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Equivalence of Z_N orbifolds and Landau–Ginzburg models*

E. J. Chun
Department of Physics and
Center for Theoretical Physics
Seoul National University, Seoul 151–742, Korea

J. Lauer
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309, USA

H. P. Nilles
Physik Department
Technische Universität München
D-8046 Garching
and
Max-Planck-Institut für Physik und Astrophysik
— Werner-Heisenberg-Institut für Physik —
P.O. Box 401212, D-8000 München (Germany)

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Abstract

We show that Landau–Ginzburg models with modality one are equivalent to Z_N orbifold models on two dimensional tori. The identification of the primary field content is given and the operator product coefficients of the twisted sectors are explicitly computed.

1. Introduction

The singularities of a $N=2$ Landau–Ginzburg (LG) theory are classified according to the form of the superpotential of a two-dimensional $N=2$ superconformal field theory (SCFT) [1, 2]. An $N=2$ LG theory with modality one and conformal anomaly $c=3$ is described by either one of the following superpotentials:

$$\begin{aligned}&\Phi_1^3+\Phi_2^3+\Phi_3^3+a\Phi_1\Phi_2\Phi_3\\ &\Phi_1^4+\Phi_2^4+a\Phi_1^2\Phi_2^2\\ &\Phi_1^3+\Phi_2^6+a\Phi_1\Phi_2^4\end{aligned}\tag{1}$$

It has been argued that those LG theories correspond to the conformal field theory (CFT) on Z_3, Z_4, Z_6 orbifolds, respectively, relying on the path-integral argumentation [2]. In addition, it has been possible to relate the modal parameter a to the complex Kähler modulus of the orbifold (given in terms of a radius and an axionic scale parameter B) and thereby to derive a duality transformation in the space of LG theories [3, 4]. At $a=0$ the above superpotential was shown to define the (super–) orbifolds on the $SU(3)$, $SU(2)\times SU(2)$, and $SU(3)$ root lattices, respectively; in addition one finds that the symmetries of the corresponding LG theories are enlarged. Because of the constraint $c=3$ there are just three possibilities to obtain them, namely (1^3), (2^2), and $(1,4)$ where the entries denote the A–series building blocks in a tensor product. At $a=0$, the orbifold theory possesses further $U(1)$ symmetries which gives rise to a rational CFT.

In this paper, we will prove that the above three tensor models are equivalent to a Z_N orbifold CFT ($N\in\{3,4,6\}$) at precisely those points where the symmetry group is enhanced. Once this is demonstrated, the equivalence for arbitrary values of the modulus follows by applying marginal deformations. The exact operator relation between the modulus of the $SU(3)$ orbifold and that of the (1^3) LG theory was already found in [5]. To prove that two CFTs are equivalent, one has to show (i) that their chiral algebras are indistinguishable, (ii) that the primary fields with respect to the chiral algebras are in one-to-one correspondence, and (iii) that the operator product expansions (OPE) involving primary fields are in agreement. For simplicity we establish these properties for the NS sector of both theories. The R sector can be treated similarly. We first have to diagonalize the primary fields of an orbifold CFT with respect to the enhanced $U(1)$ symmetries. This step is necessary because the tensor model is naturally described in the '$U(1)$-diagonal basis' while the preferred construction of an orbifold starts from the conjugacy classes of the space group thereby endowing the ground states of the twisted sectors with specific global monodromy properties. This relation between the geometric and the algebraic construction gives thus rise to an equivalence of orbifold (Calabi–Yau) compactifactions and Gepner–type constructions [6, 7].

2. The primary fields of Z_N orbifold constructions

Z_N (super–) orbifold models on a two–dimensional target space have an $N = 2$ superconformal algebra (SCA) with the fermion number operator providing a $U(1)$ charge:

$$\begin{aligned} T(z) &= -\tfrac{1}{2}\partial X^i \partial X^i - \tfrac{1}{2}\psi^i \partial \psi^i \\ G^\pm(z) &= i\psi^\pm \partial X^\mp \\ Q(z) &= i\partial B \end{aligned} \tag{2}$$

The collection of $N = 2$ super-Virasoro primary fields in the untwisted sector consists of the NS fermion

$$\psi^\pm \equiv e^{\pm iB} \tag{3}$$

which is (anti-)chiral with $(h, Q) = (\frac{1}{2}, \pm 1)$ and an infinite number of bosonic vertex operators which are Z_N–invariant:

$$\frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} V_{\theta^n \vec{p}_L, \theta^n \vec{p}_R} \tag{4}$$

where θ generates the point group. The Narain momenta are $\vec{p}_{L,R} = \vec{e}^{*i}(p_{L,R})_i$ where $(p_{L,R})_i = \frac{1}{2}m_i + (\pm g - b)_{ij} n^j$ in case of a torus compactification with period vectors $\vec{e}_1, \vec{e}_2$ ($\vec{e}^{*1}$, $\vec{e}^{*2}$ span the associated dual basis, $g_{ij} = \vec{e}_i \cdot \vec{e}_j$ and m_i, n^j denote the integer momentum and winding (quantum) numbers). We have furthermore allowed for a constant antisymmetric torsion background $B_{\mu\nu}$ (in the lattice basis its components read $b_{ij} = e_i^\mu B_{\mu\nu} e_j^\nu$).

Conversely, the momentum $\vec{p}$ and winding $\vec{v}$ are then read off from $\vec{p} = \vec{e}_i[(p_L^i + p_R^i) + b_j^i(p_L^j - p_R^j)]$, $\vec{v} = \frac{1}{2}\vec{e}_i(p_L^i - p_R^i)$. In the s–th twisted sector ground state fields are chiral primary with $(h, Q) = (\frac{1}{2}\frac{s}{N}, \frac{s}{N})$

$$\Sigma^{(\frac{s}{N})} = e^{i\frac{s}{N}B} \sigma^{(\frac{s}{N})} \tag{5}$$

This ensues from the OPE involving the bosonic twist field $\sigma^{(\frac{s}{N})}$ and $\partial X^{\pm} = (\partial X^1 \pm i\,\partial X^2)/\sqrt{2}$:

$$
\begin{aligned}
\partial X^{+}(z)\sigma^{(\frac{s}{N})}(0) &\sim z^{-(1-\frac{s}{N})}\tau^{(\frac{s}{N})}(0) \quad + \cdots \\
\partial X^{-}(z)\sigma^{(\frac{s}{N})}(0) &\sim z^{-\frac{s}{N}}\tau'^{(\frac{s}{N})}(0) \qquad + \cdots
\end{aligned}
\tag{6}
$$

Furthermore, at the special values of Z_N orbifold background moduli space, the chiral algebra (2) enlarges so as to contain the $U(1)$ currents of conformal dimension $(1,0)$ (In most formulas yet to come we will omit the right-moving part):

$$
\begin{aligned}
J_L^{\pm}(z) &= \tfrac{i}{\sqrt{3}}\sum_{k=1}^{3} V_{\mp\vec{\alpha}_k,0}(z) \\
J_L(z) &= \tfrac{i}{\sqrt{4}}\sum_{k=1}^{2}[V_{\vec{\beta}_k,0} + V_{-\vec{\beta}_k,0}](z) \\
J_L(z) &= \tfrac{i}{\sqrt{6}}\sum_{k=1}^{3}[V_{\vec{\alpha}_k,0} + V_{-\vec{\alpha}_k,0}](z)
\end{aligned}
\tag{7}
$$

for the Z_3, Z_4 and Z_6 orbifold, respectively. Here $\vec{\alpha}_1 = (\sqrt{2},0)$, $\vec{\alpha}_2 = (-\sqrt{\frac{1}{2}}, \sqrt{\frac{3}{2}})$, $\vec{\alpha}_3 = -(\vec{\alpha}_1 + \vec{\alpha}_2)$ are the positive roots of the $SU(3)$ Lie-algebra whereas $\vec{\beta}_1 = (\sqrt{2},0)$, $\vec{\beta}_2 = (0,\sqrt{2})$ are the positive roots for $SU(2)\times SU(2)$. Note that there are two (one) additional $U(1)$ symmetries for a Z_3 (Z_4, Z_6) orbifold. We can now find the finite number of primary fields in the presence of these $U(1)$ symmetries. The $U(1)$–primary fields have the OPE $J(z)V(0,0) = \frac{q}{z}V(0,0) + \{\text{regular terms}\}$ with the $U(1)$ current (q denotes the charge of the field V). The Z_N–invariant primary fields in (4) do not yet have a definite $U(1)$ charge. But it is possible to find the $U(1)$ basis as soon as the above OPE has been calculated. To this end we have to take into account the cocycle operator associated to $V_{\vec{p}_L,\vec{p}_R}$ (for clarity zero mode operators will carry a hat accent) [8]:

$$
V_{\vec{p}_L,\vec{p}_R}(z,\bar{z}) = e^{i\vec{p}_L\cdot\vec{X}_L + i\vec{p}_R\cdot\vec{X}_R} e^{i\pi\vec{v}\cdot\hat{p}}
\tag{8}
$$

