SLAC-PUB-5614
July 1991
(T/E)

THE TAU LEPTON*

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To be published in *Reports on Progress in Physics*
by IOP Publishing Limited, Bristol, England

⋆ Work supported by the Department of Energy, contract DE–AC03–76SF00515.

Abstract

This paper reviews the present experimental knowledge of the properties of the tau lepton and the tau neutrino. The conventional theory of lepton properties and interactions is summarized; and that theory is compared with experiment. Future directions for research in tau and tau neutrino physics are described.

Contents

1. Introduction

The tau, τ, is by far the most massive of the known leptons and this gives the τ special importance in this year 1991. First of all, the τ's large mass, about 1784 MeV/c^2, leads to a great number of decay modes. Weak interaction decay theory and quantum chromodynamics should predict the complete properties of these decay modes because, unlike quarks, the tau decays as a free, isolated particle. Conversely any unexplained behavior of these decay modes could lead to the discovery of a new area in particle physics. Analogous considerations apply to the production of the τ through e^+e^- annihilation or through the decay of heavier particles. As discussed in section 9.1, there are problems in obtaining agreement between measurement and theory for some decay modes of the τ.

The second reason for the τ's importance is that the τ and its neutrino, ν_τ, constitute the third and final generation (Dydak 1991) of sequential lepton pairs. There may be other kinds of leptons with masses greater than 45 GeV/c^2. But of the sequence e, μ, τ — each charged lepton being accompanied by a small or zero mass neutrino — there are no more. The old $e-\mu$ problem (Perl 1971, Perl and Rapidus 1974) was: how does the e differ from the μ other than in mass and lepton type? The new $e-\mu-\tau$ problem is how do the three leptons differ. Perhaps studies of the rich physics of the τ can give a clue to the difference, perhaps the third and final generation has special properties?

The third reason for the τ's importance is the possible role of the ν_τ in astrophysics and cosmology. For example the present upper limit on the ν_τ mass, 35 MeV/c^2, allows the proposed dark matter in the universe to consist of ν_τ's Also, a non-zero mass ν_τ might be involved in oscillation between the ν_τ and the ν_e or ν_μ (Boehm and Vogel 1987).

This review has two purposes. First to summarize our present knowledge of the physics of the τ and ν_τ. I hope this summary will be of use to readers who are not specialists in particle physics. Hence I reference rather than discuss some areas in theory and in experimental techniques. My second purpose is to look into new experimental directions for τ and ν_τ physics: the tau-charm factory (Kirkby 1987, Kirkby 1989a, Jowett 1987, Jowett 1988), and proposals for direct ν_τ detection and study (section 8.4).

In the past few years there have been several review articles, each emphasizing different aspects of tau physics. Gan and Perl (1988) discuss the tau within the larger world of the known leptons and searches for new leptons. Barish and Stroynowski (1988) give full formulas for the polarization and cross sections in $e^+ + e^- \rightarrow \tau^+ + \tau^-$. Kiesling (1988) and Burchat (1988) describe in detail experimental techniques of studying tau decay modes. Concise and up-to-date reviews have been published by Pich (1990a, 1990b) and Stoker (1991). I shall frequently refer to these reviews to keep this article to a reasonable size. Finally, unless otherwise noted, the measured properties of the τ and ν_τ are taken from the very valuable *Review of Particle Properties* (Aguilar-Benitez *et al* 1990).

The cutoff date for references used in this review was August 1, 1991.

2. History and major properties of the τ and ν_τ

2.1. *Discovery of the τ*

The discovery of the τ came from the merging of three streams of research in particle physics. One stream was experiment and speculation on the $e - \mu$ problem. For example, in the late 1960's, my colleagues and I, as well as others, had measured the cross sections for muon-proton, $\mu - p$, inelastic scattering at high energies. We hoped that comparison with electron-proton, $e - p$, inelastic scattering would show up a hidden $e - \mu$ difference (Perl 1971, Toner *et al* 1971, Entenberg *et al* 1974, Perl and Rapidus 1974). Analogous studies of $\mu - p$ elastic scattering were carried out by Kostoulas *et al* (1974). When we found no differences and also found serious systematic errors in carrying out the comparison, my thoughts turned to a second stream of research: speculations and calculation on hypothetical varieties of leptons and their properties (Bjorken and Llewellyn Smith 1973). Most speculation around 1970 was based on one or another scheme for additive lepton type conservation, therefore it is useful to review that concept. In the example of the e and its neutrino ν_e, the electron type number n_e is assigned as follows:

