Coherent Beam-Beam Interactions in Electron-Positron Colliders*

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We present the results of a new calculational technique that evaluates the beam-beam force due to an arbitrary charge distribution. We find coherent instabilities that dominate at certain operating points and depend strongly on the degree of damping in the system. We conclude that while these resonances may play a significant role for colliders with low damping, with a careful choice of operating points they should present no danger to the new generation of high luminosity heavy-quark factories under design.

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One of the factors limiting the luminosity of e^+e^- storage ring colliders is the beam-beam interaction – the effect of the electromagnetic fields of one beam on the particles of the other. The beam-beam interaction has conventionally been parameterized by the beam-beam parameter ξ (often called the beam-beam tune-shift) that is proportional to the beam current divided by the cross-sectional area. Larger values of ξ generally correspond to greater luminosity. The beam-beam interaction blows-up the beam and therefore limits ξ; the maximum value achieved amongst present-day e^+e^- colliders is $\xi = 0.06$. Although the beam-beam interaction has been studied with a wide variety of theoretical, experimental, and computational techniques, the dynamical reason for this beam-beam limit is not well understood.

This is especially true of coherent (or collective) beam-beam effects. At the 1989 ICFA Workshop [1] opinions of different working groups ranged from "It is unnecessary to invoke coherent modes in describing performance limits", to "Experimental results, which have been obtained from various machines, indicate the important role of collective effects in the beam-beam phenomenon." Dipole motion, where the beam centroids oscillate, is routinely observed in operating storage rings, but there is no evidence that it affects the luminosity. Centroid motion is easily detected, and could be removed with feedback.

The potential for performance limitations comes from effects that distort the beam shape. Such effects have been analyzed with two different types of models. In the first, of Hirata [2] and of Furman *et al.* [3], nonlinear maps for the colliding beam system are developed in the moments of the distributions. Working under different approximations they find either flip-flop solutions or higher-period solutions, respectively.

In the second type of model, of Dikansky and Pestrikov [4] and of Chao and Ruth [5], modes develop in the phase-space distributions of the two beams. The stabil-

ity of these modes is analyzed with the linearized Vlasov equation, assuming small perturbations from equilibrium. Their results are characterized by the appearance of even-order nonlinear coherent resonances of finite width in ξ_0 (the value of the beam-beam parameter obtained by using the nominal, unperturbed cross-sectional area of the beam). While these calculations indicate the potential importance of coherent beam-beam effects, there remain open questions about the approximations used, as well as about the effects of including radiation and Landau damping.

Computer simulations are an important tool in the study of beam-beam phenomena. In the usual simulations the electromagnetic fields are calculated assuming that the beam has a Gaussian distribution, even though it is observed in both experiments and simulations that the distribution is non-Gaussian in the presence of the beam-beam interaction. This is a reasonable approximation for studying one important aspect of the beam-beam interaction: nonlinear single-particle (incoherent) motion. However, constraining the fields to be those of a Gaussian distribution could inhibit coherent effects. In this Letter we report the results of a computer simulation that does not impose this constraint. We find that coherent instabilities are dominant at certain operating points.

In the simulation the storage ring was modelled as having a single interaction region, with a magnetic arc focussing and transporting particles between collisions. The collisions were head-on. The arc was linear with equal horizontal and vertical tunes Q_β. The damping and quantum excitation effects of synchrotron radiation were included. Longitudinal motion (synchrotron oscillations) was not. The beams were nominally round (equal amplitude functions β^*, and emittances ϵ, in the two transverse dimensions), but we did not constrain them to remain so [6].

The particles were assumed to be ultra-relativistic. The electromagnetic field of a beam was calculated by Lorentz transforming to its rest frame and solving numer-

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ically for the electrostatic field from the coordinates of the test particles comprising the beam [7]. Test particles were cast onto a two-dimensional polar mesh. The resulting charge distribution was first Fourier analyzed and then smoothed by statistical methods [8], and Poisson's equation was solved for each azimuthal component. The smoothing turned out to be an important step in the calculation; in its absence the results were dominated by numerical noise. It was checked that, with smoothing, the numerical solutions for sample Gaussian distributions agreed with the corresponding analytical expressions for the fields [9], and that the results presented below were independent of the granularity of the mesh and the number of test particles.

