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The CKM Matrix and CP Violation*

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ABSTRACT

The CKM picture of the quark sector is reviewed. We explain how the phenomena of quark mixing, CP violation and the absence of flavor changing neutral currents arise in the Standard Model. We describe the determination of the CKM elements from direct measurements, from unitarity and from indirect measurements. We discuss the motivation for schemes of quark mass matrices and analyze the Fritzsch scheme as an example. Finally, we list the experimental and theoretical improvements expected in the future in the determination of the CKM matrix.

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1. Introduction

The purpose of these lectures is to give a detailed and pedagogical description of the CKM picture of the quark sector [1,2]. The discussion is held within the Standard Model (SM) [3−5]. Examples of physics beyond the SM are given only to demonstrate which properties are unique to the SM and what kind of ingredients have to be superseded in order to have the predictions modified.

We first list those ingredients of the SM which are essential for the CKM picture and introduce our notations. Then we show how the quark sector becomes more and more intriguing as the number of generations increases: masses, mixing and CP violation arise in a world of one, two and three quark generations, respectively. The absence of flavor changing neutral currents is contrasted with a world of three quark flavors, and provides evidence for the existence of the top quark.

Once the theoretical picture of quark mixing and CP violation becomes clear, we turn our attention to phenomenology. We describe how various entries of the CKM matrix are measured by weak decays and deep inelastic neutrino scattering ("direct measurements"), while those that involve the top quark mixing are deduced from unitarity constraints. The unitarity triangle is presented. Further information is gained from loop processes ("indirect measurements"), *i.e.* $K-\bar{K}$ and $B-\bar{B}$ mixing. Finally, measurements of CP violating processes are discussed for both the K and the B systems.

The CKM parameters are measured in the hope of finding contradictions among various measurements that will provide evidence for physics beyond the SM. However, the numerical values of the parameters, even if consistent with each other, may provide clues to new physics which may relate quark masses and mixing parameters. We discuss the idea of schemes for quark mass matrices and describe the Fritzsch scheme as a specific example.

In the last chapter, we describe the future of the CKM picture. We list the improvements expected in experiment and theory and discuss their implications for our understanding of the quark sector and CP violation.

I. QUARK MIXING: BASICS

2. The Standard Model

To specify a model of elementary particles, one needs to give:

a. The symmetries of the Lagrangian,

b. The symmetry representations of fermions and scalars,

c. The symmetries of the vacuum (namely, the nature of the spontaneous symmetry breaking).

The Standard Model (SM) is defined as follows:

a. The gauge group is $SU(3)_C \times SU(2)_L \times U(1)_Y$. Thus there are twelve gauge bosons and the covariant derivative which determines all gauge interactions is given by

$$D^\mu = \partial^\mu + ig_s G_a^\mu L_a + igW_b^\mu T_b + ig' B^\mu Y. \tag{2.1}$$

Here L_a ($a = 1, \ldots, 8$) are the generators of $SU(3)_C$, T_b ($b = 1, 2, 3$) are the generators of $SU(2)_L$ and Y is the generator of $U(1)_Y$. There are three independent gauge couplings: g_s, g and g'.

b. Left-handed quarks are in $(3, 2)_{1/6}$ representations of the gauge group, namely triplets of $SU(3)_C$, doublets of $SU(2)_L$ and carry hypercharge $Y = 1/6$. Right-handed up-quarks are in $(3, 1)_{2/3}$ multiples while right-handed down-quarks are in $(3, 1)_{-1/3}$ multiplets. This determines the form of the various generator matrices in (2.1):

$$\begin{aligned} Q_L: &\quad L_a = \tfrac{1}{2}\lambda_a, \quad T_b = \tfrac{1}{2}\tau_b, \quad Y = 1/6, \\ u_R: &\quad L_a = \tfrac{1}{2}\lambda_a, \quad T_b = 0, \qquad Y = 2/3, \\ d_R: &\quad L_a = \tfrac{1}{2}\lambda_a, \quad T_b = 0, \qquad Y = -1/3. \end{aligned} \tag{2.2}$$

λ_a are the 3×3 Gell-Mann matrices while τ_b are the 2×2 Pauli matrices. The gauge interactions of the various quark multiplets are all derived from the kinetic energy term in the Lagrangian:

$$\mathcal{L}_{kin}(\text{fermions}) = \sum_{\text{multiplets } j} i\bar{\psi}_j \gamma^\mu D_\mu^{(j)} \psi_j. \tag{2.3}$$

The superscript (j) denotes that the various generators in the covariant derivative are in the appropriate representation. The scalar sector of the model consists of a single Higgs doublet $\phi(1,2)_{1/2}$.

c. The gauge symmetry is spontaneously broken,

$$SU(3)_C \times SU(2)_L \times U(1)_Y \;\rightarrow\; SU(3)_C \times U(1)_{EM}, \tag{2.4}$$

when the neutral member of the scalar doublet acquires a VEV:

$$\langle\phi\rangle = \left\langle \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \right\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}. \tag{2.5}$$

The two charged W bosons and one linear combination of W_3 and B, the Z boson, acquire masses, while the orthogonal combination of the neutral bosons, the photon A, remains massless:

$$\begin{aligned} W_\pm^\mu &= \frac{1}{\sqrt{2}}(W_1^\mu \mp i W_2^\mu): \quad M_W^2 = \tfrac{1}{4} g^2 v^2, \\ Z^\mu &= \cos\theta_W \; W_3^\mu - \sin\theta_W \; B^\mu : \quad M_Z^2 = M_W^2 / \cos^2\theta_W, \\ A^\mu &= \sin\theta_W \; W_3^\mu + \cos\theta_W \; B^\mu : \quad M_A^2 = 0, \end{aligned} \tag{2.6}$$

where

$$\tan\theta_W \equiv g'/g. \tag{2.7}$$

The phenomenon of quark mixing arises from the difference between the up sector and the down sector in the rotation from the interaction eigenbasis to the mass eigenbasis. To understand that, we need to study two types of quark interactions: their gauge interactions with charged vector-bosons and their Yukawa interactions with neutral scalars.

3. A World of Two Flavors: Quark Masses

We start by studying a hypothetical world of two quark flavors only. Our purpose it to follow in detail the mechanism for generating *quark masses.* However, there is no quark mixing in this world.

Assume that the spectrum of colored fermions consists of 12 degrees of freedom divided into three different multiplets:

$$Q_L^I = (3,2)_{1/6}; \quad u_R^I = (3,1)_{2/3}; \quad d_R^I = (3,1)_{-1/3}. \tag{3.1}$$

This is a one generation world. The superscript I stands for Interaction eigenstates. Mass eigenstates will carry no superscript. However, as here there is only one flavor of each electromagnetic charge, interaction eigenstates and mass eigenstates are identical. Thus, in this section we omit the superscript I.

The interactions of quarks with the $SU(2)_L$ gauge bosons are given by

$$-\mathcal{L}_W = \frac{g}{2}\overline{Q_L}\gamma^\mu \tau^a Q_L W_\mu^a. \tag{3.2}$$

The interactions of quarks with the scalar doublet are given by

$$-\mathcal{L}_Y = G\overline{Q_L}\phi d_R + F\overline{Q_L}\tilde{\phi} u_R + \text{h.c.} \tag{3.3}$$

where $\tilde{\phi} = i\sigma_2\phi^*$. Since the symmetry is spontaneously broken, $\langle\phi\rangle \neq 0$, we are able to physically distinguish among the various members within an $SU(2)_L$-multiplet. In particular, the quark doublet has two components of different charges, which we denote by

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}. \tag{3.4}$$

The charged current interactions in (3.2) are given by

$$-\mathcal{L}_W = \frac{g}{\sqrt{2}}\overline{u_L}\gamma^\mu d_L W_\mu^+ + \text{h.c.} \tag{3.5}$$

The mass terms that arise from the replacement of $\text{Re}(\phi^0) \to (v + H^0)/\sqrt{2}$ in (3.3) are given by:

$$-\mathcal{L}_M = \frac{Gv}{\sqrt{2}}\overline{d_L}d_R + \frac{Fv}{\sqrt{2}}\overline{u_L}u_R + \text{h.c.}, \tag{3.6}$$

namely

$$m_d = Gv/\sqrt{2}, \qquad m_u = Fv/\sqrt{2}. \tag{3.7}$$

4. A World of Four Flavors: Quark Mixing

The hypothetical world of four quark flavors is similar in many ways to ours. In particular, in this world *quark mixing* is simplest to understand. There is, however, one important ingredient of nature missing: CP violation.

Assume that the spectrum of colored fermions consists of 24 degrees of freedom divided into six multiplets of three different kinds:

$$\begin{aligned} Q_{L1}^I &= (3,2)_{1/6}; \quad Q_{L2}^I = (3,2)_{1/6}; \\ u_R^I &= (3,1)_{2/3}; \quad c_R^I = (3,1)_{2/3}; \\ d_R^I &= (3,1)_{-1/3}; \quad s_R^I = (3,1)_{-1/3}. \end{aligned} \tag{4.1}$$

This is a two generation world.

The interactions of quarks with the $SU(2)_L$ gauge bosons are given by

$$-\mathcal{L}_W = \frac{g}{2}\overline{Q_{L1}^I}\gamma^\mu\tau^a Q_{L1}^I W_\mu^a + \frac{g}{2}\overline{Q_{L2}^I}\gamma^\mu\tau^a Q_{L2}^I W_\mu^a. \tag{4.2}$$

The interactions of quarks with the scalar doublet are given by

$$\begin{aligned} -\mathcal{L}_Y = &G_{11}\overline{Q_{L1}^I}\phi d_R^I + F_{11}\overline{Q_{L1}^I}\tilde{\phi}u_R^I + G_{12}\overline{Q_{L1}^I}\phi s_R^I + F_{12}\overline{Q_{L1}^I}\tilde{\phi}c_R^I \\ &+G_{21}\overline{Q_{L2}^I}\phi d_R^I + F_{21}\overline{Q_{L2}^I}\tilde{\phi}u_R^I + G_{22}\overline{Q_{L2}^I}\phi s_R^I + F_{22}\overline{Q_{L2}^I}\tilde{\phi}c_R^I + \text{h.c.} \end{aligned} \tag{4.3}$$

Again, the spontaneous symmetry breaking means that the members within a doublet can be distinguished from each other, so we denote them by

$$Q^I_{L1} = \begin{pmatrix} u^I_L \\ d^I_L \end{pmatrix}; \quad Q^I_{L2} = \begin{pmatrix} c^I_L \\ s^I_L \end{pmatrix}. \tag{4.4}$$

The charged current interactions in (4.2) are given by

$$\begin{aligned} -\mathcal{L}_W =& \frac{g}{\sqrt{2}} \overline{u^I_L} \gamma^\mu d^I_L W^+_\mu + \frac{g}{\sqrt{2}} \overline{c^I_L} \gamma^\mu s^I_L W^+_\mu + \text{h.c.} \\ =& \frac{g}{\sqrt{2}} \left(\overline{u^I_L} \quad \overline{c^I_L} \right) \gamma^\mu \begin{pmatrix} d^I_L \\ s^I_L \end{pmatrix} W^+_\mu + \text{h.c.} \end{aligned} \tag{4.5}$$

The mass terms from (4.3) are

$$\begin{aligned} -\mathcal{L}_M =& \frac{v}{\sqrt{2}} \left(\overline{d^I_L} \quad \overline{s^I_L} \right) \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} d^I_R \\ s^I_R \end{pmatrix} \\ &+ \frac{v}{\sqrt{2}} \left(\overline{u^I_L} \quad \overline{c^I_L} \right) \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix} \begin{pmatrix} u^I_R \\ c^I_R \end{pmatrix} + \text{h.c.}, \end{aligned} \tag{4.6}$$

or, in a more compact notation,

$$M_d = Gv/\sqrt{2}, \qquad M_u = Fv/\sqrt{2}. \tag{4.7}$$

M_d and M_u are 2×2 matrices.

The mass eigenbasis corresponds, by definition, to diagonal mass matrices. We can always find unitary matrices V_{dL}, V_{dR}, V_{uL} and V_{uR} such that

$$V_{dL} M_D V^\dagger_{dR} = M^{\text{diag}}_D; \qquad V_{uL} M_U V^\dagger_{uR} = M^{\text{diag}}_U. \tag{4.8}$$

The M^{diag}_Q are real diagonal matrices. The mass eigenstates are then identified as

$$\begin{aligned} \begin{pmatrix} d_L \\ s_L \end{pmatrix} = V_{dL} \begin{pmatrix} d^I_L \\ s^I_L \end{pmatrix}, \quad \begin{pmatrix} d_R \\ s_R \end{pmatrix} = V_{dR} \begin{pmatrix} d^I_R \\ s^I_R \end{pmatrix}, \\ \begin{pmatrix} u_L \\ c_L \end{pmatrix} = V_{uL} \begin{pmatrix} u^I_L \\ c^I_L \end{pmatrix}, \quad \begin{pmatrix} u_R \\ c_R \end{pmatrix} = V_{uR} \begin{pmatrix} u^I_R \\ c^I_R \end{pmatrix}. \end{aligned} \tag{4.9}$$

The W-interactions (4.5) are given in the mass eigenbasis by

$$-\mathcal{L}_W = \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_L} & \overline{c_L} \end{pmatrix} \gamma^\mu (V_{uL} V_{dL}^\dagger) \begin{pmatrix} d_L \\ s_L \end{pmatrix} W_\mu^+ + \text{h.c.} \tag{4.10}$$

The matrix $(V_{uL} V_{dL}^\dagger)$ is the mixing matrix for 2 quark generations. It is a 2×2 unitary matrix. As such, it generally contains 4 parameters, of which one can be chosen as a real angle, θ_C, and 3 are phases:

$$(V_{uL} V_{dL}^\dagger) = \begin{pmatrix} \cos\theta_C \; e^{i\alpha} & \sin\theta_C \; e^{i\beta} \\ -\sin\theta_C \; e^{i\gamma} & \cos\theta_C \; e^{i(-\alpha+\beta+\gamma)} \end{pmatrix}. \tag{4.11}$$

By the transformation

$$(V_{uL} V_{dL}^\dagger) \to V = P_u (V_{uL} V_{dL}^\dagger) P_d^*, \tag{4.12}$$

with

$$P_u = \begin{pmatrix} e^{-i\alpha} & \\ & e^{-i\gamma} \end{pmatrix}, \quad P_d = \begin{pmatrix} 1 & \\ & e^{i(-\alpha+\beta)} \end{pmatrix}, \tag{4.13}$$

we eliminate the three phases from the mixing matrix. (We redefine the mass eigenstates $u_{L,R} \to P_u u_{L,R}$ and $d_{L,R} \to P_d d_{L,R}$, so that the mass matrices remain unchanged. In particular, they remain real.) Notice that there are three independent phase differences between the elements of P_u and those of P_d, and three phases in $(V_{uL} V_{dL}^\dagger)$. Consequently, there are no physically meaningful phases in V, and hence no CP violation:★

$$V = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}. \tag{4.14}$$

For two generations, V is called the Cabibbo matrix [1]. If $\sin\theta_C$ of (4.14) is different from zero, then the $W^\pm$ interactions mediate generation-changing currents.

★ In extended models, such as LRS [6] or SUSY [7], CP-violation in the quark sector is possible even with two generations only.

The W^+-boson couples to a pair of quarks $\bar{u}_i d_j$ with strength $g|V_{ij}|$. In particular, the $s \to u$ transition allows kaon decays, *e.g.* $K \to \pi e \nu$. The value of $\sin\theta_C = |V_{us}|$ can be determined from the rate of this decay, and is found to be

$$\sin\theta_C = 0.22. \tag{4.15}$$

5. A World of Six Flavors: CP Violation

It seems very likely that Nature is indeed a world of six quark flavors. From the theoretical point of view, it is not just a straightforward extension of the four quark world, because a new and important phenomenon arises: CP violation.

The spectrum of colored fermions consists of 36 degrees of freedom divided into nine multiplets of three different kinds, namely a three generation world. The multiplets additional to (4.1) are denoted by

$$Q^I_{L3} = \begin{pmatrix} t^I_L \\ b^I_L \end{pmatrix} = (3,2)_{1/6}; \qquad t^I_R = (3,1)_{2/3}; \qquad b^I_R = (3,1)_{-1/3}. \tag{5.1}$$

The derivation follows exactly the same lines as in the previous chapter. Similarly to (4.10) we get for the charged current interaction in the mass eigenbasis:

$$-\mathcal{L}_W = \frac{g}{\sqrt{2}} (\,\overline{u_L} \quad \overline{c_L} \quad \overline{t_L}\,) \gamma^\mu (V_{uL} V^\dagger_{dL}) \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W^+_\mu + \text{h.c.} \tag{5.2}$$

The matrix $(V_{uL} V^\dagger_{dL})$ is the mixing matrix for three quark generations. It is a 3×3 unitary matrix. As such, it generally contains 9 parameters, of which three can be chosen as real angles, θ_{12}, θ_{23} and θ_{13}, and six are phases. We may again reduce the number of phases in the mixing matrix V by redefining the phases of the quark mass eigenstates:

$$V = P_u(V_{uL}V_{dL}^\dagger)P_d^*. \tag{5.3}$$

The crucial point is that there are only *five* independent phase differences between the elements of P_u and those of P_d, while there are *six* phases in $(V_{uL}V_{dL}^\dagger)$. Consequently, the mixing matrix V contains one physically meaningful phase [2], δ. The standard parametrization of V is [8, 9 − 13]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{5.4}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The matrix V for the three generation mixing is called the Cabibbo – Kobayashi – Maskawa matrix or, in short, the CKM matrix.

