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# MAGNETIC MEASUREMENTS OF QUADRUPOLE FOCUSING MAGNETS AT SLAC*

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## ABSTRACT

A rotating coil technique was used to find the magnetic centers of quadrupole magnets used in the final focus of high energy electron and positron beams in the SLAC Linear Collider at the Stanford Linear Accelerator Center. These methods provide interesting applications of classical electromagnetic theory for use in the undergraduate class.

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## I. THE STANFORD LINEAR COLLIDER

The newest experiments at Stanford Linear Accelerator Center (SLAC) use the SLAC Linear Collider, the SLC. Electrons and positrons, accelerated in the two-mile-long 50 GeV linear accelerator, crash head on in a prolific burst of subnuclear particles. To help capture and study the results of these collisions, physicists at SLAC and 32 other institutions have constructed a state of the art particle detector, the SLC Large Detector, or SLD. One of the most sophisticated machines in the world using the newest technology in detectors, SLD began receiving the first beams in mid-1991.

In order to achieve the highest possible luminosity, the electron beams must be focused down to a very tiny spot at the interaction point. But the large magnetic fields (0.6 T) employed in this detector, along with the need to put instrumentation as close to the interaction point as possible, precludes the use of iron in the final focusing magnets.[1] Thus the decision was made to use superconductors in the final focusing magnets, the so-called Superconducting Final Focus. Superconductors avoid the use of iron, which would distort the detector's own magnetic field. Additionally, superconductors provide the ability to use extremely high currents to produce larger magnetic fields than can be obtained conventionally. Larger magnetic fields produce stronger focusing, so that the beam can be reduced to a much smaller spot, enhancing the probability of collisions. The

Superconducting Final Focus reduces the beams to a spot size as small as 1 to 2 μm in diameter. Such small dimensions require that the focusing magnets be aligned within extremely tight tolerances and that the magnetic features be known with high precision.

## II. QUADRUPOLE FOCUSING MAGNETS

While ordinary dipole magnets can be used to bend the beam's direction, focusing is done by quadrupole magnets.

The superconducting quadrupoles used in the Final Focus were designed and built at Fermilab,[2] and a cross section of one is shown in Fig. 1. The central aperture through which the beam pipe extends (out of the page in the figure) is 5.0 cm in diameter. Superconducting wires provide sheets of current, coming out of the diagram on the right and left of the figure, and going in at the top and bottom, generating the alternate poles of the quadrupole. The resulting magnetic field is illustrated in Figs. 1 and 2.

As can be seen in Fig. 2, the field in an ideal quadrupole is not constant, but rather is proportional to the distance from the axis as given by

$$B_x = \Gamma y \quad \text{and} \quad B_y = \Gamma x \tag{1}$$

where the gradient, $\Gamma$, is the same in both dimensions.

To see how a quadrupole focuses, one could suppose a positron to be coming out of the page (in the positive z direction) anywhere along the x-axis in Fig. 2. Such an x-axis positron will feel a force toward the center, and the farther it is from the center, the more will be its push toward the y-axis. A parallel beam of positrons entering this quadrupole in the xz-plane will be focused to a point on the z-axis. While it is not the purpose of this paper to derive an equation for the focal length of such a quadrupole magnet, the path to a solution can readily be seen. Since the convergent field produces a restoring force that is proportional to displacement, the resulting motion must be simple harmonic. Thus, the time to converge at the center line is independent of the initial displacement. This is true in the case where the electrons are highly relativistic, since there is essentially no change in the velocity down the axis.

A similar examination of positrons in the yz-plane, however, reveals that these particles will experience a force away from the center and, in fact, they will be defocused. It is the nature of quadrupole magnets to be convergent in one dimension and divergent in the other. Focusing in both dimensions is accomplished with three quadrupoles separated by distances somewhat smaller than their focal lengths. A suitably chosen triplet will focus to the same point in both dimensions. The final focus for each of the colliding electron and positron beams in the SLD consists of a triplet of superconducting quadrupole magnets, as illustrated in Fig. 3.

