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$\gamma\gamma \rightarrow \pi^0\pi^0$ in an 1/N Expansion*

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ABSTRACT

$\mathcal{M}(\gamma\gamma \rightarrow \pi^0\pi^0)$ is computed using the effective chiral lagrangian to leading order in $1/N_f$, where N_f is the number of pion fields. The resulting amplitude is chirally invariant and unitary to all orders in momenta. Although the leading order calculation involves an infinite number of unknown coefficients, the final result depends only on the π–π scattering amplitude.

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1. Introduction and Conclusion

Current algebra and partial conservation of axial–vector currents form the basis of our understanding of pion physics. The general chiral–invariant lagrangian automatically imposes these constraints and also provides a way to compute unitarity corrections to the tree–approximation[1]. Recently, the chiral perturbation theory for strongly interacting W's has also been studied[2] as a probe into the Higgs sector. The idea here is that, at energies much higher than the W mass, the longitudinal modes of the gauge bosons can be treated as the associated would–be Goldstone bosons[3]. Since the would–be Goldstone bosons obey the same chiral symmetry as the pions in the chiral limit, the effective chiral lagrangian should also model the longitudinal modes of the gauge bosons. Much of the current literature on strongly interacting gauge bosons is based on this idea.

In practice, chiral lagrangian predictions are computed in powers of external momenta, and predictions to $O(p^4/f_\pi^4)$ are good enough to understand the observed $\pi\pi$–scattering data to 0.7–0.8 GeV[4]. The momentum expansion technique, however, has two important limitations. First, although the effective lagrangian approach is general, the first few terms in the power series expansion cannot capture any resonance feature. Hence the very structures that will help us to identify the mechanisms for electroweak symmetry breaking cannot be seen in the momentum expansion. Second, even at energies much lower than 0.8 GeV, there are some meson processes for which the chiral predictions to leading order in momentum expansion deviate greatly from the data. The reaction $\gamma\gamma \to \pi^0\pi^0$ shown on figure 1 is such an example[5].

In this paper, we propose an expansion of chiral effective lagrangian predictions in powers of $1/N_f$, where N_f is the number of pion fields, as a way to overcome these limitations. Since the general effective chiral lagrangian contains an infinite number of unspecified coefficients, it is usually assumed that any calculation beyond the tree–level will necessarily involve a large number of unknown coefficients that must be measured independently. The $1/N_f$ expansion does not solve this problem, but it organizes the unknown coeffcients in a way that is easier to control. In our case, the coefficients appearing in the leading order in $1/N_f$ and M_π^2/E^2 can be organized into a single function, which can be determined in terms of the π–π scattering amplitudes. Thus, although the expression for $\mathcal{M}(\gamma\gamma \to \pi^0\pi^0)$ involves an infinite number of unknown constants, these can be replaced by a single observable function.

Our main result is that to leading order in $1/N_f$ and for on–shell photons,

$$\begin{aligned} \mathcal{A}_J(\gamma\gamma \to \pi^0\pi^0) &= \frac{-6N_+}{N_f}\frac{e^2}{(4\pi)^2}\epsilon_1 \cdot \epsilon_2 F(S) \mathcal{A}_J(\pi^+\pi^- \to \pi^0\pi^0) \\ F(S) &= 1 + \frac{M_\pi^2}{S}\left[\log\left|\frac{1+(1-4M_\pi^2/S)^{1/2}}{1-(1-4M_\pi^2/S)^{1/2}}\right| - i\pi\right]^2, \end{aligned} \tag{1}$$

where $\mathcal{A}_J$'s are the partial wave amplitudes in the total angular momentum J, $J_z = 0$ channel, N_+ is the number of charged pion fields, and ϵ_i's are the polarization vectors of the two in–coming photons. Unitarity is automatically satisfied to $O(1/N_f)$, and, since the amplitudes contain all orders of momenta, there is a hope of capturing resonance features.

