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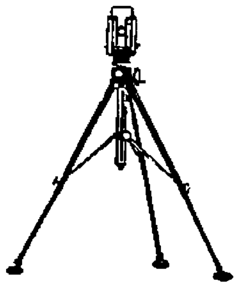

STANFORD LINEAR COLLIDER MAGNET POSITIONING *

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ABSTRACT

For the installation of the Stanford Linear Collider (SLC) the positioning and alignment of the beam line components was performed in several individual steps. In the following the general procedures for each step are outlined. The calculation of ideal coordinates for the magnets in the entire SLC will be discussed in detail. Special emphasis was given to the mathematical algorithms and geometry used in the programs to calculate these ideal positions.

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1. INTRODUCTION

This document outlines the general procedures for positioning beam line elements and calculating their ideal coordinates in the entire Stanford Linear Collider (SLC). The major goal is to position the beam line elements to their ideal positions within the final tolerances as follows:

1.) Two magnets within an achromat (defined beam line section) must point at each other with an angular accuracy of 0.04 milliradians.
2.) Transverse to the beam line (horizontal and in elevation) two magnets must be adjusted with an accuracy of 0.1 mm.
3.) The distance between two magnets must be adjusted with an accuracy of 0.5 mm.
4.) Roll must be set to within 2.0 milliradians of its ideal value.

The positioning of beam elements is done is several individual steps.
Step 1: Prepositioning of the support systems.
Step 2: Prealignment of the magnet adjustment system.
Step 3: Absolute positioning of the beam line elements.
Step 4: Smoothing of the beam line elements' position.

Special emphasis was given to the mathematical algorithms and geometry used in the programs to calculate these ideal coordinate positions.

2. DESIGN COORDINATE SYSTEMS

2.1. IDEAL COORDINATE SYSTEM

The ideal coordinate system or absolute coordinate system is given by TRANSPORT. TRANSPORT is a program used to trace the path of a charged particle or group of particles through idealized magnets. These magnets can be strung together with intervals between them to form a sequence of elements called a beam line. The initial parameters which define an incoming beam's position and orientation relative to an absolute reference frame can be specified. These initial parameters along with the input parameters of a magnet string can be used to calculate a particle's path as it traverses the line. The program provides numerous pieces of information including coordinates, orientation angles of the particles, and the physical parameters particular to the optical elements. TRANSPORT coordinate system origins (datums) change depending on the section of the accelerator.

The datums listed below are all relative to the local gravity vector at the respective origin and define right-handed rectangular cartesian coordinate systems, with Y up and parallel to the direction of the vertical, Z pointing downstream (down the beam line) and X pointing to the left while looking down the beam line. Pitch is an exception to the right-handed rule. Pitch (down) means down in respect to gravity.

The slope of the LINear ACcelerator (LINAC) with respect to gravity is documented in Appendix C.

2.1.1. DATUM FOR COLLIDER INJECTOR DEVELOPMENT (CID) AND WEST TURN AROUND (WTA) OF THE POSITRON BEAM

Datum 1: The origin is a virtual point of the beam line at coordinates

Local TRANSPORT station# = LINAC station# = 0.0

Z = 0.0

X = 0.0

Y = 0.0

Roll = Yaw = 0.0

Pitch = -0.30023[DEG] (down)

= -0.00524[RAD] (down)

The scribe line on a brass plate embedded in the floor at the beginning of sector 1 defines the Z-position of the origin. [1]

2.1.2. DATUM FOR DAMPING RING SYSTEMS

Datum 2: The origin at beam height is 59.660[inches] upstream from the end of sector 1 which is at the center of girder 1-9.[2]

LINAC station# = 003+28 [feet]

Local TRANSPORT station# = 1.3228[m] = 4.33989[feet]

---

[1] The scribe line on the brass plate is projected perpendicular to the beam line onto the beam line, rather than projecting it along the gravity vector.

[2] See MEMO from W.A. Davies-White dated Nov. 24th 1980 (Appendix B).

Z = 0.0

X = 0.0

Y = 0.0

Roll = Yaw = 0.0

Pitch = -0.29851[DEG] (down)

= -0.00521[RAD] (down)

The start of the TRANSPORT run is 1.3228[m] (4.339894865[feet]) upstream of the origin of the coordinate system. This distance is measured along the beam line (sloped distance).[3] The starting coordinates are

Local TRANSPORT station# = 0.0

Z = -1.32278[m] = -4.33984[feet]

X = 0.0

Y = 0.006892[m] = 0.02261[feet]

Roll = Yaw = 0

Pitch = -0.29851[DEG] (down)

= -0.00521[RAD] (down)

2.1.3. DATUM FOR E+ SYSTEMS IN SECTOR 19

Datum 3: The origin is a virtual point of the beam line at coordinates

LINAC station# = 060+00 [feet]

Local TRANSPORT station# = 0.0

Z = 0.0

X = 0.0

Y = 0.0

---

[3] The pitch was chosen to be 0.00521[rad] according to a MEMO from Davies-White dated Nov. 24th 1980 (Appendix B). All TRANSPORT runs for the damping rings used this pitch. According to the design the pitch should have been set to 0.00523[rad]. (Appendix C)

Roll = Yaw = 0.0

Pitch = -0.2836[DEG] (down)

= -0.00495[RAD] (down)

A scribe line in a brass plate embedded in the floor at the beginning of sector 19 defines the Z-position of the origin.

2.1.4. DATUM FOR SLC EAST

Datum 4: Origin is a virtual point of the beam line at LINAC station 100+00 with coordinates

LINAC station# = 100+00 [feet]

Local TRANSPORT station# = 0.0

Z = 0

X = 0

Y = 77.64368[m] = 254.7365[feet]

Roll = Yaw = 0

Pitch = -0.27158[DEG] (down)[4]

= -0.00474[RAD] (down)

A scribe line in a brass plate embedded in the floor at the end of sector 30, station 100+00 witnesses the Z-position of the origin.

The start of the SLC-East TRANSPORT run is 15.26977[m] upstream of the origin of the coordinate system at the beginning of the last LINAC QUAD (Q81). This distance is measured along the beam line (sloped distance). Starting coordinates are:

Local TRANSPORT station# = -15.26977[m] = -50.09767[feet]

Z = -15.26960[m] = -50.09711[feet]

X = 0

Y = 77.71606[m] = 254.97396[feet]

Roll = Yaw = 0

Pitch = -0.27158[DEG] (down)

= -0.00474[RAD] (down)

---

[4] At the beginning of the construction of the LINAC the pitch was chosen to be 0.00474[RAD]. All TRANSPORT runs for the arcs used this pitch. According to the design a pitch of 0.00476[RAD] = 0.27273[DEG] (Appendix C) should have been used.

The center of Q81 is defined as:

Local TRANSPORT station# = -15.23477[m] = -49.98284[feet]

Z = -15.23460[m] = -49.98228[feet]

X = 0

Y = 77.715897[m] = 254.973416[feet]

Roll = Yaw = 0

Pitch = -0.27158[DEG] (down)

= -0.00474[RAD] (down)

2.1.5. DATUM OFFSETS

In order to distinguish the geodetic coordinates from the rectangular TRANSPORT coordinates and in order to always work with positive coordinate values the following offsets were agreed on:

Origin	**Datum 1** Beg. sector 1	**Datum 2** SB0 Virtual element on girder 1-9 end of sector 1	**Datum 3** Beg. sector 19	**Datum 4** Station 100+00 End sector 30
Offset in X[m]	70000	70000	70000	70000
Offset in Y[m]	1100	1200	1900	2000
Offset in Z[m]	11000	12000	19000	90000

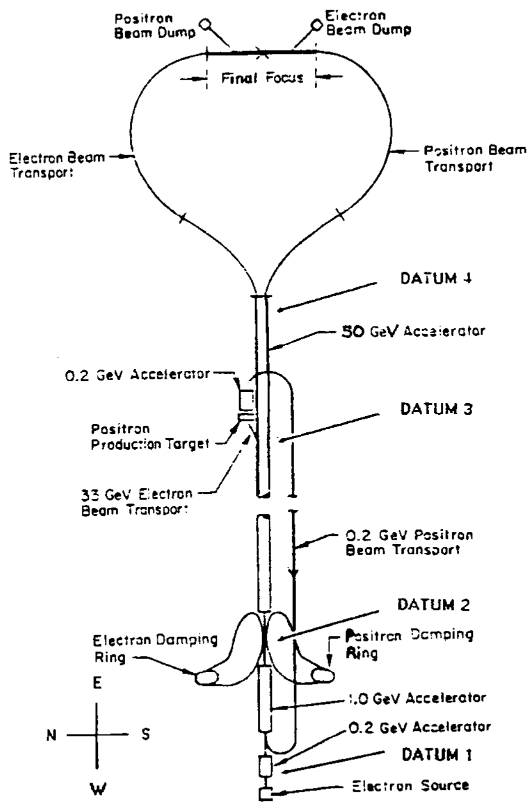


Figure 1.
Datum locations.

2.2. BEAM FOLLOWING COORDINATE SYSTEM

The beam following coordinate system is used to describe the orientation of the beam at any point along its path through the accelerator. This system remains tangent to the beam line with its positive z-axis pointing downstream. The system is rotated so that the positive x-axis generally points out from the bending arc and lies in the plane of the curve. The positive y-axis is oriented to complete the right handed coordinate system for the local beam.

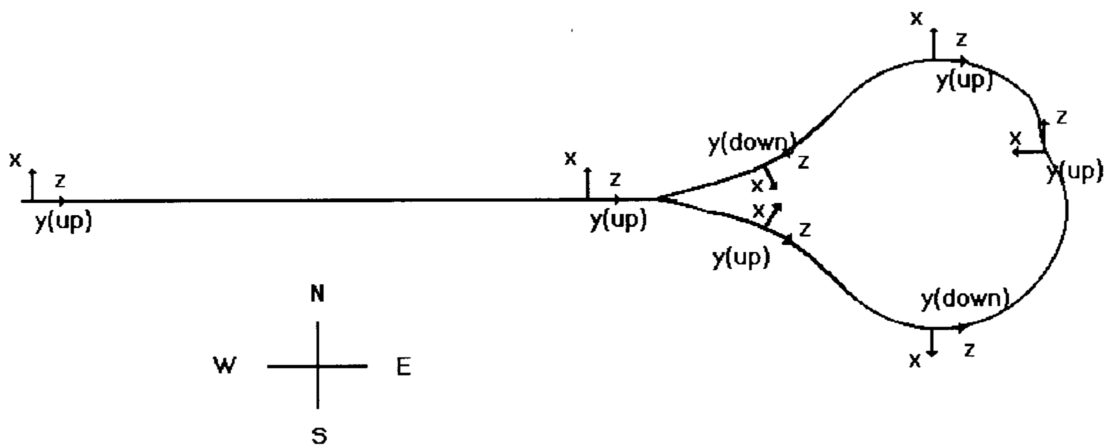


Figure 2.
Orientation of the beam following coordinate system.

To bring the absolute coordinate system into coincidence with the local system, three shifts (Z_0, X_0, Y_0) are executed first. This moves the origin along the beam line to the point of interest . Three sequential rotations are then applied which bring the axes of the shifted absolute system parallel to the axes of the beam following system. These three rotation angles are defined as follows:

yaw (θ) a rotation around the Y-axis of the shifted absolute coordinate system.

pitch (ϕ) a rotation around the once rotated X-axis.

roll (ψ) a rotation angle around the twice rotated Z-axis.

These sequential rotation angles must be applied in the order specified. All rotation angles in alignment follow the right hand rule. Picture yourself standing in the origin of the coordinate system and looking down each axis. An angle is positive if it is a clockwise rotation. However, in TRANSPORT, this is not the case. The following chart explains the sign changes:

TRANSPORT	ALIGNMENT
θ rotation (yaw, around y-axis) is positive when the z-axis turns towards the x-axis	θ rotation is positive when the z-axis turns towards the x-axis. This is the same as TRANSPORT.
ϕ rotation (pitch, around x-axis) is positive when z-axis turns towards the y-axis	ϕ rotation is positive when z-axis turns away from the y-axis. This is the opposite from TRANSPORT.
ψ rotation (roll, around z-axis) is positive when x-axis turns towards the y-axis	ψ rotation is positiv when x-axis turns towards the y-axis.This is the same as TRANSPORT.

Figure 3.
TRANSPORT versus ALIGNMENT rotation directions.

With these six transformation parameters the beam-following system is defined and is called the z_i, x_i, y_i coordinate system.

The complete orientation matrix corresponding to the above rotations is formed from three single rotation matrices [5]:

$$R = R_\psi R_\phi R_\theta$$

with

$$R_\psi = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{pmatrix}$$

$$R_\phi = \begin{pmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{pmatrix}$$

$$R_\theta = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

[5] Moffitt "Photogrammetry" 1980: see pages 596ff

$$R_\psi R_\phi = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ \sin\psi\sin\phi & \cos\psi & \sin\psi\cos\phi \\ \cos\psi\sin\phi & -\sin\psi & \cos\psi\cos\phi \end{bmatrix}$$

The product matrix is

$$R = \begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta & -\sin\phi \\ \sin\psi\sin\phi\cos\theta - \cos\psi\sin\theta & \sin\psi\sin\phi\sin\theta + \cos\psi\cos\theta & \sin\psi\cos\phi \\ \cos\psi\sin\phi\cos\theta + \sin\psi\sin\theta & \cos\psi\sin\phi\sin\theta - \sin\psi\cos\theta & \cos\psi\cos\phi \end{bmatrix}$$

The total transformation equation is as follows:

$$\begin{bmatrix} z_i \\ x_i \\ y_i \end{bmatrix} = \underline{R} \begin{bmatrix} Z_i - Z_0 \\ X_i - X_0 \\ Y_i - Y_0 \end{bmatrix}_i \quad ; \ \underline{x}_i = \underline{R}\,(\underline{X}_i - \underline{k}_i) \qquad \text{(Eqn. 2-1)}$$

Where Z_0, X_0 and Y_0 are the initial coordinates of the origin.

Since R is orthogonal, the inverse transformation can be written as follows:

$$\underline{X}_i = \underline{R}^t \underline{x}_i + \underline{k}_i \qquad \text{(Eqn. 2-2)}$$

All regular rotation matrices represent a transformation from TRANSPORT to a beam following coordinate system. All transposed rotation matrices represent a transformation from a beam-following coordinate system to TRANSPORT.

The six parameters to perform this transformation are given by TRANSPORT at the beginning and end of drift spaces and magnets along the beam. The constant shift vector $\underline{k}_i$ contains the actual beam coordinates at these points because the origin of the beam-following system lies on the beam line.

3. MAGNET GEOMETRY

Most beam lines are composed of box like magnets which guide and focus the beam through the system. However, the SLC combined function arc magnets are an exception. These magnets have a structure which greatly complicates the geometry of the system. Special section elements in this document are all beam guiding and non-beam-guiding elements which are not combined function arc magnets.

3.1. SPECIAL SECTION MAGNETS

Normally the beam passes through magnets (dipoles, quadrupoles, sextupoles) and is either bent, focused, or defocused. The most important feature of placing these magnets is that their orientation is always the same as some single point along the beam line. In the case of dipoles, quads and sextupoles, the orientation point is usually the midpoint of the beam path through the magnet. For a bending magnet, this orientation point is the midpoint of a curved path. Unlike the arc magnets, most of the dipole's body does not follow the path of the beam. To steer the particles either up or down, the bend magnets can be rolled 90 degrees to form a vertical bend, or a magnet roll angle of less than 90 degrees can be applied to obtain a combination of horizontal and vertical bending.

