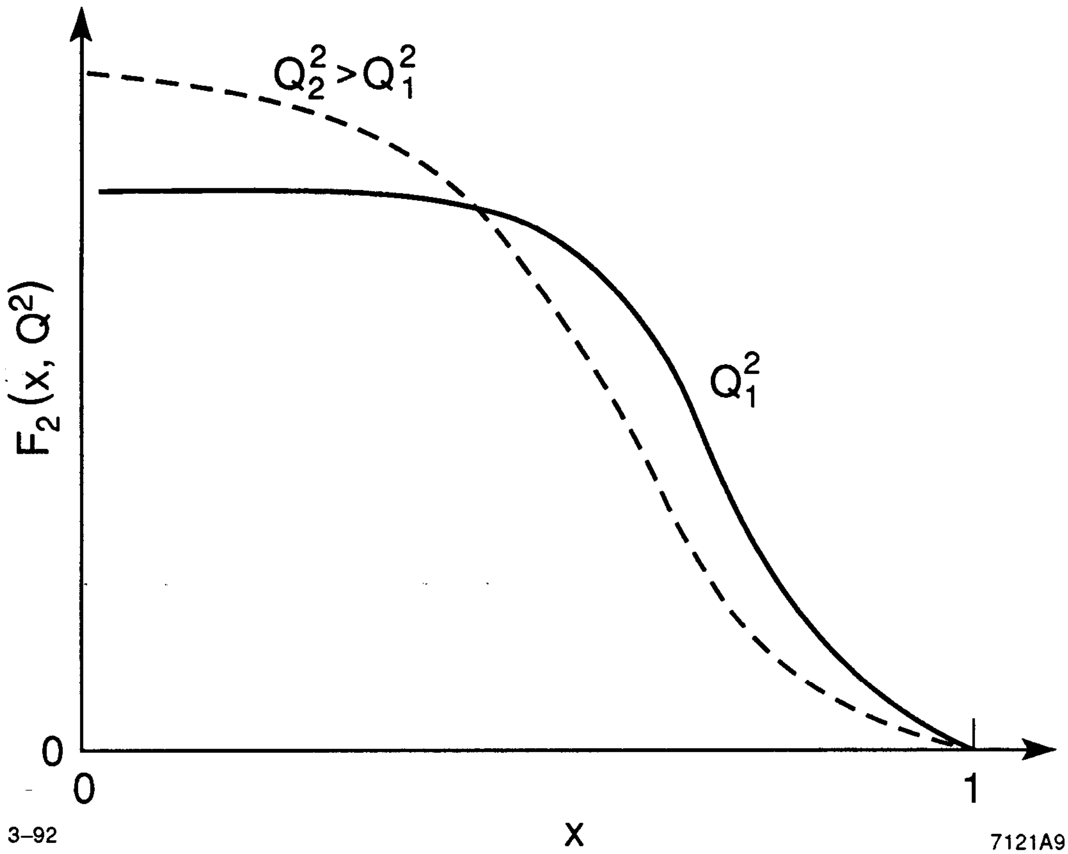

Fig. 9. Predicted deviation from scaling for increasing Q^2.

by contributions that come from near the light cone to the current commutator in Eq. (16)—i.e.,

$$\int e^{iq\cdot y}\, d^4y\ \langle \mathrm{P}\,|\,[\mathrm{J}_\mu(y), \mathrm{J}_\nu(0)]\,|\,\mathrm{P}\rangle\ . \tag{96}$$

In (96), causality restricts the invariant interval $y^2 = t^2 - \vec{y}^2 \geq 0$. Furthermore, we may expect that the dominant contributions to (96) come from the space-time region characterized by $q \cdot y \lesssim 1$, since there will be rapid oscillations in the integrand beyond this region. Recalling the definition of q^μ in the proton rest system[see Eq. (11)], this becomes

$$q \cdot y \ = \ \nu t - \sqrt{\nu^2 + Q^2}\, y_\parallel \lesssim 1\ . \tag{97}$$

Denoting by $y_\parallel$ and $y_\perp$ the components of $\vec{y}$ parallel and perpendicular to $\vec{q}$, respectively, and identifying $y_0 = t$, we find

$$t - y_\parallel \sim \frac{1}{\nu}\,, \quad y_\parallel \sim \frac{1}{Mx} \sim t\,, \quad y_\perp^2 \lesssim \frac{1}{Q^2}\,, \quad \text{and} \quad y_\mu y^\mu \sim \frac{1}{Q^2}\,, \tag{98}$$

which indicates that the dominant contribution to deep inelastic scattering comes from the region asymptotically close to the light cone. Therefore in a theory such as QCD that is asymptotically free, one expects a parton theory with scaling behavior to emerge for large Q^2.

