# Rapidity Gaps and Jets as a New Physics Signature in Very High Energy Hadron-Hadron Collisions* 

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## 1. Introduction

At SSC/LHC energies there emerges a new class of processes which are of importance in the attempt to push beyond the standard-model phenomenology. These reactions are characterized by the presence of virtual electroweak bosons in the hard subprocesses. The most familiar and perhaps important-of these [1] is the two-body scattcring of $W$ 's and $Z$ 's, with the $W$ 's and $Z$ 's treated as partons of the incoming proton beams (Fig. 1a). Closely related is the production of a Higgs boson (or other new electrowcak/Higgs-sector particle) via $W$ - $W$ fusion (Fig. 1b).


Figure 1. Basic mechanism for producing $W$ - $W$ interaction processes in high-energy $p p$ collisions, with the presence of a rapidity-gap in the final state.

At the naive, "factorized," level depicted in Fig. 1, the event-structure is atyp-

- ical. For example, in the $W$ - $W$ scattering example, let the $W$ 's decay leptonically. Then there will be a large "rapidity-gap," i.e. a region of (pseudo-) rapidity in which no hadrons are found, separating the beam-jets containing the fragments of the left-moving and right-moving projectiles.

This is the event morphology characteristic of double-diffraction, which has a large cross-section. The presence of isolated leptons, however, largely suppresses this. And if large transverse momentum is exchanged between left and right movers in the process, this double-diffraction background will itself be highly suppressed. As will be discussed further in Section 2, the signal event, as shown in Fig. 2, has the characteristic feature of "tagging-jets" at the edge of the rapidity-gap [2]. These are simply the hadronization products of the initial-state quarks that emitted the W's.


Figure 2. Even morphology in lego variables for the processes depicted in Fig. 1. The tagging jets are the hadronization products of the quarks, while for large Higgs masses, almost all of the $W$-decay products lie within the dashed circles. The remaining region, marked gap, contains on average no more than 2 or 3 hadrons.

The combination of rapidity-gaps, tagging-jets, and leptons within the gap would seem to be a strong signature for this process. Indeed even if one allows hadronic decays of the $W$ 's, the signatures still look quite good. Therefore we

- believe that the possibility of using this "underlying-event" structure should be studied seriously by theorists, phenomenologists, and experimentalists. The basic idea of utilizing the rapidity-gap signature is due to Dokshitzer, Troyan and Khoze [3]. But up to now not much has been done in developing it [4]. There are many difficult issues involved. They include the following:

1. How big must the rapidity-gaps be in order that multiplicity fluctuations do not mimic their effect?
2. How big are strong-interaction (Pomeron-exchange) backgrounds and how do they scale with energy and $p_{t}$ ?
3. What fraction of a given electroweak-boson exchange process, as defined at the parton level, really leads to a final state containing the rapidity-gap. Most of the time spectator interactions will fill in the gap present at the naive level considered above. We estimate in Section 3 that the survival probability of the rapidity gap is of order $5 \%$, but there are serious theoretical issues here which need further exploration.

To make a complete feasibility study of this strategy requires a considerable amount of serious Monte-Carlo simulation work. It is not the purpose of this paper to provide any of that. While such work is necessary, it is not sufficient. There are several fundamental theoretical issues, most having to do with the physics of rapidity-gap creation in strong processes ("Pomeron physics"), which need to be addressed before one can really assess whether the inputs to a Monte-Carlo simulation are realistic. It is the purpose of this paper to look at some of these underlying issues, and discuss how they might be addressed, both from the point of view of fundamental theory as well as from experiment.

In Section 2 we survey semi-quantitatively some typical electroweak-boson ex-
change processes in order to get some feel for the practicality of the strategy, and how they are calculated. In Section 3 we look at the physics underlying the "survival of the rapidity gap," i.e. what fraction of events retain the factorized structure containing the rapidity-gap. Section 4 considers potential backgrounds from "hard diffraction" processes, i.e. high- $p_{t}$ double or multiple diffraction. A conclusion from that section is that it is arguable that these corrections will be large. If so, these strong-interaction processes may be able to be utilized for newphysics as well. Section 5 is devoted to concluding comments, and enumeration of suggestions for further experimental and theoretical work.

## 2. Hard Collisions with Electroweak-Boson Exchange

Processes involving electroweak-boson exchanges have by now been considered at great length in connection with the high-energy hadron collider programs such as SSC and LHC. It is not our purpose to repeat any of that work here [5], but only to describe the revisions needed if one is to utilize the rapidity-gap signature. The processes we consider here are as follows:
a) Exchange of a single $\gamma, W$, or $Z$.
b) Two-boson nonresonant processes, in particular $\gamma \gamma \rightarrow \mu^{+} \mu^{-}$, and $\gamma \gamma \rightarrow X$ or $W^{+} W^{-} \rightarrow X$.
c) Resonant production of the Higgs boson and elastic scattering of $W$ 's and/or Z's.
a) Single-boson exchange:

We begin with a description of the photon-exchange process described in Fig. 3 , with final-state interactions of spectator partons temporarily disregarded. If $q^{2}$


Figure 3. Event morphology for virtual photon exchange between two protons at large $q^{2}$, with survival of the rapidity gap assumed.
is large, then the event-topology in the lego plot is as shown in Fig. 3. The jets are created by the hadronization products of the scattered quarks. A "rapidity-gap" lies between these jets (provided it is not filled in by absorption effects). It is not hard to estimate the amount of leakage into the gap [6]. For this purpose we suggest the following candidate definition of the boundary of the gap:

1. Define the tagging jets as the contents of the lego plot within a circle of radius 0.7 enveloping the jet core.
2. Define the boundary of the rapidity gap as the tangents to these circles as shown in Figs. 2-3.

Because the particle distributions of the beam jets are essentially known from deep-inelastic lepton-hadron phenomenology, it is straightforward to estimate the leakage into the gap. Only the frame of reference is non-standard (from a fixedtarget, not HERA, perspective). A simple kinematic exercise [6] leads to the estimate for the leakage per edge:

$$
\begin{equation*}
N_{\mathrm{gap}}=\frac{1}{2}\left(\frac{d N}{d \eta}\right) \epsilon^{-2 R} \lesssim 0.5 \tag{2.1}
\end{equation*}
$$

with $R=0.7$ the radius of the circle enveloping the jet. So for the signal we
typically expect no more than one hadron in the gap. Since at SSC/LHC energies

$$
\begin{equation*}
\left\langle\frac{d N}{d \eta}\right\rangle \sim 8-10 \tag{2.2}
\end{equation*}
$$

(for charged + neutral hadrons), a gap width of 3 units appears quite sufficient to reduce Poisson-like multiplicity fluctuations to a negligible level.

However, multiplicity distributions at these energies are non-Poissonian. There are large long-range rapidity correlations in the local multiplicity density $d N / d \eta$, so that it is reasonable to simply argue that only the low-multiplicity tail of the total "negative-binomial" distribution is relevant. However, for minimum bias events this component is less likely to have high- $p_{t}$ jets in the final state. Likewise ordinary hadron-hadron events containing high $p_{t}$ jets are less likely to have a lowmultiplicity component in their associated-multiplicity distribution. I therefore find little help from direct experimental information in estimating what an appropriate minimum $\Delta \eta$ should be, although 3 does seem a reasonable value.

It might be interesting to scrutinize extant samples of data on $e^{+} e^{-} \rightarrow$ hadrons to see how often rapidity-gaps this wide do appear, since there are no diffractive mechanisms available in such processes to create gaps: only fluctuations are available. Indeed, this gives rise to a rough estimate for the gap probability. Suppose there is a certain fraction of $e^{+} e^{-} \rightarrow$ hadron events containing a rapidity gap of width $\Delta \eta$ or larger. Then assume this fraction does not depend strongly on the $e^{+} e^{-} \mathrm{cms}$ energy $\sqrt{s}$. If this is true, then we may reduce $s$ until the process is quasi-elastic, i.e. to

$$
\begin{equation*}
s \sim s_{0} \sim\left(1 G e V^{2}\right) e^{\Delta \eta} . \tag{2.3}
\end{equation*}
$$

But this quasi-elastic fraction is of order $\left|F_{\pi}\left(s_{0}\right)\right|^{2}$, where $F_{\pi}$ is the elastic form
actor for $e^{+} e^{-} \rightarrow \pi^{+} \pi^{-}$, namely

$$
\begin{equation*}
\left|F_{\pi}\left(s_{0}\right)\right|^{2} \sim\left(\frac{m_{\rho}^{2}}{s_{0}}\right)^{2} \sim \frac{1}{4} e^{-2 \Delta \eta} \tag{2.4}
\end{equation*}
$$

This leads to the estimate, at any energy $s \gg s_{0}$

Fraction of events with gaps with width $\Delta \eta \approx e^{-(2 \Delta \eta+1.4)}$.