6. CP Violation from the CKM Matrix

The unremovable phase in the CKM matrix allows possible CP violation. To demonstrate that, we use a variant of an argument presented in ref. [14]. The charge conjugation matrix C fulfills

$$C\gamma_\mu C^{-1} = -\gamma_\mu^T, \quad C\gamma_5 C^{-1} = \gamma_5^T. \tag{6.1}$$

We work with representations for γ-matrices where

$$-C = C^{-1} = C^T = C^\dagger. \tag{6.2}$$

The charge conjugation transformation of spinor fields is

$$\psi \to \eta_c C\bar{\psi}^T, \quad \bar{\psi} \to \eta_c^* \psi^T C. \tag{6.3}$$

The parity transformation of spinor fields is

$$\psi(x) \to \eta_p \gamma^0 \psi(\tilde{x}), \qquad \bar{\psi}(x) \to \eta_p^* \bar{\psi}(\tilde{x})\gamma^0, \tag{6.4}$$

with $\tilde{x}^\mu = x_\mu$. Thus the CP transformation of spinor fields is

$$\psi(x) \to -\eta C \psi^*(\tilde{x}), \qquad \bar{\psi}(x) \to -\eta^* \bar{\psi}^*(\tilde{x}) C. \tag{6.5}$$

The CP transformations of scalar and left-handed currents are then

$$\begin{aligned} \bar{\psi}_i \psi_j &\to \bar{\psi}_j \psi_i, \\ \bar{\psi}_i \gamma^\mu (1-\gamma_5)\psi_j &\to -\bar{\psi}_j \gamma_\mu (1-\gamma_5)\psi_i, \end{aligned} \tag{6.6}$$

where we used

$$(\bar{\psi}_i \Gamma \psi_j)^* = -\bar{\psi}_j (\gamma_0 \Gamma^\dagger \gamma_0)\psi_i. \tag{6.7}$$

The CP transformation law for the charged vector boson is

$$W_\mu^\pm(x) \to -W^{\mp\mu}(\tilde{x}). \tag{6.8}$$

A mass term or gauge coupling can be invariant under (6.6) if the masses and couplings are real. In particular, consider the coupling of $W^\pm$ to quarks. It has the form

$$g V_{ij} \bar{u}_i \gamma_\mu W^{\mu +}(1-\gamma_5) d_j + g V_{ij}^* \bar{d}_j \gamma_\mu W^{\mu -}(1-\gamma_5) u_i. \tag{6.9}$$

The CP operation interchanges the two terms except that V_{ij} and V_{ij}^* are not interchanged. Thus, CP is a symmetry only if there is some basis in which all couplings (and masses) are real. If there were only the four quarks u, d, s and c, the $SU(2) \times U(1)$ interactions could not violate CP.

It is, of course, an important experimental question whether the KM phase δ is, in fact, the source of the observed CP violation. We will discuss this question in detail when we study K^0 physics. Note, however, that CP is not *necessarily* violated in the three generation case. If any of the following conditions held, CP would *not* be violated:

1. There is mass degeneracy among the three up-sector quarks or among the three down-sector quarks. Suppose, for example, that the u and the c quarks are degenerate in mass. Then any linear combination of the mass eigenstates u and c is still a mass eigenstate. Consequently, instead of the *diagonal* unitary matrix P_U of (5.3), we may use a more general form:

$$P_U = \begin{pmatrix} U & \\ & e^{i\phi} \end{pmatrix} \tag{6.10}$$

where U is any 2×2 unitary matrix. This allows us to remove one more phase (and an angle) from V, thus making V real.

2. Any of the mixing angles θ_{12}, θ_{23} or θ_{13} is an integer multiple of $\pi/2$.

3. The phase in the CKM matrix vanishes, $\sin\delta = 0$.

The above conditions can be summarized in one equation, which must be fulfilled if CP is to be violated:

$$(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_t^2 - m_u^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2)(m_b^2 - m_d^2)\ J \neq 0, \tag{6.11}$$

where

$$J = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta. \tag{6.12}$$

The above condition can be restated in another way [15], which is explicitly independent of the parametrization of V. Take the quark mass matrices M_D and M_U in any interaction eigenbasis. Then the LHS of (6.11) equals (up to a possible sign) $\mathrm{Im}\{\det[M_D M_D^\dagger, M_U M_U^\dagger]\}/2$. Thus CP is violated in the three generation Standard Model if and only if [15]:

$$\mathrm{Im}\{\det[M_D M_D^\dagger, M_U M_U^\dagger]\} \neq 0. \tag{6.13}$$

J, the function of the mixing angles and the phase, can also be written in a form which is explicitly parametrization-independent:

$$|J| = |\text{Im}(V_{ij}V_{lk}V_{ik}^*V_{lj}^*)| \tag{6.14}$$

for any choice of $i \neq l,\ j \neq k$.

The numerical values that we later find for the different mixing angles imply that $|J| \leq 10^{-4}$, even for $\sin\delta = 1$. This value is to be compared with the maximum possible value of $1/(6\sqrt{3})$: the SM predicts a small intrinsic value for this measure of CP-violation.

7. A World of Three Flavors: FCNC

A world of three quark flavors does not fit into a model with all left-handed quarks in doublets of $SU(2)_L$. How can we incorporate interactions of the strange quark in this picture? The solution that we now describe is wrong. Yet, it is of historical interest and, moreover, helps to understand some of the unique properties of the SM. In particular, we will see that in the three-flavor world there are flavor-changing neutral currents (FCNC) which are forbidden (at tree level) in the SM.

Assume that s_L is an $SU(2)_L$-singlet:

$$\begin{aligned} &Q_L^I = (3,2)_{1/6}; \quad s_L^I = (3,1)_{-1/3}; \\ &u_R^I = (3,1)_{2/3}; \quad d_R^I = (3,1)_{-1/3}; \quad s_R^I = (3,1)_{-1/3}. \end{aligned} \tag{7.1}$$

Consequently, s_L^I does not have gauge interactions with the charged W-bosons:

$$-\mathcal{L}_W = \frac{g}{2}\overline{Q_L^I}\gamma^\mu\tau^a Q_L^I W_\mu^a. \tag{7.2}$$

The interactions of quarks with the scalar doublet are given by

$$-\mathcal{L}_Y = G_d\overline{Q_L^I}\phi d_R^I + G_s\overline{Q_L^I}\phi s_R^I + F\overline{Q_L^I}\tilde{\phi}u_R^I + \text{h.c.} \tag{7.3}$$

An important difference from the cases discussed so far is that quark representations are no longer purely chiral. Consequently, bare mass terms appear in the

Lagrangian:

$$-\mathcal{L}_M^{\text{bare}} = m_{sd}\overline{s_L^I}d_R^I + m_{ss}\overline{s_L^I}s_R^I + \text{h.c.} \tag{7.4}$$

With the spontaneous symmetry breaking we get contributions to quark masses from both $\mathcal{L}_Y$ and $\mathcal{L}_M^{\text{bare}}$:

$$-\mathcal{L}_M = (\,\overline{d_L^I} \quad \overline{s_L^I}\,)\begin{pmatrix} m_{dd} & m_{ds} \\ m_{sd} & m_{ss} \end{pmatrix}\begin{pmatrix} d_R^I \\ s_R^I \end{pmatrix} + m_u\overline{u_L^I}u_R^I + \text{h.c.}, \tag{7.5}$$

where

$$m_{dd} = G_d v/\sqrt{2}, \quad m_{ds} = G_s v/\sqrt{2}, \quad m_u = Fv/\sqrt{2}. \tag{7.6}$$

The mass eigenbasis for the down-sector corresponds to diagonal M_d, so we find unitary 2×2 matrices V_{dL} and V_{dR} such that

$$V_{dL}\begin{pmatrix} m_{dd} & m_{ds} \\ m_{sd} & m_{ss} \end{pmatrix} V_{dR}^\dagger = \begin{pmatrix} m_d & \\ & m_s \end{pmatrix}, \tag{7.7}$$

and correspondingly

$$\begin{pmatrix} d_L \\ s_L \end{pmatrix} = V_{dL}\begin{pmatrix} d_L^I \\ s_L^I \end{pmatrix}, \quad \begin{pmatrix} d_R \\ s_R \end{pmatrix} = V_{dR}\begin{pmatrix} d_R^I \\ s_R^I \end{pmatrix}. \tag{7.8}$$

The charged current interactions are given by

$$\begin{aligned} -\mathcal{L}_W &= \frac{g}{\sqrt{2}}\overline{u_L^I}\gamma^\mu d_L^I W_\mu^+ + \text{h.c.} \\ &= \frac{g}{\sqrt{2}}\overline{u_L}\gamma^\mu (\,\cos\theta_C \quad \sin\theta_C\,)\begin{pmatrix} d_L \\ s_L \end{pmatrix} W_\mu^+ + \text{h.c.} \end{aligned} \tag{7.9}$$

(Any phases in V, the 1×2 mixing matrix, can be removed by an appropriate redefinition of the mass eigenstates.) The $s \to u$ transition still proceeds via the charged current interactions, and the same value for $\sin\theta_C = |V_{us}|$ will be measured by the $K \to \pi e \nu$ decay as in the four flavor case.

Trouble begins when we turn to neutral current interactions. The Z^0 boson, being a combination of the W_3 and B gauge bosons as given in (2.6), couples to the $(T_3 - \sin^2\theta_W Q)$-charge:

$$
\begin{aligned}
-\mathcal{L}_Z = \frac{g}{\cos\theta_W}[\ & \overline{u}^I_L\gamma^\mu(\tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_W)u^I_L + \overline{u}^I_R\gamma^\mu(-\tfrac{2}{3}\sin^2\theta_W)u^I_R \\
& + \overline{d}^I_L\gamma^\mu(-\tfrac{1}{2} + \tfrac{1}{3}\sin^2\theta_W)d^I_L + \overline{d}^I_R\gamma^\mu(+\tfrac{1}{3}\sin^2\theta_W)d^I_R \\
& + \overline{s}^I_L\gamma^\mu(+\tfrac{1}{3}\sin^2\theta_W)s^I_L + \overline{s}^I_R\gamma^\mu(+\tfrac{1}{3}\sin^2\theta_W)s^I_R\ ]\ Z_\mu.
\end{aligned}
\tag{7.10}
$$

Note that all couplings are diagonal in the interaction eigenbasis, as they should be by definition. We now concentrate on the T_3-coupling of the down-sector. All other couplings are trivially transformed into the mass eigenbasis: just omit the superscript I. However,

$$
\begin{aligned}
& \frac{g}{\cos\theta_W}\overline{d}^I_L\gamma^\mu(-\tfrac{1}{2})d^I_L Z_\mu = \\
& \frac{g}{\cos\theta_W}(\,\overline{d}_L\quad \overline{s}_L\,)\begin{pmatrix}\cos\theta_C \\ \sin\theta_C\end{pmatrix}\gamma^\mu(-\tfrac{1}{2})(\,\cos\theta_C\quad \sin\theta_C\,)\begin{pmatrix}d_L \\ s_L\end{pmatrix} Z_\mu = \\
& -\frac{g}{2\cos\theta_W}(\,\overline{d}_L\quad \overline{s}_L\,)\begin{pmatrix}\cos^2\theta_C & \cos\theta_C\sin\theta_C \\ \cos\theta_C\sin\theta_C & \sin^2\theta_C\end{pmatrix}\gamma^\mu\begin{pmatrix}d_L \\ s_L\end{pmatrix} Z_\mu.
\end{aligned}
\tag{7.11}
$$

The non-diagonal terms signal flavor changing neutral currents!

This situation is very different from the world of two generations described above. There, s^I_L is a member of a second quark doublet, see (4.4). As a result, the coupling of s^I_L to the Z^0-boson changes from (7.10) to

$$
\frac{g}{\cos\theta_W}\overline{s}^I_L\gamma^\mu(-\tfrac{1}{2} + \tfrac{1}{3}\sin^2\theta_W)s^I_L Z_\mu. \tag{7.12}
$$

(In addition there will, of course, be terms describing the c-quark couplings to the Z^0-boson.) The part of the interaction that we isolated in (7.11) is modified into

$$
\begin{aligned}
& \frac{g}{\cos\theta_W}(\,\overline{d}^I_L\quad \overline{s}^I_L\,)\gamma^\mu(-\tfrac{1}{2})\begin{pmatrix}d^I_L \\ s^I_L\end{pmatrix} Z_\mu = \\
& -\frac{g}{2\cos\theta_W}(\,\overline{d}_L\quad \overline{s}_L\,)\begin{pmatrix}1 & \\ & 1\end{pmatrix}\gamma^\mu\begin{pmatrix}d_L \\ s_L\end{pmatrix} Z_\mu.
\end{aligned}
\tag{7.13}
$$

There are no FCNC in the Standard Model. The reason for the difference between (7.11) and (7.13) can be traced back to the difference in the charged current mixing matrix. The neutral current mixing matrix $U = V^\dagger V$. When V is unitary, as in (4.10), $U = \mathbf{1}$ as in (7.13). When V is not unitary, as in (7.9) (V is a 1×2 sub-matrix of the unitary matrix $V_{dL}^\dagger$), $U \neq \mathbf{1}$ as in (7.11).

The difference in the predicted phenomenology is dramatic: the interaction (7.11) induces the decay

$$K_L^0 \to \mu^+ \mu^- \tag{7.14}$$

at a rate of the same order of magnitude as

$$K^+ \to \mu^+ \nu_\mu . \tag{7.15}$$

In fact, (7.14) is suppressed by a factor of 10^{-8} compared to (7.15)!

This problem, which was noted at the time when only three quarks were known, drove Glashow, Iliopoulos and Maiani (GIM) [16] to propose the existence of the charm quark, in an $SU(2)$ doublet with s_L^I.

8. A World of Five Flavors: Is There a Top?

If the reason for not having experimentally observed the top quark is that it does not exist, and if the gauge structure of electroweak interactions is indeed $SU(2)_L \times U(1)_Y$, then the b_L quark has to be in a singlet of $SU(2)_L$. The implications of this on phenomenology are readily understood on the basis of our analysis of the three flavor case.

The five flavor world with b_L an $SU(2)$-singlet has Z^0-mediated FCNC. In particular, there are non-diagonal couplings involving the bottom quark:

$$\begin{aligned} U_{db} &= V_{ud}^* V_{ub} + V_{cd}^* V_{cb}, \\ U_{sb} &= V_{us}^* V_{ub} + V_{cs}^* V_{cb}. \end{aligned} \tag{8.1}$$

Consequently, the decay $B \to \ell^+ \ell^- X$ will proceed in a rate comparable to the charged current decay $B \to \ell \nu_\ell X$ [17]:

$$\frac{\Gamma(B \to \ell^+\ell^- X)}{\Gamma(B \to \ell \nu_\ell X)} = [(\tfrac{1}{2} - \sin^2\theta_W)^2 + (\sin^2\theta_W)^2] \frac{|U_{db}|^2 + |U_{sb}|^2}{|V_{ub}|^2 + F_{ps}|V_{cb}|^2} \approx \frac{1}{4}. \quad (8.2)$$

(F_{ps} is a phase space factor.) Experimentally [19], this ratio is smaller than 5×10^{-4}!

Additional evidence for the existence of the top quark comes from measurements of the Z-width and of the forward-backward asymmetry in $e^+e^- \to b\bar{b}$. Using recent LEP results for these two quantities gives $T_3(b_{L,R})$ rather accurately [18]:

$$T_3(b_L) = -0.490^{+0.015}_{-0.012}; \quad T_3(b_R) = -0.028 \pm 0.056. \quad (8.3)$$

This is consistent with the SM values, $T_3(b_L) = -1/2$, $T_3(b_R) = 0$, and certainly excludes the possibility that the bottom quark is an $SU(2)_L$-singlet.

II. MEASURING THE CKM PARAMETERS: DIRECT MEASUREMENTS AND UNITARITY CONSTRAINTS

9. Direct Measurements

In direct measurements we measure processes which occur at the tree level within the SM. The assumption made here is that there are no processes from new physics which compete with SM tree-level processes. This assumption holds in most models which go beyond the SM. Thus, we expect the values of CKM matrix elements which are extracted from direct measurements to hold even if the SM is only a low-energy effective theory.

The simplest example of a model in which direct measurements would lead to wrong values for the CKM elements is a two Higgs doublet model, with a light charged Higgs. The two body decay into a charged Higgs and a quark could dominate over W-mediated decays. However, the present limits on the mass of a charged Higgs ($m_{H^{\pm}} \geq 41.7\ GeV$ [20]) allow this decay mode for only the top-quark, and thus none of the measurements described in this chapter could be affected.

As the top-quark has not been experimentally observed, its mixings, V_{ti}, cannot be directly measured. At present we have direct information on the six elements of the first two rows in the CKM matrix, and in this section we describe this information. Most of our discussion follows the lines of refs. [8, 21]. For the mixings of the b-quark, we discuss in detail the implications of the Heavy Quark Symmetry (HQS) [22] for improved measurements with future facilities [23].

Our present knowledge of the matrix elements comes from the following sources:

9.1 MEASURING $|V_{ud}|$

Nuclear beta decay, when compared to muon decay, gives [24 – 27]

$$|V_{ud}| = 0.9744 \pm 0.0010\ . \tag{9.1}$$

This includes refinements in the analysis of the radiative corrections, especially the order $Z\alpha^2$ effects, which have brought the ft-values from low and high Z Fermi transitions into good agreement.

9.2 MEASURING $|V_{us}|$

Analysis of K_{e3} decays: $K^+ \to \pi^0 e^+ \nu_e$ and $K_L^0 \to \pi^- e^+ \nu$, yields [28]

$$|V_{us}| = 0.2196 \pm 0.0023\ . \tag{9.2}$$

At the quark level the process is $s \to u e \nu_e$. However, a calculation from first principles at the quark level cannot be carried out [29]:

a. The final spectrum is completely dominated by the single pion state (the two pion final state has a branching ratio smaller by four orders of magnitude). Quark - meson duality is expected to hold when there is a dense set of final states, which is certainly not the case here.

b. There are large QCD corrections as the relevant scale for $\alpha_s(\mu)$ is $\mu = \mathcal{O}(m_s)$, but $m_s \sim \Lambda_{QCD}$ (the scale at which, by definition, $\alpha_s \sim 1$). Thus, a perturbative QCD expansion is meaningless.

c. There are large uncertainties in m_s; first, it is a running mass and we do not know the relevant scale and second, even if we knew the scale, the uncertainty in m_s is still about 30%. This is significant since the phase space for the decay depends on $(m_s)^5$.