We used this computer program to explore the consequences of allowing for unconstrained beam distributions. Dipole motion of the beams was eliminated by setting their centroids to zero after each turn, thus simulating an idealized feedback system. We chose operating tunes close to the low-order nonlinear resonances. In this Letter we focus mainly on sixth- and eighth-order resonances.

We first investigated the behavior of the $\frac{5}{6}$ resonance by running at tunes just below the resonance tune $Q_\beta = 0.83$. Figure 1(a) shows the beam-size variation at the $Q_\beta = 0.79$, with $\xi_0 = 0.10$. Here the beams typically underwent incoherent blow-up for a few thousand turns, before breaking into coherent oscillations. They stabilized rapidly in this state and continued the motion indefinitely. The final condition was therefore a stable fixed point.

It is clear from the Fig. 1(b) that there are coherent oscillations in the sizes (second-moments) of two beams. These can be described as *period-3, anti-correlated* oscillations. In physical space, on a given turn, one beam has a dense core while the other is hollow, forming a halo around the first. On the next turn the beams have comparable sizes, while on the third turn the first beam is hollow and the second dense. This three-fold pattern repeats indefinitely.

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Coherent behavior is observed only over a finite range of the nominal beam-beam parameter ξ_0. The extent of that range is a function of the distance away from $Q_\beta = 0.83$. Consequently the domain of coherent activity spans a tubular region in ξ_0 vs Q_β space. This feature is shown in Fig. 2. In regions outside this tube only incoherent blow-up of the beam is observed.

Similarly, the $\frac{7}{8}$ resonance was also investigated. The beam-sizes showed a clear period-4, anti-correlated behavior. The magnitude of the size oscillations was substantially less than in the case of the sixth-order resonance, indicating the lower strength of the higher-order resonance. This is also seen from the narrower width of the resonance tube in Fig. 2.

We emphasize that the appearance of these higher-order coherent resonances is a direct consequence of the general field calculation. They are not observed in simulations that assume a Gaussian distribution when calculating the fields created by the beams[10]. The overall character of the resonances, and in particular their finite width, is in agreement with the Vlasov-equation models discussed above, thus lending validity to the approximations made in those models. The nature of the final states arrived at by the beams, where one beam is dense and the other hollow, implies a substantial decrease in the overlap of the two beams - and consequently in the beam-beam parameter ξ and in the luminosity of the collider.

Two studies were performed to look for possible odd-order resonances. Below a tune of $Q_\beta = 0.67$ it may be argued that one expects a contribution from both, a sixth-order $\left(\frac{4}{6}\right)$ resonance as well as a possible third-order $\left(\frac{2}{3}\right)$ resonance. Below a tune of $Q_\beta = 0.83$, however, one expects only a sixth-order $\left(\frac{5}{6}\right)$ resonance. We found that the dynamics in these two regions is identical, suggesting that the $\frac{2}{3}$ resonance is not present. An effort to uncover a possible $\frac{4}{5}$ resonance at a tune of $Q_\beta = 0.77$ was unsuccessful. We conclude that, in general, odd order resonances are not present.

This agrees with the predictions of the Vlasov-equation models.

One drawback of these models is that they do not take into account the radiation damping and quantum excitation effects that play such an important role in the beam dynamics of e^+e^- colliders. Our simulation offers the opportunity to study the influences of radiation and Landau damping on these coherent resonances. Radiation damping effects are generally measured by the damping decrement δ, which is the average fractional energy radiated by a particle, per turn. We find that the widths of the resonances are strongly dependent upon δ.