$$\begin{aligned} n_e(e^-) = n_e(\nu_e) = +1 \\ n_e(e^+) = n_e(\bar{\nu}_e) = -1 \end{aligned} \tag{2.1}$$

with all other particles having $n_e = 0$. A reaction

$$a_1 + a_2 \to b_1 + b_2 + \ldots b_m \tag{2.2}$$

obeys additive electron type conservation if

$$n_e(a_1) + n_e(a_2) = n_e(b_1) + n_e(b_2) + \ldots n_e(b_m)\ . \tag{2.3}$$

Returning to the lepton speculations around 1970, one speculation proposed an excited electron, e^*, which would decay electromagnetically (Low 1965).

$$e^{*-} \to e^- + \gamma\ . \tag{2.4}$$

A similar speculation applied to the μ with

$$\mu^{*-} \to \mu^- + \gamma. \tag{2.5}$$

Another hypothesis (Bjorken and Llewellyn Smith 1973) proposed $E^\pm$ heavy leptons with $n_e(E^+) = -1$ and $n_e(E^-) = +1$. Conservation of n_e then limits decays of the so-called paraleptons to weak decays such as

$$\begin{aligned} E^- &\to e^+ + \text{ other particles} \\ E^- &\to \bar{\nu}_e + \text{ other particles} \end{aligned} \tag{2.6}$$

where the other particles have $\sum n_e = 0$.

Interest centered on the speculation that the e and μ were the first members of a sequence

$$\begin{matrix} e^- & \nu_e \\ \mu^- & \nu_\mu \\ \ell^- & \nu_\ell \\ \ell'^- & \nu_{\ell'} \end{matrix} \tag{2.7}$$

with two additional assumptions:

(i) Each pair of charged leptons and neutrinos obeys its own lepton conservation.

(ii) The charged lepton mass is larger than the neutrino mass, $m_\ell > m_{\nu_\ell}$.

In these assumptions one was simply speculating that the sequence would follow the properties of the e and the μ. The analogy to muon decay

$$\mu^- \to \nu_\mu + e^- + \bar{\nu}_e \tag{2.8}$$

would be

$$\begin{aligned} \ell^- &\to \nu_\ell + e^- + \bar{\nu}_e \\ \ell^- &\to \nu_\ell + \mu^- + \bar{\nu}_\mu \end{aligned} . \tag{2.9}$$

And given a sufficiently large $m_\ell - m_{\nu_\ell}$ difference, there would be the decays

$$\begin{aligned} \ell^- &\to \nu_\ell + \pi^- \\ \ell^- &\to \nu_\ell + \rho^- \\ \ell^- &\to \nu_\ell + \pi^- + n\pi^0 \ , \ n \geq 1 \\ \ell^- &\to \nu_\ell + \pi^- + \pi^+ + \pi^- \end{aligned} \tag{2.10}$$

and so forth.

The simplicity of these ideas allowed calculation of the relative probability of the ℓ decaying to a particular mode i, the branching fraction $B(i)$, once m_ℓ and m_{ν_ℓ} were assumed. Tsai (1971) wrote the seminal paper giving the $B(i)$ for most of the modes in (2.9) and (2.10) and considering m_ℓ up to 6 GeV. Tsai also calculated the ℓ lifetime. Another early paper was Thacker and Sakurai (1971).

The third research stream leading to the discovery of the τ was the succession of inventions and improvements which led to the construction of circular electron-positron colliders (Richter 1966, Kohaupt and Voss 1983). There were many ways to search for special kinds of hypothetical leptons (Perl and Rapidis 1974) but the electromagnetic process

$$e^+ + e^- \rightarrow \gamma_{virtual} \rightarrow \ell^+ + \ell^- \tag{2.11}$$

provided a general and comprehensive way to search for any charged lepton with $m_\ell < E_{beam}$. (Here E_{beam} is the energy of the e^+ or e^- beam in the collider.) If ℓ is a point particle, the cross section for the reaction in (2.11) is

$$\text{spin} = 0: \quad \sigma = \frac{\pi\alpha^2(\hbar c)^2\beta^3}{3s} \tag{2.12a}$$

$$\text{spin} = 1/2: \quad \sigma = \frac{4\pi\alpha^2(\hbar c)^2}{3s}\ \frac{\beta(3-\beta^2)}{2} \tag{2.12b}$$