Consequently, the OPE between two vertex operators picks up a phase

$$
\begin{aligned}
&V_{\vec{p}_L,\vec{p}_R}(z,\bar{z})V_{\vec{p}'_L,\vec{p}'_R}(0,0) = e^{i\pi\vec{v}\cdot\vec{p}'} z^{\vec{p}_L\cdot\vec{p}'_L}\bar{z}^{\vec{p}_R\cdot\vec{p}'_R} V_{\vec{p}_L+\vec{p}'_L,\vec{p}_R+\vec{p}'_R}(0,0) + \ldots \\
&\vec{v}\cdot\vec{p}' = \tfrac{1}{2}(p_L^i - p_R^i)g_{ij}(p_L'^j + p_R'^j) + \tfrac{1}{2}(p_L^i - p_R^i)b_{ij}(p_L'^j - p_R'^j)
\end{aligned}
\tag{9}
$$

This expansion enables us to find the $U(1)$ diagonal fields and their charges for the untwisted sector of Z_N orbifolds. We proceed case by case.

Z_3 orbifold: The Narain momentum $(\vec{p}_L, \vec{p}_R)$ of a $SU(3)$ torus model at the point of enhanced symmetry belongs to either one of the three conjugacy classes $(\Gamma_\epsilon, \Gamma_\epsilon)$, $\epsilon \in \{0, \pm 1\}$ where

$$\Gamma_\epsilon = \{\epsilon \vec{w}_1 + n_1 \vec{\alpha}_1 + n_2 \vec{\alpha}_2, n_1, n_2 \in Z\} \tag{10}$$

and $\vec{w}_1$ denotes a $SU(3)$ weight vector ($\vec{w}_1 \cdot \vec{\alpha}_j = \delta_{1j}, j \in \{1,2\}$). One recognizes that the conformal dimension of $V_{\vec{p}_L,\vec{p}_R}$ is $(\frac{1}{3} \mid \epsilon \mid + m_1, \frac{1}{3} \mid \epsilon \mid + m_2)$, where $m_{1,2} \in \mathbb{N}_0$. Among the infinite number of Virasoro primary fields, there are just three $U(1)$–primary fields in the class Γ_{+1} (and three in Γ_{-1}) which are the Z_3-invariant combinations of the nine fields $V_{\vec{w}_i,\vec{w}_j}$ $(i,j \in \{1,2,3\})$ where $\vec{w}_i = \theta^{i-1}\vec{w}_1$. ($\theta$ denotes a 120^0 rotation and $\vec{w}_{1,2,3}$ form the weights related to the $\underline{3}$ representation of $SU(3)$)

$$V_{\vec{w}_i} \equiv \frac{1}{\sqrt{3}} \sum_{j=0}^{2} V_{\vec{w}_{i+j},\vec{w}_{1+j}} \tag{11}$$

Here the integers appearing as subscripts are to be understood mod 3. From the OPE with the $U(1)$ currents $J^\pm$ (see (7)), we obtain the following $U(1)$ diagonal primaries (the critical background is chosen to be $\varrho := 2(\sqrt{\det g} - ib_{12}) = -i\alpha$; $\alpha = e^{\frac{2\pi i}{3}}$):

$$\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix} \begin{pmatrix} V_{\vec{w}_1} \\ V_{\vec{w}_2} \\ -V_{\vec{w}_3} \end{pmatrix} = \begin{pmatrix} \mid w_3\rangle \\ \mid w_2\rangle \\ \mid w_1\rangle \end{pmatrix} \tag{12}$$

where $\mid w_i\rangle$ means the state whose J^+–charge is $\frac{w_i}{\sqrt{2}}$. There are no $U(1)$ primary fields with $h, \bar{h} \geq 1$, whence this part of the spectrum can always be reached through further repeated action with $J^\pm_{\mathrm{L,R}}$ on $\mid w_i >$. In particular, one can verify that the vertex operators with $h, \bar{h} = 1$ are $U(1)$ descendants of the identity (e.g., $\partial X_L^+ \partial X_R^-$, $J_L^+ J_R^-$).

Z_4 orbifold: There are four conjugacy classes for the Narain momenta on a $SU(2) \times SU(2)$ torus:

$$(\vec{p}_L; \vec{p}_R) = \frac{1}{\sqrt{2}} \left(\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}; \begin{pmatrix} m_1 + 2n_1 \\ m_2 + 2n_2 \end{pmatrix} \right) \tag{13}$$

It is possible to construct two Z_4-invariant vertex operators with $h = \bar{h} = \frac{1}{4}$:

$$V_{\theta^l(10)}(z,\bar{z}) \equiv \frac{1}{2} \sum_{n=0}^{3} V_{\theta^{l+n}(\frac{1}{\sqrt{2}},0),\theta^n(\frac{1}{\sqrt{2}},0)}(z,\bar{z}) \ , \qquad l \in \{0,2\} \tag{14}$$

where θ denotes a 90 degree rotation and $\theta^2(10) \equiv (-1,0)$. The $U(1)$ diagonal primary fields w.r.t. the current $J_L(z)$ in (7) are the following ones (here $\varrho = 1$):

$$\begin{aligned} \frac{1}{\sqrt{2}}[(V_{(10)} - iV_{(-10)})] &= \mid +\tfrac{1}{2}\rangle \\ \frac{1}{\sqrt{2}}[(V_{(10)} + iV_{(-10)})] &= \mid -\tfrac{1}{2}\rangle \end{aligned} \tag{15}$$

As regards the $h = \bar{h} = \frac{1}{2}$ vertex operators, we find four Z_4-invariant fields

$$V_{\theta^l(11)} \equiv \frac{1}{2}\sum_{n=0}^{3} V_{\theta^{l+n}(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}),\theta^n(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})}(z,\bar{z}) \;, \qquad l \in \{0,1,2,3\} \tag{16}$$

with $\theta(11) \equiv (-1,1), \theta^2(11) \equiv (-1,-1), \theta^3(11) \equiv (1,-1)$. The $U(1)$ diagonal combinations read:

$$\begin{aligned} \tfrac{1}{2}[(V_{(11)} - V_{(-1,-1)}) - i(V_{(-11)} + V_{(1,-1)})] &= \mid +1\rangle \\ \tfrac{1}{2}[(V_{(11)} - V_{(-1,-1)}) + i(V_{(-11)} + V_{(1,-1)})] &= \mid -1\rangle \\ \tfrac{1}{2}[(V_{(11)} + V_{(-1,-1)}) + \ (V_{(-11)} - V_{(1,-1)})] &= \mid \ 0\rangle \\ \tfrac{1}{2}[(V_{(11)} + V_{(-1,-1)}) - \ (V_{(-11)} - V_{(1,-1)})] &= \mid \ 0\rangle \end{aligned} \tag{17}$$