The results quoted above for the sixth-order resonance were with $\delta = 1 \times 10^{-3}$, while the eighth-order resonance results were with $\delta = 1 \times 10^{-5}$. Low-energy colliders of the past have had damping decrements of the order of 10^{-5}. Existing higher-energy colliders have $\delta \sim 10^{-4}$. For the sixth-order resonance, Fig. 3 shows that with $\delta = 1 \times 10^{-4}$ the width of the resonance increases by a factor of over two. The eighth-order ($\frac{7}{8}$) resonance, on the other hand, was not observed at $\delta = 1 \times 10^{-4}$. Similarly, at $Q_\beta = 0.77$ with $\delta = 1 \times 10^{-5}$ the tenth-order ($\frac{8}{10}$) resonance was not observed. This is because damping is now strong enough to suppress the outbreak of coherent oscillations.

Damping was evident in another way. There was no sign of coherent motion at small values of ξ_0, and close to the resonance tune. For example, even with $\delta = 1 \times 10^{-5}$ the eighth-order resonance was not seen at $Q_\beta = 0.86$. This is consistent with Fig. 2 where the extrapolated widths of the resonances go to zero before the resonance tune, and is in contrast to the Vlasov-equation models where the widths are zero only at the resonance tunes.

The coherent resonances we have described could be expected to affect performance and be observed in colliders with low damping decrement. Their signature is a swift, turn-to-turn, variation in the beam-sizes. In the case of VEPP-2M there is

some speculation that coherent (possibly synchro-betatron) resonances may be the cause of performance limitations, but the evidence is by no means conclusive [11]. It is also widely believed that coherent effects were responsible for the failure of the DCI space-charge compensation experiments [12]. Since the beam-size detectors in those colliders were not sensitive to turn-by-turn variations these resonances would not have been detected, and an increase in average beam-size would have been mistaken as being due to incoherent phenomena.

The heavy-quark factories currently under design at various institutions assume flat beams. Since our work is for nearly-round beams, it is not directly applicable to these designs. However, round and flat beams were compared by Dikansky and Pestrikov [4], who find the same kinds of coherent resonances for both. We therefore believe that our work may be used to make predictions for flat beams. The heavy-quark factory designs have $\delta \sim 10^{-4}$. Our results indicate that coherent resonances beyond sixth order are not expected at this value of the damping decrement. The sixth-order resonances can easily be avoided by a suitable choice of operating tunes, and are therefore not expected to limit the beam-beam parameter ξ, and hence the luminosity, of these colliders.

In summary, our results indicate that it is critical to use general field calculations in the study of coherent beam-beam phenomena. This allows for a self-consistent calculation of the electromagnetic fields from the positions of the test particles. It results in a new class of higher-order coherent resonances, as predicted by the Vlasov-equation models. The widths of these resonances depend strongly on the degree of damping in the system, and they are not important for heavy-quark factories.

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not discussed in this Letter. We note, however, that they have no upper limit in ξ_0.

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FIGURES

FIG. 1. RMS beam-size as a function of turn number, for $Q_\beta = 0.79$, $\xi_0 = 0.10$, $\delta = 1 \times 10^{-3}$, (a) showing the onset of the instability for one of the beams, and (b) showing the period-three, anti-correlated nature of the size variations of the two beams.

FIG. 2. Onset and offset values of ξ_0 as a function of Q_β for the sixth-order (squares) and eighth-order (starbursts) resonances. In each case the region of the coherent motion is between the lines. The resonance tunes are indicated by vertical lines.

