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Review of Lattice Measurement Techniques at the SLC*

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Abstract
A technique is described for reconstructing the first order transport matrix (R) for a given beam line. Emphasis is placed on the rigorous error analysis of the data, and the use of powerful statistical techniques to estimate unknown systematic errors. The application of the technique to the measurement and subsequent correction of the SLC Arcs is briefly described.

1.0 Introduction.

In circular machines, the cyclic boundary conditions which exist uniquely define the properties of the beam: Twiss parameters become properties of the lattice. In linear (single pass) colliders, however, the beam phase space is not uniquely determined by the lattice, but is also a function of the initial conditions. Determination of the lattice properties in the SLC are generally performed by giving the beam centroid a small kick, and observing over many pulses the resulting disturbance downstream. In the case of the linear transport (**R**) matrix reconstruction described in this paper, the small kicks are angular kicks caused by small corrector dipoles, and the resulting downstream betatron oscillation the disturbance. The kick could quite have easily been a small energy kick from an accelerating cavity, in which case the downstream orbit is a measurement of the dispersion. In both cases, it is important to note that it is the lattice properties (**R** matrix) from the kick that is measured, not the beam properties.

In the following description of the **R** matrix reconstruction techniques that were used in conjunction with the SLC Arcs, emphasis is placed on the rigorous error analysis that was adopted. An understanding of the statistical nature of the measurements allows for a better choice of correction algorithm; this was the essence of the coupling corrections applied to the SLC Arcs.

2.0 Reconstruction of the 4×4 linear R matrix

2.1 Basic technique.

The first order transport of a given beam line can be measured using a technique developed by Barklow[1]. The technique relies on making small amplitude betatron oscillations using dipole correctors, and measuring the resulting orbit using downstream beam position monitors (BPM). In order to reconstruct the 4×4 **R** matrix a minimum of two betatron phases are required, ideally separated by $\pi/2$. In addition to the BPM of interest, an additional BPM (again ideally $\pi/2$ in phase away from the first) is required from which the beam angle can also be determined. Thus four correctors and four BPM measurements give the necessary minimum of 16 measurements to completely determine the **R** matrix to a given BPM. However, to gain some statistical redundancy, it is better to generate more oscillations at different phases. In the case of the SLC Arcs, a total of four correctors in each plane where used, separated by approximately $\pi/4$ in phase. Now the **R** matrix reconstruction becomes over constrained, and least squares regression techniques

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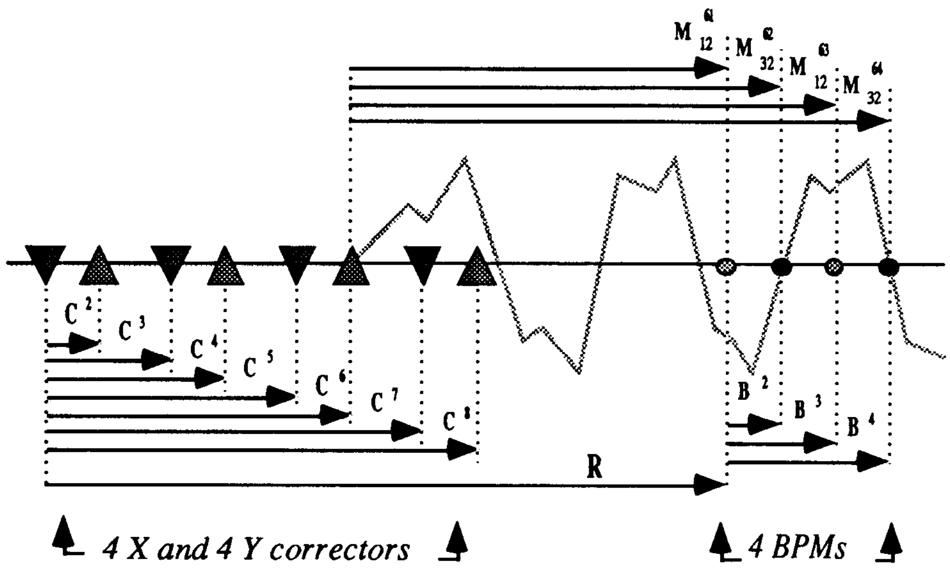

Figure 1. Schematic representation of **R** matrix reconstruction technique.

are employed.