The pair of zero charge fields turns out not to be completely degenerate if one considers the conformal dimension $(1,1)$ operators: In contrast to the case of Z_3 orbifold, there is one $U(1)$ primary field among the dimension $(1,1)$ operators. Four Z_4-invariant vertex operators with $h = \bar{h} = 1$ are

$$V_{\theta^l(20)} \equiv \frac{1}{2}\sum_{n=0}^{3} V_{\theta^{l+n}(\sqrt{2},0),\theta^n(\sqrt{2},0)}(z,\bar{z}) \;, \qquad l \in \{0,1,2,3\} \tag{18}$$

where $\theta^l(20) \equiv (02), (-20)$ and $(0,-2)$ for $l = 1, 2$ and 3, respectively. Among these, only one linear combination describes a primary field with charge zero

$$\frac{1}{2}[(V_{(20)} + V_{(-20)}) + (V_{(02)} + V_{(0,-2)})] = \mid 0\rangle \tag{19}$$

whereas the other combinations are $U(1)$-descendants of the identity, e.g.,

$$\frac{1}{2}(V_{(20)} + V_{(-20)}) - \frac{1}{2}(V_{(02)} + V_{(0,-2)}) = J_L J_R \quad . \tag{20}$$

Observe that the relative sign factor accounts for the cocycle phase in the OPE $J_L \cdot J_R$. In fact the neutral primary field (19) is the product of the neutral primary fields with dimension $(\frac{1}{2}, \frac{1}{2})$ in (17). As was the case for the Z_3 orbifold, all other vertex operators with higher dimensions are again $U(1)$ descendants of the above primary fields.

Z_6 orbifold: It is constructed by modding the Z_3 orbifold w.r.t its reflection automorphism. That is, the $U(1)$ current is $J_L = \frac{i}{\sqrt{6}}\sum_{i=1}^{3}(V_{\vec{\alpha}_i,0}+V_{-\vec{\alpha}_i,0})$. Z_6–invariant dimension $(\frac{1}{3},\frac{1}{3})$ operators are

$$V_i = \frac{1}{\sqrt{2}}(V_{\vec{w}_i} + V_{-\vec{w}_i}) , \qquad i \in \{1,2,3\} \tag{21}$$

where $V_{\pm\vec{w}_i}$ are defined in (11). Analyzing the action of $U(1)$ on the V_i's, one arrives at three diagonal states (again $\varrho = -i\alpha$):

$$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \bar{\alpha} \\ 1 & \bar{\alpha} & \alpha \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ -V_3 \end{pmatrix} = \begin{pmatrix} |\ 0\rangle \\ |-\frac{1}{\sqrt{2}}\rangle \\ |\ \frac{1}{\sqrt{2}}\rangle \end{pmatrix} \tag{22}$$

For dimension (1,1) operators, there are six Z_6 invariants

$$V_{\theta^l\vec{\alpha}_1} \equiv \frac{1}{\sqrt{6}}\sum_{n=0}^{5}(V_{\theta^{l+n}\vec{\alpha}_1,\theta^n\vec{\alpha}_1}) , \qquad l \in \{0,1,\cdots,5\} \tag{23}$$

where θ is a 60 degree rotation. While all the dimension (1,1) operators of the Z_3 orbifold are $U(1)$ descendants, three combinations of the operators in (22) become primary in the case of a Z_6–orbifold.

$$\begin{aligned}
\tfrac{1}{\sqrt{6}}[(V_{\alpha_1} - V_{-\alpha_1}) + \ (V_{\alpha_2} - V_{-\alpha_2}) - \ (V_{\alpha_3} - V_{-\alpha_3})] &= |\ 0\rangle \\
\tfrac{1}{\sqrt{6}}[(V_{\alpha_1} + V_{-\alpha_1}) + \bar{\alpha}(V_{\alpha_2} + V_{-\alpha_2}) - \alpha(V_{\alpha_3} + V_{-\alpha_3})] &= |\ \tfrac{1}{\sqrt{2}}\rangle \\
\tfrac{1}{\sqrt{6}}[(V_{\alpha_1} + V_{-\alpha_1}) + \alpha(V_{\alpha_2} + V_{-\alpha_2}) - \bar{\alpha}(V_{\alpha_3} + V_{-\alpha_3})] &= |-\tfrac{1}{\sqrt{2}}\rangle
\end{aligned} \tag{24}$$

In addition one checks that one of the other operators with dimension (1,1) is $J_L J_R = \frac{1}{\sqrt{6}}[(V_{\alpha_1} + V_{-\alpha_1}) + (V_{\alpha_2} + V_{-\alpha_2}) - (V_{\alpha_3} + V_{-\alpha_3})]$. Notice that not all of the dimension (1,1) operators are $U(1)$ descendants in the cases of Z_4 and Z_6 orbifolds. This might be attributed to the Z_2 orbifoldization of $SU(2)\times SU(2)$ and $SU(3)$ tori (90 degree and 180 degree rotation, respectively), not being an inner automorphism .

We proceed with the diagonalization of the twisted sector fields w.r.t. the $U(1)$ symmetries. It suffices to concentrate on the bosonic twist fields. The twist field $e^{i\frac{k}{N}B}$ of the k–th twisted sector of a Z_N orbifold accounts for the fermionic number charge $Q = \frac{k}{N}$. Even though we do not know the explicit form of bosonic twist fields in terms of the free field X, we can find the local expansions of $\sigma^{(\frac{k}{N})}\sigma^{-(\frac{k}{N})}$. They emerge when one factorizes a four–point function of type $\langle\sigma^+\sigma^-\sigma^+\sigma^-\rangle$ which is exactly known (cf. [9, 10]). Relying on this method it was possible to calculate the OPE coefficients for the first twisted sector of every two–dimensional Z_N orbifold (see [11], [12]). These formulae also serve to determine OPE coefficients for the higher twisted sectors. For instance, the fact that a $\frac{2}{4}$-twist ($\frac{3}{6}$-twist) gives identifications under 180 degree rotation, requires the twist fields to be Z_4– (Z_6–)invariant combinations of ordinary Z_2 twist fields [9]. Likewise, a $\frac{2}{6}$-twist field of a Z_6 orbifold must be described by a Z_6-invariant combination of Z_3 twist fields. We may restrict the range of k to $[1, \frac{N}{2}]$, due to $\sigma^{(\frac{N-k}{N})} = \sigma^{-(\frac{k}{N})} = \sigma^{(\frac{k}{N})\dagger}$. Explicit formulae for these OPE are presented in Table 1. We again discuss the cases at our disposal.

Z_3 **orbifold** (i) $\frac{1}{3}$**-twist**: There are three twist fields, $\sigma_i^{(\frac{1}{3})}$ ($i \in \{0,1,2\}$), associated to three fixed points f_i (equivalently, three conjugacy classes). The momenta and windings are also contained in coset classes $V^{i,j}$ ($i,j \in \{0,1,2\}$). Since the $U(1)$ current $J^+(J^-)$ belongs to the coset class $V^{1,2}$ ($V^{2,1}$), we find the action of J^+ on the twist fields $\sigma_i^{(\frac{1}{3})}$ with the help of the expansion for $\sigma_i^+\sigma_j^-$ whose coefficients have been determined before (see equation (3.44) in [12]).

$$J^+(z)\, \sigma_i^{(\frac{1}{3})}(0,0) = \frac{1}{3}\frac{\alpha^{i+1}}{z}\sigma_{i+2}^{(\frac{1}{3})}(0,0) + \cdots, \tag{25}$$

Then according to [4, 5] the diagonal elements are

$$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & \alpha & \alpha \\ 1 & \bar{\alpha} & 1 \\ 1 & 1 & \bar{\alpha} \end{pmatrix}\begin{pmatrix} \sigma_0^{(\frac{1}{3})} \\ \sigma_1^{(\frac{1}{3})} \\ \sigma_2^{(\frac{1}{3})} \end{pmatrix} = \begin{pmatrix} |\, \frac{1}{3}\rangle \\ |\, \frac{\alpha}{3}\rangle \\ |\, \frac{\bar{\alpha}}{3}\rangle \end{pmatrix}. \tag{26}$$