3.2. ARC MAGNETS AND ACHROMATS

The bending magnets of the SLC are combined function magnets, with magnetic properties of dipoles, quadrupoles and sextupoles. They are made by stacking and welding together 1,560 E-shaped laminations to form magnets approximately 2.5 meters long. The finished magnet is actually stacked in an arc with a sagitta of 2.75 mm.

The magnets are connected in a sausage-link fashion to form the arcs of the SLC. Figure 4 shows a typical two magnet section made up of one focusing and one defocusing magnet. It should be pointed out that the beam path is not a simple arc. It is made up of a series of curves connected by straight lines. The curves are the result of the effective bending length of a magnet. The magnetic fields actually extend beyond the physical limits of the magnet, therefore the magnetic length is longer than the length of the magnet iron. These bends are connected with straight lines where no magnetic fields affect the beam's path. The straight sections, therefore, are tangent to both the preceding and following bending arcs. This pattern of bend and drift sections is repeated 20 times to form what is known as an achromat.

An achromat is a section in the arc where the outgoing beam has the same characteristics as the incoming one. The distortions caused by a single magnet are cancelled by the time a particle bunch has traversed a complete achromat. Therefore, achromats are stacked one after the other down the beam line until the electrons and positrons reach the Final Focus. Initially each of the 20 bending magnets lie in a common plane (see section 7.0 ROLLFIX IN THE ARCS) and seem to trace out an arc on this surface. When this achromat plane is rolled the effect is to steer the beam up and over a slope as well as along an arc. Twenty-three achromats per arc are strung together to guide the beam up and down the grades while maintaining a coherent particle bunch.

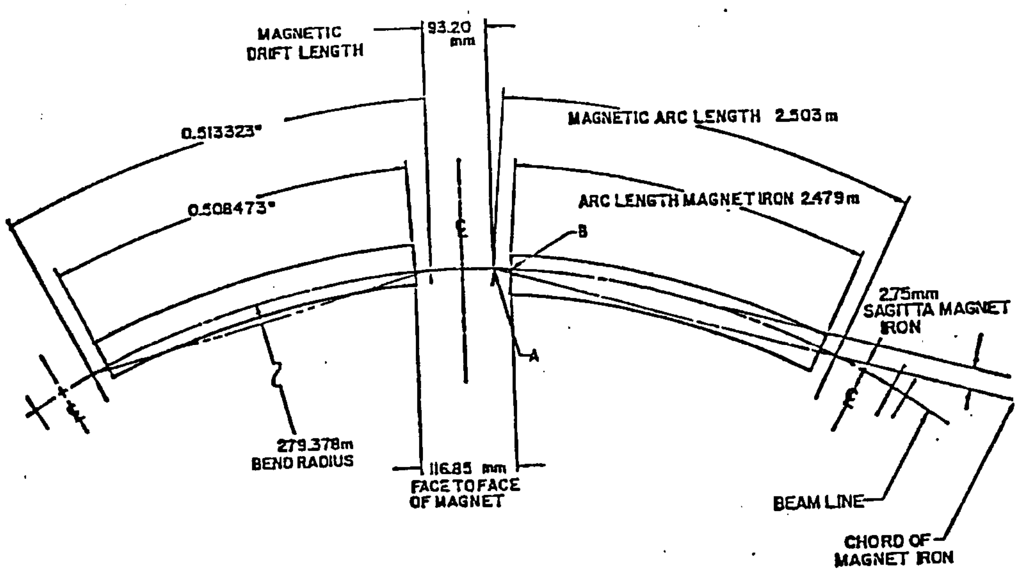


Figure 4.
Arc-magnet.

Each one of these achromat planes is rolled and pitched differently to achieve this. Therefore, the roll and pitch of the magnets with respect to the absolute coordinate system change continuously as one proceeds down the beam. These angles vary regularly within an achromat as well as between them. This can be seen by looking at the sequential rotations needed to move the absolute system to the beam-following system. The only common section of beam line between differentially rolled achromats is the linear drift section between the end magnet of the preceeding achromat and the beginning magnet of the current achromat.

This is an appropriate place to point out that TRANSPORT provides layout coordinates and rotation angles at the beginning and end of the drift sections. These points are also the beginning and end of the magnetic arcs which have a known bending radius. However, the magnetic arcs do not have a common radius point due to the drift sections between them. This makes it somewhat difficult to make computations between magnets. Therefore, additional coordinates are given at the midpoints of bending arcs and drift sections. In this paper the center of the drift section in the arcs will be referred to as the vertex point.

4. PREPOSITIONING OF SUPPORT SYSTEMS FOR THE ARCS (STEP 1 AND STEP 2)

The alignment of the magnets in the arc tunnel is performed in four steps. In the first step, the bolt locations to mount the support pedestals on the floor were surveyed and drilled. To compute the position of the vertex points projected onto the floor, TRANSPORT coordinates and rotations for the beam-following system at the vertex were used (Figure 5). The pedestals were set so that in the beam direction they were perpendicular to the pitched floor, but plumbed in the transverse direction, i.e. their pitch is equal to that of the beam line at that point while roll was adjusted to zero. Since the pedestal is pitched the location of the vertex point cannot be directly plumbed to the floor. A vector which represents the pedestal is intersected with the floor to find the exact location of the center of the base.

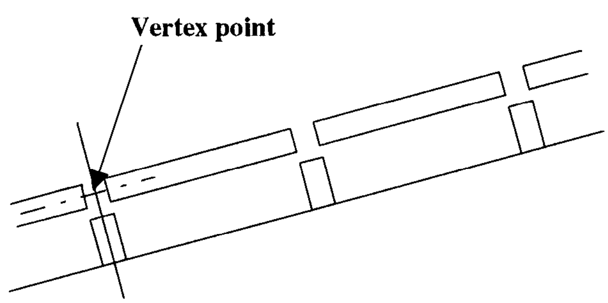


Figure 5.
Definition of vertex point for support system positioning

To obtain an exact set of coordinates for the projected vertex point one would have to measure the actual 3-dimensional location of the tunnel floor. This, however, is impractical so calculations were based on ideal floor locations. This seemed to be a reasonable assumption because the tolerance for bolt placement is ± 1.0 cm while floor uncertainties will amount to approximately 0.5 cm errors in the Z, X location of the points.

The computations were made by taking each pedestal as a rigid body with its own coordinate system. This coordinate system is equivalent to the beam-following system at the vertex point except that it is not rolled. By representing the pedestal with the y_i-axis it was possible to give the ideal floor point the coordinates of $z_i = 0$, $x_i = 0$ and y_i = negative beam height above the floor at the vertex point. Then Equation (2-2) could be applied with just yaw and pitch values inserted into the rotation matrix. When this was done, projected coordinates of the vertex were obtained in the absolute coordinate system. These coordinates could then be laid out from control points.

After the pedestals were placed over their bolts, step 1 of the alignment procedure could begin. In this step the pedestal position was refined to the 3 mm level and then it was grouted in place. For step 1, the vertex point was represented by intersecting laser beams projected through KERN E2 theodolite telescopes. These instruments occupied control points with coordinates known in the absolute system. Since the vertex point had known coordinates by measuring instrument heights, and by backsighting other control points, both horizontal and vertical angles to the vertex point could be calculated. The accuracy of step 1 procedure was ±3 mm.

In step 2 the magnet adjustment system was then positioned so that the magnets could be mounted to within .5 mm of their ideal position. Again as in step 1, the vertex point was used as the control point for positioning.

Step 2 involved the same calculations as step 1, except that now the procedure was changed slightly. Here the actual position of the vertex was measured through leveling and intersection. Its true coordinates were then compared with the ideal position and offsets calculated. The adjustment system on top of the pedestal was then used to move the vertex target to its ideal location. These motions were controlled with dial gauges connected into a computer feedback loop. This prevented mistakes when making adjustments. The new position of the vertex was then measured and the procedure was repeated if necessary to achieve the desired .5 mm level. This method could only be used as long as the vertex was visible. As soon as the magnets were mounted the vertex point and beam line were obscured.

5. ABSOLUTE POSITIONING OF THE MAGNETS (STEP 3)

Step 3 of the alignment process involved the absolute positioning of the magnets. Program STEP3 was used for the arcs and SPCLSECT for everywhere else. At this point the magnets had been mounted on the support pedestal. The major goal here is to position the magnets to their ideal positions within the final tolerances as follows:

1.) Two magnets within an achromat (defined beam line section) must point at each other with an angular accuracy of 0.04 milliradians.
2.) Transverse to the beam line (horizontal and in elevation) two magnets must be adjusted with an accuracy of 0.1 mm.
3.) The distance between two magnets must be adjusted with an accuracy of 0.5 mm.
4.) Roll must be set to within 2.0 milliradians of its ideal value.

The alignment process is much more difficult in this case than in the previous two steps, because coordinates for magnet fiducials are not provided by TRANSPORT. They must be calculated by the surveyor according to the locations of the magnet fiducial points in relation to the beam line. The three rotational elements yaw, pitch and roll must also be controlled. The yaw and pitch are set by moving fiducial points at each end of the magnet to their proper 3-dimensional positions. The roll cannot be set using the midplane of symmetry for the arc magnet. Therefore, it is necessary to calculate a roll about the beam line for one fiducial point on the magnet, and setting it by using an inclinometer. Finally, corrections for the actual magnet lengths must be taken into account for the arc magnets. Magnet fiducial points are usually represented through tooling balls on the steel surface. However, on the arc magnets no such fiducial points exist and a specially designed fixture (C-clamp) has to be used.

5.1. ARC MAGNETS

5.1.1. C-CLAMP FIXTURES

C-clamp fixtures are used for defining the position of the arc magnets. They define fiducial points with respect to the magnet's center line by clamping into the grooves of the arc magnets. The position of the fiducial points on the clamps with respect to the registration points are measured. This calibration is performed for each clamp on a CMM (Coordinate Measuring Machine) to an accuracy of better than 10 μm.

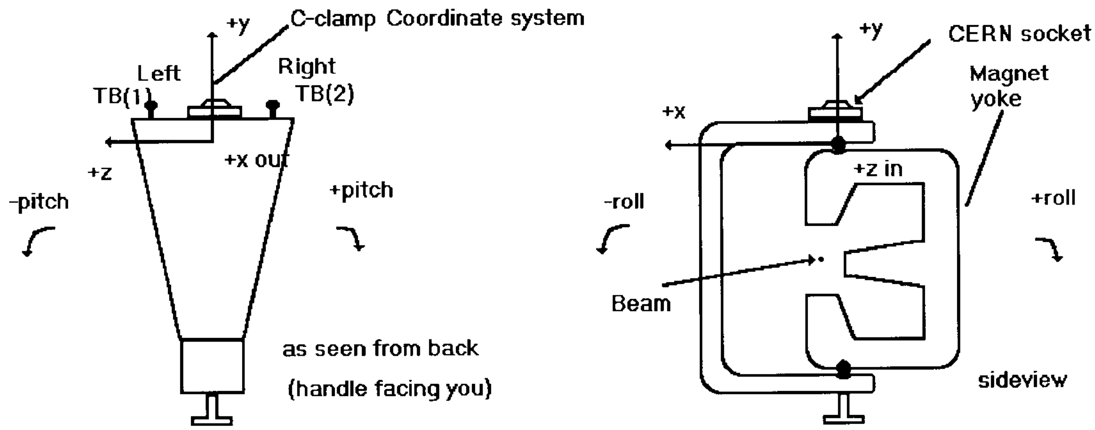


Figure 6.
C-clamp fixture.

After a C-clamp is clamped a few centimeters from the end of an arc magnet, the distance from the magnet end plate to a tooling ball on the fixture is measured with a micrometer (z-off). At the same time roll (ψ ') is measured with a Schaevitz inclinometer.

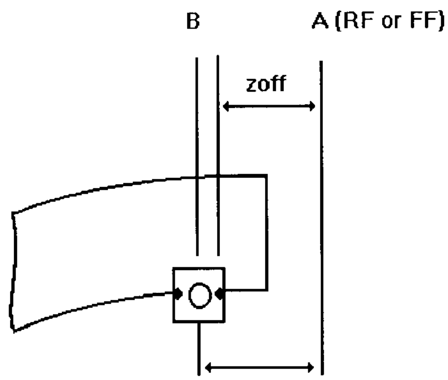


Figure 7.
C-clamp position on arc-magnet.

The chord distance from A (the magnetic edge of the magnet), to B (the CERN socket or reference point on the fixture) can be calculated by adding up:

1.) The measured offset (zoff) + half a tooling ball width.

2.) The distance from the tooling ball to the CERN socket, which is known from the C-clamp calibration.

3.) The distance from the magnet steel edge to the virtual point A (magnetic edge), which is known for each magnet and stored in a pedigree file. (See section 5.1.2 PEDIGREES).

With the chord distance and the measured roll (ψ') the ideal coordinates of the CERN socket reference point can be calculated. (See section 5.1.3 CALCULATION PROCESS).

The actual coordinates of the CERN sockets are determined using standard engineering surveying techniques. Each CERN socket is included in a horizontal direction observation set as well as a level network. Following the observations, two least squares adjustments are executed, independently for the z,x position and the y height position.
This gives the actual positions of the CERN sockets. The differences between the actual and the ideal coordinate sets are then translated into dial gauge movements. (See section 5.1.3.3 DIAL GAUGE MOTION CALCULATIONS). After applying these corrections the magnets are considered to be positioned in their absolute positions. For sign conventions for dial gauge movements see Appendix A.

5.1.2. PEDIGREES

The SLC arc magnets have fabrication and magnetic centerline errors that may often exceed in magnitude the design alignment tolerances for the beam transport system. For this reason, measured mechanical and magnetic offsets have been combined to produce a single correction factor called a "pedigree". This quantity is used in the calculation of ideal coordinates to offset the magnet from its nominal placement in order to have the actual magnetic centerline coincide with the theoretical position. This in effect results in an actual zig-zag placement of the magnets.

Figure 8 shows an example table of pedigrees for magnet XN1206. The table starts out with a summary line which shows the X and Y pedigrees at the A and B ends of the magnet. The stamped serial number, AGF750D, defines the A end of the magnet. This is also the end where the scanning of the physical dimensions of the magnet was started. From the main table, XA and XB are the first and last entries in the coordinate table. ZA and ZB represent the distance along the beam line from the magnetic edge to the physical edge of the magnet, while YA and YB are chosen so that an aligned magnet centerline will be equally split by the beam in the y-direction. This is the dimension needed to properly place the magnet in the slot provided by TRANSPORT simulations. RAB is a measure of the twist in the magnet and is not used in any alignment calculation.

PEDIGREE XN1206 AGF750D

MAGNET	ZA	XA	YA	ZB	XB	YB	RAB
AGF750D	0.4627	0.00320	-0.0008	-0.4627	0.00620	-0.0008	-0.7828

COORDINATE TABLE

N	Z(in)	X(in)	Y(in)	ROLL(mrad)
1	0.0000	0.0032		
2	0.8405	0.0002	-0.0039	0.4171
3	1.7570	0.0013	-0.0025	0.0873
4	2.6750	0.0012	-0.0020	0.0485
5	3.5925	0.0006	-0.0018	0.0776
6	4.5105	-0.0001	-0.0012	-0.3395
7	5.4290	-0.0007	0.0000	-0.4074
8	6.3470	-0.0012	0.0010	-0.6015
9	7.2660	-0.0016	0.0026	-0.6015
10	8.1870	-0.0022	0.0028	-0.7664
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
99	90.0600	0.0001	-0.0039	-0.2328
100	90.9830	0.0007	-0.0040	-0.1746
101	91.9055	0.0011	-0.0043	0.0582
102	92.8285	0.0012	-0.0039	-0.0582
103	93.7520	0.0016	-0.0030	0.1746
104	94.6755	0.0027	-0.0038	0.0970
105	95.5995	0.0040	-0.0056	0.0485
106	96.0600	0.0050	-0.0043	0.0873
107	97.1110	0.0075	-0.0057	-0.1843
108	97.6180	0.0062		

Figure 8.
Sample pedigree file.