The problem now becomes one of minimizing a χ^2 sum. For each BPM in a given plane, the raw measurement is the gradient of the beam displacement at that monitor as a function of the corrector kick: We will refer to these values as M_{ij}^{pq}, where the subscript i=1 for an X BPM, and i=3 for a Y BPM, j=2 for an X corrector and j=4 for a Y corrector (following the TRANSPORT[2] convention), and p and q index the corrector (p=1,...,8) and BPM number (q=1,...,4) respectively (figure 1). In the previous introductory discussion, it was inferred that each physical BPM could read both X and Y displacements; this is not a prerequisite however, and indeed in the Arcs the BPMs alternate X and Y.

Assuming the linear matrices between the correctors to be $\mathbf{C}^q$, and those between the BPMs to be $\mathbf{B}^p$, as depicted in figure 1, then the M_{ij}^{pq} are given by

$$M_{ij}^{pq} = \left(\mathbf{B}^p\mathbf{R}(\mathbf{C}^q)^{-1}\right)_{ij} \qquad (1)$$

where **R** is the matrix of interest. The required 16 elements of the **R** matrix can be found by minimizing the following χ^2 sum:

$$\chi^2 = \sum_{ijpq}\left(\frac{M_{ij}^{pq} - \left(\mathbf{B}^p\mathbf{R}(\mathbf{C}^q)^{-1}\right)_{ij}}{\delta M_{ij}^{pq}}\right)^2 \qquad (2)$$

where δM_{ij}^{pq} are the raw measurement errors (section 2.3). When measuring the transport of the SLC Arcs, a total of 8×4=32 measurements of the M_{ij}^{pq} elements where taken, giving 32-16=16 degrees of freedom. However, there are additional constraints due to the fact that the resulting **R** matrix must be *symplectic*.

2.2 Symplectic constraints

The symplectic constraint is given by

$$\mathbf{R}^T \mathbf{S}\ \mathbf{R} = \mathbf{S} \tag{3}$$

where **S** is the matrix

$$\mathbf{S} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \tag{4}$$

Equation 3 generates 6 non-linear constraints on the sixteen elements for the **R** matrix (hence there are only ten independent elements). The reconstructed R matrix is forced to be symplectic by minimizing the χ^2 sum given by (2) subject to the 6 symplectic constraints (3). By doing so, we not only guarantee that the resulting **R** matrix be symplectic, but we also increase the number of degrees of freedom from 16 to 22, thus gaining additional statistical redundancy. The complexity of the computation now increases, however, since the problem becomes one of non-linear regression, requiring specialized numerical algorithms and additional CPU time.

2.3 Initial analysis of M_{ij}^{pq} terms.

For each corrector, the corresponding corrector strength is stepped over a number of increments, and the BPM displacements recorded. The required M_{ij}^{pq} and δM_{ij}^{pq} are then obtained by a linear fit to the BPM data as a function of corrector kick. Experience has shown that the reconstructed **R** matrix is sensitive to the initial errors of these slopes: In particularly, systematic errors, if uncorrected, can seriously corrupt the final result. When obtaining the M_{ij}^{pq}, there are two main sources of systematic errors: i) Unknown sources of betatron oscillations upstream of the corrector of interest and ii) energy fluctuations. The latter is particular damaging with respect to the SLC Arcs since there is nominally dispersion present at each BPM. Simulations have shown that even small energy fluctuations (<0.05%) can seriously corrupt the M_{ij}^{pq} if they are not taken into account. In the case of the SLC Arcs, it was possible to fit out the effects of the energy fluctuations, since taking the average of the all 240 X BPMs in the Arc gave an accurate measurement of the relative energy error $\Delta E/E$ (when differenced with respect to some reference). Thus instead of the original *straight line* (2 parameter) fit, we now have a three parameter fit of the form:

$$X_q = (M_{ij}^{pq})\theta_p + \eta_q\left(\frac{\Delta E}{E}\right) + \text{const.} \tag{5}$$

where M_{ij}^{pq} and η_p are the dependent parameters to be determined. Figure 2 shows the χ^2 distributions for the two parameter and three parameter fits. The three parameter fit exhibits a much more well behaved χ^2 distribution, with the mean being equal to the number of degrees of freedom of the fit (in this case 1).

The effect of upstream betatron oscillations has been to a large extent solved by the addition of a feedback system at the end of the SLC LINAC. Fast (<1 Hz) jitter can be removed by averaging. There still remain unknown systematic errors (such as BPM and corrector calibrations), but their effect can be estimated by the techniques described in the following section.