Z_4 **orbifold** (i) $\frac{1}{4}$**-twist**: There are two conjugacy classes which carry the labels 0, 1. One can find an OPE similar to (25) with the $U(1)$ current contained in the class $V^{1,1}$. (It is immediately obtained by consulting equation (3.41) in [12]. We will abstain here from providing those OPE relations.) The $U(1)$–diagonal fields then read

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}\begin{pmatrix} \sigma_0^{(\frac{1}{4})} \\ \sigma_1^{(\frac{1}{4})} \end{pmatrix} = \begin{pmatrix} |\ \ \frac{1}{4}\rangle \\ |-\frac{1}{4}\rangle \end{pmatrix} \tag{27}$$

(ii) $\frac{2}{4}$**-twist**: The fields in this sector are Z_4-invariants of a Z_2 orbifold. The twisted sector of a Z_2 orbifold has four conjugacy classes labeled $(00), (01), (10), (11)$. Since a Z_4 twist mixes the classes (01) and (10), Z_4 invariant fields look as follows:

$$\begin{aligned}
\sigma_0^{(\frac{2}{4})} &= \sigma_{00}^{(\frac{1}{2})} \\
\sigma_1^{(\frac{2}{4})} &= \tfrac{1}{\sqrt{2}}(\sigma_{01}^{(\frac{1}{2})} + \sigma_{10}^{(\frac{1}{2})}) \\
\sigma_2^{(\frac{2}{4})} &= \sigma_{11}^{(\frac{1}{2})} \\
V_a^{1,1} &= \tfrac{1}{\sqrt{2}}(V^{01,01} + V^{10,10}) \\
V_b^{1,1} &= \tfrac{1}{\sqrt{2}}(V^{01,10} + V^{10,01})
\end{aligned} \tag{28}$$

It is noteworthy that there appear two types for the class $V^{1,1}$. The $U(1)$ current $J(z)$ is an element of $V_a^{1,1}$. The diagonal combinations are

$$\frac{1}{2}\begin{pmatrix} i\sqrt{2} & 1 & -1 \\ -i\sqrt{2} & 1 & -1 \\ 0 & \sqrt{2} & \sqrt{2} \end{pmatrix}\begin{pmatrix} \sigma_1^{(\frac{2}{4})} \\ \sigma_0^{(\frac{2}{4})} \\ \sigma_2^{(\frac{2}{4})} \end{pmatrix} = \begin{pmatrix} |\ \frac{1}{2}\rangle \\ |-\frac{1}{2}\rangle \\ |\ 0\rangle \end{pmatrix} \tag{29}$$

Z_6 **orbifold** (i) $\frac{1}{6}$**-twist**: There is a single fixed point; the twist field therefore being trivially diagonal has $U(1)$ charge $\frac{-1}{6\sqrt{2}}$.
(ii) $\frac{2}{6}$**-twist**: a Z_6–invariant projection of Z_3 twist fields is necessary. There are two conjugacy classes and the class $V^{1,1}$ comprises two types (labelled a, b).

$$\begin{aligned}
\sigma_0^{(\frac{2}{6})} &= \sigma_0^{(\frac{1}{3})} \\
\sigma_1^{(\frac{2}{6})} &= \tfrac{1}{\sqrt{2}}(\sigma_1^{(\frac{1}{3})} + \sigma_2^{(\frac{1}{3})}) \\
V_a^{1,1} &= \tfrac{1}{\sqrt{2}}(V^{1,1} + V^{2,2}) \\
V_b^{1,1} &= \tfrac{1}{\sqrt{2}}(V^{1,2} + V^{2,1})
\end{aligned} \tag{30}$$

With the $U(1)$ current belonging to the class $V_b^{1,1}$, one gets the following diagonalization:

$$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 & \sqrt{2}\alpha \\ \sqrt{2} & -\alpha \end{pmatrix}\begin{pmatrix} \sigma_0^{(\frac{2}{6})} \\ \sigma_1^{(\frac{2}{6})} \end{pmatrix} = \begin{pmatrix} |\ \frac{\sqrt{2}}{3}\rangle \\ |-\frac{1}{3\sqrt{2}}\rangle \end{pmatrix} \tag{31}$$

(iii) $\frac{3}{6}$**-twist**: There are two conjugacy classes one of them being a combination of three conjugacy classes of Z_2 twist fields:

$$\begin{aligned}
\sigma_0^{(\frac{3}{6})} &= \sigma_{00}^{(\frac{1}{2})} \\
\sigma_0^{(\frac{3}{6})} &= \tfrac{1}{\sqrt{3}}(\sigma_{01}^{(\frac{1}{2})} + \sigma_{10}^{(\frac{1}{2})} + \sigma_{11}^{(\frac{1}{2})}) \\
V_a^{1,1} &= \tfrac{1}{\sqrt{3}}(V^{01,01} + V^{10,11} + V^{11,10}) \\
V_b^{1,1} &= \tfrac{1}{\sqrt{3}}(V^{01,11} + V^{10,10} + V^{11,01}) \\
V_c^{1,1} &= \tfrac{1}{\sqrt{3}}(V^{01,10} + V^{10,01} + V^{11,11})
\end{aligned} \tag{32}$$

The $U(1)$ current is a member of $V_b^{1,1}$; diagonalization gives

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}\begin{pmatrix} \sigma_0^{(\frac{3}{6})} \\ \sigma_1^{(\frac{3}{6})} \end{pmatrix} = \begin{pmatrix} | \; \frac{1}{2\sqrt{2}}\rangle \\ | -\frac{1}{2\sqrt{2}}\rangle \end{pmatrix} \tag{33}$$

3. The primary fields of the $c = 3$ Landau–Ginzburg models

The conformal dimension and $U(1)$ charge for the primary fields in the NS sector of level $k-2$ minimal $N = 2$ SCFT [13] with conformal anomaly $c = \frac{3(k-2)}{k}$ are given by $(h, Q) = (\frac{l(l+2)-q^2}{4k}, \frac{q}{k})$ with $l = 0, \cdots, k-2$ and $-l \leq q \leq l$. One observes that the chiral primary fields with $q = l = s$ have the same (h, Q) as the twist fields (5) for a Z_k orbifold. This indicates that Z_k twist fields are related to chiral primary fields of level $(k-2)$. The latter are expressed in terms of order parameters of Z_{k-2} parafermionic system [14] and of a free $U(1)$ boson. Apparently, there are just three ways of combining minimal $N = 2$ SCFT's to construct tensor models having conformal anomaly $c = 3$: $(1^3), (2^2), (1, 4)$. These tensor models have three, two and two $U(1)$ currents, $i\partial\phi_j$, respectively, which are provided by each minimal model factor. To further compare tensor models with Z_k orbifolds, we must identify their total $U(1)$ current with the fermion number current of the Z_k orbifold.

$$\psi^+\psi^- = i\partial B = \frac{i}{\sqrt{M}}\sum_{j=1}^{M}\partial\phi_j \qquad M = 3\ (2) \text{ for } (1^3)\ ((2^2),\ (1,4)) \tag{34}$$

The remaining $U(1)$'s (being orthogonal to $i\partial B$), should then be identified with the additional $U(1)$ currents J_i of the multicritical Z_k orbifold. To explore the equivalences between (1^3), (2^2) and $(1,4)$ tensor models and Z_k orbifolds with $k = 3, 4, 6$ respectively, let us present the chiral algebra and field contents of tensor models.