Before we plunge into the proof of the claim, a few comments are in order:

1. Formula (1), for the realistic case of $N_f = 1$ and $N_+ = 1$, was conjectured by Donoghue, Holstein, and Lin[5] as an extrapolation of the one–loop calculation of $\gamma\gamma \to \pi^0\pi^0$. The formula in fact agrees with the data over a wide range of energy, and this suggests that the identity holds in general. In principle, the

formula can accomodate contributions from heavy mesons exchanged in the T, U, or S channel[6] as long as we work below production threshold for these heavier states.

2. The expansion based on $1/N_f$ automatically satisfies the constraints of crossing symmetry and chiral symmetry. Moreover, it is unitary to all orders in momenta. We believe that this is the first calculation of $\mathcal{M}(\gamma\gamma \to \pi^0\pi^0)$ that satisfies all three constraints. In the lowest order in momentum expansion of $\mathcal{M}(\gamma\gamma \to \pi^0\pi^0)$, the amplitude violates unitarity at $O(p^6)^5$. Morgan and Pennington[7] have constructed an alternative theory of $\gamma\gamma \to \pi^0\pi^0$ based on representing final-state correction to $\gamma\gamma \to \pi^+\pi^-$ by an Omnés function. However, this theory has an awkward chiral limit, since the Born amplitude for $\gamma\gamma \to \pi^+\pi^-$, projected onto the S-wave, becomes a δ-function at threshold as $M_\pi \to 0$.

3. Folklore has it that any prediction based on an effective lagrangians will contain a large number of unspecified constants and will therefore lead to no prediction. Our result indicates that it may be possible to assemble these constants into a small number of observable functions. Hence, for applications in which predictions based on the first few terms in the momentum expansion is inappropriate, an alternative expansion such as $1/N_f$ can lead to manageable and testable predictions.

4. Effective chiral theories with higher derivative kinetic terms do not uniquely specify the $\gamma\pi^+\pi^-$ vertex. Upon gauging the effective chiral lagrangian, this ambiguity generates operators of the form $\pi^+ F_{\mu\nu} F^{\mu\nu} \pi^-$ over which we have no control and makes it impossible to write the $\gamma\gamma \to \pi^0\pi^0$ amplitude purely in terms of the π–π scattering data. It is possible, however, to adopt a convention for gauging the theory in which these operators are not generated. This ambigu-

ity represents a limitation of our result. This is further discussed in section 3.

5. Since N_f in our calculation is small (N_f= 3), one must question the efficacy of the whole approach based on expansion in powers of $1/N_f$. In fact, it will turn out later that the expansion parameter is actually $3/N_f$. We chose the $1/N_f$ expansion, not for its convergence properties, but for the analytic properties that it satisfies at each order. In this sense, our choice of the $1/N_f$ expansion should be understood as a systematic way to resum the ordinary perturbation series so that the amplitudes obey the analytic properties demanded by the two–body unitarity. Indeed, the leading order amplitude in $1/N_f$ for the pion four–point function contains exactly the same diagrams as the self–consistent solution to the two–body unitarity, $2\text{Im}T = \phi_2 TT^*$, where ϕ_2 is the two–body phase space.

6. Two issues must be confronted when quantizing the effective chiral lagrangian containing higher derivative kinetic energy terms. . Barua and Gupta[8] showed that theories with higher derivative interactions, such as the effective chiral lagrangian, exhibit subtle cancellations among various diagrams that are automatically handled by the use of dimensional regularization. Naturally, we choose dimensional regularization. Pais and Ulenbeck[9] showed that higher derivative theories in general contain states of negative norm and, as a result, its energy spectrum is no longer positive definite. It is possible to hide these states by placing them above the cut–off for the theory. However, we find that the problem does not appear if we treat the higher derivative terms as perturbative vertices.