## III. ROTATING COIL

As integrated into the entire Final Focus system,[3] the three quadrupole magnets in the Final Focus triplet must be very accurately positioned to produce a focus at the micron level. This means that the magnetic centers of each member must be known with great precision. The method we used to determine the magnetic centers of the quadrupoles, as well as to map the field in x, y, and z, was to insert a rotating coil into the field and to use the induced EMF in the coil to determine the magnetic gradient of the quadrupole.

To allow us to plot the change in the field down the magnet, we used a relatively short coil, wound as shown in Fig. 4. Our coil had $n = 60$ turns of wire wound so that one edge of the rectangular form, with a length of $L = 0.035$ m, remained on the axis of rotation. The width of the coil, and the effective radius of rotation for the outer edge, was $r = 0.0115$ m. The coil was rotated at 405 rpm, an angular frequency of $\omega = 42.5$ rad/s. The high resolution mapping of the quadrupoles was made with the magnets warm. Thus they were normally resistive rather than superconducting. A current of 18 A was used, generating a field about 1/250th of that produced when the magnets are cryogenic.

In the usual textbook case,[4] a coil like this is rotated in a dipole field with a constant field strength, $B_0$, and an EMF is induced through Faraday's Law. Since the area perpendicular to the field is $r L \cos \omega t$, the rate of change of the flux, and thus the EMF induced is

$$\mathcal{E}_{dipole} = n B_0 r L \omega \sin \omega t \quad . \tag{2}$$

However, in a quadrupole magnet the field is not constant, but rather varies with B proportional to the distance from the center, as given in Eq. (1). In fact, B = 0 at the center of the quad.

If we look at the small section of a single winding whose area is **L** $\times$ **dr**, then the y and x components of flux passing through this area would be given by

$$d\Phi_y = \Gamma x L dx \quad \text{and} \quad d\Phi_x = \Gamma y L dy \quad . \tag{3}$$

This can be expressed in terms of r, the radial distance from the axis of the quadrupole magnet. Since $x = r \cos \omega t$, and $dx = dr \cos \omega t$, while $y = r \sin \omega t$ and $dy = dr \sin \omega t$, then,

$$d\Phi_y = \Gamma L r \, dr \cos^2 \omega t \quad \text{and} \quad d\Phi_x = \Gamma L r \, dr \sin^2 \omega t \quad , \tag{4}$$

and the total flux through the coil is found by integrating from 0 to r,

$$\Phi_y = \tfrac{1}{2} \Gamma L r^2 \cos^2 \omega t \quad \text{and} \quad \Phi_x = \tfrac{1}{2} \Gamma L r^2 \sin^2 \omega t \quad . \tag{5}$$

So the EMF induced in the coil by Faraday's Law is given by the time derivatives,

$$\mathcal{E}_y = d\Phi_y/dt = -\tfrac{1}{2} \, n \Gamma L r^2 \, (2 \sin \omega t \cos \omega t) \tag{6}$$

and

$$\mathcal{E}_x = d\Phi_x/dt = \frac{1}{2}\, n\, \Gamma\, L\, r^2\, (2 \sin \omega t \cos \omega t) \quad . \tag{7}$$

But a look at Fig. 5 will show that the EMF induced by the changing field is in the same direction for both the x and the y components of the flux. Even though $\Phi_y$ is decreasing (as shown by the negative sign in the derivative) while $\Phi_x$ is increasing, the two components traverse the coil in opposite directions so that their voltages add. And, by using the trig identity that $2 \sin \theta \cos \theta = \sin 2\theta$,

$$\mathcal{E}_{quad} = n\, \Gamma\, L\, r^2\, \omega \sin 2\omega t \quad . \tag{8}$$

End effects prevent $Lr^2$ from being calculated precisely from geometry alone; and so, its value is determined through calibration in a known field. With that, and the angular frequency given by a digital encoder attached to the motor, the gradient can be determined from a peak voltage reading. In the coil that we used (with the dimensions given earlier) a gradient of 0.50 T/m generated an EMF of 5.90 mV.