The $N=2$ SCA of level $(k-2)$ minimal SCFT is generated by

$$\begin{aligned} G^+ &= \sqrt{\tfrac{k-2}{k}}\psi_1(z)e^{i\sqrt{\frac{k}{k-2}}\phi} \\ G^- &= \sqrt{\tfrac{k-2}{k}}\psi_1^\dagger(z)e^{-i\sqrt{\frac{k}{k-2}}\phi} \\ Q &= i\sqrt{\tfrac{k-2}{k}}\partial\phi \end{aligned} \tag{35}$$

together with the stress tensor $T(z) = T_\phi(z) + T_{(k-2)}(z)$ for a free boson ϕ and a Z_{k-2} parafermionic system. The primary fields of $N=2$ minimal SCFT have been written by Qiu in [13] as a product of a parafermionic field and a bosonic field: $\Phi_q^l \exp(i\alpha_q^l\phi)$. Here we suppress the indices for a right-moving part. When needed, the same indices q for the right-moving part may be revived since we are dealing with a left-right symmetric theory. (For the complete expressions see [15].) Note that all chiral primary fields $(h,Q) = (\frac{l}{2k}, \pm\frac{l}{k})$ of a level $(k-2)$ theory have the order parameter $\Phi_{\pm l}^l$ in their parafermionic part:

$$\Phi_{\pm l}^l e^{\pm i\frac{l}{\sqrt{k(k-2)}}\phi}, \qquad l \in \{1,\cdots,(k-2)\} \tag{36}$$

By tensoring the primary fields for the level 1, 2 and 4 models, we arrive at the primary fields of $c=3$ tensor models. Table 2 lists them. Comparing the conformal dimensions and $U(1)$ charges, the primary field contents of tensor models and Z_k orbifolds is seen to coincide. One point which should be emphasized is that there exist primary fields with excited twist states different from ground state twist fields in the twisted sectors. One may express the tensor model versions of the excited twist fields in terms of parafermionic fields and $U(1)$ free bosons by calculating the OPE (6). Parafermionic descriptions of free bosons $\partial X^\pm$ on the orbifold result from identifying the stress tensor of a tensor model with the one given in (2). This will be discussed below (see also [16]).

(1^3) model: It is simply composed of three free bosons ϕ_j; a parafermionic system is not needed. The total $U(1)$ (fermion number) and the additional two $U(1)$ currents are expressed as $Q = i\partial B$, $J^\pm = i\partial H^\pm = i(\partial H^1 \pm i\partial H^2)/\sqrt{2}$. Here B and $\vec{H} = (H^1, H^2)$ are related to ϕ_j via

$$\phi_j = \frac{1}{\sqrt{3}}(B + \vec{\alpha}_j \cdot \vec{H}) \tag{37}$$

($\vec{\alpha}_j$ was defined below (7)). Then the total superstress tensor becomes

$$G^+ = \sqrt{\frac{1}{3}}\sum_{j=1}^{3} e^{i\sqrt{3}\phi_j} = \sqrt{\frac{1}{3}}e^{iB}\sum_{j=1}^{3} e^{i\vec{\alpha}_j\cdot\vec{H}} \tag{38}$$

Comparing now with (2), (7) for a Z_3 orbifold, one finds the rebosonization formulae

$$\begin{aligned} i\partial H^{\pm} &= \tfrac{i}{\sqrt{3}}\sum_j V_{\mp\vec{\alpha}_j,0}[\vec{X}] \\ i\partial X^{\pm} &= \tfrac{i}{\sqrt{3}}\sum_j e^{\mp i\vec{\alpha}_j\cdot\vec{H}} \end{aligned} \tag{39}$$

where $\vec{X}=(X^1,X^2)$. Inspection of Table 2.2 reveals that there appear three bosonic fields ($Q=0$) in the untwisted sector with $(h,Q)=(\frac{1}{3},0)$ which must be identified with (12); for example, $1/\sqrt{3}(V_{\vec{w}_1}+V_{\vec{w}_2}-V_{\vec{w}_3})=\exp(i\vec{w}_3\cdot\vec{H})$. As regards the $(\frac{1}{3})$-twist fields, one can see that $\exp(\pm i\frac{1}{3}\vec{\alpha}_j\cdot\vec{H})$ forms an eigenvector w.r.t. the application of $J^+(z)$ in (26). This was shown before in [5] where modding out by a twist has been traded for the equivalent modding out by a shift. As announced above the additional fields of the form $\exp(i\frac{1}{3}B)\exp(-i\frac{2}{3}\vec{\alpha}_i\cdot\vec{H})$ correspond to the excited twist fields of the orbifold model , which can be seen by calculating the OPE (6) (e.g., $i\partial X^+\cdot\frac{1}{\sqrt{3}}(\sigma_0^{(\frac{1}{3})}+\alpha\sigma_1^{(\frac{1}{3})}+\alpha\sigma_2^{(\frac{1}{3})})=\frac{i}{\sqrt{3}}\sum_j\exp(-i\vec{\alpha}_j\cdot\vec{H})\cdot\exp(i\frac{1}{3}\vec{\alpha}_1\cdot\vec{H})$):

$$\frac{1}{\sqrt{3}}\begin{pmatrix}1&\alpha&\alpha\\1&\bar{\alpha}&1\\1&1&\bar{\alpha}\end{pmatrix}\begin{pmatrix}\tau_0^{(\frac{1}{3})}\\ \tau_1^{(\frac{1}{3})}\\ \tau_2^{(\frac{1}{3})}\end{pmatrix}=\begin{pmatrix}e^{-i\frac{2}{3}\vec{\alpha}_1\cdot\vec{H}}\\ e^{-i\frac{2}{3}\vec{\alpha}_2\cdot\vec{H}}\\ e^{-i\frac{2}{3}\vec{\alpha}_3\cdot\vec{H}}\end{pmatrix} \tag{40}$$

($\mathbf{2^2}$) model: There are two $U(1)$ currents $Q=i\partial B$, $J=i\partial H$ arising from $\phi_{1,2}$:

$$\begin{aligned} B &= \tfrac{1}{\sqrt{2}}(\phi_1+\phi_2) \\ H &= \tfrac{1}{\sqrt{2}}(\phi_1-\phi_2) \end{aligned} \tag{41}$$

Then we express $i\partial X^{\pm}$ in terms of two Z_2 parafermions ψ_1,ψ_1' and H:

$$\begin{aligned} i\partial X^- &= \tfrac{1}{\sqrt{2}}(\psi_1 e^{iH}+\psi_1' e^{-iH}) \\ i\partial X^+ &= \tfrac{1}{\sqrt{2}}(\psi_1^{\dagger} e^{-iH}+\psi_1'^{\dagger} e^{iH}) \end{aligned} \tag{42}$$

One can also see the one-to-one correspondences between primary fields in Table 2.3 and those in (15), (17), (19), (27), (29). There occur also excited twist fields with $(h,Q)=(\frac{3}{8},\frac{1}{4})$, $(\frac{5}{8},\frac{1}{4})$, $(\frac{3}{4},\frac{1}{2})$ in Table 2.3. We choose $(\frac{3}{8},\frac{1}{4})$ fields and show how they are obtained by applying $i\partial X^{\pm}$ to the ground state twist fields:

$$
\begin{aligned}
& i\partial X^+(z)\cdot \tfrac{1}{\sqrt{2}}[\sigma_0^{(\frac{1}{4})}+i\sigma_1^{(\frac{1}{4})}](0) \\
&= \tfrac{1}{\sqrt{2}}(\psi_1 e^{-iH}+\psi_1' e^{iH})(z)\cdot(\Phi_1^1 e^{i\frac{1}{4}H})(0) \\
&= \tfrac{1}{\sqrt{2}} z^{-\frac{3}{4}}(\Phi_1^1 e^{-i\frac{3}{4}H})(0) + \text{regular terms}
\end{aligned}
\tag{43}
$$