The coordinate table lists the pedigree values at approximately 1 inch intervals along the magnet. This interval corresponds to the SAMMI (SLAC Automatic Magnet Measurement Instrument) measurement routine. SAMMI was a device built specifically to measure certain dimensions of every magnet. These data were combined with magnetic measurement data from a sample (30%) of magnets to calculate the pedigree offsets.

"N" is the point number while Z designates the distance from the serial number (A) end of the magnet to the point. The X and Y values are the pedigree offsets for these points. The roll is included but not used for calculation. Approximately 110 points are available for each magnet but this number varies from magnet to magnet.

The data also point out some interesting alignment characteristics of the magnets. First, the magnet ends tend to have the largest manufacturing errors which are reflected in the size of the pedigrees when compared to values closer to the center of the magnet. Second, the pedigree can vary greatly in size from point to point. This implies that the local variation in pedigree must be taken into account when fixtures spanning a number of sample points are mounted on the grooves.

The agreed upon coordinate system for the pedigrees has its z' axis pointing from the A to the B end of the magnet. The x' axis points outward of the bending radius and the y' axis follows the right hand rule (see Figure 9). This arrangement was violated when the y' pedigrees were passed on with the incorrect sign. The problem is corrected in the pedigree subroutines by multiplying the y' coordinate by -1 and not changing the tables. All sign conventions and calculations assume that the agreed upon coordinate system is used.

When calculating ideal coordinates for the C-clamps, the pedigrees can be thought of as a correction to the origin of the C-clamp coordinate system. The calculation procedure involves putting both C-clamp coordinates and pedigrees into the beam following coordinate system and then adding them together to obtain the local coordinates of the CERN socket to be transformed as explained below.

Sign conversions are dictated by the magnet's orientation in the beam line. To place magnets so that their curvatures match the arcs, they had to be rotated end-for-end so that either the A or B end of the magnet sees the beam first. This reverses the pedigree coordinate system relative to the beam following system and a table of sign conversions results. Appendix A details these for each section of the arcs.

In case of the C-clamps used for Steps 3 and 4 positioning, an average pedigree offset is found for the individual setup of that clamp. This is done by interpolating the value for the three registration points, two on the top and one on the bottom grooves. These offsets are then averaged to obtain the final value.

For clamps used in the magnet-to-magnet alignment (see section 7), a more elaborate treatment of the "local" pedigrees is required. These fixtures carry long arms to translate the position of one magnet to its neighbor. These long lever arms magnify the effect of the different pedigree values for the mounting points of the clamp. To contend with the problem an average offset is calculated as explained above, and the individual pedigrees for the mounting points are used in determining the "local" pedigree induced yaw and pitch orientation angles of the clamp. This modification allows the designed magnet-to-magnet alignment geometry to reflect the influence of local pedigree offsets.

During commissioning of the SLC it was discovered that the focus and defocus magnets were systematically too far apart in the x-axis. This was fixed by physically moving the magnets closer together thereby changing the effective pedigrees for each magnet. This fix was applied as follows:

- all the south arc magnets were moved 0.200mm closer to the beam line,
- achromat 21 in the north arc had all magnets moved closer to the beam line by 0.200mm,
- achromats 20, 22, and 23 in the north arc had all magnets moved closer to the beam line by 0.150mm.

A program was written to add these constants into the pedigree tables so the offsets would be reflected in all the alignment steps. A column to keep track of the old pedigree data was added for historical purposes.

5.1.3. CALCULATION PROCESS

5.1.3.1. IDEAL COORDINATES

To start these calculations, one should first look at a simple case. Assume that a magnetic fiducial point is located directly above point A (a point in space) in Figure 9. To position a fiducial point above A is of course impossible because the magnet iron ends at point B. However this is a convenient place to start because TRANSPORT coordinates are provided for the point on the beam line below the fiducial mark A. The orientation angles for the local coordinate system are also given. This makes the computation of the needed coordinates simple. One must only know the coordinates of the fiducial point in the beam-following system. This can be done by building fixtures (C-clamps) which locate the mark in a known position with respect to the magnet's center line. Then Equation (2-2) (see section 2.2 BEAM FOLLOWING COORDINATE SYSTEM) can be applied to obtain TRANSPORT coordinates of the desired point. This point only exists in space because it is located at the end of the magnetic bend arc but not on the magnet iron. To find coordinates on the actual magnet iron, the local coordinate system must be translated along the beam line to a point beneath the fiducial mark.

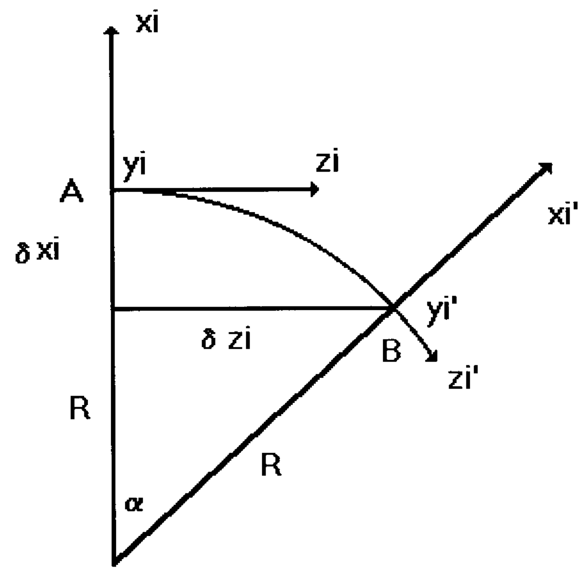


Figure 9.
Top view, idealized C-clamp position.

By referring to Figure 9 one can see how this would be done. First the local system z_i', x_i', y_i' at point B is rotated through a yaw angle to orient it to the $z_i x_i y_i$ system. The rotated system is then shifted by dz_i and dx_i so that its origin coincides with the origin of the beam-following system at point A. In doing this the z_i', x_i', y_i' coordinates of the fiducial mark which are set through fixturing are transformed into beam-following coordinates. The controlling equation is as follows:

$$\begin{pmatrix} z_i \\ x_i \\ y_i \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} z_i' \\ x_i' \\ y_i' \end{pmatrix} + \begin{pmatrix} r \sin\alpha \\ -r(1-\cos\alpha) \\ 0 \end{pmatrix} \qquad \text{(Eqn. 5-1)}$$

The angle α can be calculated by using the radius of the curve provided by T RANSPORT and either the measured length of the arc or the chord. After the z_i, x_i, y_i coordinates are computed, Equation (2-2) is again applied to find TRANSPORT coordinates of the fiducial mark above point B. This calculation can be done for any point along the bending arc of one magnet if a chord or arc length is measured from a point with known TRANSPORT coordinates.

These will not be the final coordinates of the fiducial marks since the magnet may be manufactured with a twist around its magnetic axis. If this is true the z_i', x_i', y_i' system must undergo an additional rotation to compensate for the twist. This is necessary because the coordinates of the fiducial mark are known only in a system which is twisted with respect to the z_i', x_i', y_i' system. To do this, Equation (5-1) must be modified to include a twist (γ) about the tangent to the beam line. This would result in the following equation:

$$\begin{pmatrix} z_i \\ x_i \\ y_i \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha \cos\gamma & \cos\alpha \cos\gamma & \sin\gamma \\ \sin\alpha \sin\gamma & -\cos\alpha \sin\gamma & \cos\gamma \end{pmatrix} \begin{pmatrix} z_i' \\ x_i' \\ y_i' \end{pmatrix} + \begin{pmatrix} r \sin\alpha \\ -r(1 - \cos\alpha) \\ 0 \end{pmatrix}$$

$$\underline{x}_i = \underline{B} * \underline{x}_i' + c \quad ; \quad \underline{B} = R_\gamma * R_\alpha \qquad \text{(Eqn. 5-2)}$$

Equation (2-2) is then applied to these results to find the needed coordinates. It should be pointed out that a total of five rotations are needed to translate the position of the fiducial point in a magnet coordinate system to the absolute system of TRANSPORT.

5.1.3.2. IDEAL ROLL CALCULATION

Now that these coordinates are found, a roll value (ψ') with respect to the gravity vector must be calculated. However, this is not a simple matter because of the five sequential rotations needed to transform the above coordinate systems. For this reason it is easiest to go back to point A where reference coordinates and rotations are provided by TRANSPORT. One may think that the problem is trivial at this point because a roll value is provided. This is not the case, though, because the roll given is a sequential roll. It is not measured on a plane which is parallel to gravity, but about a twice rotated Z-axis. Therefore, this number cannot be used to set a precise roll with an inclinometer which uses gravity as a reference (Figure 10).

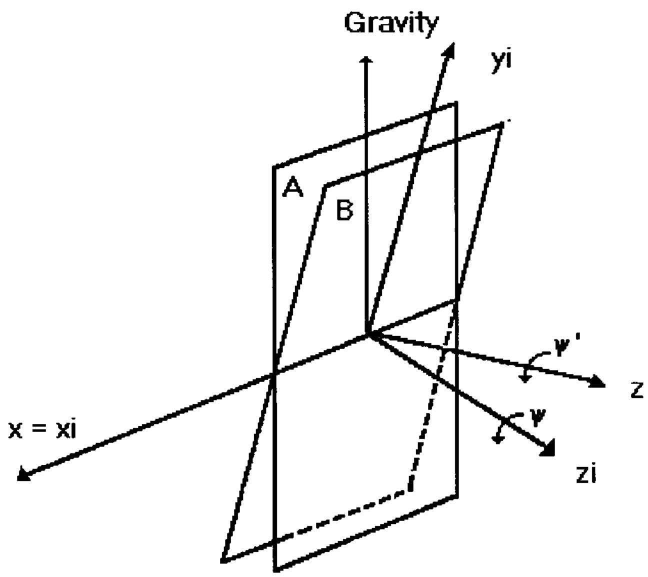


Figure 10.
Roll with and without respect to gravity.

One must understand how the inclinometer works to solve this problem. In the case of the SLC, a type of inclinometer (Schaevitz 1978) is used whose electronic axes are according to specifications not affected by tilt transverse to the direction of measurement. This means that it can be used on a magnet which is both pitched and rolled, to measure a roll angle with respect to gravity. To do this accurately and quickly it must be easy to orient the inclinometer in a convenient direction which is repeatable for every setup. In this case, it is easiest to orient in the direction of the x_i-axis of the beam-following coordinate system; i.e. perpendicular to the beam line. It has to be noted, though, that the roll and the pitch proved to be highly correlated in the present measurement setup of the Schaevitz inclinometers. For highly pitched areas this effect is distinct and easily observed. [6] Unfortunately we still don't have a solution and it requires some further investigations.

As was said above, the TRANSPORT roll angle is an angle measured about a twice rotated axis and is <u>NOT</u> the roll to be set with respect to gravity. The measured roll is a projection of the TRANSPORT roll on to the plane formed by the gravity vector and the x_i-axis. The formula to calculate the correct roll at point A can be found by using the fact that the rotation matrix for a given orientation of the beam-following coordinate system is unique, but the combination of sequential rotations is not. In other words, the values of the nine elements of the rotation matrix are fixed but these nine numbers can be calculated from several different sequential rotations. This makes it possible to equate corresponding elements of different sequential rotation matrices which define a given orientation. In this case the roll must be calculated in a system which has been yawed but not pitched. To do this, the sequence of rotations is changed to yaw, roll and then pitch. This gives the following rotation matrix <u>M</u>:

$$\begin{pmatrix} \cos\phi'\cos\theta - \sin\phi'\sin\psi'\sin\theta & \cos\phi'\sin\theta + \sin\phi'\sin\psi'\cos\theta & -\sin\phi'\cos\psi' \\ -\cos\psi'\sin\theta & \cos\psi'\cos\theta & \sin\psi' \\ \sin\phi'\cos\theta + \cos\phi'\sin\psi'\sin\theta & \sin\phi'\sin\theta - \cos\phi'\sin\psi'\cos\theta & \cos\phi'\cos\psi' \end{pmatrix} \qquad \text{(Eqn. 5-3)}$$

It should be noted that ϕ' is not equal to the TRANSPORT pitch for the same reason that ψ' is not equal to ψ . Now the m_{23} element can be equated to the r_{23} element of the TRANSPORT rotation matrix R. This gives the following formula for ψ':

$$\psi' = \sin^{-1} (\sin\psi \ * \cos\phi) \qquad \text{(Eqn. 5-4)}$$

In the worst case, the difference between the TRANSPORT roll ψ and measured roll ψ' is 0.9 milliradians. This is a significant amount and must be taken into account. Theoretically a correction for the earth's curvature needs to be applied to ψ', but it was found to be insignificant.

This procedure applies to any point along the beam line that has given TRANSPORT coordinates. If TRANSPORT coordinates have to be calculated (see section 5.1.3.1 IDEAL COORDINATES), the same procedure as above can be applied to the t_{23} element of the total

[6] Bernard Bell

rotation matrix $\underline{T}$ for the twisted magnet at point B. The $\underline{T}$ matrix is formed by multiplying $\underline{B}$ of Equation (5-2) by $\underline{R}$:

$$\underline{T} = \underline{B} * \underline{R} \qquad \text{(Eqn. 5-5)}$$

The resulting formula for ψ' is:

$$\psi' = \sin^{-1} (\sin\phi \sin\alpha \cos\gamma + \sin\psi \cos\phi \cos\alpha \cos\gamma + \cos\psi \cos\phi \sin\gamma) \qquad \text{(Eqn. 5-6)}$$

Equation (5-6) is used in two separate ways:

a) For layout roll the twist (γ) will be set to zero and no pedigree twist is applied.
The equation is solved for (ψ') .

b) If the unknown twist of the magnet is to be calculated the roll (ψ') is measured and Equation (5-6) is solved for the unknown twist (γ) by means of an iterative approach. Equation (5-6) can be seen as follows:

F = Parameter_1 * cos(x) + Parameter_2 * sin(x) - Parameter_3

x is solved for.

The resulting twist is then used as input for Equation (5-2).

5.1.3.3. DIAL GAUGE MOTION CALCULATIONS

In Steps 2, 3 and 4(smoothing) the misalignments of magnets must be translated into dial gauge motions to be applied to the adjustment systems. For Steps 2 and 3 the residual misalignments are found in the overall TRANSPORT design coordinate system and must be rotated to the individual pedestal based system corresponding to the alignment adjusters. This transformation uses equation (2-1) without the application of shifts or the roll angle. The Step 4 smoothing adjustments are transformed in a similiar manner, but the yaw angle, the roll and the shifts are not applied. This is possible because the residual misalignments are already in a system parallel to the azimuth of the beam line. See section 6.0 SMOOTHING OF THE MAGNETS for more details.

5.2. SPECIAL SECTION MAGNETS

Special section magnets usually have tooling balls whose positions are measured relative to the mechanical center of the magnet. If there are no tooling balls available, which is the case for the damping ring quadrupoles, grooves and flanges are known relative to the mechanical center of each magnet. With the coordinates of either the grooves or tooling balls in the local beam following coordinate system known, it is just a matter of applying Equation (2-2) (see section 2.2 BEAM FOLLOWING COORDINATE SYSTEM) to determine ideal coordinates for the elements. Most magnets are placed like boxes on the beam line with their center lines and orientation angles identical to the beam following coordinate system. Some cases violate this convention. One such case is bend magnet alignment. Since bending magnets are often built like boxes but the beam traces a curved trajectory through them, a decision of how to place them in yaw must be made. The agreed upon convention is to make the yaw of the magnet the same as the beam at the center of the bend. A shift equal to one-half the sagitta of the curve is added, thus off-setting the center line of the box towards the center of the curve (Figure 11). This system best uses the magnet's field. In most cases, S is subtracted from the magnet's fiducial point's x-coordinate.