For $\Delta \eta \sim 3$, the fraction would be no more than $10^{-3}$. A more careful examination lowers this number by orders of magnitude [7]. In any case it is important to study the issues experimentally.

Returning to the process of interest, we now estimate the cross-section for the photon-exchange process, differential in $q^{2}=-Q^{2}$ and the positions of the tagging-jets in the lego plot. This has the simple form

$$
\begin{equation*}
\left.\frac{d \sigma}{d \eta_{1} d \eta_{2} d Q^{2}}=\left.\frac{4 \pi \alpha^{2}}{Q^{4}} F_{2}\left(x_{1}, Q^{2}\right) F_{2}\left(x_{2}, Q^{2}\right)\langle | S\right|^{2}\right\rangle \tag{2.6}
\end{equation*}
$$

with $F_{2}$ the familiar structure function and $\left.\left.\langle | S\right|^{2}\right\rangle$ the absorption correction, discussed in Section 3. We have defined the tagging-jet rapidities ${ }^{1}$ to be

$$
\begin{equation*}
\eta_{1}=-\ell n \tan \frac{\theta_{1}}{2}>0 \quad \eta_{2}=\ell n \tan \frac{\theta_{2}}{2}<0 \tag{2.7}
\end{equation*}
$$

The parton fractions $x_{1}$ and $x_{2}$ are related to these rapidities as follows:

$$
\begin{align*}
\eta_{1} & \cong \ln \frac{2}{\theta_{1}}=\ln \frac{2 E_{1}}{Q}=\ln \frac{2 E_{1} x_{1}}{Q}=\ln \frac{\sqrt{s}}{Q}+\ln x_{1} \\
-\eta_{2} & =\ln \frac{\sqrt{s}}{Q}+\ln x_{2} . \tag{2.8}
\end{align*}
$$

[^0]We see that

$$
\begin{equation*}
\eta_{1}-\eta_{2}=\ell n x_{1} x_{2} s-\ell n Q^{2}=\ell n \frac{\widehat{s}}{Q^{2}} \tag{2.9}
\end{equation*}
$$

where $\sqrt{\hat{\beta}}$ is the cms energy of the $q q$ system undergoing the hard subprocess. A necessary condition that there be a rapidity gap between the two tagging jets is

$$
\begin{equation*}
\eta_{1}-\eta_{2}=\Delta \eta+2(0.7) \tag{2.10}
\end{equation*}
$$

with the rapidity-gap $\Delta \eta \gtrsim 3$. This implies a subenergy for the $q \bar{q}$ process bounded below by

$$
\begin{equation*}
\widehat{s} \gtrsim Q^{2} e^{(\Delta \eta+1.4)} \gtrsim 80 Q^{2} \tag{2.11}
\end{equation*}
$$

This is not much of a constraint for these processes but will be a considerably larger one for two-boson exchange processes to be discussed later.

By themselves, we do not know how interesting this class of processes would be to study. As discussed in Section 4, there probably is a large hard-diffraction background, difficult to eliminate. And the structure-function physics in general requires precision. Sufficiently accurate determination of $Q^{2}$ and of the size of absorption corrections to attain that precision might be problematic. However the values of $Q^{2}$ and $W^{2}$ available at SSC/LHC energies exceeds by far what will have been studied at HERA. This follows form the observation that, according to Eq. (2.6), the elementary parton-level cross-sections at $Q^{2} \sim 10^{5} \mathrm{GeV}^{2}$ are of order $10^{-35} \mathrm{~cm}^{2}$. With a $5 \%$ survival probability for the rapidity gap, as discussed in Section 3, this leads to a respectable event sample for any integrated luminosity in excess of $10^{38} \mathrm{~cm}^{-2}$. Note that at $Q^{2} \sim 10^{5} \mathrm{GeV}^{2}, W$ and $Z$ exchange predominate over photon exchange.
b) Two-boson processes:

There are many processes of considerable interest, but we shall begin with a quite mundane one, namely $\gamma \gamma \rightarrow \mu^{+} \mu^{-}$. Our reasons for this are that it is a simple prototype reaction and most importantly, it appears to be an excellent reaction for experimentally determining the absorption corrections, i.e. the "survival probability of the rapidity-gap." The event-structure is shown in Fig. 4 and is similar to the previous case. In calculating the cross-section for this kind of process, it is convenient to consider the hard subprocess to be

$$
\begin{equation*}
q+\bar{q} \rightarrow q+\bar{q}+\mu^{+}+\mu^{-} \tag{2.12}
\end{equation*}
$$

and compare the yield with given gap parameters to the total yicld.


Figure 4. Event structure for dilepton-plus-rapidity-gap production in $p p$ collisions.

To determine the yield when the kinematics is constrained to allow a rapiditygap, with each muon isolated within the gap, it is convenient to view the process at first in the collinear frame for which the photoproduced dimuon system has zero longitudinal momentum; this is essentially the cms system of the dimuon. In

- this frame we make a cut on the dimuon angular distribution about $90^{\circ}$; more specifically we only allow a limited rapidity separation between $\mu^{+}$and $\mu^{-}$:

$$
\begin{equation*}
\left|\eta_{+}-\eta_{-}\right|<\Delta \eta_{\mu \mu} \tag{2.13}
\end{equation*}
$$

A value $\Delta \eta_{\mu \mu}$ of 2 already covers most of " $4 \pi$ ": $|\cos \theta|<0.75$.
We now require that the edges of the rapidity-gaps, created by the taggingquark jets, are at least distances $\Delta \eta_{1}, \Delta \eta_{2}$ from the dimuons (cf. Fig. 4). This requires

$$
\begin{equation*}
\eta_{1}-\eta_{2} \geq \Delta \eta_{1}+\Delta \eta_{2}+\Delta \eta_{\mu \mu}+2(0.7) \equiv \Delta \eta_{j j} \tag{2.14}
\end{equation*}
$$

As in the previous case, the distance in the lego-plot between the tagging-jets, together with their transverse momenta $q_{t}$, determines the cms energy of the $q \bar{q}$ system:

$$
\begin{equation*}
\eta_{1}-\eta_{2}=\ln \frac{2 E_{1}}{q_{t_{1}}}+\ln \frac{2 E_{2}}{q_{t_{2}}}=\ln \widehat{s}-\ln q_{t_{1}}-\ln q_{t_{2}} \tag{2.15}
\end{equation*}
$$

To get a feel for numbers, take

$$
\begin{equation*}
q_{t_{1}} \sim q_{t_{2}} \sim 20 G \epsilon V \tag{2.16}
\end{equation*}
$$

And for the dimuon constraints, take

$$
\begin{align*}
\Delta \eta_{1} & =\Delta \eta_{2}=1 \\
\Delta \eta_{\mu \mu} & =2 \tag{2.17}
\end{align*}
$$

This gives a minimal tagging-jet separation of

$$
\begin{equation*}
\Delta \tau_{j j} \approx 5.4 \tag{2.18}
\end{equation*}
$$

and a minimum $q \bar{q} \mathrm{cms}$ energy of

$$
\begin{equation*}
\sqrt{\widehat{s}_{\text {min }}}=q_{t} t^{\frac{1}{2} \Delta \eta_{j j}} \approx 14 q_{t} \sim 300 \mathrm{GeV} . \tag{2.19}
\end{equation*}
$$

Note that the typical laboratory angles of the tagging jets in the dimuon cms frame do not exceed, for this choice of $q_{t}$,

$$
\begin{equation*}
\theta_{\max } \cong \frac{2 q_{t}}{\sqrt{\hat{s}}} \sim 50 \mathrm{mrad} \tag{2.20}
\end{equation*}
$$

This implies that in any frame, at least one tagging jet will have a production angle smaller than this amount; equivalently at least one tagging jet has a rapidity exceeding $\eta_{\min } \sim 3.7$. The minimum angle a tagging-jet can possess is, for the SSC, roughly

$$
\begin{equation*}
\theta_{\min } \sim \frac{q_{t}}{(5-10 \mathrm{TeV})} \sim 2-4 \mathrm{mrad} \tag{2.21}
\end{equation*}
$$

or $\eta_{\text {max }} \lesssim 7$.
Let us now turn to an estimation of the cross section. The usual WeiszackerWilliams method is eminently suitable, given the kinematics sketched above, which leaves the longitudinal fraction of the photon momentum relative to the parent quark small compared to unity (in order to create the rapidity-gaps).

