FOCUSING ELECTRON BEAMS AT SLAC*

RICHARD L. TAYLOR

Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94305 and J. J. Pearce High School, 1600 North Coit Rd., Richardson, TX 75080

I. Introduction

I have always heard that a beam of electrons could be focused with magnets. While that seemed to make good sense, I had no clear idea of exactly how that could be done until I began spending my summers at SLAC, the Stanford Linear Accelerator Center. SLAC is operated for the U.S. Department of Energy by Stanford University, and it is located at the base of the foothills south of San Francisco, California.

^{*} Work supported by Department of Energy contract DE–AC03–76SF00515.

My association with SLAC began in the summer of 1989, when a GIFT (Growth Initiatives For Teachers) grant from General Telephone and Electronics (GTE) allowed me to visit SLAC as part of the personal development section of my proposal. So that I could spend more time there, Bill Ash, head of the Superconducting Final Focus Group, took a chance and hired me as a Visiting Experimental Physicist, and I have been lucky to be invited back each summer since then. The time spent working at this high energy physics lab has invigorated and inspired my teaching and has provided the source of many new homework problems for my students.

Magnetic focusing came into my life because it was the job of the Superconducting Final Focus Group to produce a set of magnets that would focus high energy electron and positron beams to a tiny spot where they could collide, annihilating, and generating new particles. In the summer of 1991, our magnets were given their first beams, and we are all proud to say that ever since they have been working just beautifully.

II. SLAC and the SLC

The job of the Stanford Linear Accelerator Center is to accelerate electrons to very high energies, to crash them into things, and then to see what happens. The acceleration is accomplished with a linear accelerator, or linac, which can accelerate electrons and positrons to an energy of 50 GeV. In SI units, 1 GeV = $1000 \text{ MeV} = 1 \times 10^9 \text{ eV} = 1.6 \times 10^{-10} \text{ J}$, so $50 \text{ GeV} = 8 \times 10^{-9} \text{ J}$.

It is important to appreciate the magnitude of this energy: these electrons are superrelativistic! Since the rest mass of an electron (from $E = mc^2$) is 0.51 MeV, a

final energy of 50 GeV means that the electron's kinetic energy is 100 000 times its rest energy. When an electron emerges from the end of the linac, it's traveling with a velocity of 99.999999995% c! This differs from the speed of light by 0.015 m/s, or about the speed of a turtle!

At two miles long, the linac at SLAC is the largest linear accelerator in the world. Accelerating electrons to this energy of 50 000 MeV in a distance of 3200 m means that the electrons are gaining about 15 MeV/m. At this rate, that means that an electron reaches 99.5% the speed of light in the first *foot* of travel down the accelerator! Of course, they can never go faster than the speed of light, so they don't speed up much after that! It is apparent, then, that for relativistic electrons, the accelerator does not make the electrons go much faster, but rather it gives them more momentum and energy. At SLAC, we have "acceleration" and constant velocity at the same time!

Historically, electrons have been made to crash into things using two methods: they are shot at fixed targets, or they collide in storage rings. Collisions of electrons with stationery protons at SLAC led to the discovery of quarks, recognized by the 1991 Nobel Prize. Colliding beams in the storage ring SPEAR at SLAC produced the first tau lepton, and SPEAR also produced the charm quark which crystallized the acceptance of the Standard Model of particle interactions and produced the 1976 Nobel Prize. Colliding beams have become the rage among accelerator physicists, because, since the total momentum of the colliding particles is zero, no energy is wasted moving the center of mass after the collision; all of the energy goes to make new particles. More recently, SLAC has been engaged in the design of an entirely new type of particle collider. Rather than a circular collider like SPEAR (or like those at Fermilab near Chicago, CERN in Europe, and the SSC being built near Dallas), SLAC's proposal would have two large linear accelerators aimed precisely at one another, their particles colliding directly between the barrels of the two "guns." A noncircular collider avoids the problem of large portions of the energy being lost as synchrotron radiation, a substantial problem for low mass, highly relativistic electrons.

Figure 1: The Stanford Linear Collider.