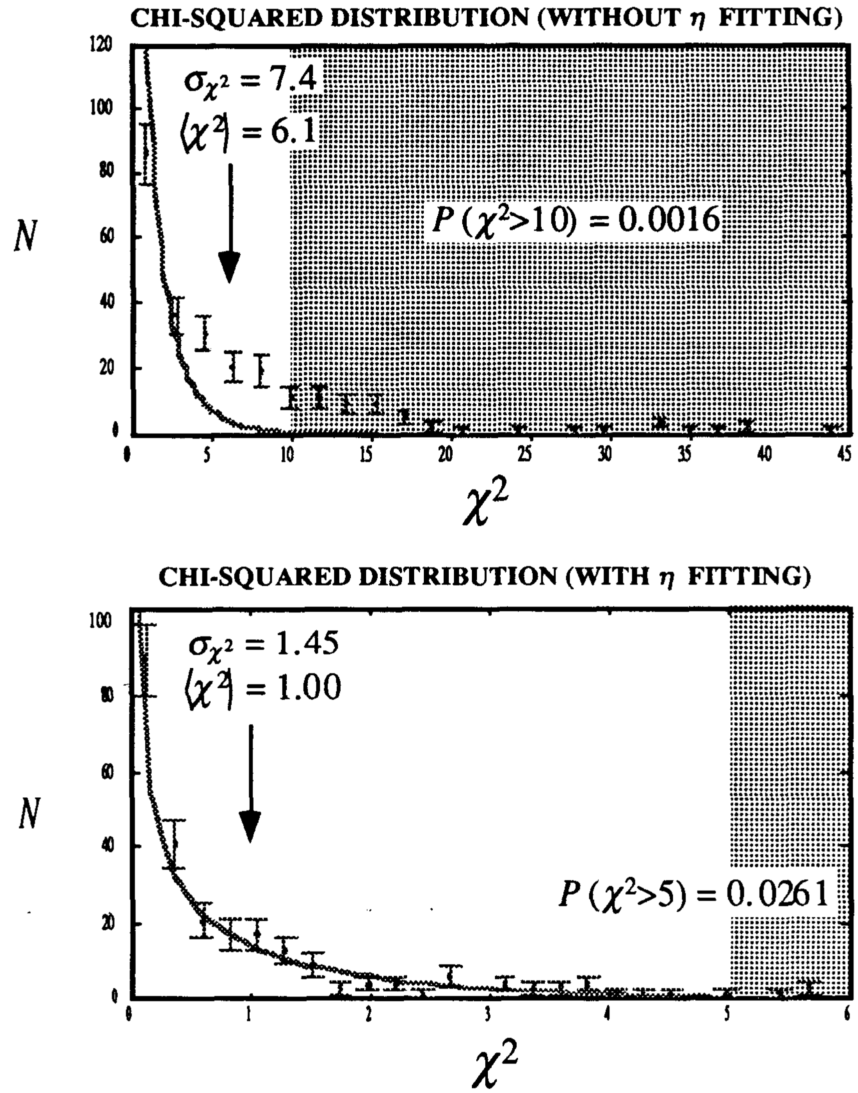

Figure 2. χ^2 distribution for the initial M_{ij}^{pq} fit, with and without fitting for the energy fluctuations. The shaded areas indicate the given probability P of a χ^2 being greater than the indicated value.

2.4 The χ^2 distribution: Estimation of systematic errors.

In the previous section it was mentioned that the distribution of the χ^2 obtained from several fits of the M_{ij}^{pq} to the raw BPM data did not exhibit the correct behavior; another way of saying this is that the model for the fit was incorrect (due to the non-inclusion of the dispersion term). For purely random errors (noise) the χ^2 distribution has a well defined form, with the mean equal to the number of degrees of freedom (N_f), and the variance equal to $2N_f$. For the **R** matrix reconstruction, we assume a global equivalent random error which represents the unknown systematics. Defining the global error as σ_{sys}, the χ^2 sum in equation 2 becomes

$$\chi^2 = \sum_{ijpq} \frac{\left(M_{ij}{}^{pq} - \left(\mathbf{B}^p\mathbf{R}(\mathbf{C}^q)^{-1}\right)_{ij}\right)^2}{\left(\delta M_{ij}{}^{pq}\right)^2 + \sigma_{sys}^2} \qquad (6)$$

For the SLC Arcs, 460 **R** matrices were reconstructed from essentially the same data set. By adjusting σ_{sys}, the mean of the χ^2 distribution formed by all 460 reconstructions was forced to be the number of degrees of freedom for the fit (22). In most cases, the value of σ_{sys} was comparable to the individual δM_{ij}^{pq} (~0.1 m). Although this rescaling of the errors does not in general alter the actual resulting elements of the **R** matrices, it does affect the associated error covariance matrix; this is important when considering correction algorithms. One other benefit of looking at the χ^2 distribution is to be able to reject BPMs at which the reconstructions are statistically bad: Again this is important when deciding which measurements to use for tuning purposes.