2. 1/N Expansion in the Limit $M_\pi^2/E^2 \rightarrow 0$

We first generalize the $SU(2) \times SU(2)/SU(2)(= O(4)/O(3))$ nonlinear σ–model to the $O(N+1)/O(N)$ nonlinear σ–model and compute various Green's functions to leading order in $1/N_f$. The lagrangian is

$$\mathcal{L} = \frac{3}{N_f}[\frac{f_\pi{}^2}{2}\partial_\mu U^\dagger \partial^\mu U + c_0 U^\dagger \partial^4 U + c_1(\partial_\mu U^\dagger \partial^\mu U)^2 + c_2(\partial_\mu U^\dagger \partial_\nu U)^2 + \cdots] + m_0^2 \mathcal{L}_m,$$

where $U = (\sqrt{1-x^2}, \vec{x})$, and $x_i = \sqrt{3/N_f}\pi_i/f_\pi$. For $N_f = 3$ and $m_0 = 0$, this reduces to the effective lagrangian for massless pions.

Many simplifications occur in the limit $M_\pi^2/E^2 \rightarrow 0$. For example, all tadpole type diagrams may be ignored in this limit: All tadpoles come from two–point functions of the type

$$< \pi(x)\pi(x) > \sim \int \frac{d^dQ}{(2\pi)^d}\frac{i}{Q^2 - m_0^2}(1 + aQ^2 + \cdots) \tag{2}$$

This integral is always proportional to m_0^2 and, since chiral symmetry guarantees that the pion mass is multiplicatively renormalized, such a quantity can only contribute to those renormalized terms that are always down by a factor of M_π^2/E^2 compared to the terms of the same order of momentum expansion that do not vanish in the massless limit.

The self–consistent equation for the leading order pion two–point function is shown on figure 2. The full pion two–point function in the absence of any tadpole contribution reduces to the sum of contributions from the higher derivative kinetic terms (figure 2). Writing the full two–point function as $G^{(2)}(p^2) \equiv i/(p^2 - m_0^2) + c_1 + c_2 p^2 + c_3 p^4 + \cdots$, the same analysis also shows that all quadratic and higher divergences in the loop diagram in figure 2a are all proportional to m_0^2 and can be ignored in the limit $M_\pi^2/E^2 \to 0$. Thus, we have the following formula:

$$\int \frac{d^d q}{(2\pi)^d} G^{(2)}(q^2) G^{(2)}((q+p)^2) V_1 V_2 = \int \frac{d^d q}{(2\pi)^d} \frac{i^2 V_1 V_2}{(q^2 - m_0^2)((q+p)^2 - m_0^2)},$$

where the V_i's are arbitrary, momentum–dependent vertex functions. Figure 3 illustrates these results.

To leading order in $1/N_f$, the four–point function is then given by the sum of bubbles shown on figure 4. For our purpose, however, it is not necessary to find the exact expression for the four–point function in terms of the coefficients that appear in the lagrangian; it is enough to note that the on–shell four–point function must be of the form $\delta_{ab}\delta_{cd}A(s,t,u) + \delta_{ac}\delta_{bd}A(t,s,u) + \delta_{ad}\delta_{bc}A(u,t,s)$, where a, b, c, and d are the isospin indices, and $A(s,t,u)$ is given by the diagrams in figure 4.

3. $\mathcal{M}(\gamma\gamma \to \pi^0\pi^0)$

For the sake of clarity, we first present a calculation of $\mathcal{M}(\gamma\gamma \to \pi^0\pi^0)$ in which certain claims are made without proof. These claims are proven at the end of the chapter.

The U(1) covariant derivative D_μ acts on the π's in the usual way and on the U as a diagonal matrix: $D_\mu U = \partial_\mu U + iA_\mu QU$. Here we have chosen, for odd values of $N = 2k+1$, charge e to k complex π–fields, and charge 0 to the remaining real π–field.

Claim 1. Equation (2) and symmetry arguments show that the leading order $\mathcal{M}(\gamma\gamma \to \pi^0\pi^0)$ in $1/N_f$ is given simply by a loop diagram(figure 5a) formed by the tree–level on–shell Compton scattering amplitude(figure 5b),

$$\mathcal{M}_{\mu\nu} = ie^2\big[2g_{\mu\nu} + \frac{(2p_a+k_1)_\mu(2p_b+k_2)_\nu}{(p_a+k_1)^2-m_\pi^2} + \frac{(2p_a+k_2)_\nu(2p_b+k_1)_\mu}{(p_a+k_2)^2-m_\pi^2}\big] + \mathcal{M}_{\mu\nu}^{higher},$$

where we have explicitly written out the Compton amplitude to the lowest order in momentum expansion, and the rest of the leading order on–shell amplitude $\mathcal{M}(\pi^+\pi^- \to \pi^0\pi^0)$.