The shift S is calculated as follows:

$$S = \text{sagitta}/2 = r(1 - \cos(\alpha / 2))/2 \qquad \text{(Eqn. 5-7)}$$

r = radius of bend magnet
α = total bending angle.

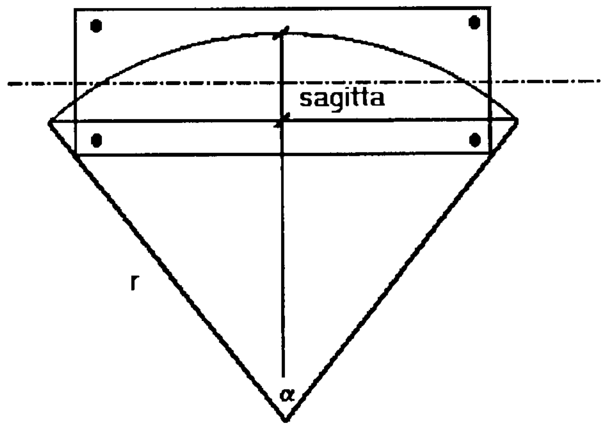


Figure 11.
Magnet sagitta.

Another exception is any component which has rings around its outside as alignment references. They might include Protection Collimators (PC), Beam Position Monitors (BPM) or vacuum flanges. Since it makes no sense to calculate a roll into the coordinates for the ring references, these components assume a roll of zero.

6. SMOOTHING OF THE MAGNETS (STEP 4)

Smoothing is the term used to describe the process of positioning beamline components around a trend curve rather than in absolute space. It is especially useful when approaching final magnet to magnet alignment tolerances since it eliminates systematic measurement errors and mathematical artifacts. Influences such as atmospheric refraction and mechanical centering of theodolites and targets can introduce systematic errors as large as the final alignment tolerances. Repetition of a given set of measurements generally results in a different configuration of these error sources and, therefore, a different result. As the size of component position adjustments approaches the size of the systematic effects, the adjustments cease to converge to zero and begin to oscillate.

Smoothing in the special sections for all the beam guiding elements is a two phase process. In the first phase a smooth curve is modelled for all beam guiding elements (Step 4 procedure). Then, after the beam guiding elements are positioned, components like BPMs, and PC's have to be aligned with respect to the new positions of their neighboring magnets. This is generally done by optical alignment technics. The roughly square profile of the magnets, the even distribution of fiducials, and the use of individual supports under each magnet allows smoothing of the beam guiding elements in a single step.

In the arcs, also, a two-phase smoothing procedure consisting of Step 4 and MTM (Magnet To Magnet) was developed. Since the upstream end and downstream end of two adjacent magnets are supported by the same pedestal, a Step 4 process is used to first align the upstream ends. In a second phase MTM is used to align each RF end (downstream) to its adjacent FF end (upstream).

Several different curve fitting techniques were considered for the smoothing (Step 4) procedure, including spline and polynomial fits. However, these approaches are not easily adapted to the three dimensional space curve formed by a string of SLC magnets. Also these methods are not robust, in that measurement errors and outliers bias the resulting curve significantly. An algorithm which is well suited for this situation is the technique of "principal curves." The technique is described in Trevor Hastie's "Principal Curves and Surfaces" (Hastie 1984) [7]. This approach produces smooth curves which pass through the "center" of a three-dimensional data set. Its goal is to minimize the sum of squared distances between the ideal points of the curve and the data set. Figures 13 through 16 demonstrate the differences between principal curves and other fitting techniqes.

The principal curve algorithm is an iterative process which will closely approach almost all points in a data set if allowed to iterate many times. Therefore, a criterium must be established to stop the process before the curve can no longer be considered smooth. Machine physicists usually provide tolerance requirements; in many cases an offset of 0.1mm between adjacent magnets is acceptable. After each iteration the offsets between adjacent magnets are calculated and compared to this threshold (Figure 12). As soon as one offset exceeds the threshold, the process is stopped. The results of the previous iteration will be used for the calculation of the x and y adjustments for each magnet (Figure 17). Adjustments greater than 0.06 mm will be flagged with a star.

[7] Hastie 1984, (SLAC-276, STAN-LCS-11)

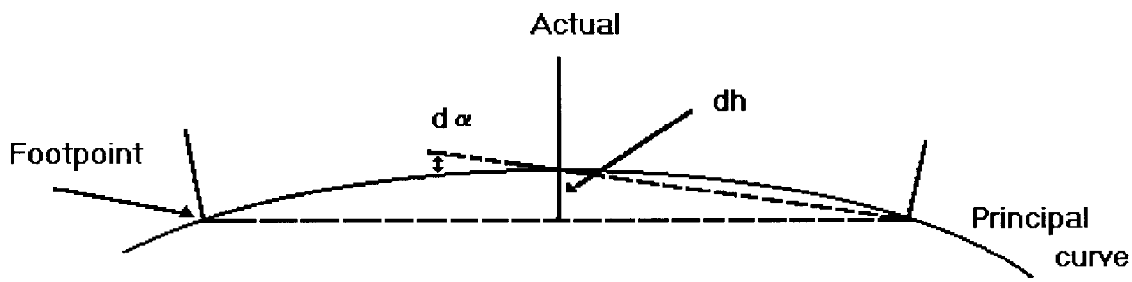


Figure 12.
Smoothness criteria in the arcs. da <= 0.1 milliradians = dh <= 0.1mm

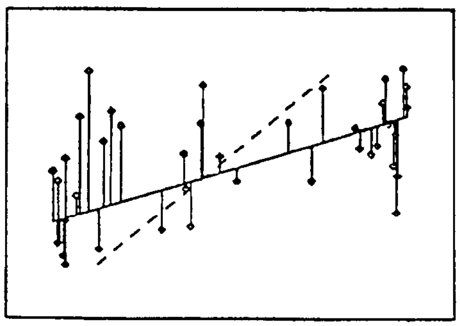


Figure 13.
The linear regression line minimizes the sum of the squared errors in the response variable.

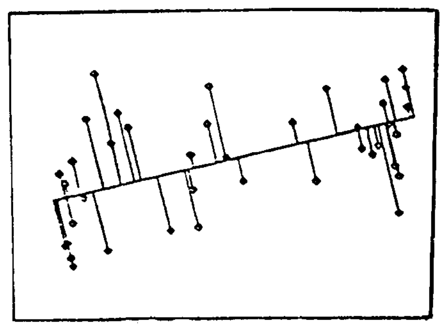


Figure 14.
The principal component line minimizes the sum of the squared errors in all the variables.

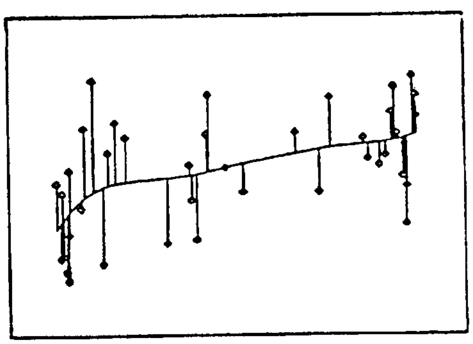


Figure 15.
The smooth regression curve minimizes the sum of squared errors in the response variable, subject to smoothness constraints.

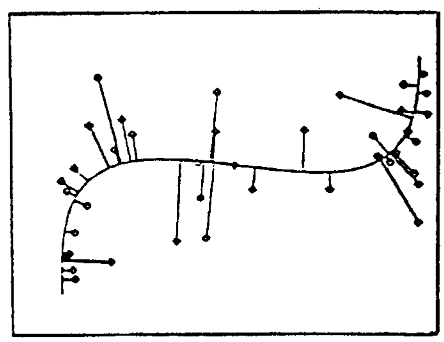


Figure 16.
The principal curve minimizes the sum of squared errors in all the variables, subject to smoothness constraints.

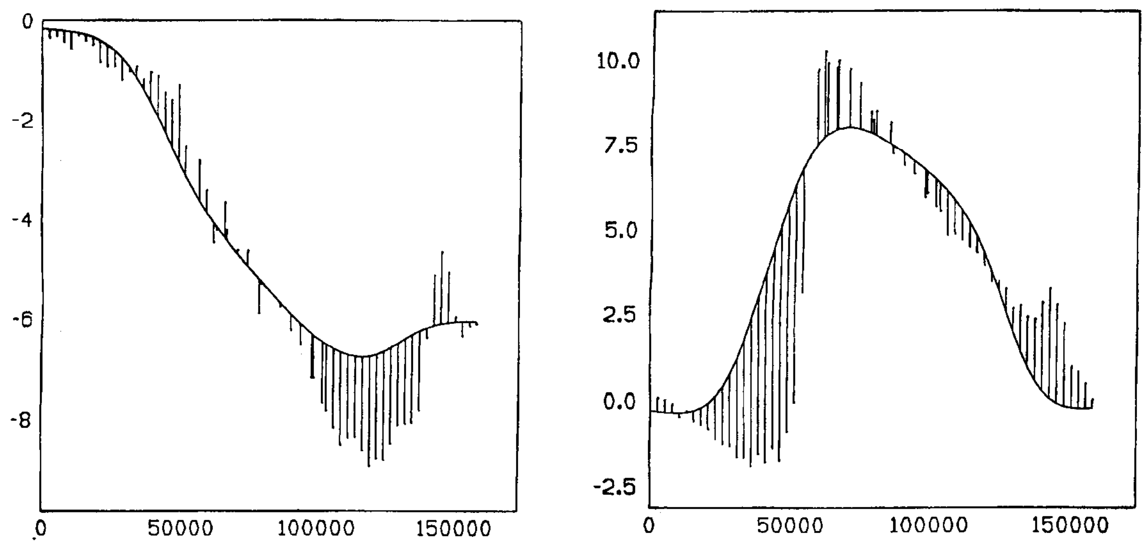


Figure 17.
PCURVE output plots.

6.1. ARC MAGNETS

6.1.1. FRONT FIDUCIALS

Step by step calculation procedure:

1.) Ideal coordinate determination.

Ideal coordinates for the C-clamp positions at the upstream end (Front Fiducial,FF) of each magnet are calculated (see section 5.1.1 C-CLAMP FIXTURES). It requires the roll measurement using a Schaevitz inclinometer and the z-offset be measured from the magnet steel edge to the C-clamp. The actual calculation of the ideal coordinates also requires knowledge about the C-clamp calibration and the pedigree data of each magnet.

2.) Actual coordinate determination.

C-clamps are placed on the FF ends of the arc magnets. Every fifth clamp is then occupied by a theodolite. Horizontal direction sets are measured to all other C-clamp sockets and invar wire distances are pulled between the C-clamp sockets. Actual coordinates are calculated by reducing the surveyed data, and processing the horizontal and vertical network separately in a least-squares adjustment. The first and the last C-clamp positions are held fixed to their ideal positions in this adjustment process. The elevation is determined by running level loops over a maximum five C-clamps before closing a loop. All loops have to be interconnected.

3.) Differences between ideal and actual coordinate sets.

The ideal and the actual coordinates are transformed into a beam following coordinate system. The C-clamp positions are standardized in the Z coordinate to the magnetic edges of the magnets (FM points). In this system the differences between ideal and actual coordinates are calculated.

4.) Elimination of Δ z.

From the difference between the ideal and actual coordinate sets a Δ z value is obtained which will be mathematically eliminated. The z-dimension is not very critical and elimination of Δ z reduces a 3-dimensional problem to a 2-dimensional problem. The ideal coordinates are then recalculated by yawing the original ideal coordinates to the actual z-locations and eliminating the Δ z offset (yaw angle $\alpha = \Delta$ z / magnet radius). This Δ z reduction can be seen as replacing the mechanical adjustment of an arc magnet in z by mathematical means.

5.) Differences between new ideal and actual coordinate sets.

The newly found ideal coordinates are now subtracted again from the actual coordinates to give the results in the TRANSPORT coordinate system.

6.) Transform differences from TRANSPORT into BFS.

At this point the results have to be adjusted for yaw to map radial deviations along a single axis. (see 4. above) This puts the coordinates parallel to the azimuth of the magnet (BFS). These deviations are the input values for PCURVE.

7.) PCURVE.

PCURVE is run.

8.) Dial gauge movements.

PCURVE's output is transformed into dial gauge movements for the arcs. The output is transformed into the coordinate system of the individual adjustment system. Sign conventions for dial gauge movements are in Appendix A.

6.1.2. REAR FIDUCIALS

The FF alignment of the magnets is completed at this point and the FF ends are considered aligned to their final positions. For aligning the downstream end (Rear Fiducial,RF) of each magnet, two MTM (Magnet To Magnet) alignment techniques are used. In both procedures the RF end is to be aligned in reference to the FF end of the adjacent magnet. The roll and pitch on special clamps is measured as well as the z-offset between the 2 magnet edges. Transformation parameter sets are calculated to reference the RF clamp fiducials in the FF clamp coordinate system.

1.) Within each achromat where there are no roll transitions special clamps are used (Figure 18). The MMAFI (Magnet to Magnet Alignment by Fixture) procedure is used.

2.) From achromat to achromat and in rollfixed areas where there are roll transitions the MMAS (Magnet to Magnet Alignment System) procedure is applied.

6.1.2.1. MTM WITHIN ACHROMAT

In the MMAFI procedure, the RF clamp coordinates are transformed into the FF clamp coordinate system. This transformation works only at magnet junctions with no roll transitions. On each clamp (RF and FF end) the roll, pitch and z-offset between the magnet edges is measured. The FF clamp and the RF clamp registration points are determined in the FF clamp coordinate system and distances between the corresponding FF and RF registration points are calculated.

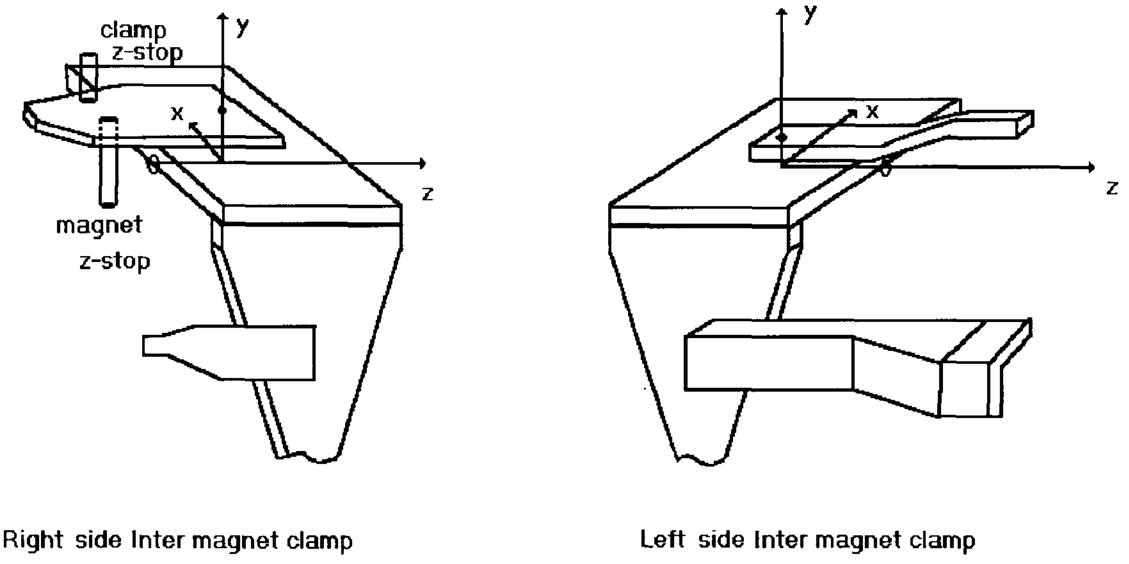


Figure 18.
Magnet to magnet fixtures. Clamps 71,72,73,74

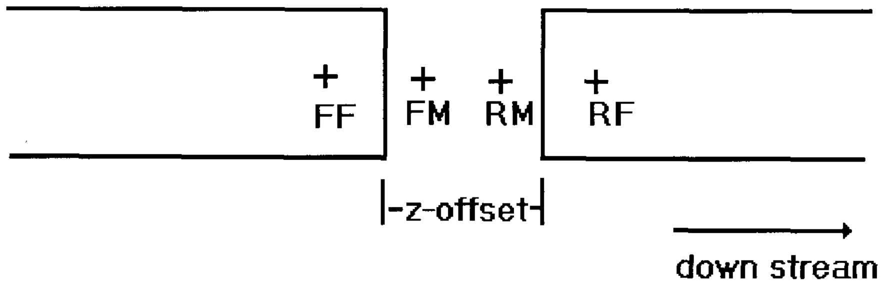


Figure 19.
Magnet ends naming convention.