As to the rate of approach to scaling, a formal, elegant formalism was developed in 1969 by K. Wilson to systematically study expansions about the light cone. A more intuitive and physical picture is given by that of the evolution equations, such as developed in the studies of the development of cosmic ray showers passing through matter. The generalization of this approach from Abelian QED to non-Abelian QCD for quarks and gluons was pioneered by Kogut and Susskind, and given an elegant formulation in terms of the master equation for evolution by Altarelli and Parisi in 1977 and by Gribov, Lipatov and collaborators. This approach will give us a very nice physical understanding of the origin of the Q^2 variation. As always for an iterative description of the interaction of quarks with gluons, we work in an infinite momentum frame with $\vec{\mathrm{P}} \to \infty$ as appropriate for the quark-parton model.

We define $q^i(x,\tau)$ to be the number density of quarks of type i; that is the probability that $(\text{quark})_i$, when probed by a current at log momentum τ has momentum fraction x. The τ is defined by

$$\tau \ \equiv \ \ell n\, \frac{Q^2}{Q_0^2}\,, \tag{99}$$

where Q_0 is an arbitrary reference scale of which physical results and predictions must be independent. Tau plays a role analogous to time in familiar time evolution problems, and henceforth, for simplicity, we shall call τ simply t. The t

dependence of $q^i(x,t)$ appears because as discussed above, what appears as a quark with x at a given t_1 may appear as a quark with only a fraction of that x accompanied by a gluon or quark pairs when viewed by a current at higher resolving power $t_2 > t_1$. Consider that a quark with x may radiate a gluon and retain a fraction $z < 1$ of its original momentum. Were there no gluon interactions, the probability density of finding a quark with a fraction $z < 1$ of momentum x would be zero, no matter what the resolving power t. Then the probability density of finding a quark would be

$$\mathcal{P}_{qq}^{(0)} = \delta(z-1) . \tag{100}$$

Figure 10 illustrates what is being described (in one-dimension, with neglect of small transverse momentum corrections). Figure 10a shows that in the absence of QCD gluon interactions the current "sees" a quark with momentum xP.

However, with gluon coupling, shown to lowest order in Fig. 10b, there is also probability density in momentum space to "see" a quark with momentum fraction z, and the amplitude for this is proportional to the running coupling constant of QCD, $\alpha_s(t)$. This contribution adds to Eq. (100) the probability density of finding a quark with fraction z when probed by a current with resolving power Δt about t. We have now

$$\mathcal{P}_{qq}^{(0)} + d\mathcal{P}_{qq}^{(0)} = \delta(z-1) + \frac{\alpha_s(t)}{2\pi} P_{qq}^{(0)}(z)\, dt , \tag{101}$$

where we have defined $[\alpha_s(t)/(2\pi)]\, P_{qq}(z)$ as the variation per unit t of the probability to find a quark with fraction z within the original one when probed at t. This introduces correspondingly the change in number density of quarks of type i

$$\begin{aligned} \frac{dq^i(x,t)}{dt} &= \frac{\alpha_s(t)}{2\pi} \int_0^1 dy \int_0^1 dz\, \delta(zy-x)\, P_{q^iq^i}(z)\, q^i(y,t) , \\ &= \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dy}{y}\, P_{q^iq^i}\left(\frac{x}{y}\right)\, q^i(y,t) . \end{aligned} \tag{102}$$

This simplest form for the evolution equation is incomplete since we must still include the contribution of gluons transiting back to quarks, and in non-Abelian theories, to gluon pairs; all three contributions shown in Fig. 11 must be