Here we used the OPE $\psi_1(z)\Phi_l^l(0)=z^{-l/s}\Phi_{l+2}^l(0)+\cdots$ from Z_s parafermionic theory where one identifies $\Phi_l^l=\Phi_{s+l}^{s-l}=\Phi_{l+2s}^l$ [14]. Thus one gets

$$
\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}\begin{pmatrix} \tau_0^{(\frac{1}{4})} \\ \tau_1^{(\frac{1}{4})} \end{pmatrix} = \begin{pmatrix} \Phi_1^1 e^{-i\frac{3}{4}H} \\ \Phi_1'^1 e^{i\frac{3}{4}H} \end{pmatrix}
\tag{44}
$$

(1, 4) model: Two $U(1)$ currents are generated by

$$
\begin{aligned}
B &= \tfrac{1}{\sqrt{3}}(\phi_1+\sqrt{2}\phi_2) \\
H &= \tfrac{1}{\sqrt{3}}(\sqrt{2}\phi_1-\phi_2)
\end{aligned}
\tag{45}
$$

where ϕ_1 belongs to the level 1 model and ϕ_2 is the $U(1)$ boson of the level 4 model. Moreover the free bosons on a Z_6 orbifold are related to H and the Z_4 parafermion ψ_1 by:

$$
\begin{aligned}
i\partial X^- &= \sqrt{\tfrac{1}{3}}e^{i\sqrt{2}H}+\sqrt{\tfrac{2}{3}}\psi_1 e^{-i\frac{1}{\sqrt{2}}H} \\
i\partial X^+ &= \sqrt{\tfrac{1}{3}}e^{-i\sqrt{2}H}+\sqrt{\tfrac{2}{3}}\psi_1^\dagger e^{i\frac{1}{\sqrt{2}}H}
\end{aligned}
\tag{46}
$$

Now we are ready to compare the field contents in Table 2.4 with what appears in (22), (24) and (31), (33). For the excited twist fields we can repeat the calculation given above.

4. The twisted sector operator product expansion — a comparison of both constructions

Finally we must calculate the OPE between the twist fields in order to confirm the identifications. In Table 3 the twist fields σ are expressed in terms of linear combinations of $U(1)$ diagonal twist fields. Since σ is written in product form and since H is a free boson the only nontrivial ingredient is (cf. [14])

$$
\langle \Phi_{l_1}^{l_1}(z_1)\Phi_{l_2}^{l_2}(z_2)\left[\Phi_{l_1+l_2}^{l_1+l_2}\right]^\dagger(z_3)\rangle = C_{l_1 l_2}\, z_{12}^{h_3-h_1-h_2}\, z_{13}^{h_2-h_1-h_3}\, z_{23}^{h_1-h_2-h_3}
$$

$$\text{where } C^2_{l_1 l_2} = \frac{\Gamma(\frac{1}{N})\Gamma(\frac{1+l_1+l_2}{N})\Gamma(\frac{N-l_1-1}{N})\Gamma(\frac{N-l_2-1}{N})}{\Gamma(\frac{N-1}{N})\Gamma(\frac{N-l_1-l_2-1}{N})\Gamma(\frac{1+l_1}{N})\Gamma(\frac{1+l_2}{N})}$$

Let us concentrate on the most singular parts and omit the worldsheet arguments. For a Z_3 orbifold we then have ($\tau \equiv i\varrho = \alpha$)

$$\begin{aligned} \sigma_i^{(\frac{1}{3})}\sigma_i^{(\frac{1}{3})} &= \tfrac{2}{\sqrt{3}}\sigma_i^{-(\frac{1}{3})} \\ \sigma_i^{(\frac{1}{3})}\sigma_j^{(\frac{1}{3})} &= -\tfrac{\alpha}{\sqrt{3}}\sigma_{-(i+j)}^{-(\frac{1}{3})} \text{if } i \neq j \end{aligned} \tag{47}$$

The ratio of these OPE coefficients is $-2\bar{\alpha}$ which was independently found by a calculation of orbifold Yukawa couplings (see [12]). The situation for a Z_4 orbifold is as follows:

$$\begin{aligned} \sigma_0^{(\frac{1}{4})}\sigma_1^{(\frac{1}{4})} &= \tfrac{1}{\sqrt{2}}\sigma_1^{(\frac{2}{4})} \\ \sigma_1^{(\frac{1}{4})}\sigma_1^{(\frac{1}{4})} &= \tfrac{\sqrt{2}-1}{2}\sigma_0^{(\frac{2}{4})} + \tfrac{\sqrt{2}+1}{2}\sigma_2^{(\frac{2}{4})} \\ \sigma_0^{(\frac{1}{4})}\sigma_0^{(\frac{1}{4})} &= \tfrac{\sqrt{2}+1}{2}\sigma_0^{(\frac{2}{4})} + \tfrac{\sqrt{2}-1}{2}\sigma_2^{(\frac{2}{4})} \end{aligned} \tag{48}$$

And the pattern for a Z_6 orbifold reads ($C_{11} = C_{12} = 2^{\frac{1}{6}}$) :

$$\begin{aligned} \sigma_0^{(\frac{1}{6})}\sigma_0^{(\frac{1}{6})} &= \sqrt{\tfrac{2}{3}}C_{11}\sigma_0^{(\frac{2}{6})} - \sqrt{\tfrac{1}{3}}\alpha C_{11}\sigma_1^{(\frac{2}{6})} \\ \sigma_0^{(\frac{1}{6})}\sigma_0^{(\frac{2}{6})} &= \tfrac{1}{\sqrt{6}}(1+\sqrt{2}C_{12})\sigma_0^{(\frac{3}{6})} + \tfrac{i}{\sqrt{6}}(1-\sqrt{2}C_{12})\sigma_1^{(\frac{3}{6})} \\ \sigma_0^{(\frac{1}{6})}\sigma_1^{(\frac{2}{6})} &= \tfrac{\bar{\alpha}}{\sqrt{6}}(\sqrt{2}-C_{12})\sigma_0^{(\frac{3}{6})} + \tfrac{i\bar{\alpha}}{\sqrt{6}}(\sqrt{2}+C_{12})\sigma_1^{(\frac{3}{6})} \\ \sigma_0^{(\frac{2}{6})}\sigma_0^{(\frac{2}{6})} &= \tfrac{2}{\sqrt{3}}\sigma_0^{-(\frac{2}{6})} \\ \sigma_0^{(\frac{2}{6})}\sigma_1^{(\frac{2}{6})} &= -\tfrac{\alpha}{\sqrt{3}}\sigma_1^{-(\frac{2}{6})} \\ \sigma_1^{(\frac{2}{6})}\sigma_1^{(\frac{2}{6})} &= -\tfrac{\alpha}{\sqrt{3}}\sigma_0^{-(\frac{2}{6})} + \sqrt{\tfrac{2}{3}}\sigma_1^{-(\frac{2}{6})} \end{aligned} \tag{49}$$

These OPEs can be shown to be consistent with space group selection rules [9] and one may check that the coefficients of the above OPEs are the same as those obtained in [12]. However it would take us too far if we were going to reproduce the explicit values of the OPE coefficients (cf. formula (3.62) in [12]). For simplicity let us therefore use the duality operation S and the discrete shift T of the axionic variable b_{12} for the twist fields (details can be found in [12]) to extract the ratios of the coefficients. We use the abbreviations Y_0 (Y_1) for the Yukawa couplings $< \sigma_j\sigma_j\sigma_j >$ ($< \sigma_0\sigma_1\sigma_3 >$). Then their ratio at the background point $\tau = \alpha$ becomes $Y_0/Y_1 = -2\bar{\alpha}$. On the other hand two-dimensional orbifold models possess the discrete symmetry group SL$(2, Z)$ w.r.t which the modulus τ, the primary fields and the correlation functions form various representations such as

$$Y_i(\alpha) = S^Y_{ij} T^Y_{jk} Y_k(-\frac{1}{\alpha} - 1) \tag{50}$$

where S^Y and T^Y are defined in [4], since the enhancement point $\tau = \alpha$ is the fixed point of the transformation ST.