The cross section is

$$
\begin{equation*}
\left.d \sigma=\left.\left(\frac{2 \alpha}{\pi}\right)^{2} F\left(x_{1}\right) F\left(x_{2}\right) \frac{d Q_{1}^{2}}{Q_{1}^{2}} \frac{d Q_{2}^{2}}{Q_{2}^{2}} \frac{d y_{1}}{y_{1}} \frac{d y_{2}}{y_{2}} d \eta_{1} d \eta_{2} \sigma_{\gamma \gamma}\left(s^{\prime}, \Delta \eta_{\mu \mu}\right)\langle | S\right|^{2}\right\rangle \tag{2.22}
\end{equation*}
$$

with the photon longitudinal fractions given by

$$
\begin{equation*}
y_{1}=\frac{k_{1}}{E_{1}} \quad y_{2}=\frac{k_{2}}{E_{2}} . \tag{2.23}
\end{equation*}
$$

The $k_{i}$ and $E_{i}$ are photon and quark energies respectively in, say, the dimuon rest

- frame. The squared $\gamma \gamma \mathrm{cms}$ energy is

$$
\begin{equation*}
s^{\prime}=y_{1} y_{2} \widehat{s}=y_{1} y_{2}\left(x_{1} x_{2} s\right) \tag{2.24}
\end{equation*}
$$

and $\Delta \eta_{\mu \mu}$ is the maximum dimuon rapidity separation allowed by the restricted integration of cms dimuon angles.

A convenient way to cast the above cross-section formula is in terms of $s^{\prime}$ and $\bar{\eta}$, the mean rapidity of the dimuon system. This means eliminating $y_{1}$ and $y_{2}$ in terms of $s^{\prime}$ and $\bar{\eta}$. In the $\gamma \gamma$ rest frame, $\bar{\eta}$ vanishes and $k_{1}=k_{2}$. Then

$$
\begin{equation*}
\ln y_{2}-\ln y_{1}=\ln \frac{E_{1}^{*}}{E_{2}^{*}} \tag{2.25}
\end{equation*}
$$

But from Eq. (2.8) and Eq. (2.15)

$$
\begin{align*}
& \eta_{1}^{*}+\eta_{2}^{*}=\ln \frac{2 E_{1}^{*}}{Q_{1}}-\ln \frac{2 E_{2}^{*}}{Q_{2}}=\ln \frac{E_{1}^{*}}{E_{2}^{*}}-\ln \frac{Q_{1}}{Q_{2}}  \tag{2.26}\\
& \eta_{1}^{*}-\eta_{2}^{*}=\ln \widehat{s}-\ln q_{1} q_{2}
\end{align*}
$$

This allows us to determine $\eta_{1}^{*}$ and $\eta_{2}^{*}$ in the $\gamma \gamma$ rest frame and thereby allow $\bar{\eta}$ to be related to them (and the $y$ 's):

$$
\begin{align*}
\bar{\eta} & =\eta_{1}-\eta_{1}^{*}=\eta_{2}-\eta_{2}^{*}=\frac{1}{2}\left(\eta_{1}+\eta_{2}\right)-\frac{1}{2}\left(\eta_{1}^{*}+\eta_{2}^{*}\right) \\
& =\frac{1}{2}\left(\eta_{1}+\eta_{2}\right)-\frac{1}{2}\left(\ln y_{2}-\ln y_{1}\right)+\frac{1}{2} \ln \frac{q_{1}}{q_{2}} \tag{2.27}
\end{align*}
$$

Thus we have the superbly simple Jacobian

$$
\begin{equation*}
d \bar{\eta} \frac{d s^{\prime}}{s^{\prime}}=\frac{d y_{1}}{y_{1}} \frac{d y_{2}}{y_{2}} \tag{2.28}
\end{equation*}
$$

The cross section is

$$
\begin{align*}
& \frac{d \sigma}{d \eta_{1} d \eta_{2} d \bar{\eta} d \ell n Q_{1}^{2} d \ell n Q_{2}^{2} d \ell n s^{\prime}} \\
& \left.\quad=\left.\left(\frac{2 \alpha}{\pi}\right)^{2} F_{2}\left(x_{1}, Q_{1}^{2}\right) F_{2}\left(x_{2}, Q_{2}^{2}\right) \sigma_{\gamma \gamma}\left(s^{\prime}, \Delta \eta_{\mu \mu}\right)\langle | S\right|^{2}\right\rangle \tag{2.29}
\end{align*}
$$

Expressed in these variables, it depends upon almost nothing except kinematic limits. Again, analogous to Eq. (2.8), we have

$$
\begin{equation*}
\eta_{1}=\ln \frac{\sqrt{s}}{Q_{1}}+\ln x_{1} \quad-\eta_{2}=\ln \frac{\sqrt{s}}{Q_{2}}+\ln x_{2} \tag{2.30}
\end{equation*}
$$

The yield as a function of $\eta_{1}$ and $\eta_{2}$ is in proportion to the product of parton densities $F\left(x_{1}, Q_{1}^{2}\right) F\left(x_{2}, Q_{2}^{2}\right)$. This is plotted in Fig. 5 for a few choices of transverse momenta $q_{i} ; q_{1}=q_{2}$ is assumed for simplicity. Also shown is the kinematic restriction for a rapidity gap, Eq. (2.15). Note that the cross-section formula breaks down when the rapidity-gap closes up too much, in particular when the approximation $y_{1}, y_{2} \ll 1$ breaks down. This case requires a better calculation, but one can expect a diminishing yield where it occurs.

For fixed $Q_{i}^{2}, s^{\prime}$, and $\bar{\eta}$, one has some idea of how much yield one gets after integrating over the tagging-jet locations $\left\{\eta_{1}, \eta_{2}\right\}$ by inspection of Fig. 5. Since $\sigma_{\gamma \gamma} \sim\left(s^{\prime}\right)^{-1}$, the $s^{\prime}$ integration is dominated by the threshold region. The $\bar{\eta}$ range of integration is again straightforward, as is, more or less, the range in $\log q^{2}$.

The dimuon yield is low, but measurable, with choices of $q$ and $\sqrt{s^{\prime}}$ in the tens of GeV . The issue will be backgrounds, not rate. These are controlled in terms of high- $p_{t}$ triple-diffraction processes. These are very uncertain to estimate, but are discussed in Section 4. Alternatively one may reduce $Q^{2}$ to $1-10 \mathrm{GeV}^{2}$ and gain in rate, but at the price of a more difficult background problem.


Figure 5 . Estimate of the parton luminosity $\mathcal{L}$, defined as

$$
\mathcal{L}=F_{2}\left(x_{1}, Q_{1}^{2}\right) F_{2}\left(x_{2}, Q_{2}^{2}\right)
$$

for $\sqrt{s^{\prime}}=100 \mathrm{GeV}$ and $\sqrt{s}=40 \mathrm{TeV}$ (SSC), as a function of the tagging-jet rapidities $\eta_{1}$ and $\eta_{2}$ : (a) $Q_{1}^{2}=Q_{2}^{2}=100 \mathrm{GeV}^{2}$; (b) $Q_{1}^{2}=Q_{2}^{2}=10^{3} \mathrm{GeV}^{2}$. Up to a factor $\ell n S / Q^{2}$ in $\sigma_{\gamma \gamma}$, $\mathcal{L}$ is proportional to the differential dilepton yield, Eq. (2.29).