Of course, the coil may not be in the center of the magnet, but being off center simply means that the magnetic field will not be zero. The EMF induced in a rotating coil off the quadrupole axis is just a linear combination of Eqs. (2) and (8). These two effects can be separated completely, in fact, by performing a Fourier analysis on the incoming waveform. The dipole component is the amplitude of the fundamental frequency, and the quadrupole component is the amplitude of the second harmonic. In fact, the dipole component gives information about where the coil is relative to the magnetic center, and the quadrupole component determines the gradient. Additionally, nonuniformity of the gradient shows up in higher harmonics in the Fourier spectrum, primarily in the third harmonic, which is associated with a sextupole component.

## IV. MAGNETIC MEASUREMENTS OF A SUPERCONDUCTING QUADRUPOLE

Figure 6(a) illustrates the sort of waveform that might be seen by an oscilloscope connected to a rotating coil that is offset from the magnetic axis of a quadrupole. (Although this graph is exaggerated—the coil would normally be positioned very nearly on axis.) This voltage signal was fed to a computer program that performed a Fourier analysis, generating a spectrum such as that shown in Fig. 6(b). The amplitude at the fundamental frequency, 6.75 Hz, is the average magnetic field strength, and it is determined solely by the distance the coil is off the axis. The amplitude of the second harmonic, at 13.5 Hz, indicates the magnitude of the gradient, $\Gamma$.

Of course, the uniformity of the gradient is an important consideration, as well as the total gradient integrated over the length of the magnet (providing the total focusing effect). A plot of the gradient down the axis of the center member of one of the triplets is shown in Fig. 7(a), and, except for the rapid drop-off at the ends, the gradient is extremely uniform. The gradient at 18 A is about 0.5 T/m. While operating cold in the detector, a typical operating current would be 4250 A, producing a gradient of 118 T/m.

The amplitude of the dipole component is caused by the coil being offset from the axis of the quadrupole field. But the quadrupole axis (where the field is actually zero) and the geometrical axis of the magnet may not coincide exactly. During the measurement, the location of the coil was monitored with an alignment telescope so that the magnetic axis could be determined relative to the geometry of the cylinder. The result is plotted in Fig. 7(b), where 0 represents the center. The x and y components of this displacement were resolved by using the relative phase of the dipole contribution. Within the region of nominal field strength, the variation is well within tolerances set by the overall optics. Based on these measurements, information about the average center of the quadrupole is used to align the triplets in the beam.

Each magnet was mapped as shown in the figures with the magnet warm and with only 18 A of current. Similar measurements, but with less resolution, were performed after the three quadrupoles were combined into a triplet and enclosed in a liquid helium cryostat. Magnetic measurements were made at a temperature of 4.3 K with currents up to 5000 A. The results were completely consistent with the warm mapping.

## V. CONCLUSIONS

The Superconducting Final Focus began accepting electron and positron beams in Spring 1991, and it has already produced focused spots smaller than any previously obtained. Precise calibration and alignment were necessary to produce this success.

While it is usual for textbooks to discuss the EMF induced in a coil rotating in a constant magnetic field, it is also instructive to show the effects from a field with a constant gradient. Most students have heard that magnets are used to focus particle beams, and they show great interest in seeing how that can be. My students have been impressed to see how the classical electromagnetism that they learn in a first-year physics class is used in high energy physics applications.

## VI. ACKNOWLEDGEMENTS

The magnetic measurement system was built by Jim Ferrie, whose analytical skepticism and concern for detail were indispensable to the success of the Final focus. Kim Cook provided the innovative design and precision construction of the coil assembly. The Fourier analysis routine was written by Rich Belcinski. Special thanks must go to Bill Ash, who leads the Superconducting Final Focus group, and who has provided much inspiration. This group has been extremely helpful during my stay at SLAC.

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4. D. Halliday and R. Resnick, *Fundamentals of Physics*, 3rd ed. (John Wiley & Sons, New York, NY, 1988), p. 760.

## FIGURE CAPTIONS

Figure 1: Cross section of SLC Superconducting Final Focus quadrupole magnet, with sample field lines.

Figure 2: Magnetic field along x- and y-axes.

Figure 3: Positrons converging, then diverging, then converging to a focus after passing through a triplet of quadrupole magnets.

Figure 4: Probe coil for determining the magnetic center of a quadrupole.

Figure 5: Wire coil rotating in a magnetic field.