Thus, $|V_{us}|$ has to be calculated at the meson level:

$$\frac{BR(K \to \pi e \nu)}{\tau(K)} = \left[\frac{G_F^2 M_K^5 C^2}{192\pi^3}\right] F_{ps}(1+r)|f_+(0)|^2|V_{us}|^2. \tag{9.3}$$

The quantities on the LHS of this equation are given by experiments with an overall accuracy of 1-1.5%. The quantities in brackets are known. The phase space factor F_{ps} depends on an experimentally-fitted parameter, which introduces a 0.5% uncertainty. The radiative corrections r can be calculated to an accuracy of about 0.3%, but an ambiguity in the way these corrections were incorporated into various data analyses adds up to 1% uncertainty in $(1+r)$. The main theoretical uncertainty is in the normalization of the form factor $|f_+(0)|$. However, in the $SU(3)$ limit $(m_u = m_d = m_s)$ we have $|f_+(0)| = 1$, and deviations from this value are only second order in the symmetry breaking parameter. The approximate symmetry allows a determination of $|f_+(0)|$ with an uncertainty of only 0.8%. The isospin violation between K_{e3}^+ and K_{e3}^0 decays has been taken into account, bringing the values of $|V_{us}|$ extracted from these two decays into agreement at the 1% level of accuracy. The analysis of hyperon decay data has larger theoretical uncertainties because of first order $SU(3)$ symmetry breaking effects in the axial-vector couplings, but due account of symmetry breaking [30] applied to the WA2 data [31] gives a corrected value of 0.222 ± 0.003 . We average these two results to obtain:

$$|V_{us}| = 0.2205 \pm 0.0018 \; . \tag{9.4}$$

9.3 MEASURING $|V_{cd}|$

The magnitude of $|V_{cd}|$ may be deduced from neutrino and antineutrino production of charm off valence d quarks. The dimuon production cross sections of the CDHS group [32] yield $\overline{B}_c \, |V_{cd}|^2 = (0.41 \pm 0.07) \times 10^{-2}$, where $\overline{B}_c$ is the semileptonic branching fraction of the charmed hadrons produced. The corresponding preliminary value from a recent Tevatron experiment [33] is $\overline{B}_c \, |V_{cd}|^2 = (0.534^{+0.052}_{-0.078}) \times 10^{-2}$. Averaging these two results gives $\overline{B}_c \, |V_{cd}|^2 = (0.47 \pm 0.05) \times 10^{-2}$. Supplementing this with measurements of the semileptonic branching fractions of charmed mesons [34], weighted by a production ratio of $D^0/D^+ = (60 \pm 10)/(40 \mp 10)$, to give $\overline{B}_c = 0.113 \pm 0.015$, yields

$$|V_{cd}| = 0.204 \pm 0.017. \tag{9.5}$$

A second method is to measure the semileptonic strangeless D decay. At present there is only one such measurement and with large uncertainties [35]:

$$BR(D^0 \to \pi^- e^+ \nu) = (3.9^{+2.3}_{-1.1} \pm 0.4) \times 10^{-3}. \tag{9.6}$$

This, together with the theoretical uncertainties in the form factor, make this method less accurate at present.

9.4 MEASURING $|V_{cs}|$

Values of $|V_{cs}|$ from neutrino production of charm are dependent on assumptions about the strange quark density in the parton-sea. The most conservative assumption, that the strange-quark sea does not exceed the value corresponding to an $SU(3)$ symmetric sea, $2S \leq \bar{U} + \bar{D}$, leads to a lower bound [32], $|V_{cs}| > 0.59$. It is more advantageous to proceed analogously to the method used for extracting $|V_{us}|$ from K_{e3} decay; namely, we compare the experimental value for the width of $D_{\ell 3}$ decay with the expression (The result for $M = 2.2$ GeV is found in ref. [36]) that follows from the standard weak interaction amplitude:

$$\Gamma(D \to \bar{K} e^+ \nu_e) = |f_+^D(0)|^2 |V_{cs}|^2 (1.54 \times 10^{11} \text{sec}^{-1}). \tag{9.7}$$

Here $f_+^D(q^2)$, with $q = p_D - p_K$, is the form factor relevant to D_{e3} decay; its variation has been taken into account with the parametrization $f_+^D(t)/f_+^D(0) = M^2/(M^2 - t)$ and $M = 2.1\ GeV/c^2$, a form and mass consistent with Mark III [35] and TPS [37] measurements. Combining data on branching ratios for $D_{\ell 3}$ decays [35, 37]

$$BR(D^0 \to K^- e^+ \nu_e) = \begin{cases} (3.4 \pm 0.5 \pm 0.4) \times 10^{-2} & \text{MARK III} \\ (3.8 \pm 0.5 \pm 0.6) \times 10^{-2} & \text{TPS} \end{cases} \tag{9.8}$$

with accurate values for τ_{D^0} from the E691 [38] and E687 [39] experiments in Fermilab,

$$\tau(D^0) = \begin{cases} (4.22 \pm 0.08 \pm 0.10) \times 10^{-13} \text{ sec} & \text{E691} \\ (4.24 \pm 0.11 \pm 0.07) \times 10^{-13} \text{ sec} & \text{E687} \end{cases} \tag{9.9}$$

gives the value $(0.85 \pm 0.10) \times 10^{11}$ sec^{-1} for $\Gamma(D \to \bar{K} e^+ \nu_e)$. Therefore

$$|f_+^D(0)|^2 |V_{cs}|^2 = 0.55 \pm 0.07. \tag{9.10}$$

A very conservative assumption is that $|f_+^D(0)| < 1$, from which it follows that $|V_{cs}| > 0.70$. Calculations of the form factor either performed [40, 41] directly at

$q^2 = 0$, or done [42] at the maximum value of $q^2 = (m_D - m_K)^2$ and interpreted at $q^2 = 0$ using the measured q^2 dependence, yield $f_+^D(0) = 0.7 \pm 0.1$. It follows that

$$|V_{cs}| = 1.06 \pm 0.18 \ . \tag{9.11}$$

The constraint of unitarity when there are only three generations gives a much tighter bound (see below). The ratio $|V_{cd}/V_{cs}|$ is free of the uncertainties in $\tau(D^0)$. Moreover, it depends on the *ratio* $|f_+^{D\to\pi}/f_+^{D\to K}|$ which is 1 in the $SU(3)$ limit, and expected to hold within 10%. We get $|V_{cd}/V_{cs}| = 0.25 \pm 0.06$.

9.5 MEASURING $|V_{ub}|$

The ratio $|V_{ub}/V_{cb}|$ can be obtained from the semileptonic decay of B mesons by fitting to the lepton energy spectrum as a sum of contributions involving $b \to u$ and $b \to c$. The relative overall phase space factor between the two processes is calculated from the usual four-fermion interaction with one massive fermion (c quark or u quark) in the final state. The value of this factor depends on the quark masses, but is roughly one-half (in suppressing $b \to c$ compared to $b \to u$). Both the CLEO [43] and ARGUS [44,45] collaborations have reported evidence for $b \to u$ transitions in semileptonic B decays. The interpretation of the result in terms of $|V_{ub}/V_{cb}|$ depends fairly strongly on the theoretical model used to generate the lepton energy spectrum, especially for $b \to u$ transitions [41,42,46]. Combining the experimental and theoretical uncertainties, we quote

$$q \equiv |V_{ub}/V_{cb}| = 0.11 \pm 0.05. \tag{9.12}$$

The heavy quark symmetry holds promise to allow a determination of $|V_{ub}|$ in a much more accurate (and less model-dependent) way [23]: the ratio $|V_{ub}/V_{cd}|$ can be achieved by a comparison of the spectra of $B \to Xe\nu$ and $D \to Xe\nu$ for some charmless final state X.

For $X = \pi$, a measurement of the spectrum from D decay will allow predictions of the kinematically similar portion of the spectrum in B decay. Of the two form factors,

$$\langle \pi | V_\mu | D \rangle = f_+ (p_D + p_\pi)_\mu + f_- (p_D - p_\pi)_\mu, \tag{9.13}$$

only f_+ is measurable because $|p_D - p_\pi| \sim m_e$. However, f_+^D and f_+^B are not simply related by mass rescaling. If the $1/M_D$ corrections are ignored, uncertainties of order 20% in the determination of $|V_{ub}|$ are introduced (the model of ref. [42] was used for this estimate). A correction for this error can be made only at the price of some model dependence.

A potentially better way is to use the final $X = \rho$ state. (This mode has been recently observed [47] with $BR(B^+ \to \rho^0 \ell^+ \nu) = (1.13 \pm 0.36 \pm 0.26) \times 10^{-3}$.) In this case an angular analysis is needed to separate the form factors for $D \to \rho e \nu$,

$$\begin{aligned} \langle \rho | V_\mu | D \rangle =& ig \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu} (p_D + p_\rho)^\lambda (p_D - p_\rho)^\sigma, \\ \langle \rho | A_\mu | D \rangle =& if \epsilon^*_\mu + a_+ (\epsilon^* \cdot p_D)(p_D + p_\rho)_\mu + a_- (\epsilon^* \cdot p_D)(p_D - p_\rho)_\mu, \end{aligned} \tag{9.14}$$

and make any predictions for $B \to \rho e \nu$. Again, one of the form factors – a_- – is not measurable. However, two of the measurable ones, f and g, can be obtained for B by simple scaling. If the a_+ form factor contributes negligibly to the total rate, then a measurement of the total $B \to \rho e \nu$ rate would suffice to accurately determine $|V_{ub}/V_{cd}|$. Various models seem to differ in their predictions of whether this is indeed the case. Fortunately, one can avoid this model dependence altogether if there is sufficient data to measure angular correlations in $B \to \rho e \nu$ decays and thus separate the f^B and g^B form factors.

Another alternative would be to fully analyze the $D \to K^* e \nu$ form factors and use flavor $SU(3)$ symmetry to predict those for $D \to \rho e \nu$. It may be best to determine $|V_{ub}|$ in all the above methods and use the spread in the results as a measure of the uncertainty.

Finally, if one could measure the differential width for inclusive charmless semileptonic B decays, then the HQS provides sum-rules that will allow model-independent determination of $|V_{ub}|$ [48]. However, the task is probably experimentally impossible.

9.6 MEASURING $|V_{cb}|$

The magnitude of V_{cb} itself can be determined if the measured semileptonic bottom hadron partial width is assumed to be that of a b quark decaying through the usual $V - A$ interaction:

$$\Gamma(b \to c\ell\bar{\nu}_\ell) = \frac{BR(b \to c\ell\bar{\nu}_\ell)}{\tau_b} = \frac{G_F^2 m_b^5}{192\pi^3}\,\eta F(m_c/m_b)|V_{cb}|^2 , \tag{9.15}$$

where τ_b is the b lifetime, η is a QCD correction factor and $F(m_c/m_b)$ is the phase space factor noted above as approximately one-half. (The spectator quark model has been shown to hold for the inclusive rate in the heavy quark symmetry limit [48].) From various measurements held at the $\Upsilon(4S)$ resonance, the semi-leptonic branching ratio is found to be [47]

$$BR(B \to \ell\nu X) = (10.3 \pm 0.2) \times 10^{-2}. \tag{9.16}$$

New results from various LEP measurements are consistent with this range [49]. The lifetime of bottom hadrons is [8]

$$\tau_b = (1.18 \pm 0.11) \times 10^{-12} \text{ sec.} \tag{9.17}$$

Adding to this average the recent LEP results yields [49] $\tau_b = 1.28 \pm 0.06$ psec. However, most of the error on $|V_{cb}|$ derived from (9.15) is not from the experimental uncertainties, but in the theoretical uncertainties in choosing a value of m_b. Instead of a quark model, we quote the value derived from the $B_{\ell 3}$ decay, $\bar{B} \to D\ell\bar{\nu}_\ell$, by comparing the observed rate with the theoretical expression that involves a form factor, $f_+^B(q^2)$. This is analogous to what gives the most accurate values for $|V_{us}|$ (from K_{e3} decay) and $|V_{cs}|$ (from $D_{\ell 3}$ decay). It avoids all questions of what masses to use, and the heavy quarks in both the initial and final states give more confidence in the accuracy of the theoretical calculations of the form factor. With account of a number of models of the form factor, the data [50 – 52] yield

$$|V_{cb}| = 0.044 \pm 0.009 \ . \tag{9.18}$$

The central value and the error are now comparable to what is obtained from the inclusive semileptonic decays [51], but ultimately, with more data and more confidence in the calculation of the form factor consistent with the heavy quark symmetry, exclusive semileptonic decays should provide the most accurate value of $|V_{cb}|$. The mode to study is $B \rightarrow D^* e\nu$ for which $1/M$ corrections vanish at the kinematic limit point (the leading model dependent corrections are of order Λ^2/M_D^2). The model dependence arises principally from the extrapolation to the end point of the spectrum and hence can be significantly reduced with a high statistics study of that region. A recent calculation [53] using a modified version of the model of ref. [41] to calculate the corrections to the HQS gives

$$|V_{cb}| = 0.045 \pm 0.007 \ . \tag{9.19}$$

In what follows, we use this range for $|V_{cb}|$.

10. Unitarity

10.1 INTRODUCTION

The requirement of unitarity can be simply stated as $V^\dagger V = \mathbf{1}$. This imposes the following conditions on the matrix elements:

$$\sum_{j=1}^{n} |V_{ij}|^2 = 1; \quad \sum_{i=1}^{n} |V_{ij}|^2 = 1; \quad \sum_{k=1}^{n} V_{ik}^* V_{kj} = 0. \tag{10.1}$$

(In the last equation $i \neq j$.) Unitarity may be used in several ways:

1. If we directly measure enough of the matrix elements, we may check whether their values are consistent with the unitarity constraint. We illustrate this in a two generation model, and explain the present situation in the three generation case.
2. Within the minimal SM, where neutrinos are all massless, the number of generations is known to be three. We may then find values (or allowed ranges) for the matrix elements which have not been directly measured.
3. In extensions of the Standard Model, where neutrinos of higher generations may be very massive, the number of generations is only known to be *at least* three. We may still give upper bounds on the unmeasured matrix elements.

10.2 TWO GENERATIONS

To demonstrate how unitarity gives a consistency check on our measurements, we now pretend to know of two generations only [54]. The mixing matrix is the 2×2 Cabibbo matrix. Direct measurements give the following range for the absolute values of its elements:

$$|V_C| = \begin{pmatrix} 0.9744 \pm 0.0010 & 0.2205 \pm 0.0018 \\ 0.204 \pm 0.017 & 1.06 \pm 0.18 \end{pmatrix}. \tag{10.2}$$

Unitarity implies that the above matrix depends on one parameter only:

$$V_C = \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \end{pmatrix}. \tag{10.3}$$

With the above measurements we have certainly overdetermined the Cabibbo angle. The test of the two generation Standard Model is the following: Can we find a range for the Cabibbo angle which is consistent with all measurements? The answer is in the affirmative if

$$0.220 \leq s_{12} \leq 0.221. \tag{10.4}$$

Thus, a two generation picture is consistent within the experimental errors on the matrix elements; we could not tell that there is a third generation if it were not for its direct observation (or from CP violation). From our knowledge about $|V_{cb}|$ and $|V_{ub}|$ we know that the third generation mixings would be probed only if we reached an accuracy level of 10^{-4} in the determination of $|V_{ui}|$ or 10^{-3} in the determination of $|V_{ci}|$ $(i = d, s)$; this is well beyond the present level of accuracy. At present, the values in (10.2) imply only the following mild bounds on the possible mixings of a third generation:

$$|V| = \begin{pmatrix} \cdot & \cdot & 0-0.07 \\ \cdot & \cdot & 0-0.51 \\ 0-0.13 & 0-0.50 & 0-1 \end{pmatrix} \tag{10.5}$$

The results derived here have some bearing on the three generation analysis. We have directly measured six elements, which should give a consistency check on the four parameters of the CKM matrix. However, due to the small values of $|V_{ub}|$ and $|V_{cb}|$ together with the present level of accuracy, four of the elements overdetermine s_{12}, just as described above for two generations, and there is no overdetermination yet ~~of~~ the other parameters.

10.3 THREE GENERATIONS

The recent measurements of the number of light neutrinos [55], $N_\nu = 2.99 \pm 0.05$, imply that, within the minimal SM and some of its extensions, the number of generations is three. This makes it very likely that the 3×3 CKM matrix exactly fulfills the unitarity constraints. Consequently, we may deduce the allowed ranges for the V_{ti} elements, and also further restrict the allowed ranges for elements which were directly measured.