3.0 Application of R matrix measurements to the SLC Arcs

The application of the **R** matrix reconstruction techniques described in section 2 to the SLC Arcs, and the use of those measurements in the subsequent first order corrections is documented elsewhere[3]. However, to emphasize the importance of rigorous error propagation to the tuning algorithms, we will repeat the essentials here.

3.1 Correction of cross plane coupling.

To facilitate terrain following, the SLC Arcs contain vertical steering which is introduced via matched roll pairs, separated by a phase advance of $6n\pi$. In this way, the cross plane coupling introduced at the first roll is cancelled by the second opposite roll. However during early commissioning it was discovered that optical errors within the achromat sections caused uncorrectable anomalous coupling, leading to large projected emittance growth in both planes[4]. It was soon realized after several attempts to tune out the coupling that it was necessary to correct the entire 4×4 **R** matrix. There were two predominant sources of the anomalous coupling: (i) phase errors between matched rolls, resulting in uncancelled coupling and (ii) anomalous skew quadrupoles within achromats. Both the phase (quad) errors and skew-quad errors were considered to accrue from systematic misalignments of the Arc mixed function dipoles, all of which contain a strong sextupole component to correct the second order optics. Since the Arc magnets run on a common bus bar, there is no freedom in adjusting the various multipole strengths individually. Hence, closed orbit bumps were used to introduce controlled amounts of normal and skew-quadrupole, again by virtue of the sextupole field present in the magnets.

Ideally, to correct the 10 independent **R** matrix elements for a given section of beam line requires 10 independent orthogonal adjustments. In the case of the Arcs, the five possible closed bumps (per plane) available in any given achromat were very close to being degenerate in terms of their perturbative effect on the optics. In order to surmount this problem it was necessary to parameterize the optical anomaly of interest, and reduce the magnitude of this parameter with a single closed bump. To this end, two χ^2 figures of merit were defined.

3.2 Definition of χ^2_c and $\chi^2_{\alpha\beta}$.

The phase errors (in-plane optics) and local coupling were dealt with separately. A single horizontal bump, which introduced a normal quadrupole component to the beam line, was used to adjust the in-plane optics, while a vertical bump (skew-quadrupole) was used to adjust the coupling. Correction was achieved by the minimization of two χ^2 sums. Defining the **R** matrix thus

$$\mathbf{R} = \begin{pmatrix} \mathbf{A}_{2\times2} & \mathbf{B}_{2\times2} \\ \mathbf{C}_{2\times2} & \mathbf{D}_{2\times2} \end{pmatrix} \tag{7}$$

where **A**, **B**, **C** and **D** are 2×2 submatrices, the χ^2 sum for the coupling correction, $\chi^2{}_c$, is defined as:

$$\chi^2{}_c = \sum_{ij} (C_i - C_i^d)(V_R)^{-1}_{ij}(C_j - C_j^d) \tag{8}$$

where C_i cycles over the four independent elements of the C submatrix, the superscript *d* referring to the design (required) values. It is important to note that the more general from of the χ^2 sum with the full covariance matrix $\mathbf{V_R}$ is used. Although the four elements of the **C** matrix are independent, their *errors* will be coupled (by virtue of the reconstruction technique), and this coupling must be taken into account when forming a figure of merit of this type. Failure to do so can lead to an adjustment which will over or under correct the error. For the in plane optics, a χ^2 is defined over the remaining 6 independent **R** matrix elements in the **A** and **D** submatrices. In this case, however, the design Twiss parameters are propagated through the beam line of interest (an achromat), and the χ^2 is formed over the difference between the design Twiss parameters, and those propagated values. Thus we define $\chi^2{}_{\alpha\beta}$ as

$$\chi^2{}_{\alpha\beta} = \sum_{ij} (\zeta_i - \zeta_i^d)(V_\zeta)^{-1}_{ij}(\zeta_j - \zeta_j^d) \tag{9}$$

where ζ_i cycles over $(\beta_x,\alpha_x,\beta_y,\alpha_y)$, $\mathbf{V}_\zeta$ is the associated 4×4 covariance matrix, and the superscript *d* refers to design values as before. $\mathbf{V}_\zeta$ is determined directly from $\mathbf{V_R}$ by

$$\mathbf{V}_\zeta = \mathbf{J}\,\mathbf{V_R}\,\mathbf{J}^T \tag{10}$$

J being the 4×10 Jacobian of the form $\partial\zeta_i/\partial R_{jk}$.