Figure 6: (a) Waveform similar to signal from coil rotating off-center in a quadrupole; (b) Fourier spectrum of signal in (a).

Figure 7: (a) Gradient down axis of quadrupole; (b) smoothness of quadrupole axis.

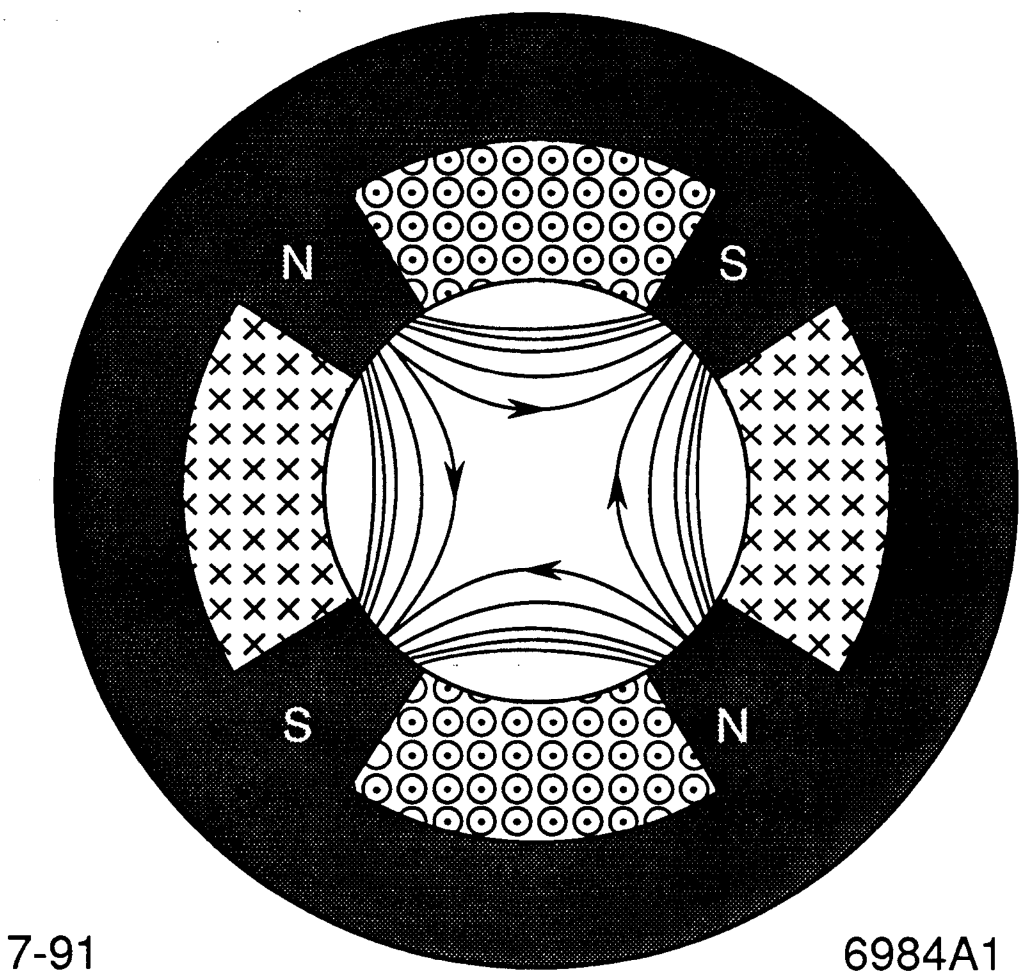


Fig. 1

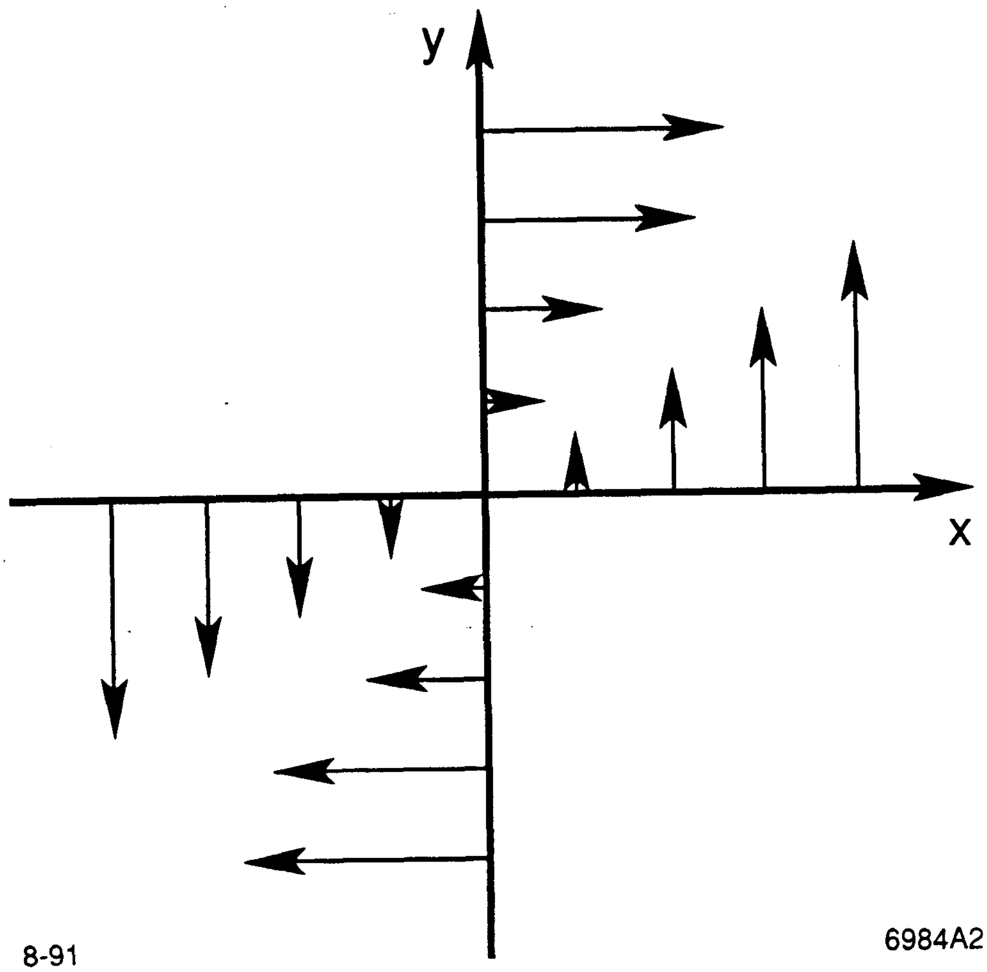


Fig. 2

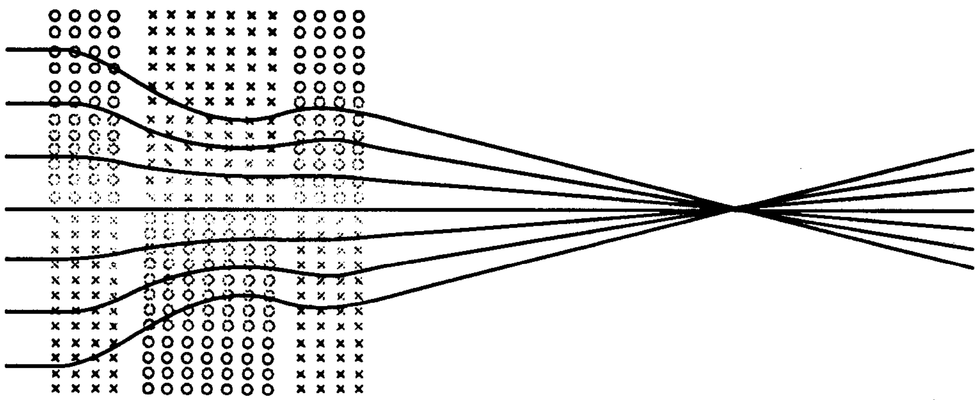


Fig. 3