1. The value of $|V_{tb}|$ is derived from

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1. \tag{10.6}$$

As both $|V_{ub}|$ and $|V_{cb}|$ are measured to be much smaller than 1, the $|V_{tb}|$ value is very close to 1:

$$0.9986 \leq |V_{tb}| \leq 0.9993. \tag{10.7}$$

2. The value for $|V_{ts}|$ is derived from

$$V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} = 0. \tag{10.8}$$

As the first term on the left hand side is much smaller than the other two, and as both $|V_{cs}|$ and $|V_{tb}|$ are very close to 1, the $|V_{ts}|$ value is very close to $|V_{cb}|$:

$$0.035 \leq |V_{ts}| \leq 0.053. \tag{10.9}$$

3. The allowed range for $|V_{td}|$ is derived from

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0. \tag{10.10}$$

As both V_{ud} and V_{tb} are very close to 1, and as $V_{cd} \approx -s_{12}$, we may approximate

(10.10) by

$$V_{ub}^* + V_{td} = s_{12} V_{cb}. \tag{10.11}$$

This gives:

$$0.002 \leq |V_{td}| \leq 0.020. \tag{10.12}$$

Full information on the ranges for the absolute values of the CKM elements (at one sigma) from both direct measurements and unitarity is summarized by:

$$|V| = \begin{pmatrix} 0.9749 - 0.9754 & 0.2187 - 0.2223 & 0.002 - 0.009 \\ 0.218 - 0.221 & 0.9735 - 0.9752 & 0.038 - 0.052 \\ 0.002 - 0.020 & 0.035 - 0.053 & 0.9986 - 0.9993 \end{pmatrix} \tag{10.13}$$

In terms of the parameters we get:

$$s_{12} = 0.2205 \pm 0.0018; \quad s_{23} = 0.045 \pm 0.007; \quad q \equiv s_{13}/s_{23} = 0.11 \pm 0.05. \tag{10.14}$$

There are no direct constraints on the phase δ. Among the three real angles, there are large uncertainties in s_{13} only. Therefore, it is useful to present the information coming from indirect measurements as constraints in the $q - \delta$ plane.

10.4 THE UNITARITY TRIANGLE

As is apparent from (10.13), the only poorly determined matrix elements are V_{td} and V_{ub}. They are related to each other by the unitarity constraint (10.10) or, to a very good approximation, (10.11). The information that we may get from indirect measurements will have to comply with this constraint. It is very convenient to present such information and to discuss further predictions by using the *unitarity triangle*, which is just a geometrical representation of the relation (10.10) in the complex plane: The three complex quantities, $V_{ub}^* V_{ud}$, $V_{cb}^* V_{cd}$ and $V_{tb}^* V_{td}$ should form a triangle, as shown in Fig. 1. Rescaling the triangle by $[1/(s_{12}|V_{cb}|)]$, the coordinates of the three vertices A, B and C become

$$A\left[\frac{\mathrm{Re}\ V_{ub}}{s_{12}|V_{cb}|}, -\frac{\mathrm{Im}\ V_{ub}}{s_{12}|V_{cb}|}\right],\ B(1,0),\ C(0,0). \qquad (10.15)$$

In the Wolfenstein parametrization [11], which is just the small mixing-angle approximation of the standard parametrization, the coordinates of the vertex A are (ρ, η).

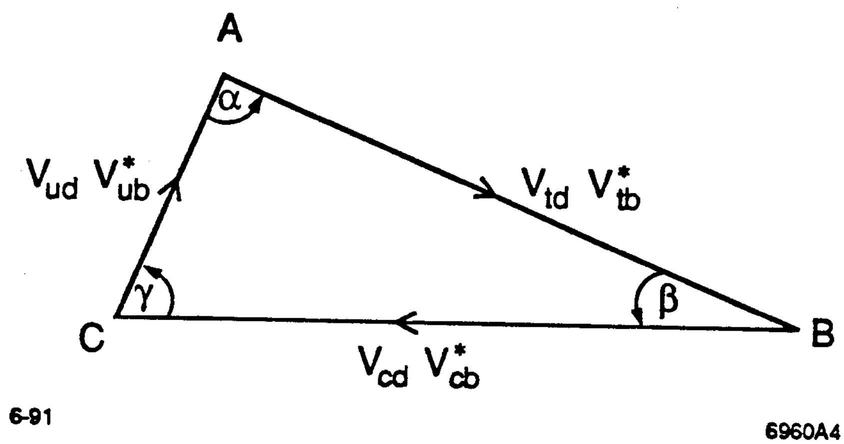

Figure 1. The unitarity triangle is a representation in the complex plane of the triangle formed by the CKM matrix elements $V_{ud}V_{ub}^*$, $V_{cd}V_{cb}^*$ and $V_{td}V_{tb}^*$. The angles α, β and γ are measurable through CP asymmetries in B decays, as explained in Chapter 16.

III. THE NEUTRAL MESON SYSTEMS: MIXING AND CP VIOLATION

11. Introduction and Notations

We consider a neutral meson P^0 and its antiparticle $\bar{P}^0$. An arbitrary neutral P-meson state

$$a\left|P^0\right\rangle + b\left|\bar{P}^0\right\rangle \tag{11.1}$$

is governed by the time-dependent Schrödinger equation

$$i\frac{d}{dt}\begin{pmatrix} a \\ b \end{pmatrix} = H\begin{pmatrix} a \\ b \end{pmatrix} \equiv \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} a \\ b \end{pmatrix}. \tag{11.2}$$

Here M and Γ are 2×2 Hermitian matrices. CPT invariance guarantees $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. $i\Gamma$, the anti-Hermitian part of H, describes the exponential decay of the P-meson system, while M, the Hermitian part, is called a mass matrix. If all the Γ's were zero, the system would evolve without decay, but the off-diagonal elements in the mass matrix would still cause mixing of P^0 with $\bar{P}^0$.

The mass eigenstates are

$$\begin{aligned} |P_1\rangle &= p\left|P^0\right\rangle + q\left|\bar{P}^0\right\rangle, \\ |P_2\rangle &= p\left|P^0\right\rangle - q\left|\bar{P}^0\right\rangle, \end{aligned} \tag{11.3}$$

with eigenvalues

$$\mu_{1,2} = M_{1,2} - \frac{i}{2}\Gamma_{1,2}. \tag{11.4}$$

Here $M_{1,2}$ and $\Gamma_{1,2}$ denote the masses and decay widths of $P_{1,2}$. Define:

$$\Delta\mu \equiv \mu_2 - \mu_1 \equiv \Delta M - \frac{i}{2}\Delta\Gamma. \tag{11.5}$$

The eigenvalue problem

$$\det\left(M - \frac{i}{2}\Gamma - \mu\mathbf{1}\right) = 0 \tag{11.6}$$

leads to the condition

$$[\Delta\mu]^2 = 4(M_{12}^* - i\Gamma_{12}^*/2)(M_{12} - i\Gamma_{12}/2). \tag{11.7}$$

The real and imaginary parts of (11.7) can be rewritten as

$$\begin{aligned}(\Delta M)^2 - (\Delta\Gamma)^2/4 =& 4(|M_{12}|^2 - |\Gamma_{12}|^2/4)\\ \Delta M \Delta\Gamma =& 4\mathrm{Re}(M_{12}\Gamma_{12}^*)\end{aligned} \tag{11.8}$$

For the ratio q/p we find:

$$\frac{q}{p} = \frac{-\Delta\mu}{2(M_{12} - i\Gamma_{12}/2)}. \tag{11.9}$$

We choose the following convention for charge conjugation:

$$C\left|P^0\right\rangle = \left|\bar{P}^0\right\rangle, \quad C\left|\bar{P}^0\right\rangle = \left|P^0\right\rangle. \tag{11.10}$$

Then, because the P's are pseudoscalars, the states with zero three-momentum satisfy

$$CP\left|P^0\right\rangle = -\left|\bar{P}^0\right\rangle, \quad CP\left|\bar{P}^0\right\rangle = -\left|P^0\right\rangle. \tag{11.11}$$

The CP even and odd states are then

$$|P_+\rangle = \frac{\left|P^0\right\rangle - \left|\bar{P}^0\right\rangle}{\sqrt{2}}, \quad |P_-\rangle = \frac{\left|P^0\right\rangle + \left|\bar{P}^0\right\rangle}{\sqrt{2}}. \tag{11.12}$$

In the absence of CP violation, there is no relative phase between M_{12} and Γ_{12}. In particular, we can choose a phase convention such that both M_{12} and Γ_{12}

are real,

$$M_{12} = M_{12}^*, \quad \Gamma_{12} = \Gamma_{12}^*, \quad (CP). \tag{11.13}$$

Then

$$\Delta M = \pm 2|M_{12}|, \quad \Delta\Gamma = \pm 2|\Gamma_{12}|, \quad q/p = \pm 1, \quad (CP). \tag{11.14}$$

The last equation in (11.14) demonstrates that in the absence of CP violation, the mass eigenstates are CP eigenstates.

12. The Neutral K System: Mixing

Much of our discussion of mixing in the K system is based on two textbooks. The theoretical discussion follows the presentation in ref. [14], while the description of the experimental aspects follows that of ref. [56]. Further reading in both references is highly recommended.

The mixing in the K^0 system depends on m_c and $\sin\theta_C$. Historically, it led Gaillard and Lee to predict the mass of the charm quark [57]. At present, both parameters are known rather accurately. This will allow us to realize how indirect measurements are useful even though the uncertainties are large. The discussion of $K^0 - \bar{K}^0$ mixing is simplest in a two generation framework. The effects of the third generation are small (as long as we do not discuss CP violation) and therefore our analysis holds even quantitatively.

12.1 PHENOMENOLOGY OF NEUTRAL KAONS

1. The neutral kaons have very different lifetimes.

In the two generation framework, CP is conserved, and the mass eigenstates coincide with the CP eigenstates. These states have very different lifetimes because their nonleptonic decay modes are radically different. Two pions, $\pi^0\pi^0$ or $\pi^+\pi^-$, in an $L = 0$ state must be CP even. On the other hand, the small Q-value (70

MeV) for the decay into three pions suggests $L = 0$, that is, the three pions are in a relative S-state. The CP-parity of $\pi^+\pi^-$ is still $+1$. The π^0 has $C = +1$ (since it decays to two γ's) and $P = -1$, and therefore $CP = -1$. So, combining the π^0 with the $\pi^+\pi^-$ system, we obtain one of $CP = -1$. For $L > 0$, both positive and negative CP eigenvalues can result, but such decays are strongly suppressed by angular-momentum barrier effects. Thus, if CP is conserved, only K_+ can decay into two π's, while K_- decays into three pions. As mentioned above, this description is exactly true in our hypothetical two generation world, but as in nature CP violation is a very small effect, our picture looks very much like the real world.

The two-pion final state has much larger phase space than the three-pion final state. Thus, K_+, which can decay into two pions, decays much faster than K_-, which cannot. The two mass eigenstates,

$$|K_S\rangle = |K_+\rangle\,, \quad |K_L\rangle = |K_-\rangle\,, \tag{12.1}$$

(S stands for Short and L stands for Long) have very different lifetimes. Experimentally

$$\tau_S = 0.9 \times 10^{-10}\ \text{sec}, \quad \tau_L = 5.2 \times 10^{-8}\ \text{sec}. \tag{12.2}$$

Along a beam of neutral K's one observes many two-pion decays close to the source while three-pion decays are further downstream, where essentially only the K_L component survives.

2. Pure-K^0 beams can be produced.

The K^0 mesons with strangeness $S = 1$ and $\bar{K}^0$ with $S = -1$ are the states that are produced by the strangeness conserving QCD strong interactions (interaction eigenstates). The K^0 can be produced by

$$\pi^- + p \to \Lambda + K^0, \tag{12.3}$$

while the $\bar{K}^0$ can be produced by

$$\begin{aligned} \pi^+ + p \rightarrow & K^+ + \bar{K}^0 + p, \\ \pi^- + p \rightarrow & \bar{\Lambda} + \bar{K}^0 + n + n. \end{aligned} \tag{12.4}$$

The threshold pion energy for (12.3) is 0.91 GeV, while for (12.4) it is much higher, 1.50 and 6.0 GeV for the respective processes. Thus it is possible to produce a pure K^0 beam by choosing incident pions of suitable energy.

3. Neutral kaons oscillate.

The amplitude of the states K_S or K_L at time t can be written as

$$\begin{aligned} a_S(t) = & a_S(0) e^{-(\Gamma_S/2 + iM_S)t}, \\ a_L(t) = & a_L(0) e^{-(\Gamma_L/2 + iM_L)t}. \end{aligned} \tag{12.5}$$

Now suppose that at $t = 0$, a beam of unit intensity consists of pure K^0. Then, from (12.1), $a_S(0) = a_L(0) = 1/\sqrt{2}$. After a time t the K^0 intensity will be

$$\begin{aligned} I(K^0) = & (a_S(t) + a_L(t))(a_S^*(t) + a_L^*(t))/2 \\ = & \frac{1}{4}[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos(\Delta M_K\, t)], \end{aligned} \tag{12.6}$$

where $\Delta M_K = M_L - M_S$. Similarly, the $\bar{K}^0$-intensity will be

$$I(\bar{K}^0) = \frac{1}{4}[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S + \Gamma_L)t/2} \cos(\Delta M_K\, t)]. \tag{12.7}$$

Thus, the K^0 and $\bar{K}^0$ intensities *oscillate* with the frequency ΔM_K. If one measures the number of $\bar{K}^0$ interaction events (*i.e.* the hyperon yield) as a function of position from the K^0 source, one can deduce ΔM_K.

4. K_S can be regenerated.

Suppose we start with a pure K^0 beam. After it coasts for say a 100 K_S mean lives, all the K_S-component has decayed and we are left with K_L only. Now, let the K_L beam traverse a slab of material and interact. The $\bar{K}^0$ component has more strong channels open and is therefore absorbed more strongly than K^0. After emerging from the slab, we shall therefore have a K^0 amplitude $f\,|K^0\rangle$ and a $\bar{K}^0$ amplitude $\bar{f}\,|\bar{K}^0\rangle$, where $\bar{f} < f < 1$. The emergent beam is not a pure K_L,

$$K_L = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle), \tag{12.8}$$

but instead

$$\frac{1}{\sqrt{2}}(f\,|K^0\rangle + \bar{f}\,|\bar{K}^0\rangle) = \tfrac{1}{2}(f+\bar{f})\,|K_L\rangle + \tfrac{1}{2}(f-\bar{f})\,|K_S\rangle\,. \tag{12.9}$$

Since $f \neq \bar{f}$, it follows that some of the K_S state has been regenerated. This regeneration of short-lived K_S-states in a long-lived K_L-beam was confirmed by experiment. It can be used to make an accurate measurement of the mass difference between K_S and K_L.

5. The mass difference between neutral kaons.

The result of such experiments is

$$\Delta M_K = M_L - M_S = (5.35 \pm 0.02) \times 10^9 \text{ sec}^{-1} = (3.52 \pm 0.01) \times 10^{-6}\, eV, \tag{12.10}$$

or a fractional mass difference

$$\frac{\Delta M_K}{M_K} = 7 \times 10^{-15}. \tag{12.11}$$

It is this number that we will use in our investigation of the quark sector.

12.2 BACK TO THEORY

We would like to calculate

$$\Delta M_K = 2\mathrm{Re}(M_{12}) \tag{12.12}$$

within the SM. While M_{11} is just the K^0 mass, M_{12} is a $\Delta S = 2$ effect and therefore $\mathcal{O}(G_F^2)$ (the weak interactions in lowest order change strangeness only by ± 1). If this were not the case, the mass difference ΔM_K would be much larger than what is observed. Thus we must work to second order in the weak interactions.

Following ref. [14], we write the Hamiltonian as a power series in G_F,

$$H = H_0 + H_1 + H_2 + \cdots \tag{12.13}$$

where H_j is proportional to $(G_F)^j$. The $|K^0\rangle$ and $|\bar{K}^0\rangle$ states are eigenstates of H_0 because they are degenerate, stable particles when the weak interactions are turned off. Therefore

$$M_{12} = \langle K^0|H_2|\bar{K}^0\rangle + \sum_n \frac{\langle K^0|H_1|n\rangle \langle n|H_1|\bar{K}^0\rangle}{M_{K^0} - E_n} + \cdots \tag{12.14}$$

The leading contributions are both second order in G_F because H_1 only changes strangeness by ± 1. In terms of states with the conventional normalization, the first term is

$$M_{12} = \frac{1}{2M_K} \langle K^0|\mathcal{H}_2(0)|\bar{K}^0\rangle \tag{12.15}$$

where $\mathcal{H}_2(0)$ is the second order weak Hamiltonian density. We are able to estimate this term, as we shall soon do. The second term in (12.14), however, depends sensitively on the details of low-energy strong interactions because it involves long distance contributions from low lying $S = 0$ mesonic states. We hide our ignorance of the long distance contribution by defining a parameter D,

$$D \cdot M_{12} \equiv \sum_n \frac{\langle K^0|H_1|n\rangle \langle n|H_1|\bar{K}^0\rangle}{M_{K^0} - E_n}. \tag{12.16}$$

We will calculate only the short distance contribution to the mass difference, $(\Delta M_K)_{S.D.} = \Delta M_K(1-D)$. Nevertheless, this estimate is interesting and important, as we will see.

The leading contribution to $\mathcal{H}_2$ comes from the box diagrams in Fig. 2.

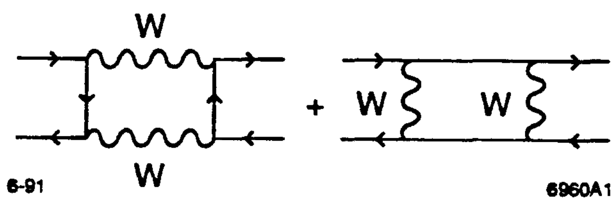

Figure 2. The box diagrams for neutral meson mixing.