The hadronic, $q \bar{q}$ final states produced in $\gamma \gamma$ collisions are described by the same expression as used for the dimuons. However, these processes are probably obscured by the aforementioned backgrounds. If one wishes to study such configurations, it is probably best to utilize hard diffraction processes to produce them.

However, there may be "new physics" which is more accessible. Promising candidates would include pair-production of charged, color-singlet systems (heavy leptons or sleptons, for example ${ }^{2}$ ) for which the absence of an underlying event might create a relatively clean signature. Another class might be particles with significant partial-width for decay into $\gamma \gamma$. We do not explore these here, but turn to similar, perhaps more promising cases, utilizing $W$ - $W$ collisions.
c) Higgs Production in $W$ - $W$ Collisions:

The two- $W$ processes are a central subject of SSC/LHC physics, and have been extensively studied. Here we very briefly sketch a simple and most interesting case-that of $W_{L}-W_{L}$ annihilation into a Higgs-boson resonant state. The analogous Weiszacker-Williams cross-section is easily constructed. For the process $q \bar{q} \rightarrow q \bar{q} H$ (via $W_{L}-W_{L}$ annihilation), we find

$$
\begin{equation*}
\left.\left.\frac{d \sigma}{d \eta_{1} d \eta_{2} d \bar{\eta}} \cong \frac{\alpha_{W}^{3}}{16 m_{W}^{2}} \Phi\left(\eta_{1}-\tilde{\eta}\right) \Phi\left(\eta_{2}-\tilde{\eta}\right)\langle | S\right|^{2}\right\rangle \tag{2.31}
\end{equation*}
$$

Here we have used the results of Chanowitz and Gaillard [8], in particular their Eqs. (4.1) and (4.4). The parameters $\bar{\eta}$ (Higgs rapidity) and $\widetilde{\eta}$ (mean tagging jet rapidity) are discussed in more detail in what follows. When the cms quark-quark energy $\sqrt{\hat{s}}$ is much larger than the Higgs mass $m_{H}$, the energies of the secondary quarks are essentially the same as the primary energies. Thus the rapidities of the

2 I thank Gordon Kane for this suggestion.
secondary-quark tagging jets are determined by their transverse momenta, which have the distribution (for longitudinal- $W$ emission)

$$
\begin{equation*}
d N \sim \frac{d p_{t}^{2}}{\left(p_{t}^{2}+m_{W}^{2}\right)^{2}} . \tag{2.32}
\end{equation*}
$$

Upon changing to the rapidity variables $\eta_{i}$, this leads to the distribution

$$
\begin{equation*}
\Phi\left(\eta^{\prime}\right)=\frac{1}{2 \cosh ^{2} \eta^{\prime}} \quad \int_{-\infty}^{\infty} d \eta^{\prime} \Phi\left(\eta^{\prime}\right)=1 \tag{2.33}
\end{equation*}
$$

where $\eta^{\prime}$ is the fluctuation about the mean rapidity $\widetilde{\eta}$ of the tagging-jets, given by the expression

$$
\begin{equation*}
\tilde{\eta}=\log \tan \frac{\sqrt{\hat{s}}}{m_{W}} \tag{2.34}
\end{equation*}
$$

which occurs for $p_{t_{i}}=m_{W}$. For $q \bar{q}$ cms energies of 1 to 10 TeV , the mean separation $\Delta \eta$ of the tagging-jets in the lego plot varies as follows:

$$
\Delta \eta \cong 2 \tilde{\eta}=2 \log \frac{\sqrt{\hat{s}}}{m_{W}}= \begin{cases}7.0 & \sqrt{\hat{s}}=1 \mathrm{TeV}  \tag{2.35}\\ 9.6 & \sqrt{\hat{s}}=10 \mathrm{TeV}\end{cases}
$$

The separation at $\sqrt{\hat{s}}=1 T \epsilon V$ is already just about sufficient for a rapidity-gap signature, while the size of the gap at $\sqrt{\hat{s}}=10 \mathrm{TeV}$ is clearly more than enough.

In any case, we see that the criterion for creating a large rapidity-gap has little to do with the properties of the Higgs-boson, and much more to do with the cms energy of the $q \bar{q}$ system. Remarkably this is true for the cross-section, Eq. (2.31), as well, which shows no dependence on Higgs mass at all! Actually the mass-dependence is buried in the dependence of the cross-section on $\bar{\eta}$, the mean rapidity of the decay products of the Higgs. (The kinematics here is the same as

- that of the previous subsection for $\gamma \gamma \rightarrow \mu \mu$, and the conclusions are similar.) The total cross-section is, according to Chanowitz and Gaillard [8],

$$
\begin{equation*}
\sigma_{\mathrm{tot}}=\frac{\alpha_{W}^{3}}{16 m_{W}^{2}}\langle\Delta \bar{\eta}\rangle \tag{2.36}
\end{equation*}
$$

with

$$
\langle\Delta \bar{\eta}\rangle=(1+x) \ln \frac{1}{x}-2(1-x) \rightarrow \begin{cases}\ln \frac{\widehat{s}}{m_{H}^{2}}-2 & \widehat{s} \gg m_{H}^{2}  \tag{2.37}\\ \frac{1}{6}\left(1-\frac{\hat{s}}{m_{H}^{2}}\right)^{3} & \widehat{s} \sim m_{H}^{2}\end{cases}
$$

and

$$
\begin{equation*}
x=\frac{m_{H}^{2}}{\widehat{s}} . \tag{2.38}
\end{equation*}
$$

In accordance with the discussion in the previous subsection, the kinematic restriction on the cross-section for producing a rapidity-gap of width $\Delta \eta$ (we expect $\Delta \eta \gtrsim 3$ to be a reasonable cut) is

$$
\begin{equation*}
\eta_{1}-\eta_{2} \geq \Delta \eta+1.4 \tag{2.39}
\end{equation*}
$$

In addition we should require (cf. Fig. 2)

$$
\begin{equation*}
|\bar{\eta}| \lesssim \frac{\Delta \eta}{2}-1.4 \tag{2.40}
\end{equation*}
$$

in order that most of the time the decay products of the Higgs boson land within the rapidity-gap. Thus

$$
\begin{equation*}
|\bar{\eta}| \lesssim \frac{1}{2}\left(\eta_{1}-\eta_{2}\right)-2.1=\widetilde{\eta}+\frac{1}{2}\left(\eta_{1}^{\prime}+\eta_{2}^{\prime}\right)-2.1 . \tag{2.41}
\end{equation*}
$$

The typical values of $\eta_{1}^{\prime}$ and $\eta_{2}^{\prime}$ are of order $\pm 1$; a safe limit for $\bar{\eta}$ should be

$$
\begin{equation*}
|\bar{\eta}| \lesssim \tilde{\eta}-3=\frac{1}{2} \ell n \frac{s}{m_{W}^{2}}-3 . \tag{2.42}
\end{equation*}
$$

This leads to a crude estimate for the cross-section for Higgs production with decay

- products going into the rapidity gap

$$
\begin{equation*}
\left.\left.\sigma_{\mathrm{gap}} \cong \frac{\alpha_{W}^{3}}{16 m_{W}^{2}}\left(\ln \frac{\widehat{s}}{m_{W}^{2}}-6\right)\langle | S\right|^{2}\right\rangle \tag{2.43}
\end{equation*}
$$

In Fig. 6, the $s$-dependence of the factor $\ell \frac{\widehat{s}}{m_{W}^{2}}-6$, which controls the cross-section behavior with gap signature, is compared with that of $\langle\Delta \bar{\eta}\rangle$, the parameter which controls the total cross-section energy dependence.


Figure 6. Approximate dependence of the Higgs-production cross-section, with the Higgsparticle landing within the gap (dashed curve), upon the cms energy $\sqrt{\hat{s}}$ of the initiating $q q$ system. Also shown (solid curves) is the $\sqrt{\hat{s}}$-dependence of the total $q q \rightarrow q q H$ cross-section (cf. Eqs. (2.36) and (2.43)).

It appears from that figure that once the Higgs mass exceeds about 500 GeV , the decay products of the Higgs resonance almost always automatically fall within the fiducial rapidity gap. Therefore the efficiency of the gap signature is controlled by the magnitude of the absorption correction $\left.\left.\langle | S\right|^{2}\right\rangle$.