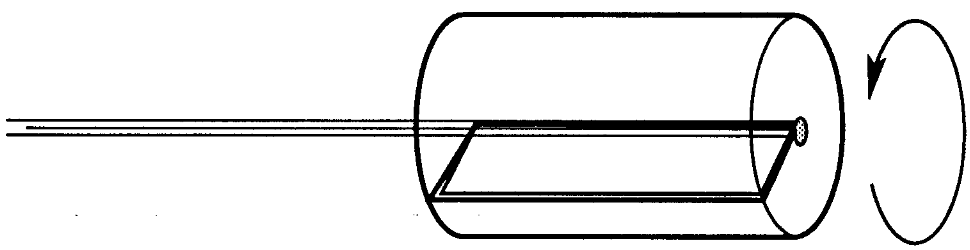

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Fig. 4

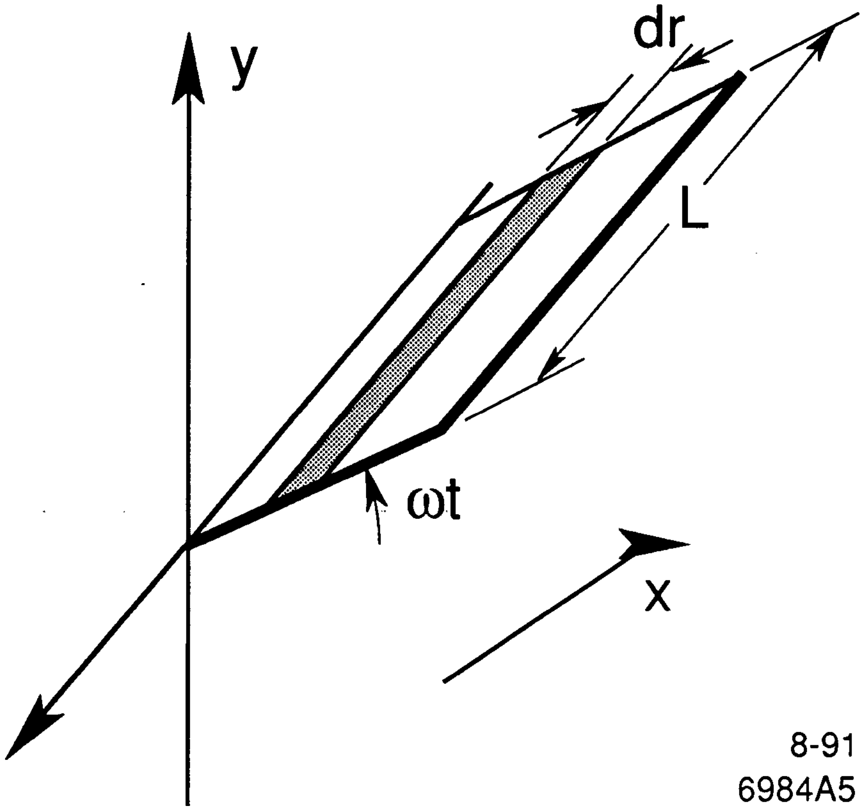
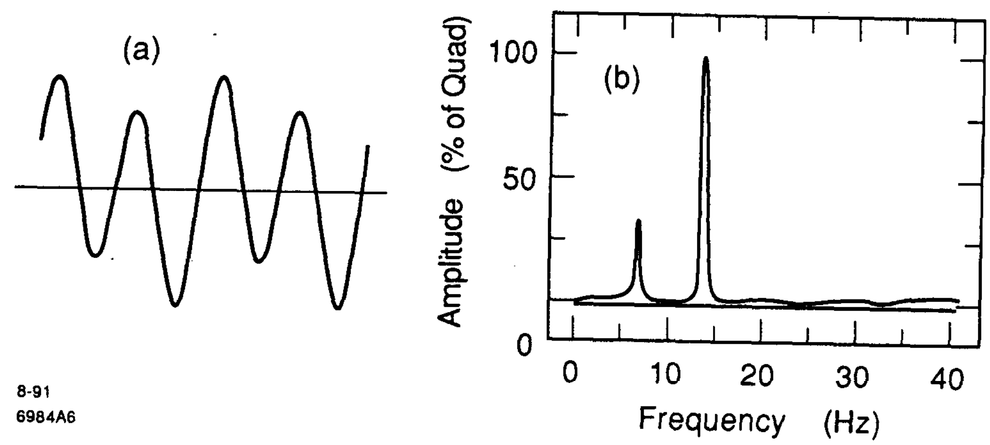