The forward rotation sequence to a MTM clamp, yawed and pitched by pedigrees, is defined as follows:

$(\theta) \rightarrow (\phi) \rightarrow (\psi) \rightarrow (\alpha') \rightarrow (\varepsilon) \rightarrow (\lambda)$

θ = TRANSPORT yaw

ϕ = TRANSPORT pitch

ψ = TRANSPORT roll

α' = yaw within a magnet plus pedigree induced yaw

ε = pedigree induced pitch, measured with an inclinometer (sequential pitch)

λ = twist

$$R_4 = R_\lambda R_\varepsilon R_{\alpha'} R_\psi R_\phi R_\theta \qquad \text{(Eqn. 6-1)}$$

$$R = R_\psi R_\phi R_\theta$$

$$R_2 = R_\varepsilon R_{\alpha'}$$

$$R_4 = R_\lambda R_3$$

with

$$R_\varepsilon = \begin{pmatrix} \cos\varepsilon & 0 & -\sin\varepsilon \\ 0 & 1 & 0 \\ \sin\varepsilon & 0 & \cos\varepsilon \end{pmatrix}$$

$$R_{\alpha'} = \begin{pmatrix} \cos\alpha' & \sin\alpha' & 0 \\ -\sin\alpha' & \cos\alpha' & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_2 = R_\varepsilon R_{\alpha'} = \begin{pmatrix} \cos\varepsilon\ \cos\alpha' & \cos\varepsilon\ \sin\alpha' & -\sin\varepsilon \\ -\sin\alpha' & \cos\alpha' & 0 \\ \sin\varepsilon\ \cos\alpha' & \sin\varepsilon\ \sin\alpha' & \cos\varepsilon \end{pmatrix}$$

$$R_3 = R_2 R$$

$$r_{11} = \cos\varepsilon\ \cos\alpha'\ \cos\phi\ \cos\theta + \cos\varepsilon\ \sin\alpha' \sin\psi \sin\phi\ \cos\theta - \cos\varepsilon\ \sin\alpha' \cos\psi \sin\theta - \sin\varepsilon\ \cos\psi\ \sin\phi\ \cos\theta - \sin\varepsilon \sin\psi \sin\theta$$

$$r_{12} = \cos\varepsilon\ \cos\alpha'\ \cos\phi\ \sin\theta + \cos\varepsilon\ \sin\alpha' \sin\psi \sin\phi\ \sin\theta + \cos\varepsilon\ \sin\alpha' \cos\psi \cos\theta - \sin\varepsilon\ \cos\psi\ \sin\phi\ \sin\theta + \sin\varepsilon \sin\psi \cos\theta$$

$$r_{13} = -\cos\varepsilon\ \cos\alpha'\ \sin\phi + \cos\varepsilon\ \sin\alpha' \sin\psi \cos\phi - \sin\varepsilon \cos\psi\ \cos\phi$$

$$r_{21} = -\sin\alpha'\ \cos\phi\ \cos\theta + \cos\alpha' \sin\psi \sin\phi\ \cos\theta - \cos\alpha' \cos\psi \sin\theta$$

r_{22} = - $\sin\alpha'$ $\cos\phi$ $\sin\theta$ + $\cos\alpha'$ $\sin\psi$ $\sin\phi$ $\sin\theta$ + $\cos\alpha'$ $\cos\psi$ $\cos\theta$

r_{23} = $\sin\alpha'$ $\sin\phi$ + $\cos\alpha'$ $\sin\psi$ $\cos\phi$

r_{31} = $\sin\varepsilon$ $\cos\alpha'$ $\cos\phi$ $\cos\theta$ + $\sin\varepsilon$ $\sin\alpha'$ $\sin\psi$ $\sin\phi$ $\cos\theta$ - $\sin\varepsilon$ $\sin\alpha'$ $\cos\psi$ $\sin\theta$ + $\cos\varepsilon$ $\cos\psi$ $\sin\phi$ $\cos\theta$ + $\cos\varepsilon$ $\sin\psi$ $\sin\theta$

r_{32} = $\sin\varepsilon$ $\cos\alpha'$ $\cos\phi$ $\sin\theta$ + $\sin\varepsilon$ $\sin\alpha'$ $\sin\psi$ $\sin\phi$ $\sin\theta$ + $\sin\varepsilon$ $\sin\alpha'$ $\cos\psi$ $\cos\theta$ + $\cos\varepsilon$ $\cos\psi$ $\sin\phi$ $\sin\theta$- $\cos\varepsilon$ $\sin\psi$ $\cos\theta$

r_{33} = - $\sin\varepsilon$ $\cos\alpha'$ $\sin\phi$ + $\sin\varepsilon$ $\sin\alpha'$ $\sin\psi$ $\cos\phi$ + $\cos\varepsilon$ $\cos\psi$ $\cos\phi$

$$R_4 = R_\lambda R_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\lambda & \sin\lambda \\ 0 & -\sin\lambda & \cos\lambda \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

Rotation to same point by 3 rotations:

$(\theta'') \rightarrow (\phi'') \rightarrow (\psi'')$

Yaw Pitch Roll

$$R_5 = R_{\psi''} R_{\phi''} R_{\theta''} \qquad \text{(Eqn. 6-2)}$$

$$R_5 = \begin{pmatrix} \cos\phi'' \cos\theta'' & \cos\phi'' \sin\theta'' & -\sin\phi'' \\ \sin\psi'' \sin\phi'' \cos\theta'' - \cos\psi'' \sin\theta'' & \sin\psi'' \sin\phi'' \sin\theta'' + \cos\psi'' \cos\theta'' & \sin\psi'' \cos\phi'' \\ \cos\psi'' \sin\phi'' \cos\theta'' + \sin\psi'' \sin\theta'' & \cos\psi'' \sin\phi'' \sin\theta'' - \sin\psi'' \cos\theta'' & \cos\psi'' \cos\phi'' \end{pmatrix}$$

To get sequential pitch from a measured pitch:

$R_5\,(1,3) = R_3\,(1,3)$

$$-\sin\phi'' = -\cos\varepsilon\ \cos\alpha'\ \sin\phi + \cos\varepsilon\ \sin\alpha'\ \sin\psi\ \cos\phi - \sin\varepsilon\ \cos\psi\ \cos\phi \qquad \text{(Eqn. 6-3)}$$

-----> solve for sequential pitch ε iteratively.

To get sequential twist from a measured roll:

$R_4(2,3) = r_{(2,3)}$ (5)

$\sin\psi'' \cos\phi'' = \cos\lambda \sin\alpha' \sin\phi + \cos\lambda \cos\alpha' \sin\psi \cos\phi - \sin\lambda \sin\varepsilon \cos\alpha' \sin\phi + \sin\lambda \sin\varepsilon \sin\alpha' \sin\psi \cos\phi + \sin\lambda \cos\varepsilon \cos\psi \cos\phi$ (Eqn. 6-4)

-----> solve for sequential twist λ iteratively.

Step by step calculation procedures:

1.) Calculation of local yaw and pitch of both clamps induced by pedigrees. (The local pitch is calculated but is not carried forward as its influence proved to be unpredictable).

2.) Calculation of the local yaw angle (α') from a point of known TRANSPORT coordinates to the fiducial point on the arc of the magnet.
(α')=(chord-length/magnet radius) + pedigree induced yaw.

3.) Calculate sequential pitch (ε) from measured pitch at FF and RF end.

4.) Calculate sequential twist (λ) from measured roll at FF and RF end.

5.) Apply sequential twist (λ), sequential pitch (ε) and local yaw (α') of RF clamp to RF registration point coordinates to transform from RF clamp BFS (beam following coordinate system) into RM (Rear Magnetic) point BFS coordinate system where the TRANSPORT parameters are known.

6.) Apply TRANSPORT orientation parameters of RM to get RF clamp registration points in TRANSPORT coordinates.

7.) Apply TRANSPORT orientation parameters of FM to above coordinates.

8.) Apply sequential twist (λ), sequential pitch (ε) and local yaw (α') of FF clamp to above. As a result you have the RF registration points in the local FF clamp BFS.

9.) Calculate distances between the corresponding FF and RF registration points and translate them into dial gauge movements.

6.1.2.2. MTM FROM ACHROMAT TO ACHROMAT (WITH ROLL TRANSITION)

In MMAS the coordinates of the RF clamp are also transformed into the FF clamp coordinate system. Specially designed clamps without registration arms are preferred to the clamps used in MMAFI. Instead of determining the ideal coordinates of the arm registration points the ideal coordinates for the RF clamp tooling balls in the FF clamp coordinate system are determined. The pitch and the roll is measured on each clamp. Using an Industrial Measurement System (e.g. ECDS, SIMS) the actual coordinates of all tooling balls are determined in the FF clamp coordinate system. The differences between the actual and the ideal coordinate sets are averaged for all tooling balls and translated into dial gauge movements. The calculation algorithms for the ideal coordinates are identical to the equations 6-1 through 6-4.

Ideal calculations:

1.) Calculation of local yaw and pitch of both clamps induced by pedigrees. (The local pitch is calculated but is not carried forward as its influence proved to be insignificant).

2.) Calculation of the local yaw angle (α') from a point of known TRANSPORT coordinates to fiducial point on the arc.
(α') = (chord-length / magnet radius) + pedigree induced yaw.

3.) Calculate sequential pitch (ε) from measured pitch at FF and RF end.

4.) Calculate sequential twist (λ) from measured roll at FF and RF end.

5.) Apply sequential twist (λ), sequential pitch (ε) and local yaw (α') of RF clamp to RF tooling ball coordinates to transform from RF clamp BFS (beam following coordinate system) into RM (Rear Magnetic) point BFS coordinate system where the TRANSPORT parameters are known.

6.) Apply TRANSPORT orientation parameters of RM to get RF clamp tooling balls in TRANSPORT coordinates.

7.) Apply TRANSPORT orientation parameters of FM to above coordinates.

8.) Apply sequential twist (λ), sequential pitch (ε) and local yaw (α') of FF clamp to above. As a result you have the RF tooling ball points in the local FF clamp BFS.

The ideal coordinate set and the actual coordinate set from ECDS are subtracted from each other, averaged for all tooling balls for error checking and translated into dial gauge movements. For sign conventions of dial gauge movements see Appendix A.

6.2. SPECIAL SECTION MAGNETS

Step by step calculation procedure:

<u>1.) Ideal coordinate determination.</u>
Calculate ideal coordinates for the tooling ball positions of each magnet. (see section 5.2 SPECIAL SECTION MAGNETS).

<u>2.) Actual coordinate determination.</u>
Horizontal direction sets are measured to all tooling balls. The heights are independently determined by means of a separate level network. Actual coordinates are calculated by reducing the surveyed data and processing the horizontal and the vertical network separately in a least-squares adjustment program. The first and the last magnet tooling ball positions are held fixed to their ideal coordinate positions in this adjustment process. No constraint is usually put on the monument stations. Reduction of the geodetic y-coordinates to rectangular coordinates is also made.

<u>3.) Differences between ideal tooling ball coordinates and actual tooling ball coordinates.</u>
Differences between ideal and actual coordinates are calculated.

4.) Elimination of Δ z.

From the difference between the ideal and actual coordinate sets a Δ z value is obtained which will be mathematically eliminated. The z-dimension is not very critical and elimination of Δ z reduces a 3-dimensional problem to a 2-dimensional problem. The ideal coordinates are then recalculated by yawing the original ideal coordinates to the actual z-locations and eliminating the Δ z offset (yaw angle $\alpha = \Delta$ z / magnet radius). This Δ z reduction can be seen as replacing the mechanical adjustment of a magnet in z by mathematical means.

5.) Differences between new ideal and actual coordinate sets.

The newly found ideal coordinates are now subtracted again from the actual coordinates and the results are obtained in the TRANSPORT coordinate system.

6.) Transform differences from TRANSPORT into BFS.

At this point the results have to be just yawed in order to get them parallel to the azimuth of the magnet (BFS). The average of the differences of the tooling balls for each element are the input values for PCURVE.

7.) PCURVE.

PCURVE is run.

8.) Dial gauge movements.

PCURVE's best fit curve output in the form of coordinate differences is then subtracted from all individual tooling ball differences (result from 6.). The remainder is transformed into dial gauge movements. For sign conventions for dial gauge movements see Appendix A.

7. ROLLFIX IN THE ARCS

Rollfix is a program designed to "feather" the roll transitions between achromats through several magnet junctions rather than the original single boundary junction. The objective is to make the arcs less prone to the cross coupling of dispersed beams caused by abrupt roll changes. It is performed on achromat sections where the roll transitions were significantly large. This one time project feathered the following achromat boundaries [8]:

[8] The magnet motions as required by the rollfix program moved the respective magnets away from their nominal TRANSPORT locations. It needs to be pointed out that TRANSPORT has not been modified up to date, to reflect these changes.

	North	South
Ritfix	0/1	0/1
	2/3	2/IS
Ritharm	11/12	10/11
	12/13	14/15
	13/14	15/16
	20/21	17/18
	21/22	18/19
	23/FF	20/21
		21/22
		23/FF

The fix consisted of a series of moves for each of 5 magnets. These moves were made up of roll rotations about the magnets' chords and shifts perpendicular to the plane of the magnet. If one achromat boundary was rollfixed it had to be countered by a similiar but opposite fix further down the beam line. If one looks carefully in the table of movements, it can be seen that each set of movements comes in pairs reflecting this requirement.

A five magnet move at an achromat boundary is known as a Ritharm junction while a single magnet move is known as a Ritfix. Two sets of junctions in the BSY region were Ritfixed to avoid the complicated alignment required in that conjested area.

7.1. ROLLFIX CALCULATIONS

For each magnetic end of the magnet TRANSPORT supplies yaw, pitch and roll. The yaw from the beam line to the chord can be calculated and the roll around the chord is supplied by physicists. The objective is to calculate new layout rolls in respect to gravity and new sequential TRANSPORT rotations for each magnet end after the magnet was rolled about its chord. The rotation matrix which defines the rotation of the magnet about its chord was derived in two separate methods and compared in order to assure the accuracy of the calculations.