FIG. 3. Average RMS beam-size (normalized to the nominal size) for two different damping decrements. The bars indicate the turn-by-turn variation in size, not the uncertainity of the average. The finite number $(10,000)$ of test particles used gives 1% variations in size. Larger variations are due to coherent oscillations.
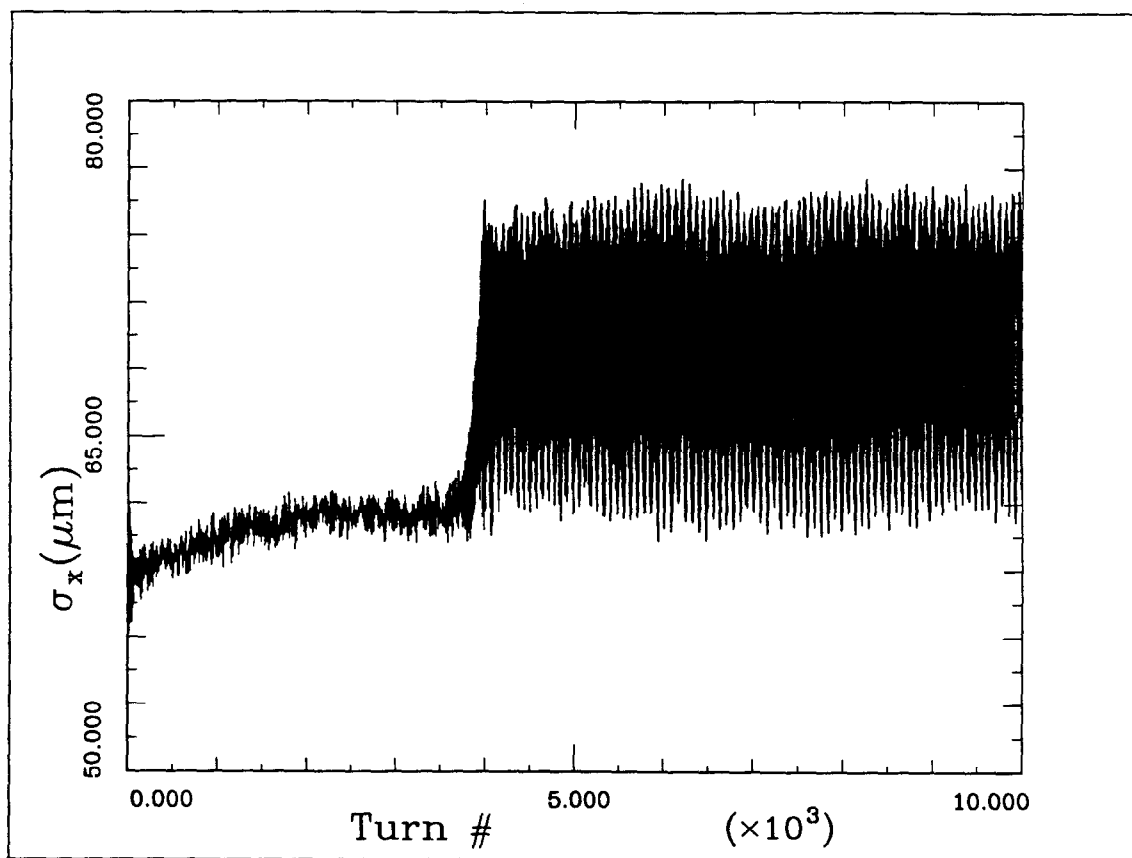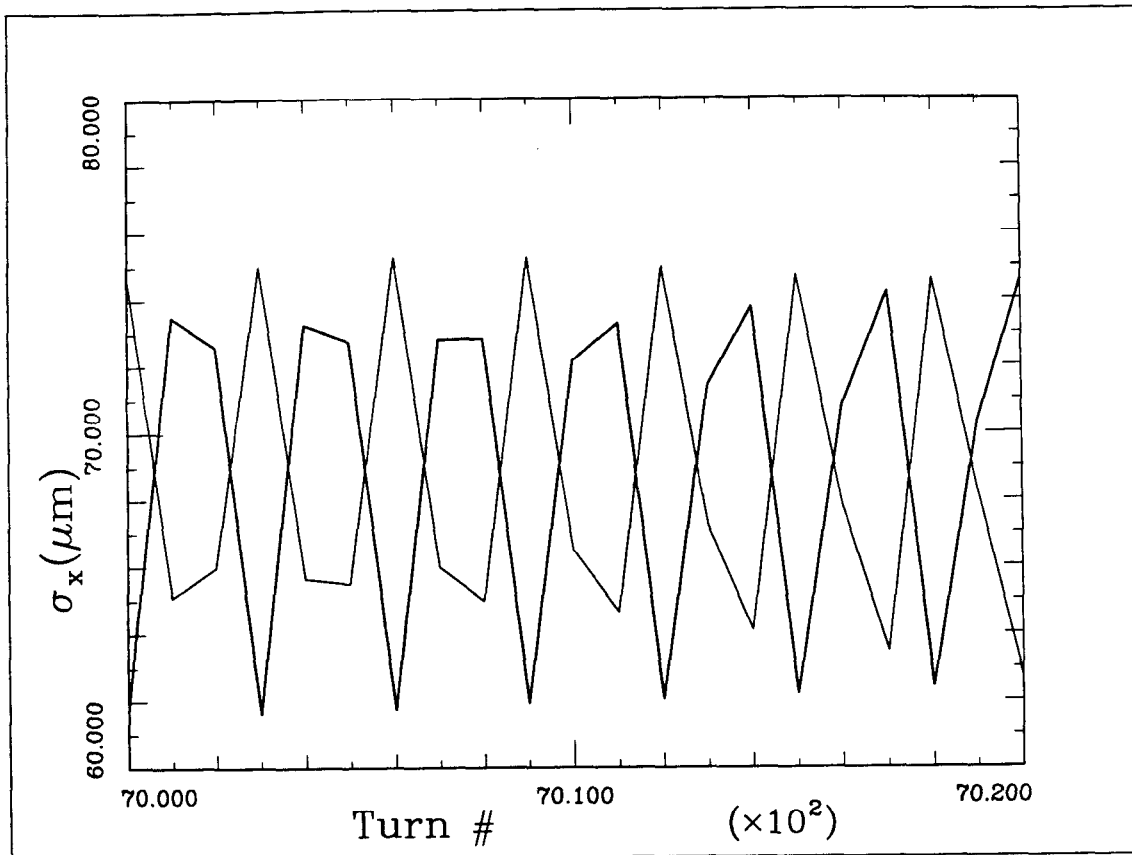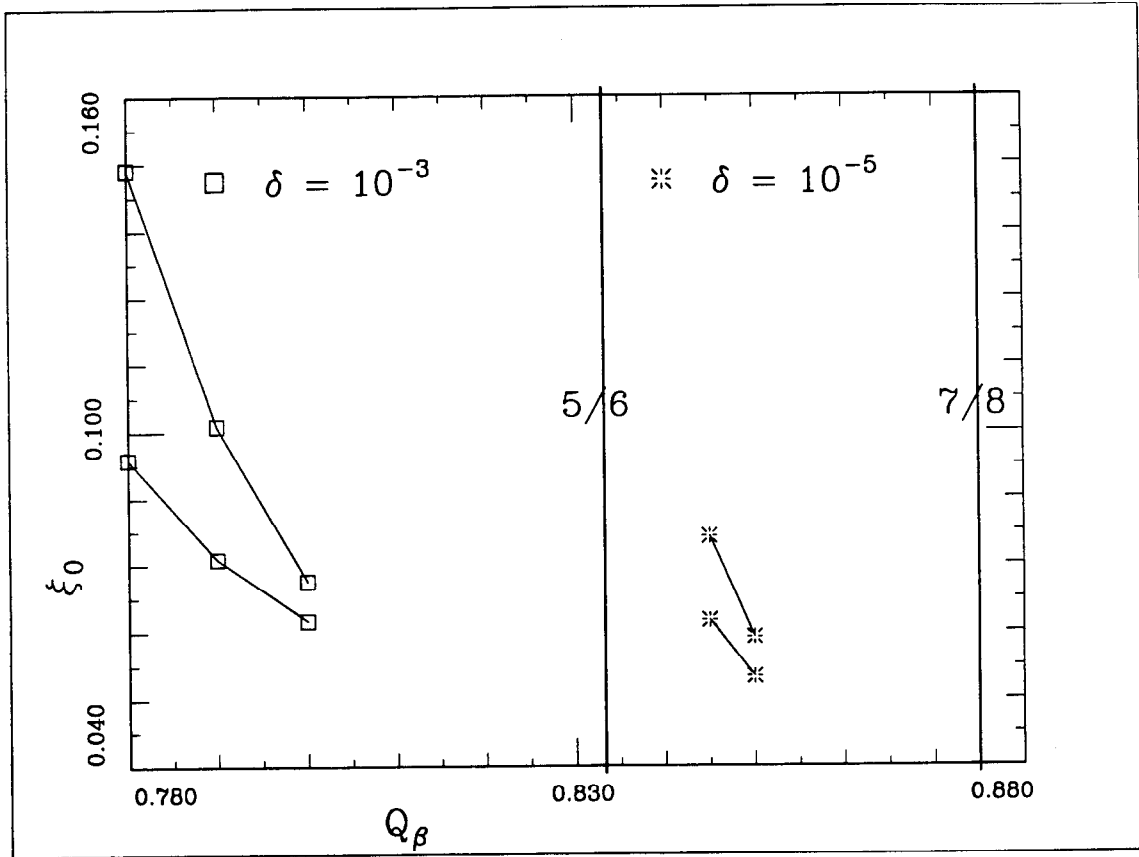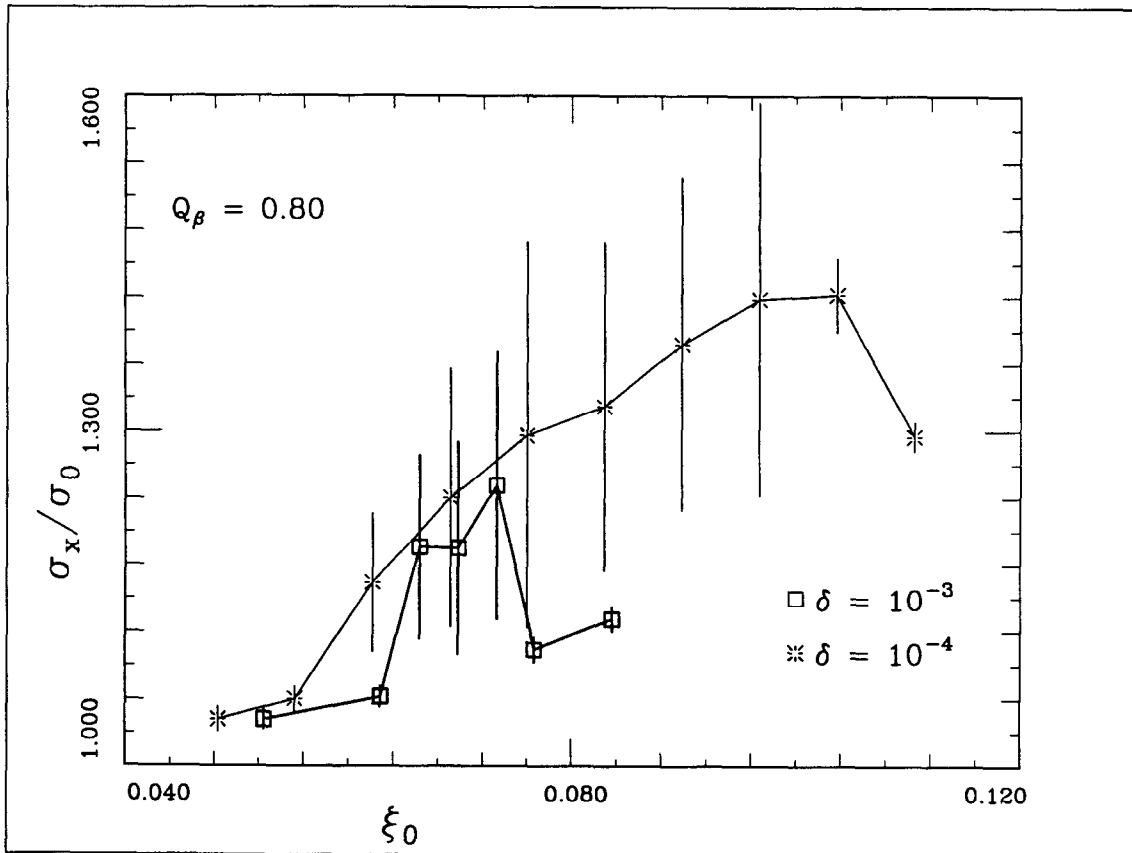
Fig. 1a

Fig. 1b

Fig. 2

Fig. 3