Claim 2. Using gauge invariance, $\mathcal{M}_{\mu\nu}^{higher}$ can be shown to vanish on shell. By projecting the out–going π^0 state on to definite angular momentum states, the leading order on–shell amplitude for the $\pi^+\pi^- \to \pi^0\pi^0$ process in the S–channel reduces to the partial wave amplitude $\mathcal{A}_J(s)$. $\mathcal{A}_J(s)$ is related to $A(S,T,U)$ by $\mathcal{A}_J(S) = \int d\Omega A(s,t,u)Y_{J0}$, where $t,u = s(1\pm\cos\theta)/2$.

Hence, we have a simple expression for $\mathcal{M}(\gamma\gamma \to \pi^0\pi^0)$, in a given angular momentum J channel:

$$2ie^2\frac{3N_+}{N}A_J(s)\int\frac{d^dq}{(2\pi)^d}\frac{(2q-k_1)_\mu(2q+k_2)_\nu - g_{\mu\nu}(q^2-m_\pi^2)}{((q-k_1)^2-m_\pi^2)(q^2-m_\pi^2)((q+k_2)^2-m_\pi^2)},$$

where, as before, N_+ is the number of the charged pion fields, and $A_J(s)$ is the partial wave amplitude in the angular momentum J, isoscalar channel. By contracting $\mathcal{M}$ with the photon polarization vectors and expanding the result to zeroth order in M_π^2/E^2, we obtain

$$\mathcal{A}_J^{(0)}(\gamma\gamma \to \pi^0\pi^0) = \frac{-6N_+}{N_f}\frac{e^2}{(4\pi)^2}\epsilon_1\cdot\epsilon_2\mathcal{A}_J(\pi^+\pi^- \to \pi^0\pi^0).$$

Notice that this is precisely the result (1) in the limit $M_\pi^2/E^2 \to 0$. Away from this limit, $\mathcal{A}_J^{(0)}$ must be replaced with $\mathcal{A}_J^{(0)}(1+\delta\mathcal{A}_J)$, where $\delta\mathcal{A}_J$ is some expression that must vanish in the limit $M_\pi^2/E^2 \to 0$. Numerically, $\delta\mathcal{A}_J$ is important in the low energy limit. But in the low energy limit, $\delta\mathcal{A}_J$ can be computed explicitly by chiral loop expansion[5]. It is

$$\frac{M_\pi^2}{S}\left[\log\left|\frac{1+(1-4M_\pi^2/S)^{1/2}}{1-(1-4M_\pi^2/S)^{1/2}}\right|-i\pi\right]^2.$$

Assembling these together, we obtain the result (1).

The first claim is proven below: The general tree–level $\gamma\pi^+\pi^-$–vertex is always antisymmetric under exchange of the two pion momenta, p_+ and p_-. On the other hand, the π^{2m}–vertices are always symmetric under exchange of any two pions on the same isospin line. Thus, all of the diagrams shown on figure 6 vanish identically. Thus, there are only four diagrams that contribute to $\mathcal{M}(\gamma\gamma\to\pi^0\pi^0)$ at the leading order in $1/N_f$. They are shown on figure 7. The loop integral diagram 7d has the form given in the equation (2) and may be ignored in the limit $M_\pi^2/E^2\to 0$.