It gives

$$\begin{aligned}\mathcal{H}_2 =& 2\left(\frac{-ig}{\sqrt{2}}\right)^4 \sum_{i,j}(V_{id}^* V_{is})(V_{jd}^* V_{js}) \int \frac{d^4p}{(2\pi)^4}\left(\frac{-i}{p^2-M_W^2}\right)^2 \\ &\times \left(\bar{d}_{La}\gamma^\mu \frac{i(\not{p}+m_i)}{p^2-m_i^2}\gamma^\nu s_L^a\right)\left(\bar{d}_{Lb}\gamma_\nu \frac{i(\not{p}+m_j)}{p^2-m_j^2}\gamma_\mu s_L^b\right).\end{aligned} \tag{12.17}$$

Thus, it is indeed $O(G_F^2)$, namely fourth order in the weak coupling. Furthermore, the contribution is proportional to $\sin^2\theta_C \cos^2\theta_C$. (In the absence of charged current mixing, $\sin\theta_C = 0$, there would be no $K^0 - \bar{K}^0$ mixing.) However, the GIM mechanism introduces an additional suppression factor $\sim m_c^2/M_W^2$. The important point is that for large loop momentum, the contributions of the u and c quarks cancel on each quark line: all strangeness-changing interactions would disappear if the charge 2/3 quarks were degenerate. Thus, each quark line in the figure is proportional to $m_c^2 - m_u^2$ and the diagram is quadratically convergent. The final result is:

$$\mathcal{H}_2 = \sin^2\theta_c \cos^2\theta_c m_c^2 \frac{G_F^2}{16\pi^2}(\bar{d}\gamma^\mu(1-\gamma_5)s)(\bar{d}\gamma_\mu(1-\gamma_5)s). \qquad (12.18)$$

(Some useful tricks in the actual evaluation can be found in ref. [58].) To proceed would require some estimate of the matrix element of the $\Delta S = 2$ operator. Gaillard and Lee [57] used the vacuum insertion approximation: one inserts a complete set of states in the middle of the operators in (12.18). This is a peculiar thing to do: the renormalized operator cannot really be considered as a product of two factors. But then in the sum over states, only the vacuum state is kept. This is an even more peculiar thing to do, but at least it makes the calculation easy:

$$\langle K^0|\bar{d}\gamma^\mu(1-\gamma_5)s|0\rangle \langle 0|\bar{d}\gamma_\mu(1-\gamma_5)s|\bar{K}^0\rangle = \frac{8}{3} f_K^2 M_K^2. \qquad (12.19)$$

Again, the peculiar assumptions are hidden by a new parameter,

$$B_K \equiv \frac{\langle K^0|\bar{d}\gamma^\mu(1-\gamma_5)s|0\rangle \langle 0|\bar{d}\gamma_\mu(1-\gamma_5)s|\bar{K}^0\rangle}{\langle K^0|\bar{d}\gamma^\mu(1-\gamma_5)s\bar{d}\gamma_\mu(1-\gamma_5)s|\bar{K}^0\rangle}. \qquad (12.20)$$

We later return to estimates of B_K. Combining (12.18) with (12.19) (and incorporating QCD corrections denoted by η_1) gives a contribution to ΔM_K,

$$(\Delta M_K)_{S.D.} = \Delta M_K(1-D) = \frac{G_F^2}{6\pi^2}\eta_1 M_K (B_K f_K^2) m_c^2 \ \mathrm{Re}[(V_{cd}^* V_{cs})^2]. \qquad (12.21)$$

We divide the parameters in (12.21) (other than m_c, V_{cd} and V_{cs}) into two categories:

1. Parameters which are known to a high level of accuracy. We collect many of them into

$$C_K \equiv \frac{6\pi^2 \Delta M_K}{G_F^2 f_K^2 M_K M_W^2} = 1.8 \times 10^{-5} \tag{12.22}$$

where we use:

$$\begin{aligned} &G_F = 1.166 \times 10^{-5} GeV^{-2}, \quad M_W = 80\ GeV \\ &f_K^2 = (0.165\ GeV)^2, \quad \Delta M_K / M_K = 7 \times 10^{-15}, \end{aligned} \tag{12.23}$$

and $\eta_1 = 0.7$.

2. Parameters with large theoretical uncertainties. The D parameter gives the relative part of long distance contributions to ΔM_K. We use

$$0 \leq D \leq 0.5. \tag{12.24}$$

The B_K parameter gives the ratio between the short distance contribution to ΔM_K and its value in the vacuum insertion approximation. We use

$$\frac{1}{3} \leq B_K \leq 1 \tag{12.25}$$

A few comments are in place:

a. If the D-parameter were very close to 1, or if the B_K-parameter were very different from 1, then our calculation would not be useful, because it would not indicate to us even the order of magnitude of ΔM_K.

b. While there is no known rigorous way to calculate D or B_K (except for, in principle, a lattice calculation of B_K), they are estimated within various models and approximations. The hope is that, if the various results do not differ dramatically, this gives a fair estimate of the parameters. Different models have very different systematic errors, and it is unlikely that all of them miss the correct value in the same direction and amount.

c. The ranges for the parameters in (12.24) and (12.25) reflect the spread of results from various models. While this seems a reasonable thing to do for the reasons explained in the previous comment, there is some danger that a single wrong model would lead to pessimistically large errors.

d. When using the results from $K-\bar{K}$ mixing, one should always carry in mind the uncertainties. The information on the CKM parameters is much less accurate than from direct measurements.

Eq. (12.21) can then be rewritten as

$$C_K \frac{(1-D)}{B_K} = \eta_1 (m_c^2/M_W^2)(V_{cd}V_{cs})^2. \tag{12.26}$$

(In the two generation case all matrix elements are real.) When the original study of $K-\bar{K}$ mixing [57] was performed, the c-quark was not yet experimentally discovered. Thus, one could use (12.26) to predict the mass of the c-quark. In the original calculation, the vacuum saturation approximation was used ($B_K = 1$), and neither long-distance contributions nor QCD corrections were taken into account ($D = 0,\ \eta_1 = 1$). This led, somewhat coincidentally, to the correct prediction [57]: $m_c = 1.5\ GeV$. With the full range of B_K and D,

$$0.9 \times 10^{-5} \leq C_K \frac{(1-D)}{B_K} \leq 5.3 \times 10^{-5}, \tag{12.27}$$

and with the correct value for η_1, one would have predicted $1.3\ GeV \leq m_c \leq 3.2\ GeV$. If, on the other hand, we use $m_c = 1.4\ GeV$, we get $0.20 \leq V_{cd}V_{cs} \leq 0.50$. This is to be compared with the constraints from direct measurements and unitarity. Note that a third generation is unnecessary in explaining $K-\bar{K}$ mixing. Thus, somewhat disappointingly, we learn that at the present level of accuracy, the combination of direct measurements, unitarity constraints and indirect measurements of <u>all</u> the parameters that do not directly involve the third generation, could not have revealed to us the existence of the third generation.

With three generations one has to take into account contributions from intermediate t-quarks. The RHS of (12.21) becomes more complicated [59] and depends also on m_t, V_{ts} and V_{td}. However, the contribution of diagrams involving the t-quark is suppressed by more than an order of magnitude compared to the c-quark contribution. With the large theoretical uncertainties, it is impossible to derive any useful information on m_t and V_{td}.

12.3 $D - \bar{D}$ MIXING

A measurement of $D - \bar{D}$ mixing is experimentally difficult and, moreover, will *not* provide us with clean information on the CKM parameters. The reasons for that are easy to understand [60] on the basis of our discussion of $K - \bar{K}$ mixing in the previous section.

a. The valence quarks in the neutral D-mesons belong to the up sector. That means that the intermediate quarks in the box diagrams are the d and the s quarks. Thus, $D - \bar{D}$ mixing would vanish in the flavor $SU(3)$ limit. Even though $SU(3)$ breaking effects are not necessarily small, $D - \bar{D}$ mixing is expected to be a very small effect.

b. It is very likely that $D - \bar{D}$ mixing is dominated by long distance contributions. In other words, the equivalent of the D-parameter of Eq. (12.16) for the D^0 system *is* expected to be very close to 1. That, as explained in the previous section, would render a calculation of the short distance contribution to ΔM_D quite useless.

13. The Neutral B System: Mixing

13.1 $B - \bar{B}$ MIXING

The discussion of mixing in the neutral K system is simplified by the fact that it is accounted for, to a good approximation, by physics of two quark generations, and that CP violation can be ignored. (The two facts are, of course, related.) Mixing in the neutral B-system involves, of course, the third generation and there is no reason to assume a small phase between M_{12} and Γ_{12}. However, here the discussion simplifies because

$$\Gamma_{12}(B^0) \ll M_{12}(B^0). \tag{13.1}$$

(Remember that $\Gamma_{12}(K^0) \sim 2M_{12}(K^0)$.) Within the SM one can explicitly calculate the two relevant quantities (assuming that a quark-level description is appropriate) and get $\Gamma_{12}/M_{12} \sim 10^{-2}$. However, this order-of-magnitude estimate holds far beyond the SM (see discussion in [61]). The argument is further supported by experimental evidence: while

$$x_d \equiv \Delta M_B/\Gamma_B = 0.66 \pm 0.11, \tag{13.2}$$

(upper limits on) branching ratios into states that contribute to Γ_{12} are at the level of 10^{-3}. When Γ_{12} can be neglected relative to M_{12}, one finds

$$\Delta M_B = 2|M_{12}|, \tag{13.3}$$

to be compared with (12.12).

Within the SM, the mixing of the neutral B's comes from box-diagrams and is completely dominated by intermediate t-quarks (see ref. [62] for a comprehensive discussion of $B - \bar{B}$ mixing in various models). The CKM-factors $|V_{qb}V_{qd}^*|^2$ are comparable for $q = u,\ c$ or t, the large mass of the top makes its contribution much larger than that of any other quark. A detailed calculation gives (similar to (12.18) and (12.19)):

$$x_d = \tau_b \frac{G_F^2}{6\pi^2} \eta M_B (B_B f_B^2) M_t^2 f_2(m_t^2/M_W^2) |V_{td}^* V_{tb}|^2 \tag{13.4}$$

where [59]

$$f_2(y) = 1 - \frac{3}{4} \frac{y(1+y)}{(1-y)^2} \left[1 + \frac{2y}{1-y^2} \ln(y) \right]. \tag{13.5}$$

The parameters in (13.4) (other than m_t and V_{td}) can be divided into:

1. Parameters which are known to a high level of accuracy. We collect them into

$$C_B \equiv \frac{6\pi^2}{G_F^2 M_B M_W^2} = 1.3 \times 10^7 \; GeV \tag{13.6}$$

where, in addition to the previously given parameters, G_F and M_W, we use

$$M_B = 5.28 \; GeV. \tag{13.7}$$

The parameter η is a QCD correction, $\eta = 0.85$. To a very good approximation $|V_{tb}| = 1$.

2. Parameters with relatively large theoretical ambiguities ($B_B f_B^2$), experimental errors (x_d), or both ($\tau_b |V_{cb}|^2$).

One should note that $|V_{cb}|$ and τ_b appear only in the combination $\tau_b |V_{cb}|^2$, which does not depend on τ_b (see eq. (9.15)). Therefore, the error on this combination is somewhat smaller than on $|V_{cb}|^2$ alone:

$$\tau_b |V_{cb}|^2 = (3.5 \pm 0.6) \times 10^9 \; GeV^{-1}. \tag{13.8}$$

The hadronic parameter B_B (analogous to B_K of the Kaon system) is believed to be close to 1. However, there is much uncertainty involved in the calculation of the B decay constant f_B. (Note that for the K, the decay constant f_K is experimentally determined.) A range of values for f_B has been derived from QCD sum rules and lattice calculations:

$$\sqrt{B_B} f_B = 0.15 \pm 0.05 \ GeV. \tag{13.9}$$

(Recent lattice calculations in the static quark approach give much higher values, $f_B \sim 0.3 \ GeV$ [63]. We have not included these calculations in (13.9).) The ARGUS and CLEO collaborations observe $B_d - \bar{B}_d$ mixing with

$$r_d = \begin{cases} 0.21 \pm 0.06 & \text{ARGUS} \\ 0.14 \pm 0.05 & \text{CLEO} \end{cases} \tag{13.10}$$

The x_d parameter is related to r_d by $r_d = x_d^2/(2 + x_d^2)$. We take the combined result of the two experiments, $r_d = 0.18 \pm 0.05$. This is very similar to an updated average quoted in ref. [47]: $r_d = 0.184 \pm 0.043$. We get

$$x_d = 0.66 \pm 0.11. \tag{13.11}$$

Eq. (13.4) can then be rewritten as

$$C_B \frac{x_d}{(\tau_b |V_{cb}|^2)(B_B f_B^2)} = \eta y_t f_2(y_t) |V_{td}/V_{cb}|^2. \tag{13.12}$$

With the standard parametrization we get

$$C_B \frac{x_d}{(\tau_b |V_{cb}|^2)(B_B f_B^2)} = \eta y_t f_2(y_t)(s_{12}^2 + q^2 - 2 s_{12} q \cos\delta). \tag{13.13}$$

From the ranges given above:

$$0.044 \leq C_B \frac{x_d}{(\tau_b |V_{cb}|^2)(B_B f_B^2)} \leq 0.35. \tag{13.14}$$

Eq. (13.12) together with the unitarity constraints on $|V_{td}|$ gives $m_t \gtrsim 50 \ GeV$, which is below the bound from direct searches, but may be useful in extensions of the Standard Model [64]. Eq. (13.12) together with the upper bound on m_t gives

$$|V_{td}| \geq 0.005. \tag{13.15}$$

For $m_t \gtrsim 185\ GeV$ the upper limit on $|V_{td}|$ that follows from (13.12) is stronger than the unitarity bound.

Finally, let us mention that the x_d value provides further evidence for the existence of the top quark [65] (see our discussion in chapter 8). If b_L were an $SU(2)_L$-singlet, then the non-diagonal Z-couplings (see Eq. (8.1)) would give a tree contribution to x_d,

$$(x_d)_{\text{tree}} = \tau_b \frac{\sqrt{2}G_F}{6} M_B(B_B f_B^2)|U_{db}|^2, \tag{13.16}$$

which is about two orders of magnitude larger than the upper limit on x_d.

13.2 $B_s - \bar{B}_s$ MIXING AND OTHER INDIRECT MEASUREMENTS

Mixing in the B_s system is not yet experimentally measured (though part of the mixing observed in hadron colliders is certainly due to B_s). Although the calculation of both x_d and x_s is subject to large uncertainties, the ratio between them is expected to be reasonably approximated by

$$x_s/x_d = |V_{ts}/V_{td}|^2. \tag{13.17}$$

Deviations from (13.17) are due to flavor $SU(3)$ breaking effects that shift the ratio f_{B_s}/f_{B_d} away from 1, its value in the symmetry limit. With the parametrization (5.4), Eq. (13.17) reads

$$x_s/x_d = 1/|s_{12} - qe^{i\delta}|^2. \tag{13.18}$$

Note that the ratio does not depend on m_t and s_{23}. It is minimized when $\delta \approx 180^o$

(the phase δ cannot be exactly 180^o because it would lead to $\epsilon = 0$),

$$x_s/x_d \geq 1/(s_{12}+q)^2 \gtrsim 7, \tag{13.19}$$

where the second inequality results from $q \leq 0.16$. This gives

$$x_s \geq 3.8 \Longrightarrow r_s \geq 0.88. \tag{13.20}$$

Thus a large $B_s - \bar{B}_s$ mixing is expected, independently of m_t. An exact determination of x_s from a measurement of r_s is difficult because r_s is near-maximal. Consequently, a time-dependent measurement of $B_s - \bar{B}_s$ oscillations is called for, but that would be an experimentally difficult task if $x_s \gtrsim 15$, in which case the oscillation length is very short.

We note that there are additional indirect measurements that may provide useful information on the CKM parameters, most noticeably $BR(K_L \to \mu\mu)$ and $BR(K \to \pi\nu\nu)$. The most recent experimental results are [66, 67]

$$\begin{aligned} BR(K_L^0 \to \mu\mu) &= (7.0 \pm 0.5) \times 10^{-9}, \\ BR(K^+ \to \pi^+\nu\nu) &\leq 5 \times 10^{-9}. \end{aligned} \tag{13.21}$$

The implications for the CKM parameters together with a careful description of the uncertainties involved can be found in ref. [68].

14. The Neutral K System: CP Violation

14.1 THE ϵ PARAMETER

The CP violating phenomena in the neutral kaon system that we study in this section arise because M_{12} and Γ_{12} in H cannot be made simultaneously real. Note that the each of $\arg(M_{12})$ and $\arg(\Gamma_{12})$ is phase-convention dependent: They would change by changing the relative phases of the K^0 and $\bar{K}^0$ states, which amounts to making a diagonal unitary transformation of H. But we cannot change the relative phase between M_{12} and Γ_{12}. It turns out that in Nature the relative phase is indeed nonzero ($M_{12}\Gamma_{12}^*$ is complex) and CP violation shows up in neutral K decays.

Our analysis here follows the one given in ref. [69]. With CP violation, the eigenstates of H are not K_+ and K_-, but

$$K_{L,S} = \frac{1}{\sqrt{2(1+|\bar{\epsilon}|^2)}} \begin{pmatrix} (1+\bar{\epsilon}) \\ \pm(1-\bar{\epsilon}) \end{pmatrix} \tag{14.1}$$

where $\bar{\epsilon}$ signifies the deviation from the CP limit. We switched notations here from p and q to $\frac{1+\bar{\epsilon}}{\sqrt{2(1+|\bar{\epsilon}|^2)}}$ and $\frac{1-\bar{\epsilon}}{\sqrt{2(1+|\bar{\epsilon}|^2)}}$, so that (11.9) now reads

$$\frac{(1+\bar{\epsilon})}{(1-\bar{\epsilon})} = 2\cdot\frac{M_{12}-i\Gamma_{12}/2}{\Delta M - i\Delta\Gamma/2} = \frac{1}{2}\frac{\Delta M - i\Delta\Gamma/2}{M_{12}^* - i\Gamma_{12}^*/2}. \tag{14.2}$$

Note that while the observable quantities ΔM and $\Delta\Gamma$ depend only on the relative phase between M_{12} and Γ_{12}, as they should (see (11.8)), the parameter $\bar{\epsilon}$ does depend on the choice of phase.