The amplitudes of the required closed bumps were determined by treating the effect of the bump on the optics as a linear perturbation. The differential effect of each possible bump on the **R** matrix was calculated using the code DIMAD[5], and another computer code selected the best bump out of the five possible to minimize the required χ^2. Details of the implementation of the bumps are given in reference 3.

3.4 Some results.

Historically, the coupling in the SLC Arcs has been characterized by the determinant of the C submatrix (det C) although in practise it is not the quantity that is tuned on. Figure 3 shows the north Arc det C as a function of BPM unit number before and after the application of the correction algorithm. Figure 4 shows histograms of the $\chi^2{}_c$ for each achromat before and after the application of vertical bumps. It is interesting to note that the careful error analysis allowed us to determine (in conjunction with montecarlo simulations of magnet misalignments using DIMAD) that the measurements were precise enough to resolve 100μm rms random magnet misalignments, the original design alignment tolerances for the SLC Arcs.

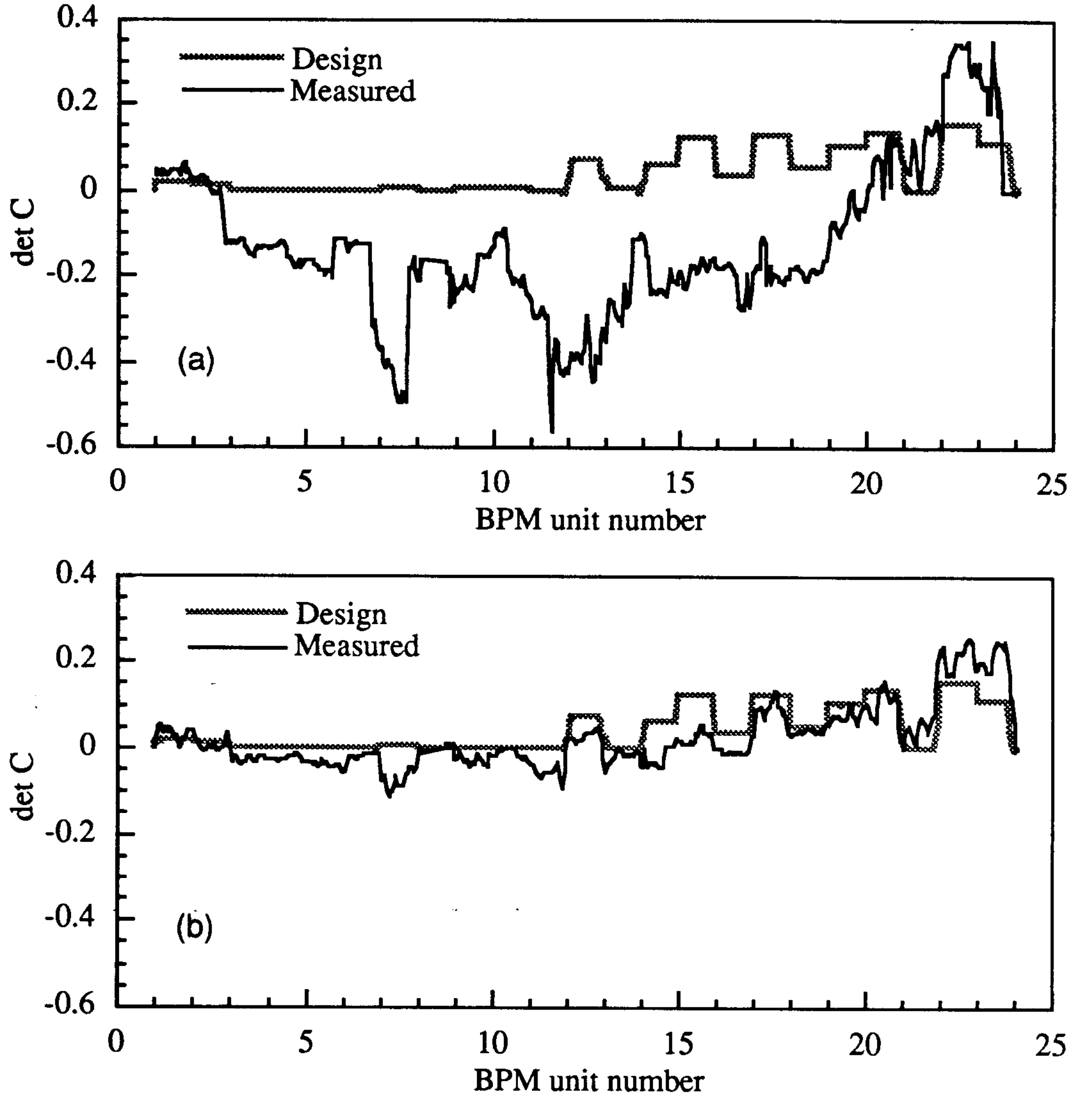

Figure 3. Det C for the north SLC Arc (a) before correction and (b) after correction.

4.0 Extension to non-linear behavior.

So far we have only described a technique for reconstructing the linear 4×4 **R** matrix. It is possible however, to extend the technique to second (or even higher) order. In the initial determination of the M_{ij}^{pq} elements, small amplitude oscillations were used (section 2). By