The loop integral in diagram 7c is

$$\int dq\frac{[e^{iq\cdot y}\ \tilde{\partial}\ \epsilon_\mu^1 e^{-ik_1\cdot y}\ \tilde{\partial}e^{-i(q-k_1)\cdot y}][e^{iq\cdot y}\ \tilde{\partial}\ \epsilon_\nu^2 e^{-ik_2\cdot y}\ \tilde{\partial}e^{-i(q-k_1)\cdot y}]}{(q^2-m_0^2)((q-k_1)^2-m_0^2)},$$

where the first bracket is from the $\gamma\pi^2$–vertex, and the second bracket is from the $\gamma\pi^4$–vertex. In terms of Feynman parameters, this is

$$\int dQ\frac{[e^{iq\cdot y}\ \tilde{\partial}\ \epsilon_\mu^1 e^{-ik_1\cdot y}\ \tilde{\partial}e^{-i(q-k_1)\cdot y}][e^{iq\cdot y}\ \tilde{\partial}\ \epsilon_\nu^2 e^{-ik_2\cdot y}\ \tilde{\partial}e^{-i(q-k_1)\cdot y}]_{q=Q+xk_1}}{(Q^2-\Delta)^2},$$

where $\Delta=m_0^2-x(1-x)k_1^2=m_0^2$ for on shell photons. We may ignore all quadratic and higher divergences as $\int dQQ^{2n}/(Q^2-m_0^2)^2\sim O(m_0^{2m})$. The logarithmic divergence is given by setting $Q = 0$ in the numerator of the integrand:

$$\int d\tilde{Q} \frac{[e^{iq\cdot y}\ \tilde{\partial}\ \epsilon^1_\mu e^{-ik_1\cdot y}\ \tilde{\partial} e^{-i(q-k_1)\cdot y}][e^{iq\cdot y}\ \tilde{\partial}\ \epsilon^2_\nu e^{-ik_2\cdot y}\ \tilde{\partial} e^{-i(q-k_1)\cdot y}]_{q=+xk_1}}{(Q^2 - m_0^2)^2}.$$

But then the polarization vector ϵ_1 must be dotted into k_1, and the logarthmic divergence identically vanishes. Thus, diagram 7c also vanishes in the limit $M_\pi^2/E^2 \to 0$.

The remaining two diagrams can be regarded as a loop formed by the tree–level Compton scattering diagram, $\mathcal{M}(\gamma\gamma \to \pi^+\pi^-)$ to all orders in momenta,with the diagram for the leading order π^4–vertex. We shall show that all terms contained in either of the two diagrams that vanish on shell gives a vanishing contribution in the limit $M_\pi^2/E^2 \to 0$. In diagram 7a, any term in either vertex that vanishes on shell must be proportional to the square of one of the momenta in the loop, and therefore leads to a quadratic or higher divergence. These are always proportional to m_0^2 and vanish in the limit $M_\pi^2/E^2 \to 0$. In diagram 7b, the vanishing terms give rise to log or higher divergences. As before, we only need to consider the logartithmic divergence. The logarithmic divergence can be isolated by setting $Q = 0$ as before. The two photon polarization vectors cannot be dotted into each other, and hence the integral vanishes once again by the virtue of the fact that these polarization vectors are orthogonal to the photon momenta.

We now prove the second claim: If we replace $\pi\partial^{2n}\pi$ by $\pi D^2 \cdots D^2\pi$, the tree–level on shell amplitude for the Compton scattering is given by

$$< \pi\pi\gamma\gamma\ \pi A^2\pi > + < \pi\pi\gamma\gamma\ \pi(\partial\cdot A + A\cdot\partial)\pi\ \pi(\partial\cdot A + A\cdot\partial)\pi >$$

$$+ < \pi\pi\gamma\gamma\ \pi(\partial\cdot A + A\cdot\partial)\tilde{\partial}^2(\partial\cdot A + A\cdot\partial)\pi >$$

$$+ < \pi\pi\gamma\gamma\ \pi(\partial\cdot A + A\cdot\partial)\partial^{2l}\pi\ \pi\partial^{2m}\pi\ \pi\partial^{2n}(\partial\cdot A + A\cdot\partial)\pi >,$$

where $l+m+n>0$, and $m \neq 1$. Evaluating this, we have

$$\begin{aligned}\mathcal{M}_{\mu\nu} =& ie^2\left[2g_{\mu\nu} + \frac{(2p_a+k_1)_\mu(2p_b+k_2)_\nu}{(p_a+k_1)^2-m_\pi^2} + \frac{(2p_a+k_2)_\nu(2p_b+k_1)_\mu}{(p_a+k_2)^2-m_\pi^2}\right] \\ &+[(2p_a+k_1)_\mu(2p_b+k_2)_\nu + (2p_a+k_2)_\nu(2p_b+k_1)_\mu]F,\end{aligned}$$

where F is a power series in the scalar products of the external momenta and the polarization vectors. Imposing current conservation on this, we see that $F \equiv 0$.

