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ON STRING THEORY AND AXIONIC STRINGS & INSTANTONS*

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ABSTRACT

Progress in understanding exact superconformal field theories of four-dimensional axionic instantons and axionic strings is reported.

Superstring theory is the first promising candidate of quantum gravity, perhaps unified with strong and electroweak interactions. Still, our current understanding is largely based on weak coupling perturbative analysis. On the other hand, most of interesting physics of superstring seem to require nonperturbative understandings: just to name a few, dynamical supersymmetry breaking, cosmological constant problem and spacetime singularity are such examples. Therefore, in this talk, I will discuss what I find one of the most interesting topics in superstring theory to the above direction: stringy solitons and instantons. Recently, considerable progress in nonperturbative aspects of noncritical strings has been made. Here, I will restrict myself to critical superstrings in ten-dimensions, suitably compactified to four dimensions keeping $N = 1$ spacetime supersymmetry.

At scales well below that of compactification, massless string excitations may be described by a low-energy supergravity theory. It is observed that, in all known string theories, there arises ubiquitously a massless gravity supermultiplet: graviton, dilaton and Kalb-Ramond (axion) fields. The other massless excitations depends largely on the way a specific compactification scheme is

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chosen. Since I would like to understand intrinsic nonperturbative aspects of superstrings, I will only keep the gravity multiplet in the low-energy whose dynamics is described by an effective action.[1]

$$S_{eff} = \int d^4x\sqrt{G}[-\mathcal{R}(\Omega) + (higher\ \ orders)]. \tag{1}$$

Here, Ω_μ denotes a general affine connection with nonzero dilaton and torsion ${\Omega_\mu}^{ab} = \omega_\mu^{ab} + \delta_\mu^{[a}\nabla^{b]}\Phi + {H_\mu}^{ab}$.

1. Axionic Instanton

After Wick rotation to Euclidean spacetime, we find axionic instantons as solutions to self-dual Einstein equation[2]

$$\mathcal{R}_\mu{}^a{}_\nu{}^b(\Omega) = \pm{}^*\mathcal{R}_\mu{}^a{}_\nu{}^b(\Omega). \tag{2}$$

This turns out to be equivalent to $d\Phi = \pm{}^*H$ and $\mathcal{R}_{\mu\nu} = 0$. We find N-instanton solution as

$$G_{\mu\nu}(x) = \delta_{\mu\nu}, \ \ e^{-\Phi(x)} = \sum_{a=1}^{N}\frac{|Q_a|}{2\pi(x-x_a)^2}, \ \ H_{\mu\nu\lambda} = \sum_{a=1}^{N}\frac{Q_a}{\pi}\epsilon_{\mu\nu\lambda\sigma}\frac{(x-x_a)^\sigma}{(x-x_a)^4}, \tag{3}$$

which is stabilized by a conserved topological charge $Q = \frac{1}{2\pi}\oint_{\partial E} H = \mathbf{Z}$.

An exact superconformal field theory of the axionic instanton is found[2] to be $N = 1$ supersymmetric $su_Q(2) \times u_{\frac{1}{\gamma}}(1) = u(2)$ WZNW model of $\hat{c} = 4$. Here, Q denotes level of $su(2)$ WZNW model and $\frac{1}{\gamma}$ a background charge of $u(1)$ theory. The self-duality equation of Eq.(2) becomes an algebraic self-duality

$$\gamma = \pm\sqrt{Q}. \tag{4}$$

In this case, the supersymmetric $u(2)$ WZNW model can be written as[3]

$$S_w = \frac{Q}{8\pi}\{\int tr(\partial\Sigma^\dagger\bar{\partial}\Sigma) + \frac{1}{3}\int tr(\Sigma^\dagger d\Sigma)^3\} + \frac{1}{8\pi}\int R^{(2)}\ln(\det\Sigma + \det\Sigma^\dagger) + (fermions) \tag{5}$$

where $\Sigma = \exp(\frac{1}{\gamma}X^o + i\sigma\cdot\mathbf{X}) \in U(2)$. It can be shown[4,3] that the instanton possesses a larger symmetry: doubly extended $N = 4$ $su_{Q-1}(2) \times su_1(2) \times u_{\frac{1}{\gamma}}(1)$

superconformal invariance. Denoting $K^o \equiv \frac{1}{\gamma}\partial X^o, \mathbf{K} \equiv \mathbf{J} + i\mathbf{\Psi} \wedge \mathbf{\Psi}$ out of the $u(2)$ currents, we find the generators of doubly extended $N = 4$ SCA to be