The fact that the gap cross-section is essentially independent of Higgs mass, and that the gap-signature is efficient for Higgs mass above 500 GeV allows us to finesse the question of convolution over parton distributions. For any Higgs mass, we have

$$
\begin{align*}
\sigma_{\mathrm{gap}}(s) & \left.\left.\cong \sigma^{\mathrm{tot}}\left(s, m_{H}=500 \mathrm{GeV}\right)\langle | S\right|^{2}\right\rangle  \tag{2.44}\\
& \left.\left.\cong\left(3 \times 10^{-36} \mathrm{~cm}^{2}\right)\langle | S\right|^{2}\right\rangle
\end{align*}
$$

Assuming no large background sources from QCD or other processes, and that the absorption factor $\left.\left.\langle | S\right|^{2}\right\rangle$ is $\sim 5 \%$, this provides an ample yield of bosons because all Higgs decay modes should be accessible.

As will be discussed further in Section IV, we have not identified any obvious sources of background for this signal other than the nonresonant and not uninteresting $W$ - $W$ two-body scattering processes themselves. The experimental procedure for isolating a signal should be straightforward no matter what is the decay mode. The most problematic case is when both final-state intermediatebosons decay hadronically. This is the only case we discuss in detail here. In fact, we shall further restrict our attention to the case of a very heavy Higgs ( $0.5-1 \mathrm{TeV}$ ) because that signature is the cleanest and easiest to consider. This occurs because the dijet decay products of the $W$ are boosted into a cone of small opening angle as a consequence of the large value of $\gamma_{t}=p_{t} / m_{W} \sim 3-6$. The event topology for the signal is shown in Fig. 7. We have used the concept of extended lego-plot [6] to describe the internal structure of this dijet system. Also shown is the lego-plot of the event when the $z$-axis is chosen to be the thrust axis for the products of the decaying Higgs-boson. Either way, one sees the existence of a transverse rapiditygap in addition to the usual longitudinal one. The width of this gap $\Delta \eta_{t}$ is, for
symmetric decays, and a 1 TeV Higgs-mass

$$
\begin{equation*}
\Delta \eta_{t} \sim 2\left[\ln \frac{m_{H}}{m_{W}}-0.7\right] \sim 3.5 \tag{2.45}
\end{equation*}
$$

large enough to be of use.


Figure 7. Event-topology for the Higgs process for a 1 TeV Higgs particle: (a) the event structure as seen in extended phase space (polar coordinates used inside the circles of radius 0.7 are transcribed into a new lego plot); (b) the event as seen in lego variables with $z$-axis taken along the thrust axis of the Higgs-particle decay products in the Higgs rest frame.

To isolate this signal with a full-acceptance detector [9], one may, for the allhadronic decay modes,

1. Select 4-jet events, with $E_{t} \gtrsim E_{t \text { min }} \geq 300 \mathrm{GeV}$, say, and $p_{t}$ of each jet $>p_{\text {tmin }} \geq 20 \mathrm{GeV}$. The jets are defined as all hadrons within appropriately chosen circles-of-radius- 0.7 in the lego plot.
2. Define the fiducial rapidity gap $\Delta \eta$ as usual (Fig. 2); cut on $\Delta \eta \geq \Delta \eta_{\min } \sim 3$.
3. Measure the total multiplicity $n_{\text {gap }}$ within the gap-but exterior to the two jets within the gap. Then cut on $n_{\text {gap }} \leq n_{\max }$, with $n_{\max } \sim 3$, say.
4. Construct the extended lego plots for the interior jets; demand in each a two-jet system with $m \cong m_{W}$.
5. Define, if possible, a transverse rapidity gap and make an additional multiplicity cut on it.

We expect this idealized procedure is in fact overkill. The event topology is an experimentalist's dream: the primary signal is two well collimated coplanar "jets" with total $E_{t}$ in the $500-1000 \mathrm{GeV}$ range, and absolutely nothing else in the remainder of a typical central detector. In addition each "jet" will consist of a jet pair with $\Delta \eta-\Delta \phi$ separation $\sim 0.2$, which in principle precisely reconstructs to the intermediate-boson mass.

And this analysis is worst-case; about half the time one of the $W$ 's decays leptonically, leaving a very isolated high $p_{t}$ track. Thus if the rapidity-gap signature does exist (i.e. $\left.\left.\langle | S\right|^{2}\right\rangle$ is not too small) and if there is no background (something not easy to concoct), it may well be that a relatively simple detector, certainly no more sophisticated than Fermilab's CDF or DØ, could suffice to find the high-mass Higgs at the SSC—and in all its decay modes. However, a good deal of additional study and simulation will be necessary to back this assertion up.

Evidently this strategy is applicable to many other processes, in particular to
continuum two-body scattering processes involving $W^{\prime}$ 's, $Z$ 's, and photons. Discussion of these, as well a further discussion of the Higgs process, seems unwarranted in the absence of Monte Carlo simulations of real events. We urge that the appropriate studies be carried out.


Figure 8. A candidate background process for the Higgs production.

The question of QCD backgrounds to the rapidity-gap signature is a difficult one. The biggest candidate that comes to mind is the tree-level process shown in Fig. 8. It is of order $\alpha_{s}^{6}$, and 3 sets of color-singlet gluon-pairs (as shown by the dashed lines) are required to provide, via conventional color-transparency arguments [10], the rapidity-gaps at the perturbative level. In order of magnitude, we guess for this process

$$
\begin{equation*}
\sigma_{B G} \sim \frac{\alpha_{s}^{6} C^{3}}{(1 T e V)^{2}} \lesssim\left(4 \times 10^{-34} \mathrm{~cm}^{2}\right) \times\left(\frac{1}{7}\right)^{6} \times\left(\frac{1}{8}\right)^{3} \lesssim 10^{-42} \mathrm{~cm}^{2} \tag{2.46}
\end{equation*}
$$

where we have taken the color singlet suppression factors $C$ to be statistical. This
should be compared with the parton-level $q \bar{q}$ cross-section, Eq. (2.43)

$$
\begin{equation*}
\left.\left.\sigma_{q \bar{q}} \sim \frac{\alpha_{W}^{3}}{16 m_{W}^{2}}\langle | S\right|^{2}\right\rangle \sim 1.5 \times 10^{-38} \mathrm{~cm}^{2} \tag{2.47}
\end{equation*}
$$

The above estimate, albeit very crude, does lend encouragement to the possibility that the backgrounds will indeed be small, especially since we have not found other background mechanisms with a smaller power of $\alpha_{s}$. But more critical examination of this point is most appropriate.

## 3. Survival of the Rapidity Gap

The claims in the previous section depend in an essential way on the estimate that the fraction of events for which spectator interactions do not fill in the rapidity gaps of interest is sizeable, of order $5 \%$. This fraction was called $\left.\left.\langle | S\right|^{2}\right\rangle$, and is estimated most naively as follows.


Figure 9. Convolution of parton densities in impact plane.

The hard collision of interest is initiated by a close collision of two partons, one from each beam. It therefore is a convolution of parton densities (cf. Fig. 9)
in the transverse impact plane.

$$
\begin{equation*}
\sigma_{\mathrm{Hard}} \cong \sigma_{0} \int d^{2} B d^{2} b \rho(b) \rho(B-b) \equiv \sigma_{0} \int d^{2} B F(B) \tag{3.1}
\end{equation*}
$$