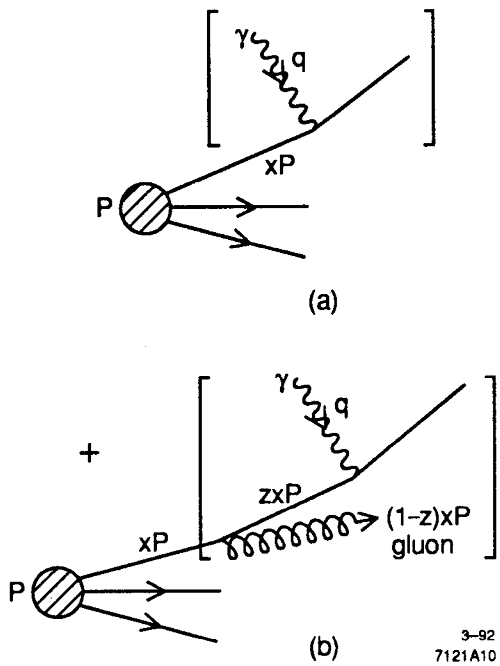

Fig. 10. The current with $\vec{q}$ sees a quark with momentum fraction x in 10(a) in the absence of gluon emission, and with a reduced fraction xz in 10(b) after gluon emission.

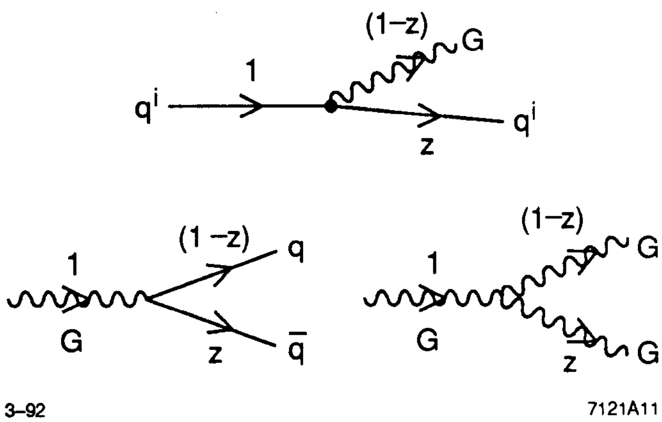

Fig. 11. The three elementary quark and gluon interactions with momentum sharing as indicated, in an infinite momentum frame.

included to order $\alpha_s(t)$. The general form of the evolution equation for i quark flavors plus gluons, as written by Altarelli and Parisi is

$$\frac{dq^i(x,t)}{dt} = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\left\{\sum_j P_{q^iq^j}\left(\frac{x}{y}\right)\ q^j(y,t) + P_{q^iG}\left(\frac{x}{y}\right)\ G(y,t)\right\}\ ,$$

$$\frac{dG(x,t)}{dt} = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y}\left\{\sum_j P_{Gq^j}\left(\frac{x}{y}\right)\ q^j(y,t) + P_{GG}\left(\frac{x}{y}\right)\ G(y,t)\right\}\ , \tag{103}$$

in an obvious notation. This system of equations is simplified by incorporating symmetry properties of QCD in the definitions of P. For example,

$$P_{q^iq^j} = \delta_{ij}\ P_{qq} \tag{104}$$

expresses the fact that the flavor does not change with gluon emission, and

$$\begin{aligned} P_{Gq^i} &= P_{Gq} \\ P_{q^iG} &= P_{qG} \end{aligned} \tag{105}$$

express the approximation of neglecting quark masses in calculating the probability of finding a gluon inside a quark (anti-quark) and vice versa. There are additional relations reflecting momentum conservation and the conservation of the difference in numbers of quarks and anti-quarks; i.e., they are only produced or annihilated in pairs.

Continuing the effort to uncover the physics without being buried in algebra, let us consider the evolution of the quark–anti-quark difference, defined as the nonsinglet number density, that is diagonal in flavor by (104) and (105) and can be written

$$\frac{d}{dt}\ \{q(x,t) - \bar{q}(x,t)\} = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y} P_{qq}\left(\frac{x}{y}\right)\ \{q(y,t) - \bar{q}(y,t)\}\ . \tag{106}$$

This result should look familiar to those of you who study the development of high-energy cosmic ray showers passing through matter. An important difference here, as we shall see shortly, is the running of the coupling constant in QCD, which can be neglected in practical applications of the evolution equation to shower development in QED. In the lowest order equivalent photon approximation in QED, Eq. (102) is valid.