$$
\begin{aligned}
T &= -\frac{1}{Q}(K^oK^o + \mathbf{K}\cdot\mathbf{K}) - (\Psi^o\partial\Psi^o + \mathbf{\Psi}\partial\mathbf{\Psi}) + \frac{1}{\gamma}\partial K^o \\
G^o &= \frac{2}{\sqrt{Q}}(\Psi^o K^o + \mathbf{\Psi}\cdot\mathbf{K} - \mathbf{\Psi}\cdot\mathbf{\Psi}\wedge\mathbf{\Psi}) + \frac{1}{\gamma}\partial\Psi^o \\
\mathbf{G} &= \frac{2}{\sqrt{Q}}(\mathbf{\Psi}K^o - \Psi^o\mathbf{K} + \mathbf{K}\wedge\mathbf{\Psi} - \Psi^o\mathbf{\Psi}\wedge\mathbf{\Psi}) + \frac{1}{\gamma}\partial\mathbf{\Psi} \\
\mathbf{I}_L &= \Psi^o\mathbf{\Psi} + \mathbf{\Psi}\wedge\mathbf{\Psi} \\
\mathbf{I}_R &= -\Psi^o\mathbf{\Psi} + \mathbf{\Psi}\wedge\mathbf{\Psi} + \mathbf{K} \\
\Psi^o &= \sqrt{Q}\psi^o, \qquad \mathbf{\Psi} = \sqrt{Q}\vec{\psi} \\
U &= \sqrt{Q}X^o.
\end{aligned}
\tag{6}
$$

If the background charge $\frac{1}{\gamma}$ is dropped off, Eq.(6) produces the original[5] doubly extended $N = 4$ SCA with KM subalgebra $su_{Q-1}(2) \times su_1(2) \times u(1)$. The central charge is $c = \frac{6k_1k_2}{k_1+k_2} = 6 - \frac{6}{Q}$. In our model, $u(1)$ background charge $\frac{1}{\gamma}$ contributes an additional central charge $\frac{6}{\gamma^2}$. Therefore, if the algebraic self-duality Eq.(4) is satisfied, the total central charge adds up to 6. Wonderfully, the doubly extended $N = 4$ SCA remains to hold even after we twist $u(1)$ algebra! Therefore, axionic instantons in type II string and in heterotic string in which gauge connection is identified with spin connection possess doubly extended $(4, 4)$ worldsheet supersymmetry.[4,3] *This hidden $N = 4$ superconformal symmetry guarantees a nonrenormalization of the axionic instanton solution to all orders in worldsheet perturbation expansions.* Patching holomorphic and antiholomorphic sectors into a modular invariant partition function seems to be a difficult step to analyze even though significant progress has been achieved recently.[6,7]

In the axionic instanton background, it was shown[2] that there exist only two chiral fermion zero modes (as opposed, e.g., to four nonchiral zero modes in Eguchi-Hanson instantons) to the lowest order in worldsheet perturbation expansions. The exact SCFT of axionic instanton enables us to prove this to all orders as well. The spacetime supersymmetry is generated by contour integrals of holomorphic $(1, 0)$ primary fields. Denoting zero modes of $N = 4$ supercurrents as $G_o^a \equiv \oint T_F^a, \mathbf{G}_o \equiv \oint \mathbf{T}_F$, $N=1$ spacetime supersymmetry is guaranteed only if they annihilate the Ramond vacuum $|0>_R$. First, it is easy to show that $\{G_o^a, G_o^b\}|0>_R = \delta^{ab}(L_o - \frac{\hat{c}}{16})|0>_R = 0$, in which contribution from $su(2)$ WZNW model cancels that from $u(1)$ with a background charge. On the other hand, imposing that G_o^a annihilate $|0>_R$, we have

$$G_o^a|0>_R = (\frac{1}{\sqrt{Q}}\epsilon^a{}_{bcd}\Gamma^b\Gamma^c\Gamma^d + \frac{1}{\gamma}\Gamma^a)\,|0>_R = 0. \qquad (7)$$

Since the Ramond ground state is GSO projected, $\Gamma_{11}|0>_R = +|0>_R$, Eq.(7) implies that $(1 \pm \Gamma_5)\,|0>_R = 0$. *Therefore, there arises only two chiral fermionic zero modes if and only if the algebraic self-duality* $\gamma = \pm\sqrt{Q}$ *is satisfied.* Indeed, this is yet another proof of the aforementioned *nonrenormalization theorem.* It also entails a very significant low-energy implications[2] to dynamical supersymmetry breaking[8] since only when there exist precisely two chiral fermion zero modes, gravitino can acquire nonzero mass, thus breaks local supersymmetry.