In preparation for this next-generation machine, SLAC has built the Stanford Linear Collider, or SLC, that accelerates electrons and positrons in a single 50 GeV linac, and then bends them around to crash head on in a prolific burst of subnuclear particles. Completed in the spring of 1989, SLC has been a tremendous success as a demonstration of the feasibility of the linear collider concept. In SLC, the collisions can be made to occur at the rest energy of the Z boson, first discovered in the colliding proton beams at CERN. It is at SLC that Z's were first made in large enough numbers to give the first indication that all of the generations of quarks and leptons have indeed already been discovered.¹ This result was quickly verified at the 27 km circumference Large Electron Positron collider at CERN.

III. Dipoles Bend the Beam

An ordinary bar magnet is a dipole magnet; it has two poles, north and south. If the magnet is bent around on itself like a horseshoe, and designed carefully, then the magnetic field in the center between the poles can be very nearly uniform. A uniform magnetic field is often referred to as a dipole field, and it is this nice uniform dipole field that we usually refer to when we have our students work problems with magnetic fields.

Of course, a nice uniform dipole field can also be generated by an electric current,. In fact, a convenient way to get a uniform magnetic field is to use two coils of wire arranged so that they are in parallel planes, much like Helmholtz coils. A beam traveling between these two coils experiences a highly uniform magnetic field.

Figure 2: Permanent magnet with uniform field between poles.

Figure 3: Pair of coils forming a dipole bending magnet.

Just as we tell our students,² a magnetic field exerts a force on a beam of charged particles according to the relationship

$$\mathbf{F} = \mathbf{q}\mathbf{v} \times \mathbf{B} \ . \tag{1}$$

The cross product indicates that the magnetic force is always perpendicular to the direction of motion, and so this force accelerates the particles by changing their direction rather than by changing their speed. The magnetic force bends a beam of charged particles in an arc. In fact, the magnetic field provides a centripetal force, so

$$qvB = mv^2/r.$$
 (2)

The beam curves around in a circle. Using the momentum, p = mv, the radius can be found by

$$\mathbf{r} = \mathbf{p} / \mathbf{q} \mathbf{B} \ . \tag{3}$$

At SLAC, the charge q is, of course, the elementary charge of the electron, so $q = 1.6 \times 10^{-19}$ C. In this form, this equation continues to hold for relativistic particles.

What *is* different about relativistic particles is that the relationship between energy and momentum must be determined using the expression

$$E^2 = (mc^2)^2 + (pc)^2, \qquad (4)$$

where m is the invariant mass, sometimes called the rest mass. In the research at SLAC, the particles are electrons, so the mass is $m = 0.51 \text{ MeV}/c^2 = 9.11 \times 10^{-31} \text{ Kg}$. Using the energy of the electron as it comes out of the linac, 50 GeV = 8×10^{-9} J, the momentum can be determined, from

$$p = \sqrt{(E/c)^2 - (mc)^2}$$
, (5)

to be $p = 2.67 \times 10^{-17}$ Ns.

In SLC, when the electrons and positrons emerge from the linac, dipole magnets are used to curve the beams around so that they can collide head on. Construction plans³ indicate that the radius of curvature in the arcs is $r_{arc} = 280$ m. From equation 3, then, we can calculate the magnetic fields used in the dipoles,

$$\mathbf{B}_{\mathrm{arc}} = \mathbf{p} / (\mathbf{q} \mathbf{r}_{\mathrm{arc}}) \ . \tag{6}$$

Calculation gives the magnetic field, B = 0.597 T. In fact, 0.597 T was the design field for the bending magnets.

Designers of the Superconducting Super Collider (SSC) know that the largest field that can reasonably be expected from mass produced superconducting dipoles is about 6 T. So what size must the ring be if this field is to hold 20 TeV (20 000 GeV) protons? The proton's mass is 1.66×10^{-26} Kg, so equation 5 indicates that a proton's momentum at this energy would be 1.067×10^{-14} Ns. Equation 3, then, tells us that the radius of the SSC must be at least 1.11×10^4 m, which corresponds to a circumference of 43 miles. It is reported that the design is, in fact, for a "circumference" of 54 miles. The difference arises because the SSC is really oval shaped, and the 54 mile figure includes two long straight sections for the detectors.