increasing the amplitude of the measurements significantly, it is possible to observe non-linear behavior. Now two parameters are required from the initial fit: the initial slope (M_{ij}^{pq}) and the quadratic parameter (W_{ij}^{pq}). For the SLC Arcs, such a fit would take the form:

$$X_q = \left(M_{ij}^{pq}\right)\theta_p + \left(W_{ij}^{pq}\right)\theta_p^2 + \eta_q\left(\frac{\Delta E}{E}\right) + \text{const.} \tag{11}$$

Clearly there are four parameters to be determined, requiring at least 4 data points. The coefficients M_{ij}^{pq} are the same as those previously defined, and can be used in exactly the same manor to recon-

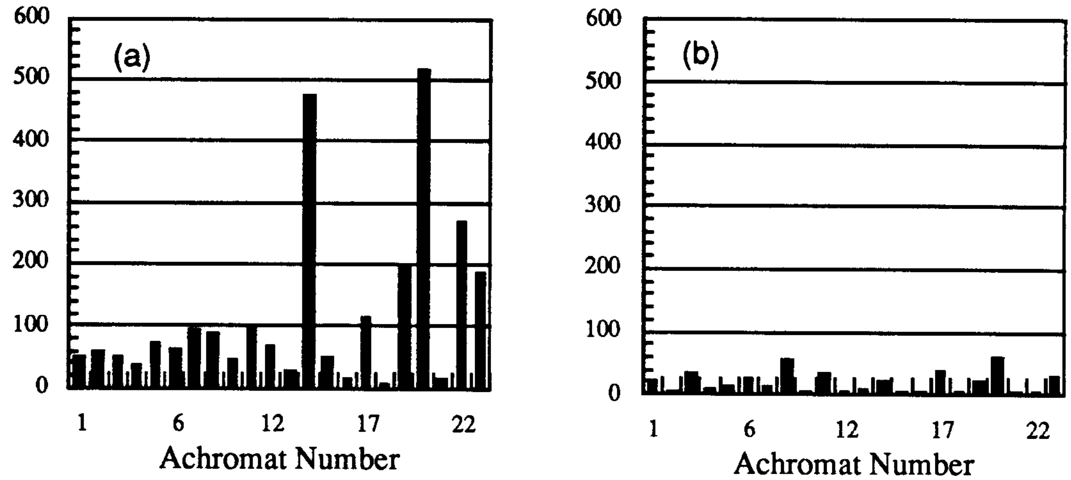

Figure 4. χ^2_c per achromat number for the north SLC Arc (a) before any correction and (b) after application of vertical closed bumps.

struct the **R** matrix. The quadratic coefficients (W_{ij}^{pq}) can then be used to reconstruct the second order Taylor expansion coefficients T_{ijk} (as defined by Brown[2]). The exact details of the reconstruction are too complex for this review, however it is important to note that exactly the same techniques described for the linear reconstruction are directly applicable to the second order calculations, including a set of symplectic constraints on the resulting T_{ijk} terms. At the present time, such a reconstruction of the second order terms has not been attempted, although large amplitude oscillation data for the SLC Arcs has been recorded, and it is our intention to apply these techniques to those data.

5.0 References.

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