For experimental analysis it is most convenient to separate the dependence of the matrix elements on momentum transfer t from the x variation of the density distributions. To do this in (106), one simply takes a Mellin transformation to form the moments of the distributions ($q^{NS} \equiv q - \bar{q}$)

$$M_n^{NS}(t) \equiv \int_0^1 dx\, x^{n-1}\, q^{NS}(x,t)\ . \tag{107}$$

For the nth moment as defined, (106) becomes

$$\frac{d}{dt} M_n^{NS}(t) = \frac{\alpha_s(t)}{2\pi} \int_0^1 dy \int_0^1 dz \underbrace{\int_0^1 dx\, x^{n-1}\, \delta(yz - x)}_{(yz)^{n-1}}\ P_{qq}(z)\, q^{NS}(y,t)\ ,$$

$$= \frac{\alpha_s(t)}{2\pi} M_n^{NS}(t) \underbrace{\left\{ \int_0^1 dz\, z^{n-1}\, P_{qq}(z) \right\}}_{\text{Physics input}} \equiv \frac{\alpha_s(t)}{2\pi} A_n^{NS}\, M_n^{NS}(t)\ , \tag{108}$$

which expresses the t variation of $M_n(t)$ in terms of the physics in A_n^{NS} and the running of $\alpha_s(t)$. If we introduce the leading log approximation to $\alpha_s(t)$,

$$\alpha_s(t) = \frac{\alpha_s}{1 + b\alpha_s t}\ , \tag{109}$$

we can integrate (108) to give the t dependence of the nth moment.

$$M_n^{NS}(t) = M_n^{NS}(0)\, (1 + b\alpha_s t)^{A_n^{NS}(t)/2\pi b}\ . \tag{110}$$

This displays the logarithmic dependence of the matrix elements in Q^2 as claimed earlier. Physics lies in the power of the log as well as the slope b, which reflects the symmetry properties—i.e., number of colors and flavors via its definition

$$b = \frac{11N - 2f}{12\pi}\ , \tag{111}$$

where N is the number of colors [= 3 for SU(3) of color], and f is the number of flavors (= 3 for three generations).

Note that if the coupling constant didn't run, i.e., $b \to 0$ in (109) and (110), the moments would vary with a fractional power of Q^2 rather than of its log

$$\frac{M_n^{NS}(t)}{M_n^{NS}(0)} \underset{(b\to 0)}{\Longrightarrow} \left(\frac{Q^2}{Q_0^2}\right)^{\frac{\alpha_s}{2\pi} A_n^{NS}}\ . \tag{112}$$

Detecting such a difference is a difficult but not impossible challenge for an experimentalist. Figure 12 shows what is required for typical numbers for the