Fig. 5


(a)
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(b)
Amplitude (% of Quad)
100
50
0
0
10
20
30
40
Frequency (Hz)

Fig. 6

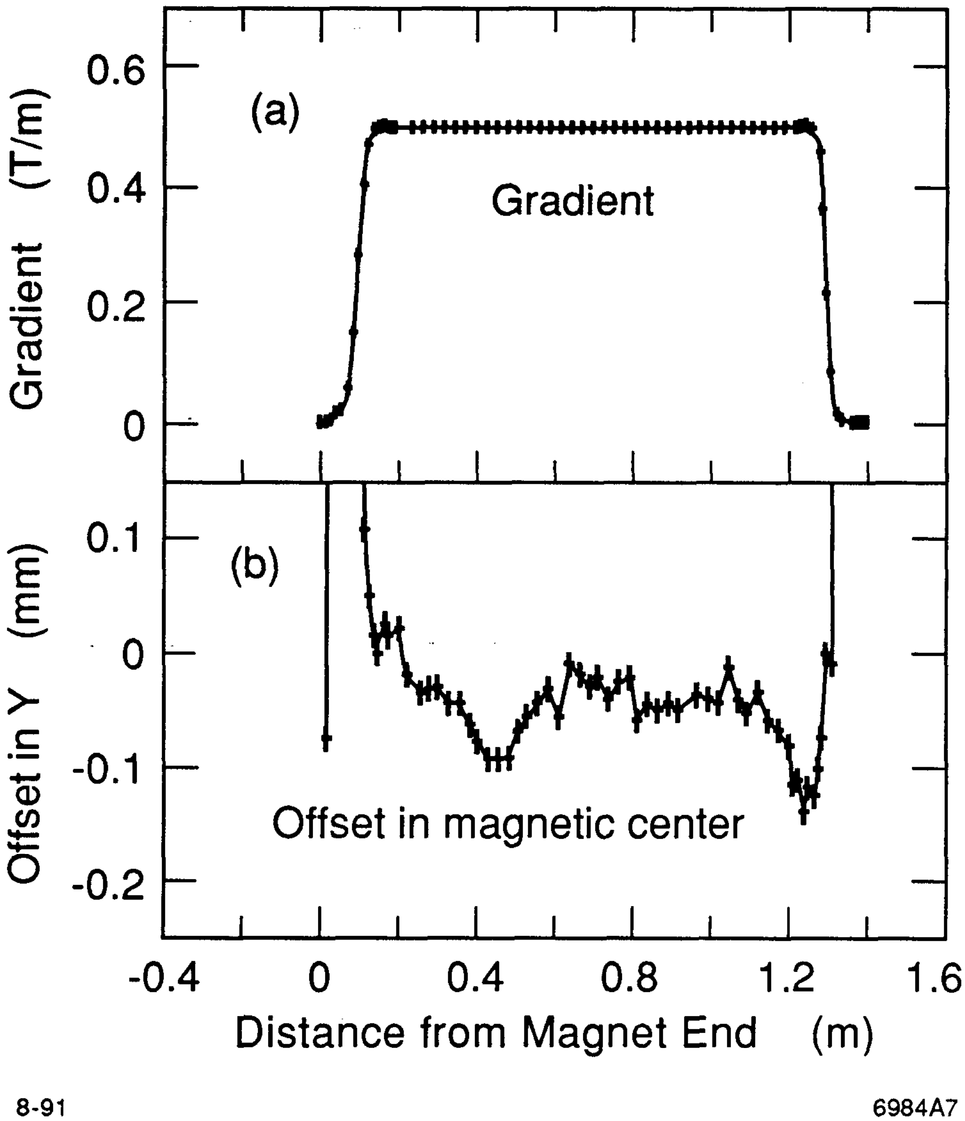
(a)
Gradient
Gradient (T/m)
0.6
0.4
0.2
0
(b)
Offset in Y (mm)
0.1
0
-0.1
-0.2
Offset in magnetic center
-0.4
0
0.4
0.8
1.2
1.6
Distance from Magnet End (m)
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Fig. 7