2. Axionic Strings

It is also possible to find a superconformal field theory of axionic strings. Recall that the axionic instanton was a source of quantized Kalb-Ramond magnetic field. That is why compact, $su_Q(2)$ WZNW model was relevant for the axionic instanton. Furthermore, integer-valued level Q of $su_Q(2)$ WZNW model was correctly interpreted as quantized magnetic charge of Kalb-Ramond field. Fundamental string is an axionic string, thus a source of Kalb-Ramond electric field. Therefore, we expect a relevant SCFT to be noncompact, $su_{-Q}(1,1)$ WZNW model with metric signature $(++-)$. In this case, the level Q takes a continuous value, and interpreted as a Kalb-Ramond electric charge. Indeed, for axionic string, we find[3,9] $\tilde{su}_{-Q,\frac{1}{\gamma}}(1,1) \times u(1)$ and $su_{-Q}(1,1) \times u(1)_{\frac{i}{\gamma}}$ WZNW models as exact SCFTs. The first consists of a $su(1,1)$ WZNW model twisted by $\frac{1}{\gamma}$ along radial direction and a compact $u(1)$ model, while the second a $su_{-Q}(1,1)$ WZNW model and a Feign-Fuks $u(1)$ model with a background charge $\frac{i}{\gamma}$ which may be interpreted as a wrong sign $u(1)$ model with a background charge $\frac{1}{\gamma}$. Thus, they have spacetime signature $(-+++)$ and $(--++)$ respectively. The first[3] describes correct dilaton and Kalb-Ramond field backgrounds of a known lowest order solution[10] but aparently different global geometry. The second is intimately related to the axionic instanton through transformations[9]: $su_Q(2) \rightarrow su_{-Q}(1,1)$ and $u_{\frac{1}{\gamma}}(1) \rightarrow u_{\frac{i}{\gamma}}(1)$ (The reader is warned that this is not equivalent to $u(1,1)$). The total central charge is $\hat{c}_{tot} = (3 + 4/Q) + (1 - 4/\gamma^2)$. This equals to the critical six if an algebraic self-duality condition $\gamma = \pm\sqrt{Q}$ is satisfied. In fact, the doubly extended $N = 4$ algebra Eq.(5) remains to hold in the axionic string as well, and the axionic strings are spacetime supersymmetric to all orders in the worldsheet perturbation expansions! Furthermore, its spacetime interpretation is that the manifold is a solution of $(++--)$ signature self-dual Einstein equation with a generalized connection. Therefore, ths axionic string constitutes a nontrivial soliton solution of $N = 2$ gauged string theory[11] at critical dimension! My motivations of studying the axionic string are two-fold.

(1) Recently, two-dimensional black hole in critical bosonic string theory was shown to be described by $su(1,1)/u(1)$ gauged WZNW model,[12] which appears to be intimately connected to one-dimensional noncritical strings. One can generalize it to an $N = 1$ supersymmetric black hole, which is described by a manifestly $N = 1$ supersymmetry $su(1,1)/u(1)$ gauged WZNW model. If one choose Landau gauge, the black hole SCFT consists of $su(1,1)$ WZNW model, negative metric free field theory and (b,c) ghosts. They contribute central charges $\hat{c} = (3 + 4/Q), 1$ and -2 respectively. The first two combines to a SCFT of $su_{-Q}(1,1) \times u(1)$ WZNW model with central charge $\hat{c} = 4 + 4/Q$. Furthermore, it can be shown that the WZNW model possess a doubly extended $N = 4$ superconformal symmetry as well! This is almost identical to the above SCFTs of axionic strings. The $N = 4$ superconformal theories being intimately related to Harmonic superspace and twistor theory, the above unexpected connections between the 2-D black hole and the axionic strings might shed light on uncovering hidden W_∞ symmetry in $c = 1$ quantum gravity.

(2) Even though conceptually simple, I found it technically hard to investigate topological sectors of $N = 1$ superstrings. The gauged $N = 2$ string theory is very simple since string field consists only of one scalar field, the Kähler potential, yet rich enough to allow soliton sectors. Therefore, I do expect that many technical questions such as collective coordinates, moduli space, definitions of topological charge etc. and other nonperturbative aspects of string theory are easier to investigate and to understand in gauged $N = 2$ string theory.

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