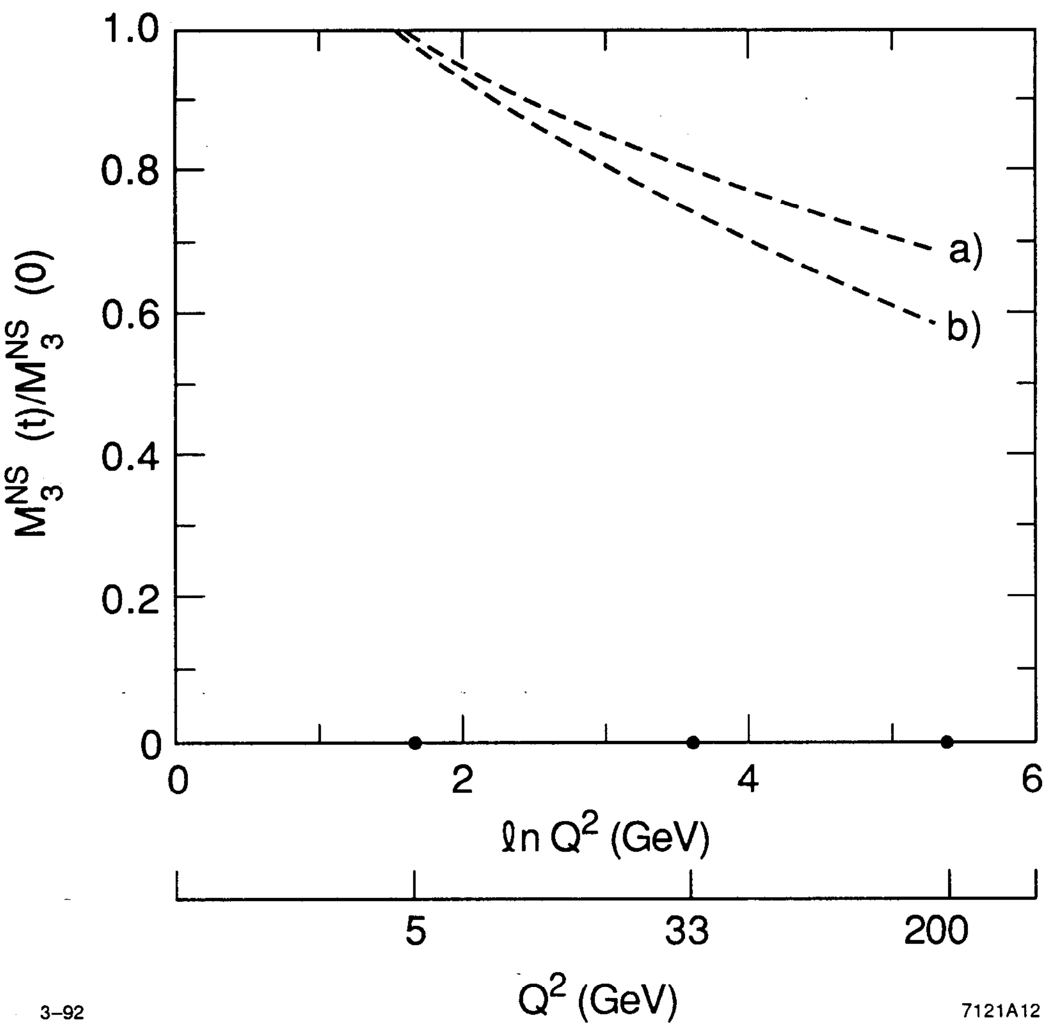

Fig. 12. Schematic representation of the difference in Q^2 variation of the third moment $M_3^{NS}(t)$ depending on whether the coupling constant runs (a) [Eq. (110)], or is constant (b) [Eq. (112)].

third moment to distinguish these two behaviors. Since the different moments are related to one another through A_n and $\alpha_s(t)$, their t variation gives information about the physics. One can calculate the A_n using the simple QCD vertices in the infinite momentum frame and thereby predict the t, or log Q^2 variation for experimental testing. For example the A_n for gluons of spin-0 differ from the values for spin-1, and accurate data confirms their spin-1 character as gauge bosons. The calcuations of A_n, while not trival, are straightforward. One calculates $P_{qq}(z)$ in the $\rightarrow \infty$ frame. This is the first graph in Fig. 11, and one finds

$$\begin{aligned} A_n^{NS} &\equiv \int_0^1 dz\ z^{n-1}\ P_{qq}(z) \\ &= C_2(R)\left[-\frac{1}{2} + \frac{1}{n(n+1)} - 2\sum_{j=2}^{n}\frac{1}{j}\right] , \end{aligned} \tag{113}$$

for $n > 1$ ($A_1^{NS} = 0$), where the color Casimir $C_2(R)$ is given by

$$C_2(R) = \frac{1}{N}\sum_a t^a t^a = \frac{N^2-1}{2N} = \frac{4}{3} \text{ for SU(3) color .} \tag{114}$$

For detailed calculations see the treatises by R. Field and C. Quigg.

This concludes the discussion of the logarithmic approach to Bjorken scaling—and I am just about out of time.

There are a number of things that I haven't talked about that are very important; in particular model building; the transparency of nuclear matter with atomic number $A > 1$; the limiting behavior for $x \rightarrow 0$ and tieing in with Reggie theory. There is a lot of very good technology being developed here and it is very important technology. Ask Stan Brodsky while you're here at the school, because he knows so much about the nucleon wave functions that are constructed to interpret the vast body of data that cannot be summarized into Sum Rules. I have slighted this work not because it is not important. Simply, I chose to concentrate my two hours on the approach to scaling and on sum rules that I consider so basic . I really hope that the Bjorken sum rule will be proved wrong. That would cause quite a stir!

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