The simplest estimate of survival probability of the gap, i.e. that no other interactions occur except the hard collisions of interest, is simply to multiply the above integrand by $|S(B)|^{2}$, the probability that the two projectiles pass through each other without any interaction when they arrive with impact parameter $B$. This estimate will be reasonable if the relevant parton densities are uncorrelated in the transverse, "impact-plane" coordinates. Whether this assumption itself is reasonable can be questioned, and will be discussed a little more later on. But setting that issue aside for the moment, we then can write for the survival probability of the gap the expression

$$
\begin{equation*}
\left.\left.\langle | S\right|^{2}\right\rangle=\frac{\int d^{2} B F(B)|S(B)|^{2}}{\int d^{2} B F(B)} \tag{3.2}
\end{equation*}
$$

which justifies the notation.
A traditional estimate of $|S|^{2}$ is given by the eikonal picture [11]

$$
\begin{equation*}
|S(B)|^{2}=\exp -\nu \chi(B) \tag{3.3}
\end{equation*}
$$

where $\chi$ is itself a convolution of parton densities, and is chosen such that

$$
\begin{equation*}
\chi(0)=1 . \tag{3.4}
\end{equation*}
$$

For simplicity, suppose that both the function $\chi$ and the hard-collision convolution $F$, are chosen to be Gaussian. Then the exponential can be expanded and the
integrals performed. The answer is simple

$$
\begin{equation*}
\left.\left.\langle | S\right|^{2}\right\rangle=\int_{0}^{\infty} \frac{d B^{2}}{R_{0}^{2}} e^{-B^{2} / R_{0}^{2}} \sum_{n=0}^{\infty} \frac{(-\nu)^{n} e^{n B^{2} / R^{2}}}{n!}=\sum_{n=0}^{\infty} \frac{(-\nu)^{n}}{n!\left(1+\left(n R_{0}^{2} / R^{2}\right)\right)} \tag{3.5}
\end{equation*}
$$

This in turn can be summed, for example by constructing the differential equation this sum satisfies and then solving it:

$$
\begin{equation*}
\left.\left.\langle | S\right|^{2}\right\rangle=\frac{a}{\nu^{a}} \int_{0}^{\nu} d u u^{a-1} e^{-u} \approx \frac{\Gamma(a+1)}{\nu^{a}} \quad(\nu \gg 1) \tag{3.6}
\end{equation*}
$$

where

$$
\begin{equation*}
a=\frac{R^{2}}{R_{0}^{2}} \tag{3.7}
\end{equation*}
$$

A crucial parameter is the central absorption $\nu$. Even more crucial is the ratio of the interaction area $\pi R_{0}^{2}$ for the hard parton-parton collision to that of the soft collisions, $\pi R^{2}$, controlling the total cross section. We may keep the former radius fixed by considering the process for differing center-of-mass energies, but always at the same $x_{1}$ and $x_{2}$ of the quarks (i.e. we keep the fraction of energy in the quark-quark subsystem fixed). But the radius associated with the total cross section behavior clearly rises with energy, and should be regarded as somewhat larger than the hard-collision radius.

We may also question whether the result depends sensitively on the choice of Gaussian distributions. Repeat of the calculation with exponential distributions leads to the result

$$
\begin{align*}
\left.\left.\langle | S\right|^{2}\right\rangle & =\int_{0}^{\infty} \frac{B d B}{R_{0}^{2}} \sum_{n=0}^{\infty} \frac{(-\nu)^{n}}{n!} e^{-n B / R}=\sum_{n=0}^{\infty} \frac{(-\nu)^{n}}{n!\left(1+\left(n R / R_{0}\right)\right)^{2}} \\
& =-a^{2} \frac{\partial}{\partial a} \frac{1}{\nu^{a}} \int_{0}^{\nu} d u u^{a-1} e^{-u} \approx \frac{\Gamma(a+1)}{\nu^{a}}\left[a \log \nu-\frac{\Gamma^{\prime}(a)}{\Gamma(a)}\right] . \tag{3.8}
\end{align*}
$$

- We again see a similar behavior, but with a somewhat larger survival probability. This is probably a consequence of the longer tail of the distribution at large impact parameters, leading to more peripherality. We conclude that the survival probability $\left.\left.\langle | S\right|^{2}\right\rangle$ depends on the central absorption $\nu$ in $p p$ collisions roughly as an inverse power, with the exponent probably between 1 and 2 , and with some sensitivity to the assumed shapes of the distributions.


Figure 10. (a) The quantity $\nu \chi(B)=-\log |S(B)|^{2}$ as a function of impact parameter $B$ for elastic $p p$ scattering at $\sqrt{s}=40 \mathrm{TeV}$. Also shown is the function $F(B)$ defining the impactparameter dependence of hard-collision luminosity. (b) This is replotted versus $b^{2}$, along with $F(B)|S(B)|_{S S C}^{2}$. The curves are taken from the analysis of Block et al. (Ref. 12).

Another way to estimate $\left.\left.\langle | S\right|^{2}\right\rangle$, perhaps the most reasonable, is to use the absorption factor determined experimentally from $p p$ elastic scattering data. These are provided in a convenient form by Block et al. [12] We use for the hard collision impact-parameter distribution a shape distribution which fits the total cross section data for $\nu \chi(b)$ at moderate energies, before the rise in total cross section sets in and the elastic scattering distribution shows significant shrinkage. [Actually, as one can see from Fig. 10a, the shape changes very little with energy.] The central absorption $\nu$ is estimated as 3-4 at ISR energies, about 5 at $\sqrt{s}=2 \mathrm{TeV}$ and nearly 10 at $\sqrt{s}=40 \mathrm{GeV}$. The results are exhibited in Fig. 10 b and lead to an estimate, via numerical calculation, of

$$
\begin{equation*}
\left.\left.\langle | S\right|^{2}\right\rangle=0.10 \tag{3.9}
\end{equation*}
$$

There is an additional uncertainty stemming from the assumption of uncorrelated parton distributions in the impact plane. It may be that there is more probability of absorption in a hard collision than the estimates above because of clustering of the distributions of the relevant partons around the valence quarks. We may consider an extreme case of this in terms of the additive quark model. We consider the constituent quarks to be small, rather black structures with a radius of order 0.2-0.3 fermi, chosen to give the approximate relation

$$
\begin{equation*}
\sigma_{p p} \approx 9 \sigma_{q q} \quad \sigma_{q q} \approx 2 \pi r_{q}^{2} \approx 4 m b \tag{3.10}
\end{equation*}
$$

In this picture, close collisions of these constituent quarks are supposed to contribute a sizeable fraction of the total proton-proton cross section. But, these collisions cannot alone produce the expected large value of the central absorption $\nu$ at SSC energies. There must be a big contribution from the clouds around these
quarks as well, one which is growing with energy. However, none of these considerations precludes the possibility of a large value of absorption in a central collision (zero impact parameter) of two constituent quarks. And again the preceding discussion can be carried over to this case. We may write, ignoring the shadowing of one constituent quark by another [13],

$$
\begin{align*}
\left.\left.\langle | S\right|^{2}\right\rangle & \cong \frac{\int d^{2} B F(B)\left|S_{p p}(B)\right|^{2} \int d^{2} b^{\prime} \sigma_{q q}^{\mathrm{Hard}}\left(b^{\prime}\right)\left|S_{q q}\left(b^{\prime}\right)\right|^{2}}{\int d^{2} B F(B) \int d^{2} b^{\prime} \sigma_{q q}^{\mathrm{Hard}}\left(b^{\prime}\right)}  \tag{3.11}\\
& \left.\left.=\left.\langle | S\right|^{2}\right\rangle\left._{p p} \cdot\langle | S\right|^{2}\right\rangle_{q q}
\end{align*}
$$

It is not at all clear what to take for the additional $q q$ survival probability, which is a simple multiplier (in this simplified case) to the previous estimate. We here only note that the expectation from perturbative QCD is that the quark-quark interaction, at any fixed scale of momentum transfer $t$, is expected to become strong as $s$ tends to infinity [14]. Therefore a significant additional diminution to the overall survival probability from this source must be considered seriously. However, in the light of our previous estimates it seems unreasonable to assess more than an order of magnitude loss from this source.

Hereafter, I take for the estimate of $\left.\left.\langle | S\right|^{2}\right\rangle$

$$
\begin{equation*}
\left.\left.\langle | S\right|^{2}\right\rangle \cong 0.05 \tag{3.12}
\end{equation*}
$$

with a factor 3 uncertainty in either direction. I can only conclude that this unhappy situation needs a lot more expertise and detailed consideration than I can here provide.

The best answer to this problem is to determine the survival probability experimentally. The $\gamma \gamma \rightarrow \mu^{+} \mu^{-}$process discussed in the previous section is an ideal

- way to do this. Another may be highly inelastic double diffraction at large $t$, if the theoretical estimates of the underlying hard subprocess can be made precise enough. This will be discussed in the next subsection.