IV. Quadrupoles Focus the Beam

While ordinary dipole magnets can be used to bend the beam's direction, focusing is done by quadrupole magnets. As the name implies, quadrupole magnets have four poles, and the poles alternate: north, south, north, south. An ideal quadrupole might be made as a permanent magnet, as shown in figure 4a, with the pole faces shaped hyperbolically.

Alternatively, a quadrupole can be formed from sheets of current. As shown in figure 4b, current coming out of the top creates field lines going to the right (from the right hand rule) within the central section. On the right side, current going in creates field lines pointing up. The upper right corner, then, behaves like a south pole, since it is the site of converging field lines. Similar reasoning follows for the other poles.

Figure 4: Quadrupole fields from (a) permanent magnets and from (b) current sheets.

Figure 5: Magnetic field along x- and y-axes.

When a quadrupole magnet is constructed in this way, the field is not constant as it would be in a dipole. Indeed, in the center the effect from all the poles cancels, and the field is zero. But the field increases away from the center, and is, in fact, proportional to the distance from the axis as given by

$$B_x = \Gamma y \text{ and } B_y = \Gamma x ,$$
 (7)

where the gradient Γ is the same in both dimensions. This is illustrated in figure 5.

To see how a quadrupole can be used to focus a beam of charged particles, one could suppose a positron to be coming out of the page (in the positive z direction) anywhere along the x-axis in figure 5. Such an x-axis positron will feel a force toward the center, and the farther it is from the center ,the more will be its push toward the axis. A parallel beam of positrons entering this quadrupole in the xz-plane will be focused to a point on the z-axis. (SLC collides positrons and electrons, and this discussion could refer to either, of course. I will stick with positrons, because the positive charge seems less confusing.)

A similar examination of positrons in the yz-plane, however, will reveal that these particles experience a force away from the center and, in fact, will be defocused. It is the nature of quadrupole magnets to be convergent in one dimension, but divergent in the other. Focusing in both dimensions must be accomplished through the use of combinations of quadrupoles.

V. Finding the Focal Length

The way that magnets bend a beam is reminiscent of the way that light beams are bent by optical systems. In fact, the process of modeling the beam path through magnets is referred to as optics, and each component has an optical analogue. Dipoles behave like prisms, for example, because they merely bend the beam.

But quadrupole magnets behave like lenses, converging and diverging the beams, and a quadrupole is said to have a focal length. While the previous discussion illustrated that all the positrons entering a quadrupole field along the x-axis would be forced toward the center, it is perhaps surprising that they should all meet at the same point independent of their initial distance from the axis. That this is so can be seen by analogy with simple harmonic motion, where the time to cross the equilibrium position is the same regardless of initial displacement. The key lies in the fact that the convergent field produces a restoring force that is proportional to displacement, just like a spring. We can even use this analogy to determine the location of the focal point of the magnetic lens.

First, something must be said about the relativistic nature of the electron beam. The magnetic field exerts a force on the electrons, so we will use Newton's Second Law to analyze the motion. It is clear, however, that F = ma breaks down for extremely high velocities where, even with an enormous force, the velocity can not increase very much. In fact, what Newton really said was that force changes momentum, and the law in that form is still useful. But, while at low velocities momentum is approximately the product of mass and velocity, at relativistic speeds this must be replaced by

$$\mathbf{p} = \gamma \,\mathrm{m}\,\mathbf{v}\,,\tag{8}$$

where m is the invariant mass (or rest mass), and

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad (9)$$

For the 50 GeV electrons at SLAC, $\gamma = 100\ 000$, approximately.