We would now like to relate $\bar{\epsilon}$ to measurable quantities. We study the decays of neutral kaons into two pions. We define the amplitudes:

$$\langle(\pi\pi)_I|H_W|K^0\rangle \doteq a_I e^{i\delta_I}, \quad \langle(\pi\pi)_I|H_W|\bar{K}^0\rangle = a_I^* e^{i\delta_I}, \tag{14.3}$$

where $I = 0$ or 2 is the isospin of the two pion system, and δ_I are the strong interaction phases. Using (14.1) we find

$$a_{I,S} = \langle(\pi\pi)_I|H_W|K_S\rangle = \frac{e^{i\delta_I}}{\sqrt{2(1+|\bar{\epsilon}|^2)}}[(1+\bar{\epsilon})a_I + (1-\bar{\epsilon})a_I^*],$$
$$a_{I,L} = \langle(\pi\pi)_I|H_W|K_L\rangle = \frac{e^{i\delta_I}}{\sqrt{2(1+|\bar{\epsilon}|^2)}}[(1+\bar{\epsilon})a_I - (1-\bar{\epsilon})a_I^*]. \tag{14.4}$$

With

$$\langle\pi^0\pi^0| = \langle(\pi\pi)_{I=0}|\sqrt{\tfrac{1}{3}} - \langle(\pi\pi)_{I=2}|\sqrt{\tfrac{2}{3}},$$
$$\langle\pi^+\pi^-| = \langle(\pi\pi)_{I=0}|\sqrt{\tfrac{2}{3}} + \langle(\pi\pi)_{I=2}|\sqrt{\tfrac{1}{3}}, \tag{14.5}$$

we get

$$A_{00,S(L)} = \langle\pi^0\pi^0|H_W|K_{S(L)}\rangle = \sqrt{\tfrac{1}{3}}a_{0,S(L)} - \sqrt{\tfrac{2}{3}}a_{2,S(L)},$$
$$A_{+-,S(L)} = \langle\pi^+\pi^-|H_W|K_{S(L)}\rangle = \sqrt{\tfrac{2}{3}}a_{0,S(L)} + \sqrt{\tfrac{1}{3}}a_{2,S(L)}. \tag{14.6}$$

We further define

$$\epsilon \equiv a_{0,L}/a_{0,S}; \tag{14.7}$$

$$t_I = \mathrm{Im}(a_I)/\mathrm{Re}(a_I). \tag{14.8}$$

Then, using (14.4) we find

$$\epsilon = \frac{\bar{\epsilon} + it_0}{1 + i\bar{\epsilon}t_0}. \tag{14.9}$$

The standard convention is to choose the phase of K^0 and $\bar{K}^0$ states to remove the phases from their $\Delta I = \frac{1}{2}$ decay amplitudes except for the effect of the final state interactions between the pions. This means that a_0 of eq. (14.3) is real and thus

$$\epsilon = \bar{\epsilon}. \tag{14.10}$$

In this basis, M_{12} and Γ_{12} would be real if there were no CP violation. Then because CP is a small effect, the phases of M_{12} and Γ_{12} are small, and we can usefully work to first order in ϵ (we shall soon find that $|\epsilon| \sim \mathcal{O}(10^{-3})$). Then (14.2) implies

$$\epsilon \approx \frac{i\mathrm{Im}M_{12} + \mathrm{Im}\Gamma_{12}/2}{\Delta M - i\Delta\Gamma/2}, \tag{14.11}$$

$$\Delta M \approx 2\mathrm{Re}(M_{12}), \quad \Delta\Gamma \approx 2\mathrm{Re}(\Gamma_{12}). \tag{14.12}$$

In the standard basis, we expect $\mathrm{Im}(\Gamma_{12})$ to be much smaller than $\mathrm{Im}(M_{12})$. This follows because $t_0 = 0$ implies that the contribution to Γ_{12} from the $(\pi\pi)_{I=0}$ states that dominate the decay is real. Thus, on top of the usual suppression of CP violating effects, $\mathrm{Im}(\Gamma_{12})$ should have an additional suppression of at least a few hundred (the ratio of the $(\pi\pi)_{I=0}$ final state to everything else). The phase of ϵ is then determined by the phase of the denominator in (14.11),

$$\Delta M \approx -\Delta\Gamma/2 \implies \arg(\Delta M - i\Delta\Gamma/2) \approx \pi/4. \tag{14.13}$$

It has become standard to use these empirical relations to simplify the expression for ϵ,

$$\epsilon \approx \frac{e^{i\pi/4}}{2\sqrt{2}}\frac{\mathrm{Im}M_{12}}{\mathrm{Re}M_{12}} \approx \frac{e^{i\pi/4}\mathrm{Im}M_{12}}{\sqrt{2}\Delta M}. \tag{14.14}$$

The value of $\mathrm{Re}(\epsilon)$ (or, equivalently when the phase convention is fixed, $|\epsilon|$) can be determined in various ways. In particular, let us define the following three observables:

$$\begin{aligned} |\eta_{+-}| &= \left[\frac{BR(K_L \to \pi^+\pi^-)}{\tau_L}\frac{\tau_S}{BR(K_S \to \pi^+\pi^-)}\right]^{1/2}, \\ |\eta_{00}| &= \left[\frac{BR(K_L \to \pi^0\pi^0)}{\tau_L}\frac{\tau_S}{BR(K_S \to \pi^0\pi^0)}\right]^{1/2}, \\ \delta &= \frac{\Gamma(K_L \to \pi^-\ell^+\nu) - \Gamma(K_L \to \pi^+\ell^-\nu)}{\Gamma(K_L \to \pi^-\ell^+\nu) + \Gamma(K_L \to \pi^+\ell^-\nu)}. \end{aligned} \tag{14.15}$$

To derive the relation between the $|\eta|$'s and ϵ, we write them in terms of the amplitudes of eq. (14.6),

$$|\eta_{+-}| = |A_{+-,L}/A_{+-,S}|, \quad |\eta_{00}| = |A_{00,L}/A_{00,S}|. \tag{14.16}$$

Experiment shows that the two quantities

$$\epsilon_2/\epsilon \equiv a_{2,L}/a_{0,L}, \quad \omega \equiv a_{2,S}/a_{0,S}, \tag{14.17}$$

are very small and we may neglect them for our purposes here. Then (14.16) gives

$$|\eta_{+-}| \approx |\epsilon|, \quad |\eta_{00}| \approx |\epsilon|. \tag{14.18}$$

As for δ, eq. (14.1) implies

$$\delta = \frac{|1+\epsilon|^2 - |1-\epsilon|^2}{|1+\epsilon|^2 + |1-\epsilon|^2} \approx 2\text{Re } \epsilon. \tag{14.19}$$

The first demonstration of CP violation was in 1964 in an experiment by Christenson, Cronin, Fitch and Turlay [70], who showed that K_L could also decay to $\pi^+\pi^-$ (with a branching ratio of order 10^{-3}). At present, all three observables have been measured. The results quoted in [8] are

$$\begin{aligned} |\eta_{+-}| =& (2.268 \pm 0.023) \times 10^{-3}, \\ |\eta_{00}| =& (2.253 \pm 0.024) \times 10^{-3}, \\ \delta =& (3.27 \pm 0.12) \times 10^{-3}. \end{aligned} \tag{14.20}$$

They all give

$$|\epsilon| \approx 2.26 \times 10^{-3}. \tag{14.21}$$

We can now use this measurement to get a potentially interesting constraint on the CP violating phase in the CKM matrix. One gets [59]:

$$|\epsilon| = \frac{G_F^2}{12\pi^2}\frac{M_K}{\sqrt{2}\Delta M_K}(B_K f_K^2)M_W^2$$
$$\times\left\{\eta_1 y_c \mathrm{Im}\left[(V_{cd}^* V_{cs})^2\right] + \eta_2 y_t f_2(y_t)\mathrm{Im}\left[(V_{td}^* V_{ts})^2\right] + 2\eta_3 f_3(y_t)\mathrm{Im}\left[V_{cd}^* V_{cs} V_{td}^* V_{ts}\right]\right\} \tag{14.22}$$

where

$$f_3(y_t) = \ln\left(\frac{y_t}{y_c}\right) - \frac{3}{4}\frac{y_t}{1-y_t}\left[1 + \frac{y_t}{1-y_t}\ln(y_t)\right]. \tag{14.23}$$

The parameters (other than those in the curly brackets) are divided into:

1. Well-known parameters, which we collect into

$$C_\epsilon \equiv \sqrt{2}|\epsilon|C_K = 5.6\times 10^{-8}. \tag{14.24}$$

2. Parameters with large uncertainties. The long distance contribution to ϵ is small and introduces an uncertainty smaller than 5%. Thus, the large uncertainty is in B_K, which we have already encountered above.

Eq. (14.22) can be rewritten as

$$\frac{C_\epsilon}{B_K} = -|V_{cb}|\mathrm{Im}(V_{td})\left\{[\eta_3 f_3(y_t) - \eta_1]y_c|V_{cd}| + \eta_2 y_t f_2(y_t)|V_{cb}|\mathrm{Re}(V_{td})\right\}, \tag{14.25}$$

or, using the standard parametrization,

$$\frac{C_\epsilon}{B_K} = (s_{23})^2 q\sin\delta\left\{[\eta_3 f_3(y_t) - \eta_1]y_c s_{12} + \eta_2 y_t f_2(y_t)(s_{23})^2(s_{12} - q\cos\delta)\right\}. \tag{14.26}$$

For the terms in the curly brackets we use $m_c = 1.4\ GeV$ and [71] $\eta_1 = 0.7$; $\eta_2 = 0.6$; $\eta_3 = 0.4$ (the η_i are QCD corrections). The constraint on the phase δ depends on the yet-unknown parameter m_t. Thus, the uncertainties are large, and we find

$$20^o \lesssim \delta \lesssim 178^o. \tag{14.27}$$

14.2 THE ϵ' PARAMETER

There is actually another CP-violating parameter in the neutral kaon system,

$$\epsilon' \equiv \frac{\epsilon_2 - \omega\epsilon}{\sqrt{2}}, \tag{14.28}$$

where ϵ_2 and ω are defined in (14.17). The exact and convention-independent expression for ϵ' is

$$\epsilon' = \frac{i}{\sqrt{2}}\left(\frac{\text{Re } a_2}{\text{Re } a_0}\right)(1-\bar{\epsilon}^2)e^{i(\delta_2-\delta_0)}\frac{t_2 - t_0}{(1+i\bar{\epsilon}t_0)^2}. \tag{14.29}$$

In the phase convention used above where $t_0 = 0$, this simplifies to

$$\epsilon' \approx \frac{i}{\sqrt{2}}\left(\frac{\text{Re } a_2}{\text{Re } a_0}\right)t_2 e^{i(\delta_2-\delta_0)}. \tag{14.30}$$

When we neglect ω compared to one (we know experimentally that $|a_2/a_0| \approx 1/20$), we get instead of the approximate expressions in (14.18)

$$\begin{aligned} \eta_{+-} &\approx \epsilon + \epsilon', \\ \eta_{00} &\approx \epsilon - 2\epsilon'. \end{aligned} \tag{14.31}$$

Thus $\epsilon' \neq 0$ signifies CP violation in the decay processes when amplitudes of different phases interfere. This is called *direct CP* violation, in contrast to $\epsilon \neq 0$ which signifies CP violation in mixing.

The most recent measurements give [72]

$$\epsilon'/\epsilon = \begin{cases} (2.3 \pm 0.7) \times 10^{-3} & \text{NA31} \\ (0.6 \pm 0.7) \times 10^{-3} & \text{E731} \end{cases} \tag{14.32}$$

The extraction of constraints on the CKM parameters from (14.32) (see ref. [73]) is a complicated theoretical task, as large hadronic uncertainties are involved. While there is a strong theoretical effort in this direction (see, for example, ref. [74] and references therein), at present the value of ϵ'/ϵ is useful to test our understanding of hadronic physics rather than for CKM fit.

15. Constraints from Indirect Measurements

We studied the various constraints on the CKM parameters for

$$89 \leq m_t \leq 200 \ GeV. \quad (15.1)$$

The lower bound comes from the direct search for the top in CDF [75 – 76]. The upper bound is a conservative upper bound from electroweak precision measurements. Ref. [77], for example, quotes $m_t \leq 182 \ GeV$ at the 95% C.L.

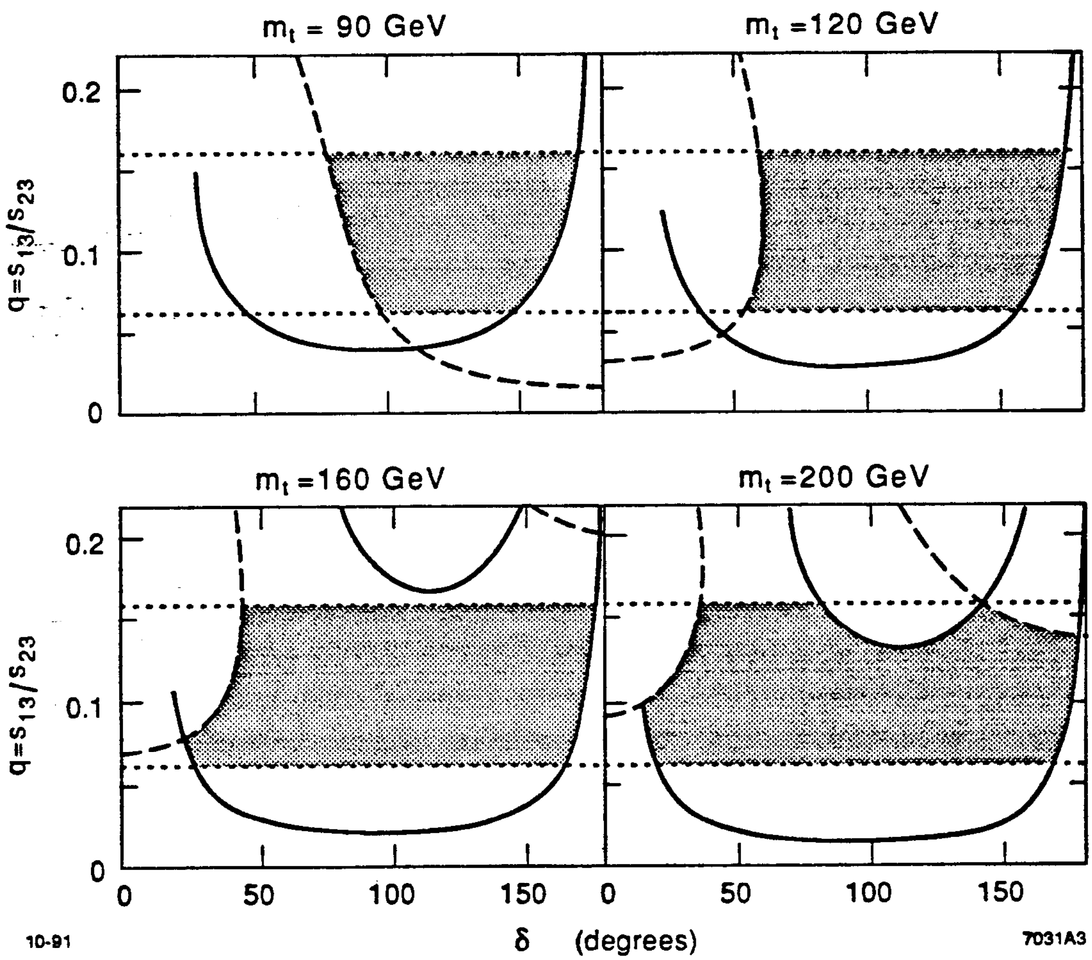

Figure 3. Constraints from $|V_{ub}/V_{cb}|$ (dotted lines), x_d (dashed curves) and ϵ (solid curves) on the parameters $q = s_{13}/s_{23}$ and δ for $m_t = 90$, 120, 160 and 200 GeV. The shaded region is the finally allowed range.

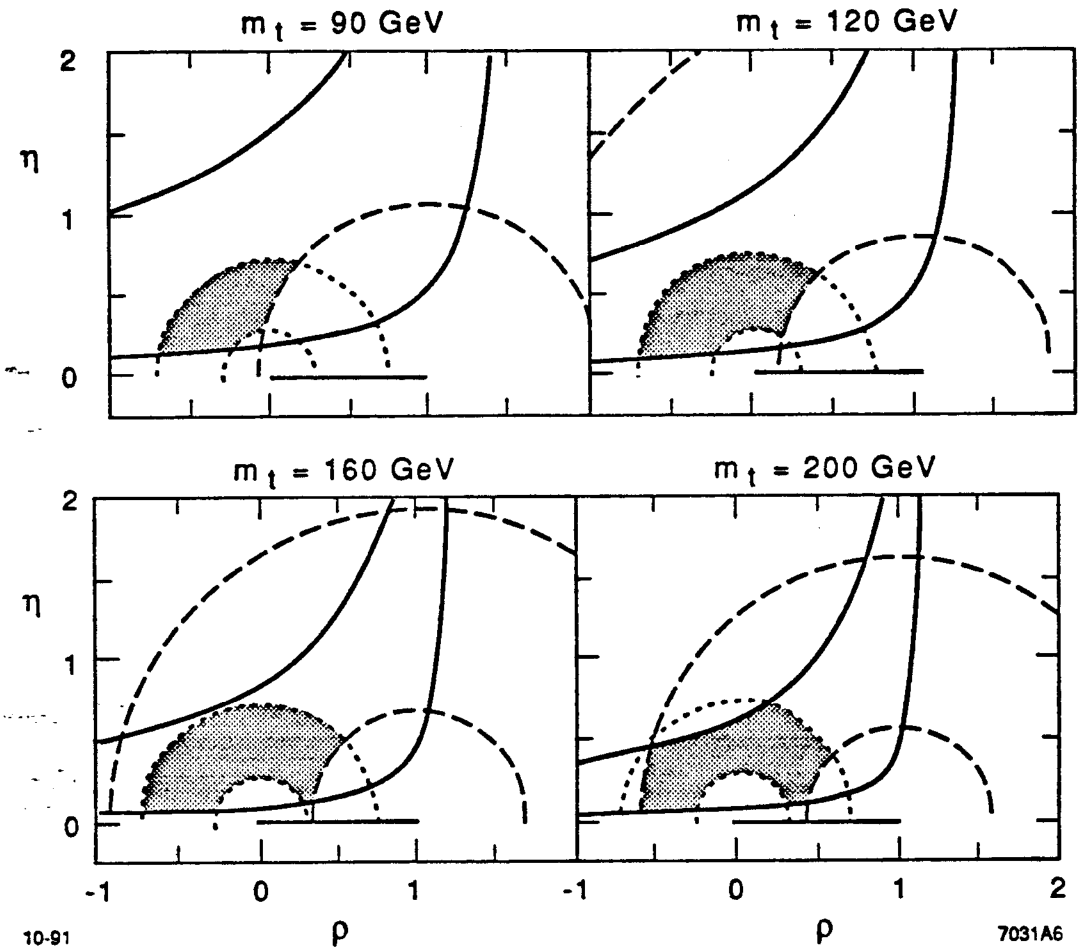

Figure 4. Constraints from $|V_{ub}/V_{cb}|$ (dotted lines), x_d (dashed curves) and ϵ (solid curves) on the rescaled unitarity triangle for $m_t = 90,\ 120,\ 160$ and $200\ GeV$. The shaded region is that allowed for the vertex $A(\rho, \eta)$.