7.1.1. SEQUENTIAL ROTATIONS

The forward rotation sequence is defined as follows:

$(\theta) \rightarrow (\phi) \rightarrow (\psi) \rightarrow (\theta_1) \rightarrow (\psi_1) \rightarrow (\theta_2)$

θ = TRANSPORT yaw

ϕ = TRANSPORT pitch

ψ = TRANSPORT roll

θ_1 = yaw BL (beam line) to chord

ψ_1 = roll about chord

θ_2 = yaw chord to BL

$$R_{TOT} = R\,\theta_2\;R\psi_1\;R\,\theta_1\;R \qquad \text{(Eqn. 7-1)}$$

$$R = \begin{pmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta & -\sin\phi \\ \sin\psi\sin\phi\cos\theta - \cos\psi\sin\theta & \sin\psi\sin\phi\sin\theta + \cos\psi\cos\theta & \sin\psi\cos\phi \\ \cos\psi\sin\phi\cos\theta + \sin\psi\sin\theta & \cos\psi\sin\phi\sin\theta - \sin\psi\cos\theta & \cos\psi\cos\phi \end{pmatrix}$$

from Eqn. 2-1

$$R_5 = \begin{pmatrix} \cos\phi'\cos\theta - \sin\phi'\sin\psi'\sin\theta & \cos\phi'\sin\theta + \sin\phi'\sin\psi'\cos\theta & -\sin\phi'\cos\psi' \\ -\cos\psi'\sin\theta & \cos\psi'\cos\theta & \sin\psi' \\ \sin\phi'\cos\theta + \cos\phi'\sin\psi'\sin\theta & \sin\phi'\sin\theta - \cos\phi'\sin\psi'\cos\theta & \cos\phi'\cos\psi' \end{pmatrix}$$

from Eqn. 5-3

$$R_6 = R\,\theta_2\;R\psi_1 = \begin{pmatrix} \cos\theta_2 & \sin\theta_2 & 0 \\ -\sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\psi_1 & \sin\psi_1 \\ 0 & -\sin\psi_1 & \cos\psi_1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta_2 & \sin\theta_2 \cos\psi_1 & \sin\theta_2 \sin\psi_1 \\ -\sin\theta_2 & \cos\theta_2 \cos\psi_1 & \cos\theta_2 \sin\psi_1 \\ 0 & -\sin\psi_1 & \cos\psi_1 \end{pmatrix}$$

$$R_7 = R_6 \, R\,\theta_1 = \begin{pmatrix} \cos\theta_2 & \sin\theta_2 \cos\psi_1 & \sin\theta_2 \sin\psi_1 \\ -\sin\theta_2 & \cos\theta_2 \cos\psi_1 & \cos\theta_2 \sin\psi_1 \\ 0 & -\sin\psi_1 & \cos\psi_1 \end{pmatrix} \begin{pmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta_2\cos\theta_1 - \sin\theta_2\cos\psi_1\sin\theta_1 & \cos\theta_2\sin\theta_1 + \sin\theta_2\cos\psi_1\cos\theta_1 & \sin\theta_2 \sin\psi_1 \\ -\sin\theta_2\cos\theta_1 - \cos\theta_2\cos\psi_1\sin\theta_1 & -\sin\theta_2\sin\theta_1 + \cos\theta_2\cos\psi_1\cos\theta_1 & \cos\theta_2 \sin\psi_1 \\ \sin\psi_1\sin\theta_1 & -\sin\psi_1 \cos\theta_1 & \cos\psi_1 \end{pmatrix}$$

$R_{TOT} = R_7 \ R$

$R_{TOT}(1,1) = \cos\phi \cos\theta(\cos\theta_2\cos\theta_1 - \sin\theta_2\cos\psi_1\sin\theta_1) + (\sin\psi\sin\phi\cos\theta - \cos\psi\sin\theta)$
$(\cos\theta_2\sin\theta_1 + \sin\theta_2\cos\psi_1\cos\theta_1) + (\cos\psi\sin\phi\cos\theta + \sin\psi\sin\theta)(\sin\theta_2\sin\psi_1)$

$R_{TOT}(1,2) = \cos\phi \sin\theta(\cos\theta_2\cos\theta_1 - \sin\theta_2\cos\psi_1\sin\theta_1) + (\sin\psi\sin\phi\sin\theta + \cos\psi\cos\theta)$
$(\cos\theta_2\sin\theta_1 + \sin\theta_2\cos\psi_1\cos\theta_1) + (\cos\psi\sin\phi\sin\theta - \sin\psi\cos\theta)(\sin\theta_2\sin\psi_1)$

$R_{TOT}(1,3) = -\sin\phi\ (\cos\theta_2\cos\theta_1 - \sin\theta_2\cos\psi_1\sin\theta_1) + \sin\psi\cos\phi\ (\cos\theta_2\sin\theta_1 + \sin\theta_2\cos\psi_1$
$\cos\theta_1) + \cos\psi\cos\phi\ \sin\theta_2\sin\psi_1$

$R_{TOT}(2,3) = \sin\phi\ (\sin\theta_2\cos\theta_1 + \cos\theta_2\cos\psi_1\sin\theta_1) + \sin\psi\cos\phi\ (-\sin\theta_2\sin\theta_1 + \cos\theta_2\cos\psi_1$
$\cos\theta_1) + \cos\psi\cos\phi\ \cos\theta_2\sin\psi_1$

Layout roll :

$R_5(2,3) = R_{TOT}(2,3)$

$\psi' = \sin^{-1}(R_{TOT}(2,3))$ (Eqn. 7-2)

Sequential pitch:

$R(1,3) = R_{TOT}(1,3)$

$\phi\ (\text{sequential}) = -\sin^{-1} R_{TOT}(1,3)$ (Eqn. 7-3)

Sequential roll:

$R(2,3) = R_{TOT}(2,3)$

$\psi\ (\text{sequential}) = \sin^{-1}\ (1/(\cos\phi\ (\text{sequential})) * \sin\ \psi')$ (Eqn. 7-4)

Sequential yaw with sinus:

$R(1,2) = R_{TOT}(1,2)$

θ (sequential) $= \sin^{-1}$ (1/(cos ϕ (sequential)) * R_{TOT} (1,2)) (Eqn. 7-5)

Sequential yaw with cosinus:

$R(1,1) = R_{TOT}(1,1)$

θ (sequential) $= \cos^{-1}$ (1/(cos ϕ (sequential)) * R_{TOT} (1,1)) (Eqn. 7-6)

To determine quadrant of yaw see Figure 20.

sin ϕ (sequential)	cos ϕ (sequential)	ϕ (sequential)
>0	>0	arcsinϕ (sequential)
>0	<0	pi - arcsinϕ (sequential)
<0	>0	arcsinϕ (sequential)
<0	<0	- pi - arcsinϕ (sequential)

Figure 20.
Quadrant of yaw.

7.1.2. ROTATION ABOUT DIRECTED LINE

A check to the previous method can be made by performing a rotation of the original beam following coordinate system about a vector representing the chord of the magnet. This vector is the directed line to be rotated about.

$$M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \lambda^2(1-\cos\alpha)+\cos\alpha & \lambda\mu(1-\cos\alpha)-\nu\sin\alpha & \lambda\nu(1-\cos\alpha)+\mu\sin\alpha \\ \lambda\mu(1-\cos\alpha)+\nu\sin\alpha & \mu^2(1-\cos\alpha)+\cos\alpha & \mu\nu(1-\cos\alpha)-\lambda\sin\alpha \\ \lambda\nu(1-\cos\alpha)-\mu\sin\alpha & \mu\nu(1-\cos\alpha)+\lambda\sin\alpha & \nu^2(1-\cos\alpha)+\cos\alpha \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

(Eqn. 7-7)

α = angle of rotation about the directed line

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \text{rollfixed coordinate system}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \text{Original beam following coordinate system (BFS)}$$

$$\begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix} = \text{Direction cosines of directed line in original coordinate system}$$

Direction cosines of a line along chord:

$A^{->}_R = \begin{pmatrix} Z \\ X \\ Y \end{pmatrix}_R =$ Coordinates of RM end of magnet in the BFS at the FM end (before magnet is rollfixed)

$\lambda = Z / | A^{->}_R |$; $\mu = X / | A^{->}_R |$; $\nu = Y / | A^{->}_R |$

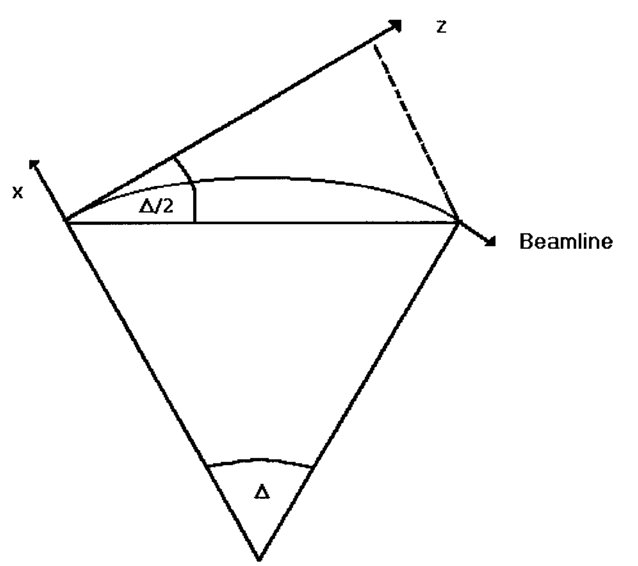


Figure 21.
Magnet geometry for direction vectors

R = 279.378 [m]

Δ/2 = 0.2566617 [DEG]

chord = 2 R * sin $\Delta/2$ = 2.502990385 [m]

Z = cos (0.2566617 [DEG]) * 2.502990385 [m] = 2.502965271 [m]

X = sin (0.2566617 [DEG]) * 2.502990385 [m] = 0.011212338 [m]

Y = 0

$| A^{->}{}_{R} | = \sqrt{Z^2 + X^2 + Y^2} = 2.502990385$ [m]

$\lambda = \cos\ \Delta/2$

$\mu = \cos\ (90\ [DEG] + \Delta/2) = -\sin\ \Delta/2$

$\nu = \cos 90\ [DEG] = 0$

$m11 = (\cos\Delta/2)^2\ (1-\cos\alpha) + \cos\alpha = (\cos\Delta/2)^2\ + (-(\cos\Delta/2)^2 + 1)\cos\alpha =$

$(\cos\Delta/2)^2\ + (\sin\Delta/2)^2 \cos\alpha$

$m12 = (\cos\Delta/2)\ (-\sin\Delta/2)(1-\cos\alpha) - \cos 90\ [DEG]\ \sin\alpha = -\sin\Delta/2 \cos\Delta/2 + \cos\Delta/2 \sin\Delta/2 \cos\alpha$

$m13 = (\cos\Delta/2)\ (0)(1-\cos\alpha) - \sin\Delta/2 \sin\alpha \Rightarrow$ (RHR) $= \sin\Delta/2 \sin\alpha$

$m\ 21 = -(\cos\Delta/2)(\sin\Delta/2)(1-\cos\alpha) + 0 \sin\alpha = -\cos\Delta/2 \sin\Delta/2 + \cos\Delta/2 \sin\Delta/2 \cos\alpha$

$m\ 22 = (\sin\Delta/2)^2\ (1-\cos\alpha) + \cos\alpha = (\sin\Delta/2)^2\ + (-(\sin\Delta/2)^2 + 1)\cos\alpha =$

$(\sin\Delta/2)^2\ + (\cos\Delta/2)^2 \cos\alpha$

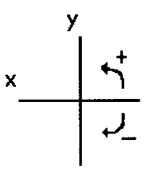
$m\ 23 = -(\sin\Delta/2)\ (0)(1-\cos\alpha) - \cos\Delta/2 \sin\alpha = -\cos\Delta/2 \sin\alpha \Rightarrow$ (RHR) $= \cos\Delta/2 \sin\alpha$

$m\ 31 = (\cos\Delta/2)\ (0)(1-\cos\alpha) - (-\sin\Delta/2) \sin\alpha = \sin\Delta/2 \sin\alpha \Rightarrow$ (RHR) $= -\sin\Delta/2 \sin\alpha$

$m\ 32 = -(\sin\Delta/2)\ (0)(1-\cos\alpha) + \cos\Delta/2 \sin\alpha = \cos\Delta/2 \sin\alpha \Rightarrow$ (RHR) $= -\cos\Delta/2 \sin\alpha$

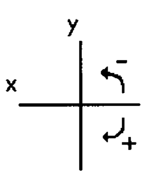
$m\ 33 = (0)^2(1-\cos\alpha) + \cos\alpha = \cos\alpha$

The sign conventions do not follow the right hand rule as they do in sequential rotations.



For rotation about directed line.

Directed line into paper.



For sequential rotations

7.1.3. COMPARISON OF ROTATIONS

R7 is derived by sequential rotations

M is derived by rotations about a directed line

The values of (θ_1) and (θ_2) are numerically equivalent, as the yaw is always seen in the magnet plane. But since all rotations for M are right handed you have to watch for the sign of $\Delta/2$. $\Delta/2$ brings chord to Beamline. Therefore (θ_1) = ($-\Delta/2$) and (θ_2) = ($\Delta/2$).

Also (ψ_1) = (α)

Elements of R7:

$r11 = (\cos\Delta/2)^2 + (\sin\Delta/2)^2 \cos\alpha = m11$

$r12 = -\sin\Delta/2 \cos\Delta/2 + \cos\Delta/2 \sin\Delta/2 \cos\alpha = m12$

$r13 = \sin\Delta/2 \sin\alpha = m13$

$r\,21 = -\cos\Delta/2 \sin\Delta/2 + \cos\Delta/2 \sin\Delta/2 \cos\alpha = m21$

$r\,22 = (\sin\Delta/2)^2 + (\cos\Delta/2)^2 \cos\alpha = m22$

$r\,23 = \cos\Delta/2 \sin\alpha = m23$

$r\,31 = -\sin\Delta/2 \sin\alpha = m31$

$r\,32 = -\cos\Delta/2 \sin\alpha = m32$

$r\,33 = \cos\alpha = m33$

GLOSSARY

Achromat: A section in the arc where the outgoing beam has the same characteristics as the incoming one.

Actual coordinates: Magnet positions as installed, determined using standard engineering surveying techniques.

Arcs: The north and south sections of the SLC after the two mile linear section.

C-clamp: A C-shaped fixture designed for the alignment of the arcs.

Datum: A coordinate system origin.

Downstream: In the direction in which the particle beam is assumed to go.

Fiducial: Reference point on a beam element in form of a Tooling Ball (TB).

Ideal Coordinate System: Design coordinate system derived from a beam simulation program (TRANSPORT).

Inclinometer: An instrument to measure the inclination of a surface in respect to gravity.

LINAC: Two mile linear accelerator section before north and south arcs.

PCURVE: A program used for SMOOTHING (STEP 4).

Pedigree: A magnet correction factor.

Pitch: Rotation around x-axis.

Roll: Rotation around z-axis.

ROLLFIX: Program designed to feather the roll transitions between achromats through several magnet junctions.

SAMMI: A magnet measurment device measuring the gap, roll, sagitta and height of a magnet in two minutes.

SPCLSECT: Program designed to calculate ideal coordinate positions.

TRANSPORT: A program used to simulate a particle beam path.

Theodolite: A precision angle measurement tool used for surveying.

Twist: Some magnet are manufactured with a twist around their magnetic axis.

Upstream: Opposite the direction in which the particle beam is assumed to go.

Vertex point: Center of drift section between two arc magnets.

Yaw: Rotation around y-axis.