Newton's Law, then, says that $\mathbf{F} = \Delta \mathbf{p} / \Delta t$. But, in our case of a magnetic force which is always perpendicular to the velocity, the speed and energy do not change; only the direction of the velocity changes. So γ remains constant, as does the mass. Thus, for our special case where the force is perpendicular to the velocity,

$$\mathbf{F} = \gamma \, \mathbf{m} \, \frac{\Delta \mathbf{v}}{\Delta t} = \gamma \, \mathbf{m} \, \mathbf{a} \quad . \tag{10}$$

For positrons coming out of the page on the x-axis, as discussed above, the magnetic force will produce an acceleration back toward the origin. Combining equations 7 and 10, the acceleration can be written

$$a_{\rm X} = -\frac{{\rm qc}}{\gamma m} \ \Gamma \ {\rm x} \ , \tag{11}$$

where the speed of light has been inserted for the velocity of the electrons. The minus sign indicates that the acceleration is in the opposite direction to the displacement. (For electrons, or for positrons on the y-axis, there would be no minus sign, of course.) The solution to this equation can be inferred by recognizing its similarity to the solution for a mass on a spring, $a = -\omega^2 x$. For that motion, the kinematic solutions are

$$\mathbf{x} = \mathbf{x}_0 \cos\left(\ \mathbf{\omega} \mathbf{t}\ \right) \tag{12}$$

and

$$\mathbf{v} = -\mathbf{x}_0 \,\,\omega \sin\left(\,\,\omega \,t\,\right) \,\,, \tag{13}$$

the well-known equations of simple harmonic motion when x_0 is the initial displacement with no initial velocity in x. The path of the positron beam through a quadrupole focusing magnet must be similar for a parallel beam of positrons, but with $\omega^2 = qc\Gamma/\gamma m$. To determine the position of the beam as it travels down the axis of the magnet *z*, we will depend on the fact that the electrons are traveling so close to the speed of light that, although they are acted on by a magnetic force, the time to travel down the beam pipe is essentially unaffected and is approximately t = z/c. Thus, the motion of the positrons in the magnet is given by

$$\mathbf{x} = \mathbf{x}_0 \cos \mathbf{Error!} \mathbf{z}) \,. \tag{14}$$

The first thing that can be deduced by this result is that, just as with simple harmonic motion, the time it takes for the positrons to reach x = 0 is independent of the initial displacement. A beam of positrons entering a quadrupole will be

bent, and all particles will reach the axis at the same time and at the same position along the z-axis—the focal point z_0 . A quadrupole magnet focuses the beam.

Secondly, we can actually determine the point z_0 at which the particles will meet, where x = 0. This must occur when the argument of the cosine function is $\pi/2$; thus

$$z_0 = \frac{\pi}{2} \sqrt{\frac{\gamma mc}{q\Gamma}} \quad . \tag{15}$$

An actual focusing magnet would be shorter than z_0 , and the focal length would be longer. Once the beam begins converging, the gradient can be removed (by the beam leaving the magnet), and the particles will continue traveling in converging straight lines. As the particles leave the magnet with length $L < z_0$, their x position is determined by equation 14, with z = L. The converging slope, analogous to equation 13, would be

$$\frac{\Delta x}{\Delta z} = x_0 k \sin(k L), \qquad (16)$$

where, for brevity, the square root term has been replaced with k (i.e., $k^2 = q\Gamma/\gamma mc$). Given the slope m and the y intercept b of a straight line, the x intercept is just b/m. So, the distance from the end of the quadrupole magnet to the focal point would be

$$d = \frac{1}{k \tan(k L)} \quad . \tag{17}$$

This is not exactly the focal length in the same sense as in optics, since this is the distance from the end of the magnet, rather than from the center. However, for moderate strength magnets, where the length of the magnet is short compared to the focal length, a quadrupole magnet can be treated as a thin lens⁴ whose focal length is given by equation 17.

This discussion has involved only the converging aspect of the quadrupole magnet; however, a quadrupole that converges in the x-axis must be diverging in the y-axis. But if L « z_0 , so that the thin lens approximation is applicable, then the focal length in the diverging dimension is just the negative of the focal length in the converging dimension.

Figure 6: Beam of positrons (a) focused in the xz-plane and(b) defocused in the yz-plane by a quadrupole

Focusing in both dimensions is accomplished with a combination of quadrupole magnets, one after the other. Some of the magnets in the group focus in one dimension, while others are given opposite gradient so that they will focus in the other dimension. The final focus for each of the colliding electron and positron beams in the SLC consists of a triplet of three superconducting quadrupole magnets.