Unfortunately, the preceding argument is not complete because of an ambiguity in ordering the covariant derivatives. For example, we have

$$0 = U^\dagger \partial_\mu \partial_\nu \partial_\mu \partial_\nu U - U^\dagger \partial_\nu \partial_\mu \partial_\mu \partial_\nu U \to U^\dagger [D_\mu D_\nu - D_\nu D_\mu] D_\mu D_\nu U = \frac{1}{2} U^\dagger F_{\mu\nu} F_{\mu\nu} U.$$

This ambiguity introduces a free parameter in $\mathcal{M}(\gamma\gamma \to \pi^+\pi^-)$ at $O(p^4)$ in addition to those already present in the ungauged theory. This is the ambiguity referred to in point 4 of section 1. Chiral symmetry alone does not resolve this ambiguity, and this represents a limitation of chiral symmetry relations. However, there is no need for nonminimal operators in the $J_z = 0$ channel calculation of $\gamma\gamma \to \pi^0\pi^0$.

4. Testing the Minimally Gauged Effective Chiral Lagrangian

We use the π–π phase shift analysis of Estabrooks and Martin[10] to predict $\sigma(\gamma\gamma \to \pi^0\pi^0)$ using the formula

$$\begin{aligned}\sigma_J(\gamma\gamma \to \pi^0\pi^0) =& \frac{1}{4}\frac{1}{16\pi s}[|\mathcal{A}_J^{++}|^2 + |\mathcal{A}_J^{--}|^2 + 2|\mathcal{A}_J^{+-}|^2] \\ =& \frac{\alpha^2}{8\pi^2}\sigma_J(\pi^+\pi^- \to \pi^0\pi^0)\end{aligned}$$

Comparison with the data from the Crystal Ball Experiment[11] is on figure 1. In this figure, we compare the $I=0$ $J_z=0$–wave contribution to the process

$\gamma\gamma \rightarrow \pi^0\pi^0$ to the total cross-section for $\gamma\gamma \rightarrow \pi^0\pi^0$. The large resonance in the data above $E_{CM} = 0.9$GeV is the $f_2(1270)$ resonance, which is mostly in the $J_z = 2$ channel[12]. Comparison at low energies is complicated by the fact that the there is no direct measurement of π–π scattering amplitudes at energies below ~ 0.6GeV. The theoretical curve shown on figure 1 at these energies is based on an extrapolation of the CERN–Munich data due to Aston[13]. Other extrapolations[14], together with a detailed analysis of the data from the Crystal Ball experiment and comparison with other theoretical predictions for $\gamma\gamma \rightarrow \pi^0\pi^0$ will be presented elsewhere[15]. Nevertheless, the apparent success of our formula in reproducing the Crystal Ball data below about 1GeV suggests that it would be worthwhile to apply the formula to the problem of predicting $\gamma\gamma \rightarrow W_L W_L$ and $\gamma\gamma \rightarrow Z_L Z_L$. The results of this investigation will be presented in a future publication[16].