ACRONYMS

BFS: Beam Following coordinate System

BPM: Beam Position Monitor

BSY: Beam Switch Yard

CEBAF: Continuous Electron Beam Accelerator Facility

CERN: French acronym for European Center for Nuclear Research

CID: Collider Injector Development

CMM: Coordinate Measuring Machine

ECDS: Electronic Coordinate Determination System

ETA: East Turn Around

FM: Front Magnetic

FF: Front Fiducial (Upstream end)

LINAC: LINear ACcelerator

MMAFI: Magnet to Magnet Alignment by Fixture

MMAS: Magnet to Magnet Alignment System

MTM: Magnet To Magnet

PC: Profile Collimator

RF: Rear Fiducial (Downstream end)

RM: Rear Magnetic

SAMMI: SLAC Automatic Magnet Measurement Instrument

SIMS: SLAC Industrial Measurement System

SLAC: Stanford Linear Accelerator Center

SLC: Stanford Linear Collider

SLD: SLC Large Detector

WTA: West Turn Around

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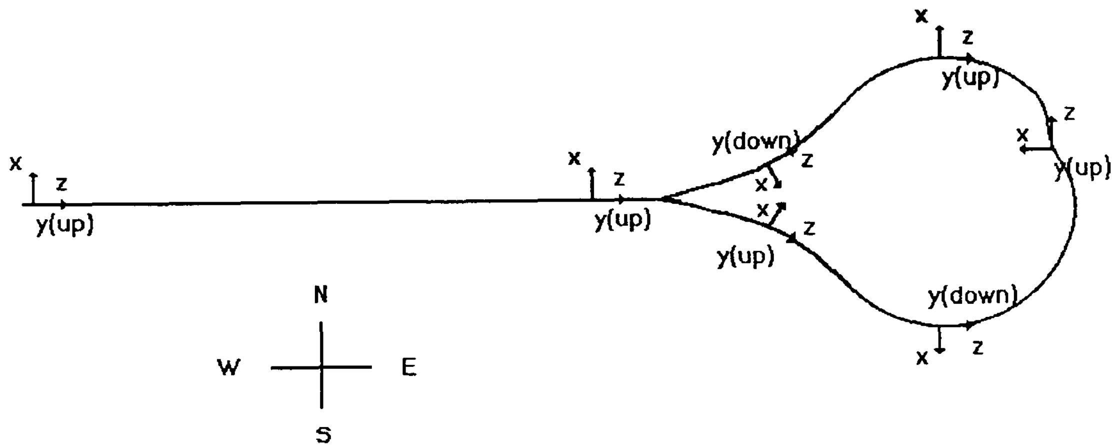
APPENDIX A

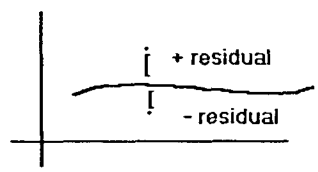
SIGN CONVENTIONS

Special Section Smoothing Movement Signs



NOTE: x-axis always opposite the radial center.

Beam following system flips at the center of reverse bends.



PCURVE output

Motions done in vertical with level
Motions done in horizontal with transit parallel to beamline
Misalignment = Residual + [(Actual - Ideal) − Average Δ]

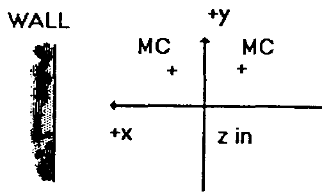
Y-coordinates

AREA	MISALIGNMENT +	ADJUSTMENT −*down*	MISALIGNMENT −	ADJUSTMENT +*up*
SBSY	too high	move down	too low	move up
NBSY	too hign	move down	too low	move up
SIS	too high	move down	too low	move up
SRB	too high	move down	too low	move up
NRB	too high	move down	too low	move up
SFF	too high	move down	too low	move up
NFF	too high	move down	too low	move up
SSPECT	too high	move down	too low	move up
NSPECT	too hign	move down	too low	move up

X-coordinates

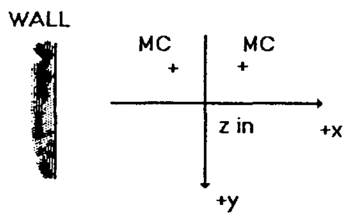
AREA	MISALIGNMENT +	ADJUSTMENT −	MISALIGNMENT −	ADJUSTMENT +
SBSY	too far north	away C-beam (−)	too far south	toward C-beam (+)
NBSY	too far north	toward C-beam (−)	too far south	away C-beam(+)
SIS	too close to aisle	toward wall (−)	too close to wall	toward aisle (+)
SRB	too close to aisle	toward wall (−)	too close to wall	toward aisle (+)
NRB	too close to wall	toward aisle (−)	too close to aisle	toward wall (+)
SFF	too close to aisle	toward wall (−)	too close to wall	toward aisle (+)
NFF	too close to wall	toward aisle (−)	too close to aisle	toward wall (+)
SSPECT	too close to wall	toward aisle (+)	too close to aisle	toward wall (−)
NSPECT	too close to aisle	toward wall (+)	too close to wall	toward aisle (−)

EFFECTIVE PEDIGREE MOVEMENTS FOR THE NORTH ARC



NARC RB -> FF

If X pedigree is **positive** the magnet moves toward the **wall**
If X pedigree is **negative the magnet moves toward** the **aisle**



NARC BSY -> RB

If X pedigree is **positive the magnet moves towards** the **aisle**
If X pedigree is **negative** the magnet moves toward the **wall**

Magnet-to-Magnet Alignment with MAS Signs for Dial Gage Motions Using the Jaws Fixture to Apply the Motions (Electronic Gages Used)

1) Extend plunger = negative
2) Depressing plunger = positive
3) MMAF gages mounted on FF end on South side
4) MMAF gages mounted on RF end on North side
5) Differences taken in the FF clamp coord. system
6) Subtraction is Actual – Ideal

Y MOTION

South (Gages on FF end)			
RF end	Sign of Difference	Movement	Gage Sign
Too high	+	– (down)	Extend –
Too low	–	+ (up)	Depress +

North (Gages on RF end)			
RF end	Sign of difference	Movement	Gage Sign
Too high	+	– (down)	Depress +
Too low	–	+ (up)	Extend –

X MOTION

South (Gages on FF end)			
RF End	Sign of Difference	Movement	Gage Sign
Too close to aisle	+	– (to wall)	Extend –
Too far from aisle	–	+ (to aisle)	Depress +

North (Gages on RF end)			
RF end	Sign of Difference	Movement	Gage Sign
Too close to aisle	+	– (to wall)	Depress +
Too far from aisle	+	+ (to aisle)	Extend –

Table to Put Pitch Signs for BPM Measurements into the Right Hand Rule for the Standard BPM System

	South		North	
	BSY → RB1	RB → FF2	BSY → RB3	RB - FF4
Focus	−1	+1	+1	−1
Defocus	+1	−1	−1	+1

Table to put BPM Actual Coordinates into the Standard BPM System. Z along Beam Y always up and X to the left (Signs 4)

	South		North	
	BSY → RB	RB → FF	BSY → RB	RB → FF
Z	1	1	1	1
X	1	−1	−1	1
Y	1	−1	−1	1

Table to Show Side of BPM Probed as Facing Down Stream

	South		North	
	BSY → RB1	RB → FF2	BSY → RB3	RB → FF4
Focus	Left	R	R	L
Defocus	Right	L	L	R

Table to Put Reduced Measured Pitch into the B-F-S

South		North	
BSY → RB	RB → FF	BSY → RB	RB → FF
+1	−1	−1	+1

Table of Signs to Put Magnet-to-Magnet Clamp (45, 46) into Beam Following System

[Assuming they are always mounted from the aisle (used for Magnet-to-Magnet calculations) or to do the inverse conversion]

	South		North	
	BSY → RB1	RB → FF2	BSY → RB3	RB → FF4
Z	+1	+1	−1	−1
X	+1	−1	+1	−1
Y	+1	−1	−1	+1

Table of Signs to Put Magnet-to-Magnet Roll Measurement into the Right-Hand Rule for the Beam Following System

(Assuming clamps are always put on from the aisle side)

	South	North
Roll	1	−1

Table of Signs to Put Magnet-to-Magnet Pitch Measurement into the Right-Hand Rule for the Beam Following System

	South		North	
	BSY → RB1	RB → FF2	BSY → RB3	RB → FF4
Pitch	−1	−1	+1	+1

Table of Signs to Put Inclinometer Sensitive Axis Angular Misalignment into Beam - Following System

	South Arc		North Arc	
	BSY → RB(1)	RB → FF (2)	BSY → RB (3)	RB → FF (4)
F	+1	−1	−1	+1
D	+1	−1	−1	+1

Table of Z locations of C-Clamps on Magnet A or B-End

	South Arc		North Arc	
End	BSY → RB(1)	RB → FF(2)	BSY → RB(3)	RB → FF(4)
FF	B	A	A	B
RF	A	B	B	A

Magnet-to-Magnet " Mic'd" T/B (clamps 45, 46)
Always Mounted from Aisle

	South Arc	North Arc
FF	2	1
RF	1	2

Table of Signs to Convert
C-Clamps Fixture Coordinates and Roll Signs
to the Beam Following System

Multiply by Factor to Get Beam Following System

		South Arc		North Arc	
		BSY → RB(1)	RB → FF(2)	BSY → RB(3)	RB → FF(4)
z	F	1	−1	−1	1
z	D	1	1	1	1
x	F	1	1	1	1
x	D	−1	−1	−1	−1
y	F	1	−1	−1	1
y	D	1	−1	−1	1
Roll	F	1	−1	−1	1 ⋆
Roll	D	−1	1	1	−1 ⋆

⋆ used in subroutine "ROLLSI"

Table of Signs to Convert Pedigree Offsets to the Beam Following Coordinate System

(Multiply by factor to get beam following system)

	South Arc		North Arc	
	BSY → RB	RB → FF	BSY → RB	RB → FF
z	0	0	0	0
x	1	−1	−1	1
y	−1	−1	−1	−1
Roll	−1	1	1	−1

Table to Determine the Tooling Ball Micrometered for the "Z" Placement of the C-Clamp

		South Arc		North Arc	
		BSY → RB	RB → FF	BSY → RB	RB → FF
F	FF	Right	Left	Left	Right
	RF	Left	Right	Right	Left
D	FF	Left	Right	Right	Left
	RF	Right	Left	Left	Right

Table to Determine Arc Magnet Type and Location from Name (XLOCAT Subroutine)

South Arc				North Arc			
1 → RB		RB → FF		BSY → Beginning 9		9 → FF	
F	D	F	D	F	D	F	D
Even	Odd	Odd	Even	Odd	Even	Even	Odd

Exceptions:
Achromat 00 – First magnet is F and odd.
Achromat IS – Third magnet is F and odd.

Achromat 8 order

South	8B then 8A
North	8A then 8B

Table of Signs to Put STEP 4 Alignment Differences into a Standard Coordinate System

Standard System

+Z	down beam
+X	to left facing down beam
+Y	up facing down beam

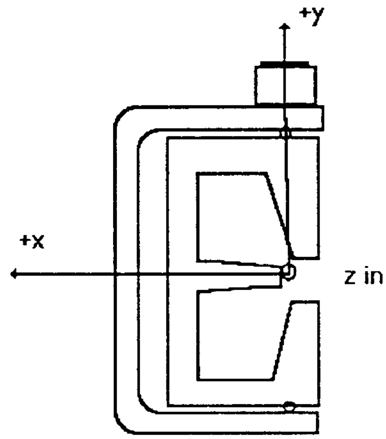
Multiply differences in beam following system to get standard system

	South Arc		North Arc	
	BSY → RB	RB → FF2	BSY → RB3	RB → FF4
X	1	−1	−1	1
Y	1	−1	−1	1

⋆ Positive difference after application of above factors means actual position is too high or too far left.

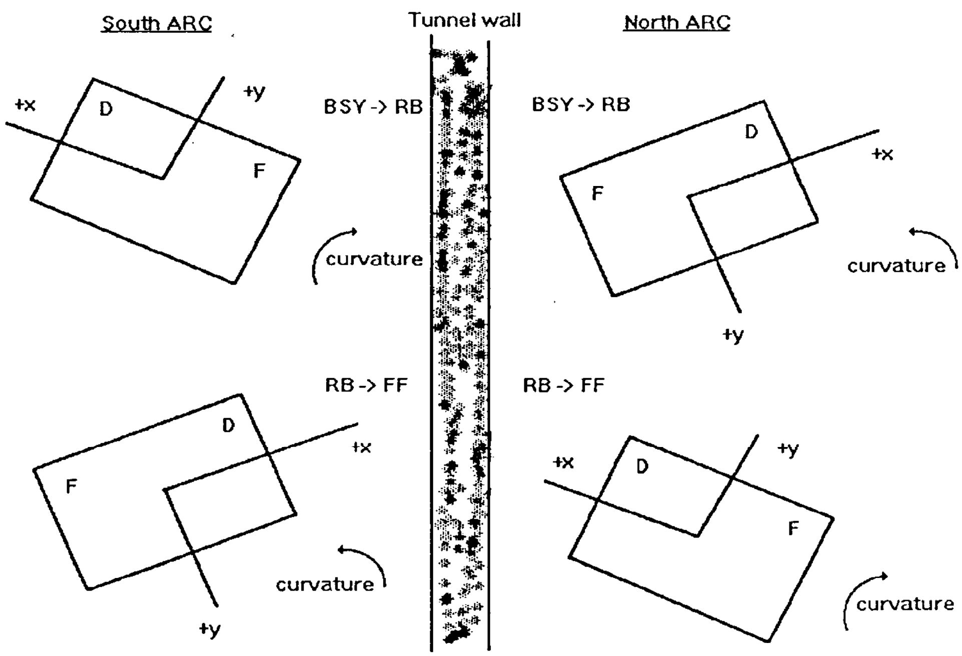
Subroutine SIGNS

This subroutine assigns the correct signs to the fixture offsets for STEP3. The sign convention for the fixture are as follows:



The z coordinate of the fixture should always be entered into the XP array as a positive since this coordinate will be used to determine ARC length. (If the z fixture coordinate is non-zero).

Local Beam Line Systems



W. Milner

DATE: November 24, 1980

TO : SLC Distribution

FROM : W.A. Davies-White

SUBJECT: SLC Damping Ring Coordinate System

I hope the following memo will clarify rather than confuse participants in the SLC project.

It is well to note that at the centre of the drift section 1-9, between girders 1-8 and 2-1, the accelerator has a slope from west to east of 17.91 minutes of arc or .00521 Radians.

It is intended that the transport line to the damping ring, the damping ring and the compressor arc all be in a horizontal plane as defined by the gravity vector at the centre of 1-9 drift section.

With the above in mind, I propose to define the origin Z = 0, Y = 0, X = 0 for the damping ring as follows.

Z_z orthogonal to the gravity vector: Z = 0 at a point -59.660 inches from the end of Sector 1 (the centre of drift section 1-9).