VI. The Superconducting Final Focus

In the SLC, electrons and positrons are accelerated in the two mile long linear accelerator to crash head on and generate new particles. To help capture and study the results of these collisions, physicists at SLAC and 32 other institutions have constructed a state of the art particle detector the called the SLAC Large Detector or SLD. One of the most sophisticated machines in the world which uses the newest technology in detectors, SLD began receiving the first beams in mid-1991. The superconducting quadrupole magnets that I have worked on focused the electron beams for analysis by SLD.

In spite of being called an electron "beam," the output of the linac consists of individual "bunches" of electrons and positrons—as many as 120 bunches per second. Each bunch that enters the focusing triplet is approximately 1 mm long and about 1 mm in diameter, containing about 10¹¹ electrons. This may seem like a pretty crowded bunch, but in a bunch this size the electrons are separated from each other by over 300 atomic diameters, or about 3 million times their own wavelengths! This is sort of like a "crowd" of people filling the moon's orbit, with individuals each over a hundred miles apart—not exactly a typical urban environment. Two such "crowds" could pass right through each other, and a collision would be extremely unlikely.

Figure 7: Positrons focused by quadrupole triplet in (a) vertical and (b) horizontal plane.

To enhance the probability of collision, the electron and positron beams must be focused down to a much smaller spot where they meet and collide at the interaction point. This requires extremely strong quadrupole magnets with very short focal lengths. However, the short focal length means that the quadrupoles must be put in so close that any iron in the magnets would saturate from the magnetic field used by the detector.^{5,6} Therefore, the SLD made the decision to use iron-free superconducting quadrupole magnets in the final focus triplet, the Superconducting Final Focus. SLD's superconducting final focus reduces the beams to a spot size as small as 1 to 2 μ m in cross section, increasing the electron density by a factor of a million.

The superconducting quadrupoles used in SLC's final focus were built as an extrapolation of the design used in the Tevatron at Fermilab, and they were constructed at the magnet factory there. SLC's quadrupoles, however, contain improved superconducting wire, have a smaller aperture, and use no iron.

Figure 8: Cross section of SLC superconducting Final Focus quadrupole magnet, with sample field lines.

A cross section of one of the superconducting quadrupoles is shown in figure 7. The beam pipe in the drawing would extend out of the page through the 5.0 cm diameter aperture shown in the center. Superconducting wires provide the sheets of current shown coming out of the diagram on both the right and left of the figure, and shown going in at the top and bottom. These current sheets generate the alternate poles of the quadrupole. The three quadrupole magnets of the triplet are connected in series, so that each has the same current and the same size gradient. However, for the triplet to focus in both dimensions, the middle quad has the current reversed in order to make it converging in the dimension for which the others are diverging, and vice versa.

The superconducting cable itself is made of many individual wires. Each individual superconducting wire consists of 570 filaments of 19 μ m diameter NbTi type II superconductor immersed in a 0.68 mm diameter copper matrix. The superconducting cable has 23 of these wires bundled together into a flat ribbon that averages 7.8 mm wide and 1.2 mm thick. This ribbon has the same cross section as a round wire that is 3.5 mm in diameter. The quadrupoles are housed in a liquid helium cryostat that maintains their temperature at just above absolute zero, about 4.3 K.

Each of the four current sections in figure 8 contains 48 superconducting cables. So imagine this: each of the 48 individual 3.5 mm equivalent diameter wires carries as much as 5000 A of current! It is amazing to see: this huge current from the power supply is brought to the cryostat by ten copper cables, each of them 25 mm in diameter, and they then transfer all their current to this tiny wire ribbon. At the typical operating current of 4250 A, the large copper cables get quite warm from the i²R heating; but R = 0 for the superconductor, so the wire ribbon remains cool. And I am still impressed when I look at the meters and see that 4250 A comes from a power supply providing only 5 V! Most of the power goes to heating up those ten one inch cables to over 80°C.