From this eigenvector equation we infer $Y_0/Y_1 = -2\bar{\alpha}$ once more. As regards the Z_4 orbifold we deduce from above that

$$\begin{aligned} &< \sigma_0^{(\frac{1}{4})} \sigma_0^{(\frac{1}{4})} \sigma_0^{(-\frac{2}{4})} > : < \sigma_0^{(\frac{1}{4})} \sigma_0^{(\frac{1}{4})} \sigma_2^{(-\frac{2}{4})} > : < \sigma_0^{(\frac{1}{4})} \sigma_1^{(\frac{1}{4})} \sigma_1^{(-\frac{2}{4})} > \\ &= (\sqrt{2} + 1) \, : \, (\sqrt{2} - 1) \, : \, \sqrt{2} \end{aligned} \tag{51}$$

Another way to obtain this ratio makes use of the invariance of the Yukawa–couplings under a duality transformation S:

$$< S(\sigma_i^+) S(\sigma_j^+) S(\sigma_k^{--}) >_{-1/\tau} = < \sigma_i^+ \sigma_j^+ \sigma_k^{--} >_\tau \tag{52}$$

At the fixed point $\tau = i$ of S this identity reduces to a homogeneous system of linear equations for the Yukawa couplings present in (51). The solution vector is seen to be subject to (51). Along the same lines one may also analyze the ratio of certain Yukawa couplings in the case of a Z_6 orbifold at a critical point in background space. There is again complete agreement with the results based on the explicit forms of the primary fields available in a Landau–Ginzburg CFT.

5. Conclusion

In this paper, we have shown the equivalence of Z_N (super–) orbifold models on two-dimensional tori at multicritical values of the characteristic background parameter τ and LG models with modality one. The primary field contents of both models have been shown to be in one-to-one correspondence by calculating the untwisted OPE relations between the $U(1)$ currents and various vertex operators. Finally, the OPE coefficients of the twisted sectors are computed to verify the equivalence of the two types of models.

Table 1: Operator products involving ground states from oppositely twisted sectors

Table 1.1: $\frac{2}{4}$ twist sector (We have omitted the coupling strength $2 \times 16^{-\frac{h+\bar{h}}{2}}$)

$$
\begin{aligned}
\sigma_0^{+(\frac{2}{4})}\sigma_0^{-(\frac{2}{4})} &= V^{0,0} + V^{1,0} + V^{2,0} \\
\sigma_1^{+(\frac{2}{4})}\sigma_1^{-(\frac{2}{4})} &= V^{0,0} + V^{0,2} - V^{2,0} - V^{2,2} \\
\sigma_2^{+(\frac{2}{4})}\sigma_2^{-(\frac{2}{4})} &= V^{0,0} - V^{1,0} + V^{2,0}
\end{aligned}
$$

$$
\begin{aligned}
\sigma_0^{+(\frac{2}{4})}\sigma_1^{-(\frac{2}{4})} &= \tfrac{1}{\sqrt{2}}(V^{0,1} + iV_a^{1,1} + V_b^{1,1} + iV^{2,1}) \\
\sigma_1^{+(\frac{2}{4})}\sigma_2^{-(\frac{2}{4})} &= \tfrac{1}{\sqrt{2}}(V^{0,1} + iV_a^{1,1} - V_b^{1,1} - iV^{2,1}) \\
\sigma_0^{+(\frac{2}{4})}\sigma_2^{-(\frac{2}{4})} &= V^{0,2} + iV^{1,2} + V^{2,2}
\end{aligned}
$$

Table 1.2: $\frac{2}{6}$ twist sector (omitted factor: $\sqrt{6} \times 27^{-\frac{h+\bar{h}}{2}}$)

$$
\begin{aligned}
\sigma_0^{+(\frac{2}{6})}\sigma_0^{-(\frac{2}{6})} &= V^{0,0} + V^{1,0} \\
\sigma_1^{+(\frac{2}{6})}\sigma_1^{-(\frac{2}{6})} &= V^{0,0} - \tfrac{1}{2}V^{1,0} + \tfrac{1}{2}V^{0,1} + \tfrac{1}{2}V_a^{1,1} + \tfrac{1}{2}V_b^{1,1} \\
\sigma_0^{+(\frac{2}{6})}\sigma_1^{-(\frac{2}{6})} &= \tfrac{1}{\sqrt{2}}(V^{0,1} + \bar{\alpha}V_a^{1,1} + \alpha V_b^{1,1}) \\
\sigma_1^{+(\frac{2}{6})}\sigma_0^{-(\frac{2}{6})} &= \tfrac{1}{\sqrt{2}}(V^{0,1} + \alpha V_a^{1,1} + \bar{\alpha}V_b^{1,1})
\end{aligned}
$$

Table 1.3: $\frac{3}{6}$ twist sector (omitted factor : $\sqrt{6} \times 16^{-\frac{h+\bar{h}}{2}}$)

$$
\begin{aligned}
\sigma_0^{+(\frac{3}{6})}\sigma_0^{-(\frac{3}{6})} &= V^{0,0} + V^{1,0} \\
\sigma_1^{+(\frac{3}{6})}\sigma_1^{-(\frac{3}{6})} &= V^{0,0} + \tfrac{2}{3}V^{0,1} - \tfrac{1}{3}V^{1,0} - \tfrac{2}{3}V_c^{1,1} \\
\sigma_0^{+(\frac{3}{6})}\sigma_1^{-(\frac{3}{6})} &= \tfrac{1}{\sqrt{3}}(V^{0,1} + iV_a^{1,1} + iV_b^{1,1} + V_c^{1,1}) \\
\sigma_1^{+(\frac{3}{6})}\sigma_0^{-(\frac{3}{6})} &= \tfrac{1}{\sqrt{3}}(V^{0,1} - iV_a^{1,1} - iV_b^{1,1} + V_c^{1,1})
\end{aligned}
$$

Table 2: Nontrivial primary fields with eigenvalues (h, Q) for tensor models in the NS sector ($Q \geq 0$)

Table 2.1: level 1,2,4 models

level 1;

$$(\frac{1}{6}, \pm\frac{1}{3}) = e^{\pm i\frac{1}{\sqrt{3}}\phi}$$

level 2;

$$\begin{aligned}(\tfrac{l}{8}, \pm\tfrac{l}{4}) &= \Phi^l_{\pm l} e^{\pm i\frac{l}{\sqrt{8}}\phi} \quad l = 0,1,2 \\ (\tfrac{1}{2}, 0) &= \Phi^2_0\end{aligned}$$

level 4;

$$\begin{aligned}(\tfrac{l}{12}, \pm\tfrac{l}{6}) &= \Phi^l_{\pm l} e^{\pm i\frac{l}{\sqrt{24}}\phi} \quad l = 0,1,2,3,4 \\ (\tfrac{1}{3}, 0) &= \Phi^2_0 & (1,0) &= \Phi^4_0 \\ (\tfrac{5}{6}, \pm\tfrac{1}{3}) &= \Phi^0_{\pm 2} e^{\pm i\frac{2}{\sqrt{24}}\phi} & (\tfrac{7}{12}, \pm\tfrac{1}{6}) &= \Phi^3_{\pm 1} e^{\pm i\frac{1}{\sqrt{24}}\phi}\end{aligned}$$

Table 2.2: (1^3) model

$$(\tfrac{1}{3}, 0) = e^{\pm i\vec{w}_i \cdot \vec{H}} \quad i = 1,2,3$$

$$\begin{aligned}(\tfrac{1}{6}, \tfrac{1}{3}) &= e^{i\frac{1}{3}B} e^{i\frac{1}{3}\vec{\alpha}_i \cdot \vec{H}} & (\tfrac{1}{3}, \tfrac{2}{3}) &= e^{i\frac{2}{3}B} e^{-i\frac{1}{3}\vec{\alpha}_i \cdot \vec{H}} \\ (\tfrac{1}{2}, \tfrac{1}{3}) &= e^{i\frac{1}{3}B} e^{-i\frac{2}{3}\vec{\alpha}_i \cdot \vec{H}}\end{aligned}$$