Here α is the fine structure constant, $\hbar$ is Planck's constant h divided by 2π, c is the velocity of light. Also $\beta = v/c$ where v is the velocity of the ℓ. Sometime I will give formulas in elementary particle physics units which set $\hbar = 1,\ c = 1$. The total energy in the center-of-mass is E_{tot} and

$$s = E_{tot}^2$$

Unless otherwise noted, all e^+e^- collision formulas in this article are based on the e^+ and e^- having equal energies, E_{beam}, and exactly opposite momenta. Thus

$$E_{tot} = 2\ E_{beam}\ ,\ \ s = 4\ E_{beam}^2$$

If the lepton is not a point particle then the cross section in (2.12) has to be multiplied by the square of an unknown from factor F(s) and the cross section becomes uncertain. But most thinking in the early 1970's assumed the lepton would be a point particle, hence one could depend on the cross section in (2.12). Since the event rate in a collider is

$$N = \sigma\ \mathcal{L}$$

where $\mathcal{L}$ is the collider luminosity in $\text{cm}^{-2}\ \text{s}^{-1}$, with large enough E_{beam} and $\mathcal{L}$ one could make a definite search for any charged lepton.

With electron-positron storage rings being constructed and the production mechanism understood, the final question was the detection. Since the decay

$$K^- \rightarrow \ell^- + \bar{\nu}_\ell$$

analogous to

$$K^- \rightarrow \mu^- + \bar{\nu}_\mu$$

had not been seen, the ℓ if it existed had a mass greater than about 490 MeV/c^2. Then the ℓ lifetime would be too short (Tsai 1971, Thacker and Sakurai 1971) for detecting the ℓ before it decayed. The ℓ had to be identified through its decay modes!

The first detectors were built or being built for the ADONE, DORIS, and SPEAR e^+e^- storage rings. With respect to these first detectors, there was little experience on how well electrons, muons, and charged pions could be separately identified. Therefore for several reasons the best way to search for an ℓ was to look for $e\mu$ events from

$$\begin{gathered} e^+ + e^- \rightarrow \ell^+ + \ell^- \\ \text{and} \\ \ell^+ \rightarrow \bar{\nu}_\ell + e^+ + \nu_e \\ \ell^- \rightarrow \bar{\nu}_\ell + \mu^- + \bar{\nu}_\mu \\ \text{or} \\ \ell^+ \rightarrow \bar{\nu}_\ell + \mu^+ + \nu_\mu \\ \ell^- \rightarrow \nu_\ell + e^- + \bar{\nu}_e \end{gathered} \tag{2.13}$$

First, the decay theory was most certain for these purely leptonic decays. Second, from (2.13) the directly observed event would be

$$e^+ + e^- \rightarrow e^\pm + \mu^\mp + \text{missing energy} \tag{2.14}$$

since the four neutrinos would not be detected. This would be a most unusual event since it would have a substantial fraction of energy missing and would appear to violate lepton conservation. Third, the efficiency of the detector for e and μ selection could be measured using the reactions

$$\begin{gathered} e^+ + e^- \rightarrow e^+ + e^- \\ e^+ + e^- \rightarrow \mu^+ + \mu^- \end{gathered} \tag{2.15}$$

Finally the sought events in (2.14) could be distinguished from the large number of events from (2.15) by requiring the e and μ not to be collinear.

These search ideas were used by two groups of experimenters at the ADONE e^+e^- storage ring at Frascati and by the Mark I group at SPEAR. The search results of one Frascati group led by M. Bernardini and A. Zichichi (Alles-Borelli *et al* 1970, Bernardini *et al* 1973) are shown in figure 1. One search region applies to a lepton with the sequential decay modes in (2.9) and (2.10); the other search region assumes a lepton with only the leptonic decay modes in (2.9). The other search at ADONE was conducted by Orioto *et al* (1974). Their search region also reached somewhat above 1 GeV/c^2.

The lepton search ideas described above were incorporated in the Mark I Proposal SP-2 of December, 1971 to the Stanford Linear Accelerator Center. A few words about SPEAR and the Mark I Collaboration: SPEAR was built under the leadership of B. Richter and J. Rees. The Mark I experimenter in this proposal (Table 1) came from SLAC Group C, led by B. Richter, SLAC Group E led by M. Perl, and a Lawrence Berkeley Laboratory group led by W. Chinowsky, G. Goldhaber and G. Trilling. When the Mark I experimenters first began taking data in 1973, the detector had no μ detection and could not be used for searching for $e\mu$ events. Under the leadership of G.Feldman a crucial μ detecting tower was added.