All of this is possible, of course, because the quadrupole magnets are immersed in liquid helium at a temperature of 4.3 K, allowing the NbTi alloy to become superconductive. The triplet of three magnets rests in a single cryostat, essentially a large aluminum "thermos" bottle. The very small amount of heat generated by nonsuperconducting connections inside the magnets is removed by the liquid helium turning into a gas and and returning to a compressor and heat exchanger. A clever design aspect of this cryostat has the cold gas returning through a pipe that surrounds the liquid helium, thus insulating the liquid and helping to keep it cold. Of course, just as in any "thermos" bottle, the whole thing is surrounded by a vacuum to keep heat from entering the system. One of the special problems associated with using superconducting magnets is that there might be some sudden change that could cause the superconductors to go normal. Then, suddenly and unexpectedly, there would be thousands of amperes of current in a wire with nonzero resistance, and i²R would no longer be zero. If only a small section went normal, all of the energy would concentrate at that one spot, and enough heat could be generated there to destroy the wire—a catastrophic event. For this reason special protection devices and circuits have been installed.

At the first electronic hint of something going wrong, such as increased voltages across the coils or a drop in the level of liquid helium in the cryostat, the power supply is disconnected from the quads by a fast acting SCR (silicon controlled rectifier). The removal of the power supply forces the current from the magnets through a large low resistance dump resistor, and dissipates the magnet's energy safely outside of the cryostat. An alternative backup system that could be used would turn on a set of heaters near the coils of the magnets. This would force large sections of the coils to go normal, spreading out the energy and saving the system. These protective devices have been tested many times.

The following table summarizes the operation of the superconducting final focus. The quads are numbered from the interaction point (the IP). The beam enters Q3 first and exits through Q1.

Typical Values for Superconducting Triplet Focusing 50 GeV Electrons			
	Q3	Q2	Q1
Length:	66.5 cm	121.0 cm	66.5 cm
Separation:	from Q2 = 36.3 cm	from Q1 = 36.3 cm	from $IP = 221 \text{ cm}$
Total length of			
superconductor:	176 m	270 m	176 m
Turns:	96	96	96
Gradient:	117 T/m	117 T/m	117 T/m
Current:	4250 A	4250 A	4250 A
Tested critical current:	~ 6800 A	~ 6800 A	~ 6800 A

Of course, there must be two sets of triplets in the collider. The triplet focusing the electrons and the triplet focusing the positrons are identical.

VII. Sextupole Magnets

A dipole magnet produces a constant magnetic field. A quadrupole magnet produces a field which changes with a constant gradient. It seems reasonable that adding another pair of poles, thus making a sextupole magnet, would result in a magnet in which the *gradient* changes at a constant rate; and indeed, this is so. This gives the effect of having a lens whose focusing becomes more powerful away from the center. Outside portions of the beam would be focused close in, while inner portions of the beam would be focused only weakly. This may seem odd, but the "lineless" bifocals that I wear work in much the same way: A "strength gradient" runs vertically so that the bottom of the lens is strong for reading, and the strength gradually changes to a negative value at the top of the lens to accommodate my nearsightedness. Sextupole magnets are used with electron beams for much the same reason as I wear bifocals (*not* because of old age!); beams that would otherwise focus in different places need to be "fixed" so they will all focus at the same spot.

We calculated the focal length of a quadrupole and found it to be independent of initial displacement. However, the focal length that we determined does depend on γ , and thus on the energy. No beam could be entirely monochromatic, with all the electrons at exactly the same energy, so, quadrupole lenses have dispersion—they focus different energies in different places. Sextupole magnets can be used as chromaticity correctors.

As a beam of different energy particles enters a quadrupole, the lower energy particles are separated from the higher energy ones, with the higher energy particles bending less and moving to the outside of the packet. The electrons won't all focus to one point, because the higher energy particles aren't bent enough—they need more help. So, when this dispersed group enters a sextupole, it is further focused. But the higher energy particles enter farther from the center—they see a stronger gradient and get an extra boost that just makes up for the bending they missed in the quadrupole. All energy particles can be made to meet at the same focus.

VIII. Conclusions

A uniform magnetic field is a dipole field, and a dipole field can be used to bend a beam of particles. A quadrupole magnet uses four poles or four current sheets to produce a field with a constant gradient. A quadrupole magnet is used to focus a beam of particles. Sextupole magnets have a varying gradient, and so, they are used to counteract the energy dispersion of other magnets.