Table 2.3: (2^2) model

$$\begin{aligned}(\tfrac{1}{4}, 0) &= \Phi^1_1 \Phi'^1_1 e^{\pm i\frac{1}{2}H} & (\tfrac{1}{2}, 0) &= \Phi^2_0,\ \Phi'^2_0,\ e^{\pm iH} \\ (1,0) &= \Phi^2_0 \Phi'^2_0\end{aligned}$$

$$\begin{aligned}(\tfrac{1}{8}, \tfrac{1}{4}) &= e^{i\frac{1}{4}B}\Phi^1_1 e^{i\frac{1}{4}H},\ e^{i\frac{1}{4}B}\Phi'^1_1 e^{-i\frac{1}{4}H} & (\tfrac{1}{4}, \tfrac{1}{2}) &= e^{i\frac{1}{2}B} e^{\pm i\frac{1}{2}H},\ e^{i\frac{1}{2}B}\Phi^1_1\Phi'^1_1 \\ (\tfrac{3}{8}, \tfrac{3}{4}) &= e^{i\frac{3}{4}B}\Phi^1_1 e^{-i\frac{1}{4}H},\ e^{i\frac{3}{4}B}\Phi'^1_1 e^{i\frac{1}{4}H} \\ (\tfrac{3}{8}, \tfrac{1}{4}) &= e^{i\frac{1}{4}B}\Phi^1_1 e^{-i\frac{3}{4}H},\ e^{i\frac{1}{4}B}\Phi'^1_1 e^{i\frac{3}{4}H} & (\tfrac{5}{8}, \tfrac{1}{4}) &= e^{i\frac{1}{4}B}\Phi^1_1\Phi'^2_0 e^{i\frac{1}{4}H},\ e^{i\frac{1}{4}B}\Phi^2_0\Phi'^1_1 e^{-i\frac{1}{4}H} \\ (\tfrac{3}{4}, \tfrac{1}{2}) &= e^{i\frac{1}{2}B}\Phi'^2_0 e^{i\frac{1}{2}H},\ e^{i\frac{1}{2}B}\Phi^2_0 e^{-i\frac{1}{2}H}\end{aligned}$$

Table 2.4: (1,4) model

$$
\begin{array}{lcllcl}
(\frac{1}{3},0) & = & \Phi^2_0,\ \ \Phi^2_{\mp 2}e^{\pm i\frac{1}{\sqrt{2}}H} & (1,0) & = & \Phi^4_0,\ \ \Phi^0_{\mp 2}e^{\pm i\frac{1}{\sqrt{2}}H}
\end{array}
$$

$$
\begin{array}{lcllcl}
(\frac{1}{12},\frac{1}{6}) & = & e^{i\frac{1}{6}B}\Phi^1_1 e^{-i\frac{1}{6\sqrt{2}}H} & (\frac{1}{6},\frac{1}{3}) & = & e^{i\frac{1}{3}B}e^{i\frac{\sqrt{2}}{3}H},\ \ e^{i\frac{1}{3}B}\Phi^2_2 e^{-i\frac{1}{3\sqrt{2}}H} \\
(\frac{1}{4},\frac{1}{2}) & = & e^{i\frac{1}{2}B}\Phi^1_{\pm 1} e^{\pm i\frac{1}{2\sqrt{2}}H} & (\frac{1}{3},\frac{2}{3}) & = & e^{i\frac{2}{3}B}e^{-i\frac{\sqrt{2}}{3}H},\ \ e^{i\frac{2}{3}B}\Phi^2_2 e^{i\frac{1}{3\sqrt{2}}H} \\
(\frac{5}{12},\frac{5}{6}) & = & e^{i\frac{5}{6}B}\Phi^1_{-1} e^{i\frac{1}{6\sqrt{2}}H} & & & \\
(\frac{1}{4},\frac{1}{6}) & = & e^{i\frac{1}{6}B}\Phi^1_{-1} e^{i\frac{5}{6\sqrt{2}}H} & (\frac{5}{12},\frac{1}{6}) & = & e^{i\frac{1}{6}B}\Phi^1_{-1} e^{-i\frac{7}{6\sqrt{2}}H} \\
(\frac{7}{12},\frac{1}{6}) & = & e^{i\frac{1}{6}B}\Phi^1_{-3} e^{-i\frac{1}{6\sqrt{2}}H} & (\frac{1}{2},\frac{1}{3}) & = & e^{i\frac{1}{3}B}e^{-i\frac{2\sqrt{2}}{3}H},\ \ e^{i\frac{1}{3}B}\Phi^2_0 e^{i\frac{\sqrt{2}}{3}H} \\
(\frac{5}{6},\frac{1}{3}) & = & e^{i\frac{1}{3}B}\Phi^0_{-2} e^{-i\frac{1}{3\sqrt{2}}H} & (\frac{7}{6},\frac{1}{3}) & = & e^{i\frac{1}{3}B}\Phi^4_0 e^{i\frac{\sqrt{2}}{3}H} \\
(\frac{3}{4},\frac{1}{2}) & = & e^{i\frac{1}{2}B}\Phi^1_{-3} e^{i\frac{1}{2\sqrt{2}}H} & (1,\frac{2}{3}) & = & e^{i\frac{2}{3}B}\Phi^0_{-2} e^{i\frac{1}{3\sqrt{2}}H} \\
(\frac{3}{4},\frac{1}{6}) & = & e^{i\frac{1}{6}B}\Phi^1_3 e^{i\frac{5}{6\sqrt{2}}H} & & &
\end{array}
$$

Table 3: Relations between twist fields of orbifolds and tensor products of minimal SCFT's

Table 3.1: Z_3 twist

$$
\begin{pmatrix} \sigma_0^{(\frac{1}{3})} \\ \sigma_1^{(\frac{1}{3})} \\ \sigma_2^{(\frac{1}{3})} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ \bar{\alpha} & \alpha & 1 \\ \bar{\alpha} & 1 & \alpha \end{pmatrix} \begin{pmatrix} e^{i\frac{1}{3}\vec{\alpha}_1\cdot\vec{H}} \\ e^{i\frac{1}{3}\vec{\alpha}_2\cdot\vec{H}} \\ e^{i\frac{1}{3}\vec{\alpha}_3\cdot\vec{H}} \end{pmatrix}
$$

Table 3.2: Z_4 twist

$$
\begin{pmatrix} \sigma_0^{(\frac{1}{4})} \\ \sigma_1^{(\frac{1}{4})} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \begin{pmatrix} \Phi^1_1 e^{i\frac{1}{4}H} \\ \Phi'^1_1 e^{-i\frac{1}{4}H} \end{pmatrix}
$$

$$
\begin{pmatrix} \sigma_0^{(\frac{2}{4})} \\ \sigma_2^{(\frac{2}{4})} \\ \sigma_1^{(\frac{2}{4})} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \Phi^1_1\Phi'^1_1 \\ e^{i\frac{1}{2}H} \\ e^{-i\frac{1}{2}H} \end{pmatrix}
$$

Table 3.3: Z_6 twist

$$\sigma_0^{(\frac{1}{6})} = \Phi_1^1 e^{-i\frac{1}{6\sqrt{2}}H}$$

$$\begin{pmatrix} \sigma_0^{(\frac{2}{6})} \\ \sigma_1^{(\frac{2}{6})} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2}\bar{\alpha} & -\bar{\alpha} \end{pmatrix} \begin{pmatrix} e^{i\frac{\sqrt{2}}{3}H} \\ \Phi_2^2 e^{-i\frac{1}{3\sqrt{2}}H} \end{pmatrix}$$

$$\begin{pmatrix} \sigma_0^{(\frac{3}{6})} \\ \sigma_1^{(\frac{3}{6})} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \begin{pmatrix} \Phi_1^1 e^{i\frac{1}{2\sqrt{2}}H} \\ \Phi_3^3 e^{-i\frac{1}{2\sqrt{2}}H} \end{pmatrix}$$

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