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REFERENCES

1. S. Weinberg, Physica **96A**(1979) 327;
   J. Gasser, H. Leutwyler, Ann. Phys.,**158**(1984)142
2. Chanowitz *et al.*, Phys. Rev. **D36**(1987)1490;
   S. Dawson, G. Valencia, Nucl. Phys. **B352** (1991)27
3. J. Bagger and C. Schmidt, Phys. Rev. **D41**(1990)264
4. J. F. Donoghue, C. Ramirez, G. Valencia, Phys. Rev. **D38**(1988) 2195
5. J. F. Donoghue, B. R. Holstein, Y. C. Lin, Phys. Rev. **D37**(1988) 2423;
   J. Bijnens and F. Cornet, Nucl. Phys. **B296**(1988)557
6. Pyungwon Ko, Phys. Rev. **D41** (1990) 1531;
   D. Morgen and M.R. Pennington, Univ. of Durham preprint, DTP–91/44
7. D. Morgan and M. R. Pennington, Phys. Lett. **B192**(1987) 207
8. D. Barua and S. N. Gupta, Phys. Rev. **D16**(1977)413
9. A. Pais and G. E. Uhlenbeck, Phys. Rev. **79**(1950)145
10. P. Estabrooks and A. Martin, Nucl. Phys. **B95**(1975)322
11. Marsiske *et al.*, Phys. Rev. **D41** (1990) 3324
12. D. Morgan and M.R. Pennington, Z. Phys. **C37**(1988)431
13. D. Aston, private communication
14. N. Cason *et al.*, Phys. Rev. **D28**(1983)1586
    M. David *et al.*, Phys. Rev. **16** (1977)2027
15. Carl J.–C. Im, University of Michigan preprint, in preparation
16. C. J.–C. Im and M. E. Peskin, SLAC preprint, in preparation

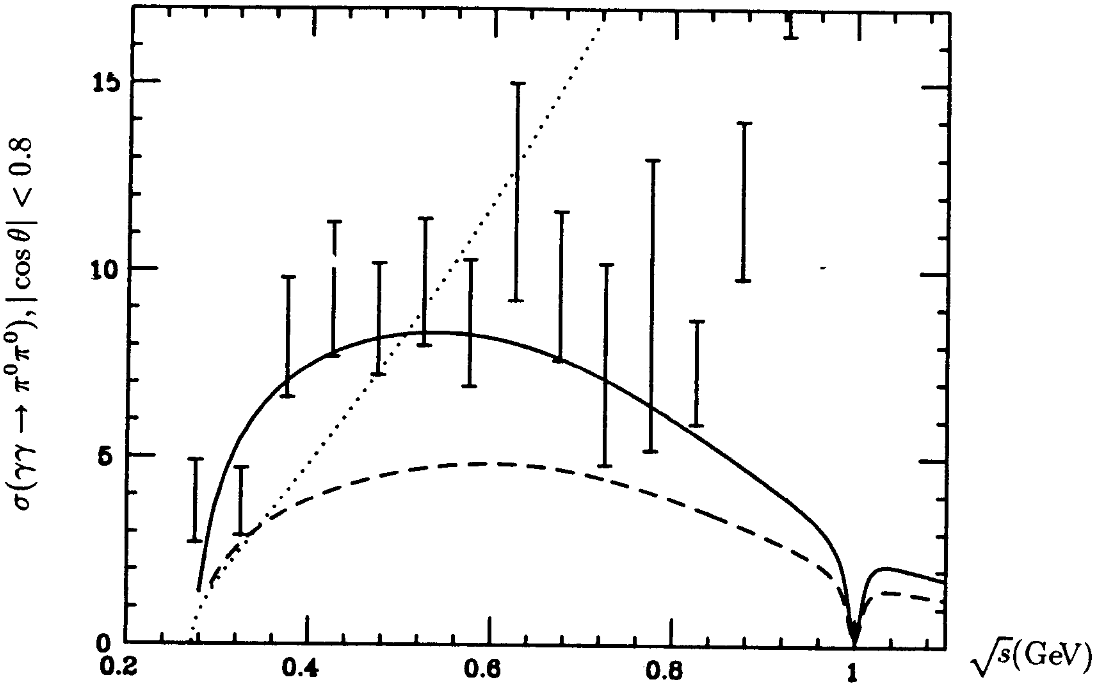

Figure 1. Cross-section for $\gamma\gamma \rightarrow \pi^0\pi^0$, as measured by the Crystal Ball experiment[11]: Our prediction for $\sigma_{(J_z=0)}$(solid line), the leading order chiral perturbation theory prediction for $\sigma_{(J_z=0)}$(dotted line), and the leading order $1/N_f$ prediction for $\sigma_{(J_z=0)}$ without finite mass effects(dashed line) are also shown for comparison. The broad peak in the data beginning at $\sim$ 1GeV is the $f_2(1270)$ in the $J_z = 2$ channel. The small absorption peak at 1GeV comes from the $K\bar{K}$ threshold in the π–π scattering data that was used to produce the plot. This signals the break down of the calculation. Inclusion of kaons should remedy this problem.