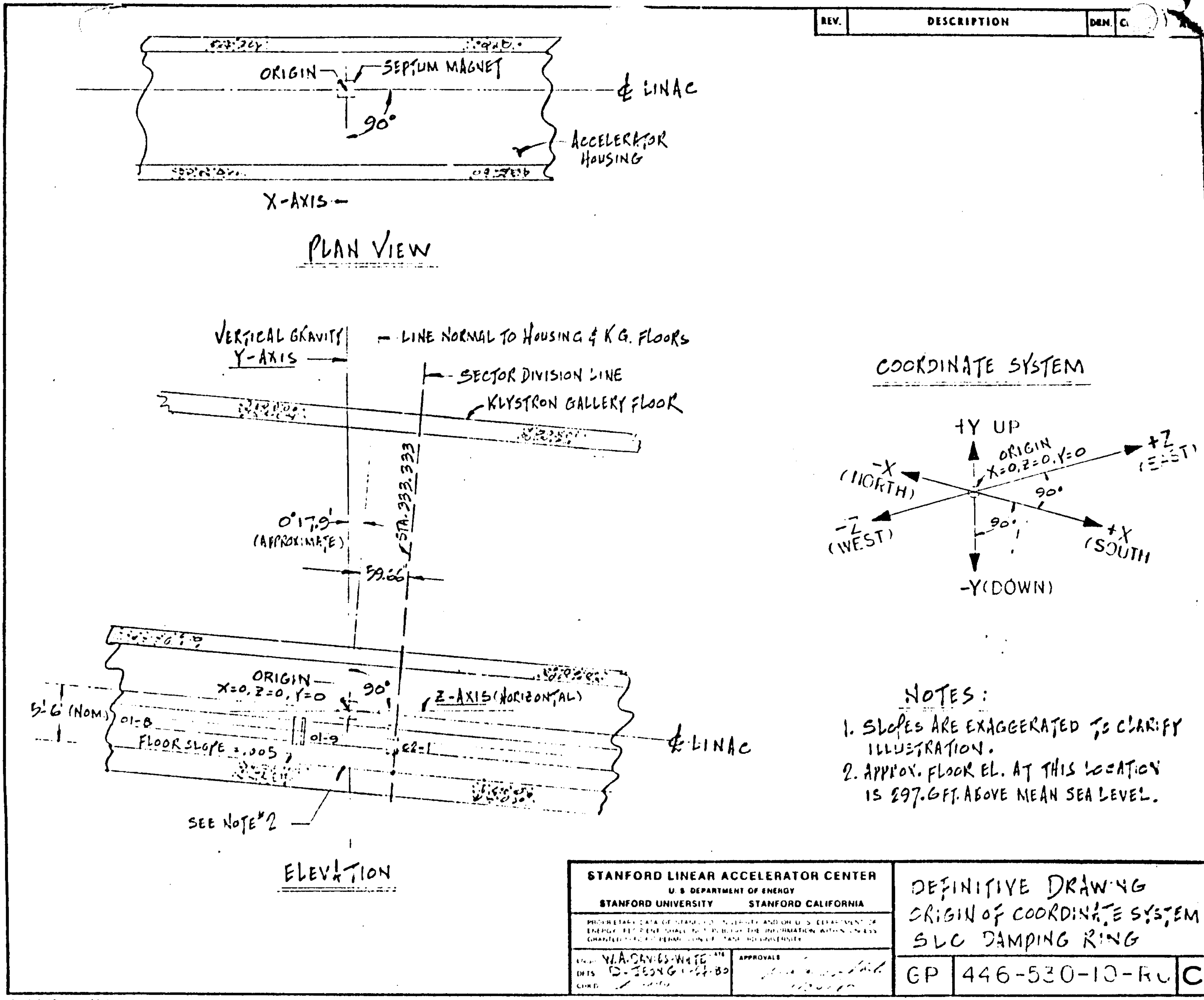
Z_y along the gravity vector, passing through Z = 0, as defined above with Y = 0 at the beam centerline.

Z_x normal to the accelerator at Z = 0 as defined above with X = 0 at the beam centerline.

By inspection of the attached drawing, one readily sees that the septum is placed at (0,0,0).

At the present time, I understand the accelerator is aligned by using two fixed points, one located at Sector 30 and the other at Sector 10 (the positron source). I would like to point out that with the addition of the damping ring and given our very close alignment tolerances, it would appear unwise to move the accelerator at Sector 1 to align to the axis as defined by the points (30-10).

If this is not feasible, then I suggest we give some serious thought, in the future, as to how we monitor motions and/or movements of the centre of Drift Section 1-9 so that we can steer the outgoing and incoming beams appropriately.


REV.
DESCRIPTION
ORIGIN
SEPTUM MAGNET
90°
℄ LINAC
ACCELERATOR HOUSING
X-AXIS
PLAN VIEW
VERTICAL GRAVITY
Y-AXIS
LINE NORMAL TO HOUSING & K.G. FLOORS
SECTOR DIVISION LINE
KLYSTRON GALLERY FLOOR
STA. 333.333
0°17.9'
(APPROXIMATE)
59.66
ORIGIN
X=0, Z=0, Y=0
90°
Z-AXIS (HORIZONTAL)
5'-6 (NOM.)
01-8
01-9
02-1
FLOOR SLOPE =.005
℄ LINAC
SEE NOTE #2
ELEVATION
COORDINATE SYSTEM
+Y UP
ORIGIN
X=0, Z=0, Y=0
+Z
(EAST)
-X
(NORTH)
90°
90°
-Z
(WEST)
+X
(SOUTH)
-Y (DOWN)
NOTES:
1. SLOPES ARE EXAGGERATED TO CLARIFY ILLUSTRATION.
2. APPROX. FLOOR EL. AT THIS LOCATION IS 297.6 FT. ABOVE MEAN SEA LEVEL.
STANFORD LINEAR ACCELERATOR CENTER
U. S. DEPARTMENT OF ENERGY
STANFORD UNIVERSITY
STANFORD CALIFORNIA
APPROVALS
DEFINITIVE DRAWING
ORIGIN OF COORDINATE SYSTEM
SLC DAMPING RING
GP
C

AETRON-BLUME-ATKINSON • ARCHITECT - ENGINEER - MANAGER • PALO ALTO

JOB NO. | JOB Stanford Linear Accelerator | BY L.G. | DATE 2/2?
SUBJECT Accel. Housing Finish Floor Profile at ℄ | CHK'D | DATE

See Yellow sheets (4 pages) for computation - details

Spirit level elevations defined by the formula
℄ housing grade Finish

$$H_x = 299.2400 - 5244.03 \times 10^{-6}(x) + 250.09 \times 10^{-10}(x^2) - 6.06 \times 10^{-14}(x^3)$$

x' = distance from Sta. 0+00 up to Sta. 100+00
(line extended to 105+80)

In the above formula the spheroid curvature and geoid height are combined. See transcript of Geodesy Division, Coast and Geodetic Survey 2/26/63 for reference in file.

The straight line elevations are, ℄ housing grade, Fin. Floor

Sta. 0+00 297.300 ft + 1.94 = 299.2400 ft
Sta 100+00 247.300 ft + 1.94 = 249.2400 ft
(line latter extended from 0+00 to -2+00 and from 100+00 to 105+...

Station	Elevation in ft.	Slope %
0+00	299.2400	
+50	298.9779	0.524
1+00	298.7159	.524
+50	298.4540	.524
2+00	298.1922	.524
+50	297.9306	.523
3+00	297.6690	.523
+50	297.4076	.523
4+00	297.1464	.522
+50	296.8853	.522
5+00	296.6242	.522
+50	296.3633	.522

SHEET N

JOB NO. JOB BY DATE

SUBJECT CHK'D DATE

6+00	296.1026	0.521
+50	295.8419	.521
7+00	295.5814	.521
+50	295.3210	.521
8+00	295.0608	.520
+50	294.8006	.520
9+00	294.5406	.520
+50	294.2807	.520
10+00	294.0209	.520
+50	293.7613	.519
11+00	293.5018	.519
+50	293.2424	.519
12+00	292.9831	.519
+50	292.7239	.518
13+00	292.4649	.518
+50	292.2060	.518
14+00	291.9472	.518
+50	291.6886	.517
15+00	291.4300	.517
+50	291.1716	.517
16+00	290.9133	.517 67
+50	290.6552	.516
17+00	290.3971	.516 2
+50	290.1392	.516
18+00	289.8814	.516 57
+50	289.6238	.515
19+00	289.3662	.515 2
+50	289.1088	.515
20+00	288.8515	.515 47
+50	288.5943	.514
21+00	288.3373	.514 2
+50	288.0803	.514

SHEET N
2 of

JOB NO. JOB BY DATE

SUBJECT CHK'D DATE

22+00	287.8235	0.514
+50	287.5669	.513
23+00	287.3103	.513
+50	287.0532	.513
24+00	286.7975	.513
+50	286.5414	.512
25+00	286.2853	.512
+50	286.0293	.512
26+00	285.7735	.512
+50	285.5178	.511
27+00	285.2623	.511
+50	285.0068	.511
28+00	284.7515	.511
+50	284.4963	.510
29+00	284.2412	.510
+50	283.9862	.510
30+00	283.7214	.510
+50	283.4766	.510
31+00	283.2221	.509
+50	282.9676	.509
32+00	282.7132	.509
+50	282.4590	.508
33+00	282.2049	.508
+50	281.9509	.508
34+00	281.6970	.508
+50	281.4433	.507
35+00	281.1897	.507
+50	280.9362	.507
36+00	280.6828	.507
+50	280.4295	.507
37+00	280.1764	.506
+50	279.9234	.506

SHEET 3 of

38+00	279.6705	0.506
+50	279.4177	.506
39+00	279.1651	.505
+50	278.9125	.505
40+00	278.6601	.505
+50	278.4079	.504
41+00	278.1557	.504
+50	277.9037	.504
42+00	277.6517	.504
+50	277.4000	.503
43+00	277.1483	.503
+50	276.8967	.503
44+00	276.6453	.503
+50	276.3940	.503
45+00	276.1428	.502
+50	275.8917	.502
46+00	275.6408	.502
+50	275.3899	.502
47+00	275.1392	.501
+50	274.8826	.501
48+00	274.6382	.501
+50	274.3878	.501
49+00	274.1376	.500
+50	273.8875	.500
50+00	273.6375	.500
+50	273.3276	.500
51+00	273.1379	.499
+50	272.8883	.499
52+00	272.6388	.499
+50	272.3894	.499
53+00	272.1401	.499
50	271.8910	.498

JOB NO. JOB BY DATE

SUBJECT CHK'D DATE

54+00	271.6420	0.498	
+50	271.3931	.498	
55+00	271.1443	.498	
+50	270.8956	.497	
56+00	270.6471	.497	
+50	270.3986	.497	
57+00	270.1503	.497	
+50	269.9022	.496	
58+00	269.6541	.496	
+50	269.4063	.496	
59+00	269.1583	.496	
+50	268.9106	.495	
60+00	268.6630	.495	← Good [illegible]
+50	268.4156	.495	
61+00	268.1682	.495	
+50	267.9210	.494	
62+00	267.6739	.494	
+50	267.4269	.494	
63+00	267.1801	.494	
+50	266.9333	.494	
64+00	266.6867	.493	
+50	266.4402	.493	
65+00	266.1938	.493	
+50	265.9475	.493	
66+00	265.7014	.492	
+50	265.4553	.492	
67+00	265.2094	.492	
+50	264.9636	.492	
68+00	264.7180	.491	
+50	264.4724	.491	
69+00	264.2270	.491	
+50	263.9816	.491	

JOB NO. JOB BY DATE

SUBJECT CHK'D DATE

70+00	263.7364	0.490
+50	263.4914	.490
71+00	263.2464	.490
+50	263.0016	.490
72+00	262.7568	.490
+50	262.5122	.489
73+00	262.2677	.489
+50	262.0234	.489
74+00	261.7791	.489
+50	261.5350	.488
75+00	261.2910	.488
+50	261.0471	.488
76+00	260.8033	.488
+50	260.5596	.487
77+00	260.3161	.487
+50	260.0727	.487
78+00	259.8294	.487
+50	259.5862	.486
79+00	259.3431	.486
+50	259.1001	.486
80+00	258.8573	.486
+50	258.6146	.485
81+00	258.3720	.485
+50	258.1295	.485
82+00	257.8872	.485
+50	257.6449	.485
83+00	257.4028	.484
+50	257.1608	.484
84+00	256.9189	.484
+50	256.6771	.484
85+00	256.4354	.483
+50	256.1939	.483

JOB NO. JOB BY DATE

SUBJECT CHK'D DATE

86+00	255.9525	0.483
+50	255.7112	.483
87+00	255.4700	.482
+50	255.2289	.482
88+00	254.9879	.482
+50	254.7471	.482
89+00	254.5064	.481
+50	254.2658	.481
90+00	254.0253	.481
+50	253.7249	.481
91+00	253.5447	.480
+50	253.3045	.480
92+00	253.0645	.480
+50	252.8246	.480
93+00	252.5848	.480
+50	252.3451	.479
94+00	252.1056	.479
+50	251.8661	.479
95+00	251.6268	.479
+50	251.3876	.478
96+00	251.1485	.478
+50	250.9096	.478
97+00	250.6707	.478
+50	250.4320	.477
98+00	250.1933	.477
+50	249.9548	.477
99+00	249.7164	.477
+50	249.4782	.476
100+00	249.2400	.476
+50	249.0020	.476
101+00	248.7640	.476
+50	248.5262	.476

102+00	248.2885	0.475
+50	248.0509	.475
103+00	247.8135	.475
+50	247.5761	.475
104+00	247.3329	.474
+50	247.1018	.474
105+00	246.8648	.474
+50	246.6279	.474
+80	246.4859	.473

Straight line EL. from 105+80 to 112+50
(Original horiz. straight line was defined from 100+00 to 108+00)
The spirit level elevations defined by the formula

$$H_x^I = 246.4859 - \cancel{18.50}\overset{?}{}\ 15.43 \times 10^{-6}(X) + 0.02312 \times 10^{-6}(X^2)$$

$X = 0$ at Sta 105+80

with 15.43 and 0.02303 elevations are the same.

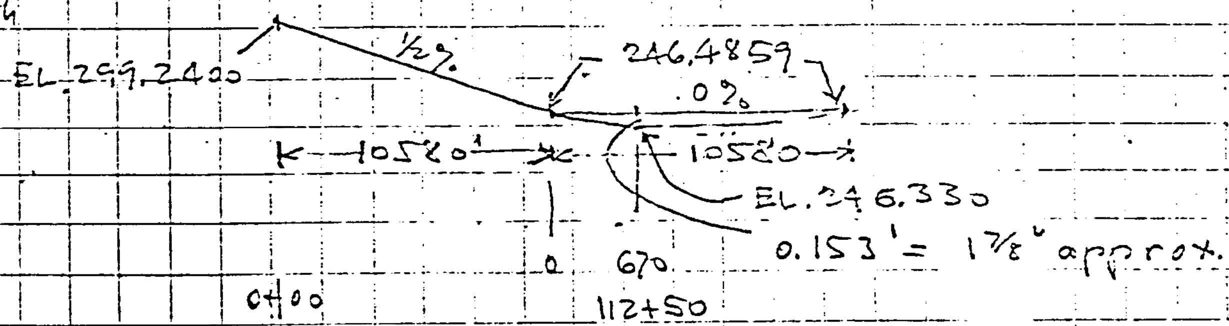
%

105+80	246.4859		
106+00		56	-0.0015
+50		49	-.14
107+00		44	-.10
+50		39	-.10
108+00		36	-.0006
+50		34	-.4
109+00		33	-.2
+50		33	+.00
110+00		35	+.2
+50		37	+.4
111+00		41	+.8
+50		46	+.10
112+00		52	+.12
+50	246.4859		+.0014

Extension of ½% slope from 0+00 to −2+00

Sta.	Elev. ft.	Slope %
0+00	299.2400	
−(0+50)	299.5025	0.525
−1+00	299.7650	.525
−(1+50	300.0275	.525
−2+00	300.2900	.525

Using proper ½% slope and level horizontal with equal length



The formula for the 10580' level distance is

$$x^2 = 670^2$$

$$H_x^1 = 246.4859 - 243.657\times10^{-6} \times \overset{x}{670} + 0.02303 \times 10^{-6} \times 448,900 =$$

0.02303 x 10580

$$= 246.3330'$$