## 4. Jets and Gaps in Strong Interactions

As we have already mentioned, there is reason to believe that there are also strong interaction mechanisms which can lead to event topologies containing both jets and rapidity gaps [15]. The simplest mechanism is just two gluon exchange between partons, with the restriction that the pair of gluons be in a color singlet state, and that the physics is short-distance dominated. The consideration of this physics has been in the lore for a long time [16]. In the interest of being reasonably self-contained, we review the calculations as simply as possible in the following.

To begin, consider quark-quark scattering at the parton level via photon exchange. The amplitude is

$$
\begin{equation*}
T_{Q E D}(q)=\frac{e^{2}}{q^{2}}\left(2 p_{1}\right) \cdot\left(2 p_{2}\right) Q Q^{\prime} \equiv \frac{8 \pi \alpha s}{t} Q Q^{\prime} \tag{4.1}
\end{equation*}
$$

where $Q$ and $Q^{\prime}$ are the charges of the relevant partons. Our normalizations are such that

$$
\begin{align*}
\frac{d \sigma}{d t} & =\frac{1}{16 \pi s^{2}}|T|^{2} \\
\operatorname{Im} T & =\frac{1}{16 \pi s} \int|T|^{2} d t+\text { inelastic contributions } \tag{4.2}
\end{align*}
$$

We work in the high energy limit at fixed but large momentum transfer. Helicity is conserved, and there will be absolutely no complications due to spin. We will be interested in the next order, two-photon exchange contribution. In QED, this leads

- mainly to acquisition of a Coulomb phase, best seen by working in the impactplane, i.e. making a Fourier transform in the transverse-momentum variables. The canonically conjugate impact-parameter variables are in fact constants of the motion (angular momenta). With

$$
\begin{equation*}
T(q)=\int d^{2} b e^{i q \cdot b} \widetilde{T}(b) \tag{4.3}
\end{equation*}
$$

we find

$$
\begin{equation*}
\widetilde{T}_{Q E D}(b)=\int \frac{d^{2} q}{(2 \pi)^{2}} e^{-i q \cdot b} \cdot \frac{8 \pi \alpha s}{q^{2}} Q Q^{\prime} \approx 2 \alpha s Q Q^{\prime} \log \frac{R^{2}}{b^{2}} \quad\left(b^{2} \ll R^{2}\right) \tag{4.4}
\end{equation*}
$$

In real life the infrared divergence in this expression is removed by screening contributions provided one has neutral projectile colliding with each other. The full amplitude will have the form

$$
\begin{align*}
\widetilde{T}_{Q E D}(b)= & \int d^{2} b_{1} \ldots d^{2} b_{n} \rho\left(b_{1} \ldots b_{n}\right) \delta^{(2)}\left(\Sigma b_{i}\right) d^{2} b_{1}^{\prime} \ldots d^{2} b_{n}^{\prime} \delta^{(2)}\left(\Sigma b_{j}^{\prime}\right) \rho\left(b_{1}^{\prime} \ldots b_{n}^{\prime}\right) \\
& \times 2 \alpha s \sum_{i j} Q_{i} Q_{j}^{\prime} \log \frac{R^{2}}{\left(b-b_{i}+b_{j}^{\prime}\right)^{2}} \tag{4.5}
\end{align*}
$$

which eliminates in principle the dependence of the cutoff $R^{2}$. If only one pair of partons $\left\{i_{0}, j_{0}\right\}$ have a close collision, we recover the simplified form of Eq. (4.4).

Hereafter we do not include the niceties, but simply cut off the divergence at a scale appropriate to hadronic size.

The unitarity condition in impact space is diagonal

$$
\begin{equation*}
\operatorname{Im} \widetilde{T}(b)=\frac{1}{4 s}|\widetilde{T}(b)|^{2} \tag{4.6}
\end{equation*}
$$

and provides an instant estimate of the dominant, imaginary part of the two-
photon-exchange amplitude

$$
\begin{equation*}
\delta \widetilde{T}_{Q E D}(b)=i \alpha^{2} s Q Q^{\prime} \log ^{2} \frac{R^{2}}{b^{2}} \tag{4.7}
\end{equation*}
$$

or

$$
\begin{equation*}
T_{Q E D}(b) \cong 2 \alpha s Q Q^{\prime} \log \frac{R^{2}}{b^{2}}\left[1+\frac{i \alpha}{2} Q Q^{\prime} \log \frac{R^{2}}{b^{2}}+\ldots\right] \tag{4.8}
\end{equation*}
$$

In QED, the contribution we have calculated exponentiates in higher orders to a phase factor, leaving the lowest order cross section unmodified. However in QCD this cannot occur. The color-singlet-exchange contribution first occurs in the two-gluon-exchange amplitude, hence cannot be a harmless additional phase on the lowest order amplitude. To be sure, higher orders can be significant, and the problems this presents will be mentioned again below.

The modification of the above estimate in the case of QCD is now a matter of inclusion of the color factors. We write

$$
\begin{equation*}
\widetilde{T}_{Q C D}(b)=2 \alpha_{s} s \log \frac{R^{2}}{b^{2}} \sum_{a=1}^{8} T_{a} \cdot T_{a}^{\prime} \tag{4.9}
\end{equation*}
$$

The imaginary part of the two-gluon exchange amplitude is

$$
\begin{equation*}
\operatorname{Im} \widetilde{T}_{Q C D}(b)=\left(\alpha_{s}^{2} s \log ^{2} \frac{R^{2}}{b^{2}}\right) \sum_{a, b=1}^{8}\left(T_{a} T_{b}\right) \cdot\left(T_{a}^{\prime} T_{b}^{\prime}\right) \tag{4.10}
\end{equation*}
$$

The color singlet piece is extracted using the identity

$$
\begin{equation*}
T_{a} T_{b}=\frac{1}{6} \delta_{a b} 1+\text { octet } \tag{4.11}
\end{equation*}
$$

leading to

$$
\begin{equation*}
\sum_{a, b=1}^{8}\left(T_{a} T_{b}\right) \cdot\left(T_{a}^{\prime} T_{b}^{\prime}\right)=\frac{2}{9} \tag{4.12}
\end{equation*}
$$

and

$$
\begin{equation*}
\left.\operatorname{Im} \widetilde{T}(b)_{Q C D}\right|_{t \text { channel singlet }}=\frac{2}{9} s \alpha_{s}^{2} \log ^{2} \frac{R^{2}}{b^{2}} \tag{4.13}
\end{equation*}
$$

We now can estimate the ratio of the one-photon-exchange amplitude to the two-gluon-exchange, color-singlet amplitude

$$
\begin{equation*}
\frac{T_{Q E D}(b)}{\operatorname{Im} \widetilde{T}_{Q C D}(b)} \approx Q Q^{\prime}\left(\frac{9 \alpha}{\alpha_{s}^{2} \log \left(R^{2} / b^{2}\right)}\right) . \tag{4.14}
\end{equation*}
$$

This gives for the cross-section ratio

$$
\begin{equation*}
\frac{\sigma_{Q E D}}{\sigma_{Q C D}}=\left[\frac{9\left\langle Q^{2}\right\rangle \alpha}{\alpha_{s}^{2} \log \left(R^{2} / b^{2}\right)}\right]^{2} \tag{4.15}
\end{equation*}
$$

To get a feel for the numbers, let us remove the logarithm in the QCD amplitude by using the running of the QCD coupling constant

$$
\begin{equation*}
\frac{1}{\alpha_{s}} \cong \frac{33-2 n_{f}}{12 \pi} \log \frac{q^{2}}{\Lambda^{2}} \tag{4.16}
\end{equation*}
$$

Assuming that this logarithm and the two-gluon-exchange logarithm are the same, something that becomes more and more accurate at higher momentum transfers, we get

$$
\begin{equation*}
\frac{\sigma_{Q E D}}{\sigma_{Q C D}} \approx\left(\frac{\alpha}{\alpha_{s}}\right)^{2}\left[3\left(\frac{33-2 n_{f}}{4 \pi}\right)\left\langle Q^{2}\right\rangle\right]^{2} \tag{4.17}
\end{equation*}
$$