In a quadrupole field, the force on a moving charge is proportional to the particle's displacement from the center. The equations of simple harmonic motion can be used to justify the conclusion that all of the particles which enter a quad parallel to the axis will end up at the same point, independent of their initial displacement. But quadrupoles focus only in one dimension, diverging the beam in the other. A combination of quadrupoles must be used to focus a particle beam to a point.

At the SLC, a pair of superconducting quadrupole triplets focuses electron and positron beams to an interaction point as small as 2 μ m in diameter, thus increasing the probability of collisions. Superconductors allow a very high current without the presence of field-distorting iron. The wire in the superconducting triplets is maintained at the temperature of liquid helium.

The magnetic bending and the focusing of electron beams involve so many different concepts of physics that I find I can use my experiences at SLAC to add interest to many different topics in my high school physics classes. While I would not ask my first year physics students to determine the focal length of a quadrupole, they nonetheless find it reasonable to expect a quadrupole to have a focal length once they have seen the connection between the magnetic field gradient and Hooke's Law for a spring. I like never to let an opportunity go by to show connections between apparently disconnected topics.

IX. Acknowledgements

The superconducting final focus measurement system was built by Jim Ferrie whose analytical skepticism and concern for detail were indispensable to the success of the final focus and to my growth as a scientist. Bill Burgess designed and implemented the unique cryogenic system. Special thanks must go to Bill Ash, who leads the Superconducting Final Focus group and who has provided much inspiration. This group has been extremely helpful during my stay at SLAC.

References

- J. Rees, "The Stanford Linear Collider," *Scientific American*, Vol. 261, No. 4, October 1989.
- D. Halliday and R. Resnick, *Fundamentals of Physics*, 3rd ed., John Wiley & Sons, New York, NY, 1988, p. 688.
- SLC Design Handbook, Stanford Linear Accelerator Center, Stanford, CA 94305, December 1984.
- 4. S. Penner, "Calculations of Properties of Magnetic Deflection Systems," *The Review of Scientific Instruments,* Vol. 32, No. 2, February 1961.

- W. Ash, "B Factory Final Focus System Using Superconducting Quadrupoles," SLAC–PUB–5127, October 1989. Invited talk presented at the Workshop Toward Establishing a B Factory, Syracuse, NY, 1989.
- R. Erickson, T. Fieguth, and J. J. Murray, "Superconducting Quadrupoles for the SLC Final Focus," SLAC–PUB–4199, January 1987; also *Proceedings of the* 1987 IEEE Particle Accelerator Conference, Washington, D.C., March 16–19, 1987, p 142.
- A. D. McInturff, J. A. Carson, H. E. Fisk, and R. A. Erickson, "The Magnetic Properties of the SLAC Intersection Region Superconducting Quadrupole Triplets," SLAC–PUB–1478, revised May 1988; also Tenth International Conference on Magnet Technology, Boston, MA, September 23–26, 1987.

Figure Captions

- Figure 1: The SLAC Linear Collider.
- Figure 2: Permanent Magnet with uniform field between poles.
- Figure 3: Pair of coils forming a dipole bending magnet.
- Figure 4: Quadrupole fields from (a) permanent magnets and from (b) current sheets.
- Figure 5: Magnetic field along x- and y-axes.
- Figure 6: Beam of positrons (a) focused in the xz-plane and (b) defocused in the yz-plane by a quadrupole.
- Figure 7: Positrons focused by quadrupole triplet in (a) vertical and (b) horizontal plane.
- Figure 8: Cross section of SLC Superconducting Final Focus Quadrupole Magnet, with sample field lines.

FOCUSING ELECTRON BEAMS AT SLAC*

RICHARD L. TAYLOR

Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94305 and J. J. Pearce High School, 1600 North Coit Rd., Richardson, TX 75080

I. Introduction

I have always heard that a beam of electrons could be focused with magnets. While that seemed to make good sense, I had no clear idea of exactly how that could be done until I began spending my summers at SLAC, the Stanford Linear Accelerator Center. SLAC is operated for the U.S. Department of Energy by Stanford University, and it is located at the base of the foothills south of San Francisco, California.

Submitted to The Physics Teacher.

^{*} Work supported by Department of Energy contract DE–AC03–76SF00515.







Figure 2







6989A4



8-91



Figure 5





6989A6

Figure 6



6989A7