We present our results for the CKM parameters in two equivalent ways:

1. Allowed regions in the $q - \delta$ plane. We use (13.13) involving x_d and (14.26) involving ϵ. For each relation, we use the full range of parameters. For a fixed top quark mass, we get allowed bands in the $q - \delta$ plane. The final allowed region is within the two bands and within the direct limits on q. We show the constraints for $m_t = 90,\ 120,\ 160$ and $200\ GeV$ in Fig. 3.

2. Allowed region for the vertex A of the unitarity triangle. The analysis is done using the x_d relation as given in (13.12) and the ϵ relation as given in

(14.25). The constraints are shown in Fig. 4.

16. The Neutral B System: CP Violation

16.1 FORMALISM

We consider a neutral meson B^0 and its antiparticle $\bar{B}^0$ [78, 79]. The two mass eigenstates are B_H and B_L (H and L stand for Heavy and Light respectively):

$$\begin{aligned} |B_L\rangle =& p\,|B_0\rangle + q\,|\bar{B}^0\rangle\,, \\ |B_H\rangle =& p\,|B_0\rangle - q\,|\bar{B}^0\rangle\,. \end{aligned} \tag{16.1}$$

We neglect the tiny difference in width between B_H and B_L:

$$\Gamma_H = \Gamma_L \equiv \Gamma. \tag{16.2}$$

($\Delta\Gamma \ll \Gamma$ because it is produced by channels with branching ratios of $\mathcal{O}(10^{-3})$ which contribute with alternating signs [80].) We define:

$$M \equiv (M_H + M_L)/2, \quad \Delta M \equiv M_H - M_L. \tag{16.3}$$

With $\Gamma_{12} \ll M_{12}$ (see discussion in chapter 13), we have

$$|q/p| = 1. \tag{16.4}$$

The amplitudes for the states B_H or B_L at time t can be written as

$$\begin{aligned} a_H(t) =& a_H(0)e^{-(\Gamma/2+iM_H)t}, \\ a_L(t) =& a_L(0)e^{-(\Gamma/2+iM_L)t}. \end{aligned} \tag{16.5}$$

The proper time evolution of an initially ($t = 0$) pure B^0 ($a_L(0) = a_H(0) = 1/(2p)$)

or $\bar{B}^0$ ($a_L(0) = -a_H(0) = 1/(2q)$) is given respectively by

$$\begin{aligned} \left|B^0_{\rm phys}(t)\right\rangle =& g_+(t)\left|B_0\right\rangle + (q/p)g_-(t)\left|\bar{B}^0\right\rangle, \\ \left|\bar{B}^0_{\rm phys}(t)\right\rangle =& (p/q)g_-(t)\left|B_0\right\rangle + g_+(t)\left|\bar{B}^0\right\rangle, \end{aligned} \tag{16.6}$$

where

$$\begin{aligned} g_+(t) =& \exp(-\Gamma t/2)\exp(-iMt)\cos(\Delta Mt/2), \\ g_-(t) =& \exp(-\Gamma t/2)\exp(-iMt)i\sin(\Delta Mt/2). \end{aligned} \tag{16.7}$$

We are interested in the decays of neutral B's into a CP eigenstate f_{CP}. We define the amplitudes for these processes as

$$A \equiv \left\langle f_{CP}|\mathcal{H}|B^0\right\rangle, \quad \bar{A} \equiv \left\langle f_{CP}|\mathcal{H}|\bar{B}^0\right\rangle. \tag{16.8}$$

We further define

$$\lambda \equiv \frac{q}{p}\frac{\bar{A}}{A}. \tag{16.9}$$

Then

$$\begin{aligned} \left\langle f_{CP}|\mathcal{H}|B^0_{\rm phys}(t)\right\rangle =& A[g_+(t) + \lambda g_-(t)], \\ \left\langle f_{CP}|\mathcal{H}|\bar{B}^0_{\rm phys}(t)\right\rangle =& A(p/q)[g_-(t) + \lambda g_+(t)]. \end{aligned} \tag{16.10}$$

The time-dependent rates for initially pure B^0 or $\bar{B}^0$ states to decay into a final CP eigenstate at time t is given by:

$$\begin{aligned} \Gamma(B^0_{\rm phys}(t) \to f_{CP}) =& |A|^2 e^{-\Gamma t}\left[\frac{1+|\lambda|^2}{2} + \frac{1-|\lambda|^2}{2}\cos(\Delta Mt) - {\rm Im}\lambda\sin(\Delta Mt)\right], \\ \Gamma(\bar{B}^0_{\rm phys}(t) \to f_{CP}) =& |A|^2 e^{-\Gamma t}\left[\frac{1+|\lambda|^2}{2} - \frac{1-|\lambda|^2}{2}\cos(\Delta Mt) + {\rm Im}\lambda\sin(\Delta Mt)\right]. \end{aligned} \tag{16.11}$$

We define the time dependent CP asymmetry as

$$a_{f_{CP}}(t) \equiv \frac{\Gamma(B^0_{\rm phys}(t) \to f_{CP}) - \Gamma(\bar{B}^0_{\rm phys}(t) \to f)}{\Gamma(B^0_{\rm phys}(t) \to f_{CP}) + \Gamma(\bar{B}^0_{\rm phys}(t) \to f)}. \tag{16.12}$$

Then

$$a_{f_{CP}}(t) = \frac{(1-|\lambda|^2)\cos(\Delta M t) - 2\mathrm{Im}\lambda \sin(\Delta M t)}{1+|\lambda|^2}. \tag{16.13}$$

If, in addition to (16.4), $|A/\bar{A}| = 1$ so that $|\lambda| = 1$, then (16.13) simplifies considerably:

$$a_{f_{CP}}(t) = -\mathrm{Im}\lambda \sin(\Delta M t). \tag{16.14}$$

The quantity $\mathrm{Im}(\lambda)$ which can be extracted from $a_{f_{CP}}$ is theoretically very interesting since it can be directly related to the CKM matrix elements.

16.2 MEASURING THE ANGLES OF THE UNITARITY TRIANGLE

The measurement of the CP asymmetry (16.12) will determine $\mathrm{Im}\lambda$ through (16.13). If $|A/\bar{A}| = 1$ (in which case the simpler expression (16.14) holds), then $\mathrm{Im}\lambda$ depends on electroweak parameters only, without hadronic uncertainties. The condition which guarantees $|A/\bar{A}| = 1$ is easy to find [81]. In the general case, A and $\bar{A}$ can be written as sums of various contributions:

$$\begin{aligned} A &= \sum_i A_i e^{i\delta_i} e^{i\phi_i}, \\ \bar{A} &= \sum_i A_i e^{i\delta_i} e^{-i\phi_i}, \end{aligned} \tag{16.15}$$

where A_i are real, ϕ_i are CKM phases and δ_i are strong phases. Thus, $|A| = |\bar{A}|$ if all amplitudes that contribute to the decay have the same CKM phase, which we will denote by ϕ_D. In such a case

$$\bar{A}/A = e^{-2i\phi_D}. \tag{16.16}$$

As mentioned above, for $\Gamma_{12} \ll M_{12}$

$$q/p = \sqrt{M_{12}^*/M_{12}} = e^{-2i\phi_M}, \tag{16.17}$$

where ϕ_M is the CKM phase in the $B - \bar{B}$ mixing. Thus

$$\lambda = e^{-2i(\phi_M+\phi_D)} \implies \mathrm{Im}\lambda = -\sin 2(\phi_M + \phi_D). \tag{16.18}$$

(Note that each of ϕ_M and ϕ_D is convention dependent, but the sum $\phi_M + \phi_D$ is not.) Indeed, Imλ depends on CKM parameters only. In what follows, we discuss only those processes which, within the SM, are dominated by amplitudes that have a single CKM phase.

There are two systems of neutral B mesons. For mixing in the B_d $[B_s]$ system $M_{12} \propto (V_{tb}V_{td}^*)^2$ $[(V_{tb}V_{ts}^*)^2]$. Consequently,

$$\left(\frac{q}{p}\right)_{B_d} = \frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}; \qquad \left(\frac{q}{p}\right)_{B_s} = \frac{V_{tb}^*V_{ts}}{V_{tb}V_{ts}^*}. \tag{16.19}$$

There are several types of relevant decay processes. We concentrate on tree decays. For decays via quark subprocesses $b \to \bar{u}_i u_i d_j$

$$\frac{\bar{A}}{A} = \frac{V_{ib}V_{ij}^*}{V_{ib}^*V_{ij}}. \tag{16.20}$$

Thus, for B_{d_j} decaying through $\bar{b} \to \bar{u}_i u_i \bar{d}_j$,

$$\mathrm{Im}\lambda = \sin\left[2\arg\left(\frac{V_{ib}V_{ij}^*}{V_{tb}V_{tj}^*}\right)\right]. \tag{16.21}$$

For decays with a single K_S (or K_L) in the final state, $K - \bar{K}$ mixing is essential because $B^0 \to K^0$ and $\bar{B}^0 \to \bar{K}^0$, and interference is possible only due to $K - \bar{K}^0$ mixing. For these modes

$$\lambda = \left(\frac{q}{p}\right)\left(\frac{\bar{A}}{A}\right)\left(\frac{q}{p}\right)_K, \quad \left(\frac{q}{p}\right)_K = \frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}. \tag{16.22}$$

Note that sign(Imλ) depends on the CP transformation properties of the final state. The analysis above corresponds to CP-even final states. For CP-odd states, Imλ has the opposite sign. In what follows, we give Imλ of CP-even states, regardless of the CP assignments of specific hadronic modes discussed.

CP asymmetries in B^0 decays into CP eigenstates provide a way to measure the three angles of the unitarity triangle independently of each other and without hadronic uncertainties. The three angles of this triangle (see Fig. 1) are defined by

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \quad \beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \quad \gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right). \quad (16.23)$$

The aim is to make enough independent measurements of the sides and angles that this triangle is overdetermined and thus check the validity of the SM. We now give three explicit examples for asymmetries that measure the three angles α, β and γ:

(i) Measuring $\sin(2\beta)$ in $B \to \psi K_S$.

The ψK_S mode is the only CP-eigenstate that has been experimentally observed so far. The average of ARGUS and CLEO results is [47]

$$BR(B^0 \to \psi K_S) = (3.0 \pm 1.5) \times 10^{-4}. \quad (16.24)$$

The mixing phase in the B_d system is given in Eq. (16.19), $(q/p)_{B_d} = \frac{(V_{tb}^*V_{td})}{(V_{tb}V_{td}^*)}$. With a single final kaon, one has to take into account the mixing phase in the K system given in Eq. (16.22), $(q/p)_K = (V_{cs}V_{cd}^*)/(V_{cs}^*V_{cd})$. The decay phase (16.20) in the quark subprocess $b \to c\bar{c}s$ is

$$\frac{\bar{A}}{A} = \frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}. \quad (16.25)$$

We get

$$\lambda(B \to \psi K_S) = \left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)\left(\frac{V_{cs}^*V_{cb}}{V_{cs}V_{cb}^*}\right)\left(\frac{V_{cd}^*V_{cs}}{V_{cd}V_{cs}^*}\right) \Longrightarrow \text{Im}\lambda = -\sin(2\beta). \quad (16.26)$$

(As ψK_S is a $CP = -1$ state, there is an extra minus sign in the asymmetry which we ignore here.)

(*ii*) Measuring $\sin(2\alpha)$ in $B \to \pi^+\pi^-$.

The mixing phase in the B_d system is given in Eq. (16.19). The decay phase (16.20) for the quark subprocess $b \to u\bar{u}d$ is

$$\frac{\bar{A}}{A} = \frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}}. \tag{16.27}$$

We get

$$\lambda(B \to \pi^+\pi^-) = \left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)\left(\frac{V_{ud}^*V_{ub}}{V_{ud}V_{ub}^*}\right) \Longrightarrow \text{Im}\lambda = \sin(2\alpha). \tag{16.28}$$

(*iii*) Measuring $\sin(2\gamma)$ in $B_s \to \rho K_s$.

The mixing phase in the B_s system is given in Eq. (16.19), $(q/p)_{B_s} = \frac{(V_{tb}^*V_{ts})}{(V_{tb}V_{ts}^*)}$. Due to the final K_S, the mixing phase for the K system has to be taken into account. The quark subprocess is, again, $b \to u\bar{u}d$. We get

$$\lambda(B_s \to \rho K_S) = \left(\frac{V_{tb}^*V_{ts}}{V_{tb}V_{ts}^*}\right)\left(\frac{V_{ud}^*V_{ub}}{V_{ud}V_{ub}^*}\right)\left(\frac{V_{cs}^*V_{cd}}{V_{cs}V_{cd}^*}\right) \Longrightarrow \text{Im}\lambda = -\sin(2\gamma). \tag{16.29}$$

The three examples that we gave above demonstrate that the three angles of the unitarity triangle can in principle be measured independently of each other. The SM predictions for the three asymmetries are described in refs. [82, 73].

IV. RELATIONS AMONG QUARK MASSES AND MIXING PARAMETERS

17. Schemes of Mass Matrices

Within the SM, the quark sector is described by ten free parameters. In the *physical* (mass) basis, these are six quark masses, three mixing angles and one phase. These parameters can all be experimentally determined. Whatever their experimental values are, the SM remains self-consistent.

In the *interaction* basis, our parameters are entries of the yet undiagonalized mass matrices. If we had some theoretical principle from which we could determine the mass matrices, we would predict the values of the physical parameters. In several schemes of mass matrices, the number of independent entries of the mass matrices is less than ten; either some entries vanish or there are relations among the non-vanishing entries. These schemes provide us with relations among quark masses, angles and phases.

The motivation to consider relations among quark masses and mixing angles comes from several sources:

1. There are "too many" parameters in the SM. The CKM picture of the quark sector has ten independent parameters and one would like to find a theory where this number is reduced.

2. Various quantities in the SM diverge (in lowest order) when quark masses are taken to infinity (for example, the x_d parameter of $B - \bar{B}$ mixing). Thus, arbitrarily heavy quarks do not decouple from the physics of low energy. If mixing angles were inversely proportional to the mass of the heavy quark, (for example, $|V_{td}| \propto m_t^{-1/2}$) then these observables would remain finite.

3. Quark masses and mixing angles have a common "origin", the mass matrices in the interaction eigenbasis. It is not unlikely that these matrices indeed have less than ten independent parameters.

4. It was noticed that the numerical values of the two generations quark sector parameters fulfill quite accurately the relation

$$\sin\theta_C = \sqrt{\frac{m_d}{m_s}}. \tag{17.1}$$

It was further recognized that such a relation would follow if the mass matrix were of the form

$$M_d = \begin{pmatrix} 0 & a \\ a & b \end{pmatrix}. \tag{17.2}$$

Most schemes for quark mass matrices, even in the three generation case, try to retain this relation.

One should check that the various relations are consistent with the experimental data. If the relations suggested by a certain scheme are not compatible with the experimental constraints, then either the scheme is incorrect, or the use of its predictions should await the finding of additional new physics.

We note that the existence of new physics beyond the SM is inherent in the suggestion of schemes for quark mass matrices. The validity of our discussion lies in the assumption that this new physics itself does not significantly contribute to CP-violation and $B-\bar{B}$ mixing. This is the case, for example, if the new physics takes place at a sufficiently large energy scale.

18. Quark Masses

The masses of the quarks, which are the eigenvalues of the mass matrices to be discussed, are not the *physical* masses but parameters in the Lagrangian. This means that they are *running* masses which should all be taken at a single energy scale. In the different schemes, the three mixing angles and the phase depend on *mass ratios* rather than on the masses themselves. As mass ratios are, to a good approximation, independent of the energy scale, the scale itself can be arbitrarily chosen. We use [83]

$$\begin{aligned} m_d/m_s =& 0.051 \pm 0.004, \\ m_u/m_c =& 0.0038 \pm 0.0012, \\ m_s/m_b =& 0.033 \pm 0.011. \end{aligned} \tag{18.1}$$

The relations that we will get involve the undetermined mass ratio m_c/m_t. In order to confront these relations with the x_d and ϵ bounds, involving the physical mass of the t quark $m_t^{\rm phys}$, we must

(*a*) Specify m_c at a certain energy scale μ. We take $\mu = 1\ GeV$:

$$m_c(\mu = 1\ GeV) = 1.35 \pm 0.05\ GeV. \tag{18.2}$$

(*b*) Translate the relations involving m_c/m_t into relations which depend on the running top mass $m_t(\mu = 1\ GeV)$.