Putting in

$$
\begin{equation*}
n_{f}=3 \quad\left\langle Q^{2}\right\rangle=0.25 \tag{4.18}
\end{equation*}
$$

gives the bottom line

$$
\begin{equation*}
\frac{\sigma_{Q E D}}{\sigma_{Q C D}} \approx 2.6\left(\frac{\alpha}{\alpha_{s}}\right)^{2} \tag{4.19}
\end{equation*}
$$

It is also interesting to normalize this to the single-gluon exchange cross-section,
which is

$$
\begin{equation*}
\frac{d \sigma}{d t} \approx \frac{8 \pi \alpha_{s}^{2}}{9 t^{2}}=\frac{2}{9}\left(\frac{\alpha_{s}}{\alpha}\right)^{2} \cdot\left(\frac{d \sigma_{Q E D}}{d t}\right) \cdot \frac{1}{\left\langle Q^{2}\right\rangle^{2}} \tag{4.20}
\end{equation*}
$$

Thus for each generic two jet final state generated by a quark-quark collision, we expect roughly the fraction

$$
\begin{equation*}
\left.\left.\frac{\sigma_{\text {Jet }}(\text { gap })}{\sigma_{\text {Jet }}}=\left.\frac{1}{2}\left(\frac{4 \pi}{33-2 n_{f}}\right)^{2}\langle | S\right|^{2}\right\rangle\left.\approx 0.1\langle | S\right|^{2}\right\rangle \tag{4.21}
\end{equation*}
$$

to contain a rapidity-gap signature. Note that here we must also include the factor $\left.\left.\langle | S\right|^{2}\right\rangle$ for the survival-probability of the rapidity gap. The fraction of the gap events which are photon-exchange are here estimated to be

$$
\begin{equation*}
\frac{\sigma^{\mathrm{Gap}}(2-\text { gluon })}{\sigma^{\mathrm{Gap}}(\text { photon })} \approx 2.6\left(\frac{\alpha}{\alpha_{s}}\right)^{2} \sim 10^{-2}-10^{-3} \tag{4.22}
\end{equation*}
$$

The same exercise can be repeated for quark-gluon and gluon-gluon collisions; the only change is the computation of the color factors. The result is

$$
\begin{equation*}
\left.\operatorname{Im} T_{g g}\right|_{\text {singlet }}=\left.\left(\frac{9}{4}\right) \operatorname{Im} T_{g q}\right|_{\text {singlet }}=\left.\left(\frac{9}{4}\right)^{2} \operatorname{Im} T_{q q}\right|_{\text {singlet }} \tag{4.23}
\end{equation*}
$$

Since the one-gluon-exchange cross-sections obey the same conditions

$$
\begin{equation*}
\frac{d \sigma_{g g}}{d t}=\left(\frac{9}{4}\right) \frac{d \sigma_{g q}}{d t}=\left(\frac{9}{4}\right)^{2} \frac{d \sigma_{q q}}{d t} \tag{4.24}
\end{equation*}
$$

it follows that the fraction of two-jet events containing gaps should not depend on whether the initiating partons are quarks or gluons.

In the above considerations, we have uncritically assumed that higher orders in $\alpha_{s}$ do not significantly change this result. This is naive, and a proper estimate


Figure 11. Important corrections to the naive two-gluon exchange amplitude at very large $s$.
should include at the least the ladders of exchanged gluons (Fig. 11), soft radiation therefrom, and virtual-loop corrections which create damping of the tree contributions. This is more properly described by the BFKL evolution equation [17], a subject beyond the scope of this paper and the competence of its author. Qualitatively, the result of these additional contributions is an increase in strength of the $q q$ interaction at very large $s$, as well as an increase in the relative importance of the color-singlet exchange contribution. The interested reader is encouraged to consult the paper of Mueller and Navelet [18] and references therein for an overview of this phenomenon.

With respect to the considerations here, it would be reassuring to see that the addition of soft-gluon emission to the two-gluon-exchange, color singlet amplitude (Fig. 12) is suppressed if the rapidity of the gluon is in the gap region, and unsuppressed in the remaining beam-jet regions. If a simple demonstration of this exists in the literature, this author would like to know about it.

In any case, the estimate we have made, Eq. (4.21), is large. Somewhere between $0.1 \%$ and $1 \%$ of two-jet events should contain a rapidity gap between


Figure 12. Amplitude for emission (a) of a soft gluon into the rapidity gap (b). This process should be highly suppressed.
them. This should be amenable to experimental tests without much difficulty. We urge that such studies be carried out.


Figure 13. Hard triple diffraction, induced by two-gluon, color-singlet exchanges.

If the above considerations are reliable, clearly one photon exchange and even single- $W$ exchange amplitudes will be swamped by the above QCD process in all circumstances. Of more interest are the two-photon and two- $W$ processes discussed in Section 2. The strong-interaction analogue is triple hard diffraction (Fig. 13). It seems reasonable that the process as shown in Fig. 14 should be estimated via


Figure 14. Lego plot for the triple hard diffraction process shown in Fig. 13.
a "factorization" ansatz. The production of the right-hand gap $R$ can be viewed in a frame where zero eta is located in the middle of that gap. In that frame, the dynamics associated with production of the left-hand gap $L$ appears to be irrelevant to the estimate, via unitarity, we previously made. That allowed the two exchanged gluons to be viewed as a single quasi-photon being exchanged instead. By boosting into a frame in which zero eta is located in the middle of the left-hand gap, the same consideration can then be applied to the left-hand exchange as well, leading to the desired result.


Figure 15. Higgs production via a triple hard diffraction mechanism. Again the gluon pairs are required to be color singlet.

More subtle is the case of Higgs production via $g g$ annihilation plus an extra gluon exchange, as shown in Fig 15. This case has already been discussed in the literature quite a bit [19]. The simple unitarity approach we have used is no longer
so simple. This can be appreciated by going to the frame where the Higgs particle is at rest. A mix of transverse gluons and Coulomb gluons are present, something which seems to be the case in any frame of reference one might entertain. This distinguishes this situation from the ones which were discussed above. Therefore, despite its importance, we do not try to analyze this case further here, although we hope to return to it in the future.

## 5. Concluding Remarks

The physics which might be accessed using the signature of rapidity gaps, jets, and isolated leptons is unquestionably superb. But real event simulations and careful creation of optimal event selection algorithms are an essential next step in order to be ready to assess candidate background processes. For the "flagship" processes of 2-body electroweak-boson interactions at $E_{\text {cms }} \lesssim 1 \mathrm{TeV}$, I find it hard to come up with a competitive background. But every effort must be made to find and evaluate the best candidates for such background.

The candidate background processes probably will emerge from hard-diffraction physics. The phenomenology and even fundamental theory for this subject is in a primitive condition. There are at least two distinct lines which need to be followed, both theoretically and experimentally. The first is the Ingelman-Schlein program [20] of determination of the "structure function of the Pomeron," both in hadron-hadron collisions and lepton-hadron collisions. The second is the study of the frequency of rapidity gaps (in the lego plot) between coplanar jets, as discussed in Section 4. Both these test the issue of what range of momentum transfers $t$ (if any) in hard diffraction are describable in terms of two-gluon exchange, with or without BFKL enhancements.

The survival-probability of the rapidity gap $\left.\left.\langle | S\right|^{2}\right\rangle$ is not well understood, and the best answer is data. The processes mentioned above are sensitive to this, provided the underlying hard subprocess can be understood, at least semiquantitatively. Perhaps a large enough data set would be sufficient to create enough confidence in the phenomenology to allow $\left.\left.\langle | S\right|^{2}\right\rangle$ to be extracted. In the absence of that option, a safe but more infrequent process is the production of dileptons which lie within a "hard-diffraction" rapidity gap.

It is very likely that hard-diffraction processes do exist and, as emphasized by Mueller and Navelet, are enhanced by orders of magnitude from the most naive two-gluon-exchange mechanisms when the initial-state parton-parton center-of-mass momentum is sufficiently high, say of order $1-10 \mathrm{TeV}$. If the hard-diffraction production mechanisms do indeed exist, they can be utilized for new-physics processes, with improved signatures in comparison to the normal situation.

It is therefore hard to avoid the conclusion that the physics of rapidity-gaps and jets should be of great importance in the coming decades.

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[^0]:    1 IIereafter we do not distinguish between true rapidity $y$ and pseudorapidity $\eta$.