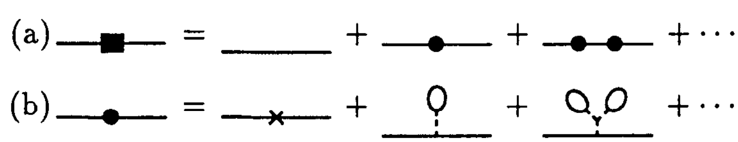

Figure 2. (a) To leading order in $1/N_f$, the pion two point function is a chain of the 1PI functions. (b) The 1PI function is the sum of all the contributions from the higher derivative kientic terms and the tadpole contributions.

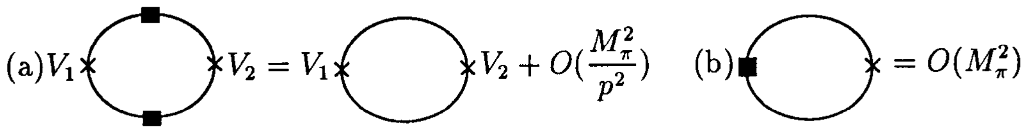

Figure 3. Some of the simplifications that occur in the limit $M_\pi^2/E^2 \to 0$. (a) In a one–loop diagram formed by two arbitrary vertices V_1 and V_2, the full propagator can be replaced by the bare propagators to leading order in M_π^2/p^2. (b) All tadpole diagrams vanish in the limit $M_\pi^2/E^2 \to 0$.

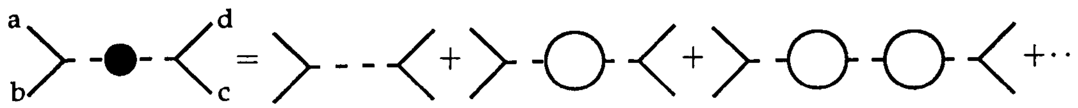

Figure 4. The leading order pion four–point function in the S–channel(Pions on the same sold lines have the same isospin index; Solid lines joined by dashes lines come from the same local vertex)

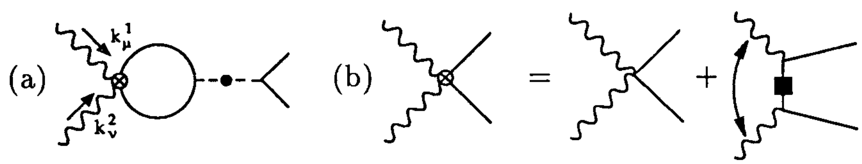

Figure 5. (a) The diagram for $\gamma\gamma \to \pi^0\pi^0$ in the limit $M_\pi^2/E^2 \to 0$. (b) The Compton scattering amplitude $\gamma\gamma \to \pi^+\pi^-$ in the diagram in 5a.

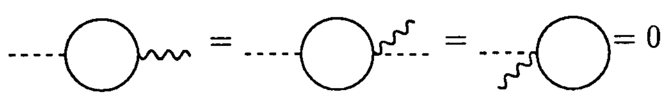

Figure 6. These diagrams identically vanish by symmetry considerations.

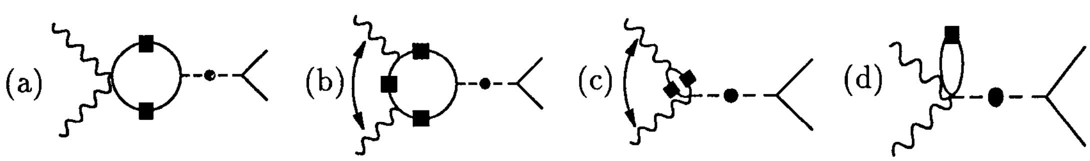

Figure 7. Diagrams that contribute to $\gamma\gamma \to \pi^0\pi^0$ to leading order in $1/N_f$.