(*c*) Write these relations in terms of the physical mass of the t quark. The relation between the physical mass and the running mass, including a first order QCD correction, is

$$m_t^{\rm phys} = m_t(\mu = m_t)\left[1 + \frac{4}{3\pi}\alpha_S(m_t)\right]. \tag{18.3}$$

In order to relate $m_t(\mu = m_t)$ to $m_t(\mu = 1\ GeV)$ we use the usual equation for the running mass

$$m(\mu) = \bar{m}\left(1 - \frac{2\beta_1\gamma_0}{\beta_0^3}\frac{\ln L + 1}{L} + \frac{8\gamma_1}{\beta_0^2 L}\right)\left(\frac{L}{2}\right)^{-2\gamma_0/\beta_0}, \tag{18.4}$$

where

$$\begin{aligned} \beta_0 &= 11 - \frac{2}{3}N_f, \quad \gamma_0 = 2, \\ \beta_1 &= 102 - \frac{38}{3}N_f, \quad \gamma_1 = \frac{101}{12} - \frac{5}{18}N_f, \\ L &= \ln(\mu^2/\Lambda^2), \quad \Lambda = 0.1\ GeV, \end{aligned} \tag{18.5}$$

and $\bar{m}$ is the renormalization group invariant mass. A good approximation for

$m_t^{\rm phys}$ in the interesting range between 90 and 200 GeV is $m_t^{\rm phys} \sim 0.6 m_t(\mu = 1\ GeV)$. In what follows we will denote the physical top mass by m_t.

19. The Fritzsch Scheme for Quark Masses

The best known scheme for quark mass matrices is due to Fritzsch [84]. We discuss it here in detail as an example to the type of predictions and tests suggested by quark mass matrices. As we shall soon see, the heavier the top quark is known to be, the more difficult it becomes for the Fritzsch scheme to be consistent with the CKM picture [85].

We study mass matrices of the form

$$M^u = \begin{pmatrix} 0 & a^u & 0 \\ a^u & 0 & b^u \\ 0 & b^u & c^u \end{pmatrix}, \quad M^d = \begin{pmatrix} 0 & a^d e^{i\phi_1} & 0 \\ a^d e^{-i\phi_1} & 0 & b^d e^{i\phi_2} \\ 0 & b^d e^{-i\phi_2} & c^d \end{pmatrix}. \tag{19.1}$$

(Either M^u or M^d of the Fritzsch form can always be made real without any effect on low-energy parameters.) The six real parameters can be expressed in terms of the six quark masses

$$\begin{aligned} a^u &\approx \sqrt{m_u m_c}, \quad b^u \approx \sqrt{m_c m_t}, \quad c^u \approx m_t, \\ a^d &\approx \sqrt{m_d m_s}, \quad b^d \approx \sqrt{m_s m_b}, \quad c^d \approx m_b. \end{aligned} \tag{19.2}$$

The approximation is good to $\mathcal{O}(m_d/m_s) \sim 1/20$. The two phases can be expressed in terms of the quark masses and the two known mixing angles s_{12} and s_{23}

$$\begin{aligned} s_{12} &\approx \left| \sqrt{\frac{m_d}{m_s}} - e^{-i\phi_1} \sqrt{\frac{m_u}{m_c}} \right|, \\ s_{23} &\approx \left| \sqrt{\frac{m_s}{m_b}} - e^{-i\phi_2} \sqrt{\frac{m_c}{m_t}} \right|. \end{aligned} \tag{19.3}$$

Thus, all eight parameters of the Fritzsch scheme are expressible in terms of seven known parameters (five quark masses and two mixing angles), and the yet unknown mass of the top quark. Consequently, for every selected value of m_t, we get predictions for the poorly-determined mixing angle s_{13}

$$s_{13} \approx \left| \frac{m_s}{m_b}\sqrt{\frac{m_d}{m_b}} + e^{-i\phi_1}\sqrt{\frac{m_u}{m_c}}\left(\sqrt{\frac{m_s}{m_b}} - e^{-i\phi_2}\sqrt{\frac{m_c}{m_t}}\right)\right|, \tag{19.4}$$

and phase δ

$$\frac{\sin\delta}{s_{12}s_{23}/s_{13} - \cos\delta} \approx \frac{\sin\phi_1}{\cos\phi_1 - \sqrt{(m_d m_c)/(m_s m_u)}}. \tag{19.5}$$

Eq. (19.3) gives a lower limit on the unknown mass ratio m_c/m_t,

$$\frac{m_c}{m_t} \geq \left(\sqrt{\frac{m_s}{m_b}} - s_{23}\right)^2 . \tag{19.6}$$

Using $m_s/m_b \geq 0.022$ and $s_{23} \leq 0.052$ we get

$$\frac{m_c}{m_t} \geq 0.009 \to m_t(\mu = 1\ GeV) \leq 151\ GeV. \tag{19.7}$$

When the bound (19.7) is translated into a bound on the physical top mass, we find that only a very narrow range if left,

$$89\ GeV \leq m_t \leq 91\ GeV. \tag{19.8}$$

(The lower limit is the direct CDF limit [75 – 76].) This is the main difficulty for the Fritzsch scheme at present: the value of s_{23} is much smaller than $(m_s/m_b)^{1/2}$. Fine tuning between m_s/m_b and m_c/m_t is required to allow such a small s_{23}-value. This fine tuning is impossible if the top is too heavy. Note, however, the upper bound on m_t is sensitive to the upper bound on s_{23}. For example, if we relax the bounds in (9.19) to $s_{23} \leq 0.055$ then $m_t \leq 97\ GeV$ is allowed. If, on the other hand, $s_{23} \leq 0.50$ than the Fritzsch scheme is excluded.

A detailed comparison between the predictions of the Fritzsch scheme and the allowed range for the CKM parameters shows that, in order to get a consistent solution: (a) the mass ratio m_s/m_b has to be close to its lower limit, $m_s/m_b \sim 0.022$, (b) the mixing angle s_{23} should be close to its upper limit, $s_{23} \sim 0.052$, (c) the $B-\bar{B}$ mixing parameter has to be close to its lower limit, $x_d \sim 0.55$, (d) the B decay constant f_B should be close to the upper limit of its theoretical range, $B_B f_B^2 \sim (0.20\ GeV)^2$, and ($e$) the B_K constant should be close to the upper limit of its theoretical range, $B_K \sim 1$. Only if all these conditions are simultaneously fulfilled will there be a narrow range of (m_t, s_{13}, δ) space which is consistent with both the Fritzsch relations and the experimental data. The following values are predicted for the various parameters:

$$\begin{aligned} m_t &\sim 90\ GeV, \\ |s_{13}/s_{23}| &\sim 0.07, \\ \delta &\sim 100^o. \end{aligned} \tag{19.9}$$

These constraints give many specific predictions that can be tested in the near future [86]. In particular, an improvement in the lower bound on m_t may soon exclude the Fritzsch scheme.

20. Outlook

In this series of lectures, we have described in much detail the determination of the CKM elements from direct measurements, three-generation unitarity and indirect measurements. The only poorly determined elements are V_{ub} and V_{td}. Equivalently, in the standard parametrization there are two poorly determined parameters, s_{13} and δ. The constraints on these parameters are presented in Figs. 3 and 4. To give a better picture of the uncertainties involved, we present in Fig. 5 the constraints on the unitarity triangle for any top mass in the range 89 to 200 GeV.

Let us now summarize the prospects for improvement in the determination of the CKM matrix:

a. The mass of the top is likely to be measured in Fermilab. This will make the information on the CKM elements from $B - \bar{B}$ mixing and from the ϵ-parameter much more accurate.

b. The value of the B_K parameter is likely to be better determined by lattice calculations. This will provide more accurate information from ϵ.

c. The value of the f_B decay constant is likely to be better determined. The improvement may come from lattice calculations; from a measurement of f_D and the use of the Heavy Quark Symmetry to relate the two; or, somewhat less likely, from an actual measurement of $BR(B \to \tau\nu_\tau)$ in a B-factory. This will provide more accurate determination of $|V_{td}|$ from $B - \bar{B}$ mixing.

d. The value of $|V_{cb}|$ is likely to be better determined from higher statistics measurements in CLEO or in a B factory and from a model independent interpretation of the results using the Heavy Quark Symmetry (see discussion in chapter 9).

e. The value of $|V_{ub}|$ is likely to be better determined by, again, a combination of higher statistics experiments and better theoretical understanding, as discussed in detail in chapter 9.

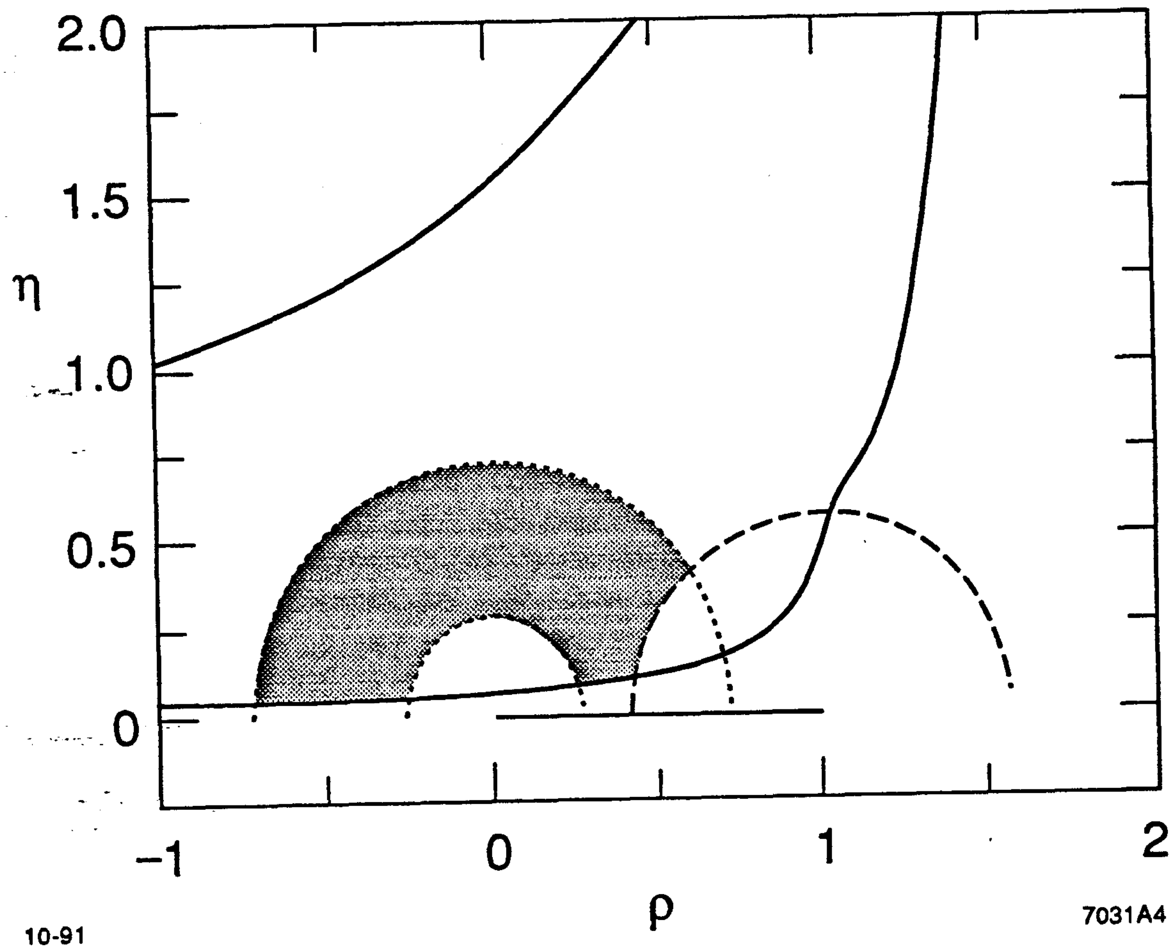

Figure 5. Constraints from $|V_{ub}/V_{cb}|$ (dotted lines), x_d (dashed curves) and ϵ (solid curves) on the rescaled unitarity triangle for $89 \leq m_t \leq 200\ GeV$. The shaded region is that allowed for the vertex $A(\rho, \eta)$.

f. The value of x_s, the $B_s - \bar{B}_s$ mixing parameter, may be determined in a B-factory (or in a Z-factory). The ratio $|V_{td}/V_{ts}|$ will thus be known rather accurately from x_d/x_s. Its extraction will be independent of the mass of the top, and depends on f_{B_d}/f_{B_s} which is much better known than f_B itself.

g. The values of the angles of the unitarity triangle will be measured in a B factory. The angle β can be determined from $B \rightarrow \psi K_S$, the angle α from $B \rightarrow \pi\pi$ and, somewhat less likely, the angle γ from $B_s \rightarrow \rho K_S$.

To demonstrate the combined power of all these significant improvements, we

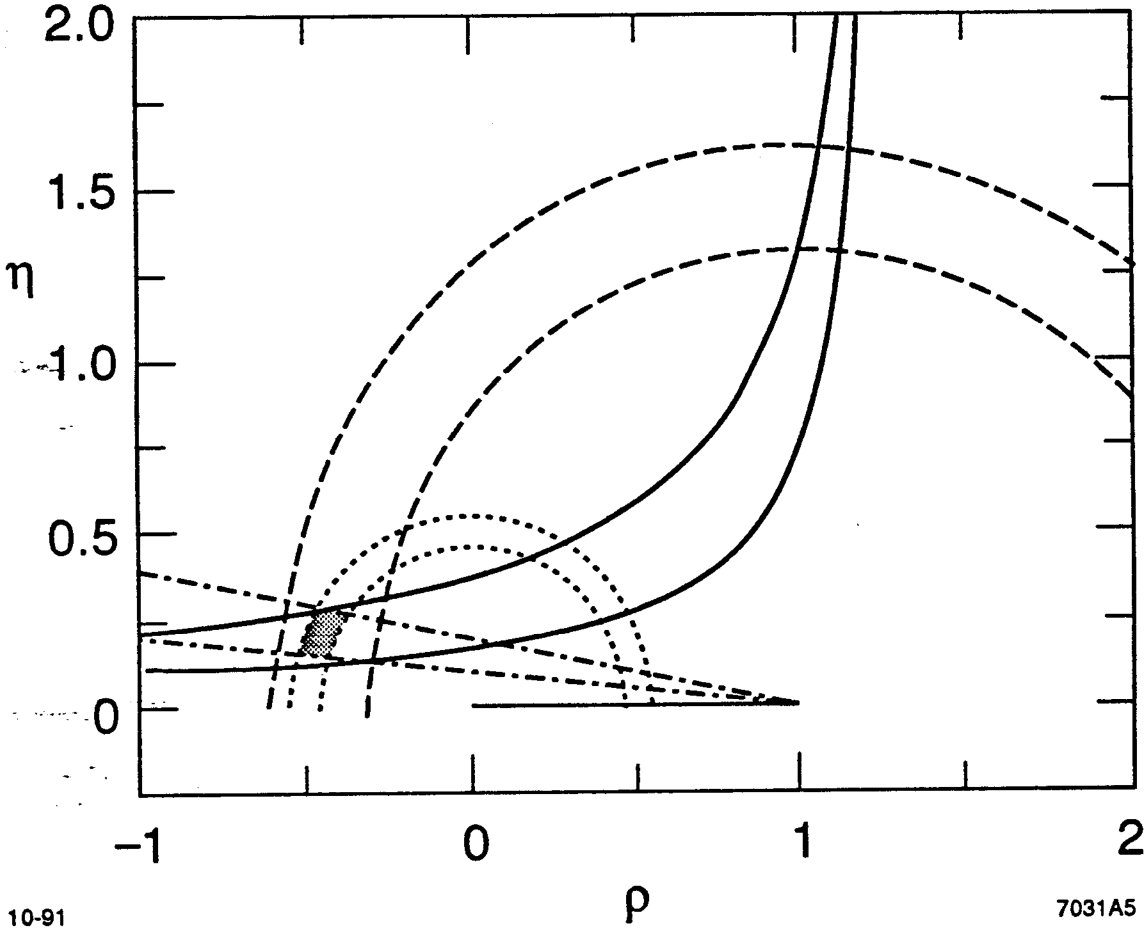

Figure 6. Future constraints from $|V_{ub}/V_{cb}|$ (dotted lines), x_d/x_s (dashed curves), ϵ (solid curves) and the CP-asymmetry in $B \rightarrow \psi K_S$ (dot-dashed curves) on the rescaled unitarity triangle. The ranges for the various parameters are given in Eq. (20.1). The shaded region is that allowed for the vertex $A(\rho, \eta)$.

show in Fig. 6 how the constraints on the CKM parameters will look like if all the above measurements are made. To make this Figure, we used the following values and uncertainties:

$$
\begin{aligned}
m_t =& 160 \pm 8 \; GeV, \\
B_K =& 0.7 \pm 0.1, \\
f_B =& 0.12 \pm 0.02 \; GeV, \\
|V_{cb}| =& 0.047, \\
|V_{ub}/V_{cb}| =& 0.11 \pm 0.01, \\
|V_{td}/V_{ts}| =& 0.32 \pm 0.04, \\
\sin 2\beta =& 0.28 \pm 0.09.
\end{aligned}
\tag{20.1}
$$

If the final picture of the various measurements indeed looks as in Fig. 6, and in particular if the measurements of ϵ and $\text{Im}\lambda(\psi K_S)$ are consistent with each other and with all other information, then the CKM explanation of CP violation will at last be tested and confirmed. Of course, one would hope that the beautiful consistency imagined in this Figure will *not* realize and a window to New Physics will be opened.

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