In 1974 we began to find events fitting the reaction

$$e^+ + e^- \to e^\pm + \mu^\mp + \text{missing energy} \tag{2.16}$$

when the total energy was above about 4 GeV. Figure 2 is an example. As such events accumulated in our Mark I data we began to believe that we had discovered the pair production of a new particle of mass $\lesssim$ 2 GeV/c^2, the production rate arguing for the particle having a point nature. It was not clear from the first data whether the particle was a lepton with the decays

$$\begin{aligned} \ell^- &\to \nu_\ell + e^- + \bar{\nu}_e \\ \ell^- &\to \nu_\ell + \mu^- + \bar{\nu}_\mu \end{aligned} \tag{2.17}$$

or the particle was a boson X with decays.

$$\begin{aligned} X^- &\to e^- + \bar{\nu}_e \\ X^- &\to \mu^- + \bar{\nu}_\mu \end{aligned} \tag{2.18}$$

I have described elsewhere (Perl 1990) the atmosphere of excitement, uncertainty, and confusion involved in the discovery and understanding of the τ in the years 1974–1978.

By the middle of 1975 the Mark I experimenters believed we had found a new particle, and we published in Physical Review Letters (Perl *et al* 1975). The final paragraph read:

> "We conclude that the signature $e-\mu$ events cannot be explained either by the production and decay of any presently known particles or as coming from any of the well-understood interactions which can conventionally lead to an e and a μ in the final state. A possible explanation for these events is the production and decay of a pair of new particles, each having a mass in the range of 1.6 to 2.0 GeV/c^2.

Thus ended the first period in the history of the τ, the initial discovery.

2.2. *History of the τ and ν_τ: 1975–1991*

In this section the history of research on the τ and ν_τ is outlined, providing perspective for the main subject of this review – the present knowledge of the τ and ν_τ.

It took about three years, 1975 through 1978, to confirm the discovery of the τ, to settle that the τ was a lepton, and to elucidate the major properties of the τ. This research used the SPEAR e^+e^- storage ring at the Stanford Linear Accelerator Center and the Doris ring at the Deutsches Elektronen-Synchrotron (DESY). Very important was the finding of the so-called anomalous μ events.

$$e^+ + e^- \rightarrow \mu^\pm + \text{hadrons} + \text{missing energy} \tag{2.19}$$

where the μ comes from the decay of one τ, the hadrons from the other τ. Crucial papers came from the Mark I experiment (Feldman *et al* 1977) and another SPEAR experiment (Cavalli-Sforza *et al* 1976), and from the PLUTO experiment at DORIS (Burmester *et al* 1977a). The PLUTO experimenters also confirmed the existence of $e - \mu$ events (Burmester *et al* 1977b). Of similar importance was the identification of e-hadron events. Much of the early work used the new DELCO detector (Bacino *et al* 1978) and the Mark I detector modified with a "lead glass wall", (Barbaro-Galtieri *et al* 1977) both at SPEAR.

The 1975–1978 period, the second period in τ history, ended with two major accomplishments. One, the decay mode

$$\tau^- \rightarrow \pi^- + \nu \tag{2.20}$$

which had been difficult to find was finally identified and its branching fraction measured (Feldman 1978). Two, the mass of the τ was measured carefully by the DASP experimenters at DORIS (Brandelik *et al* 1978) and by the DELCO experimenters (Bacino *et al* 1978).

The discovery and confirmation of the existence and leptonic nature of the τ involved the indirect discovery of the ν_τ. To this day there has been no direct detection of the ν_τ by observing the interaction of the ν_τ with nucleons (section 8.4), as has been done for the ν_e and ν_μ. Nevertheless as τ research proceeded, more and more was learned about the ν_τ (sections 2.4 and 8), and all deduced properties of the ν_τ are consistent with the ν_τ being a conventional neutrino.

The third period of τ research, 1979–1985, was dominated by the new higher energy e^+e^- storage rings, PETRA at DESY and PEP at SLAC. The pre-1979 τ studies at DORIS and SPEAR were limited to total energies below 8 GeV, these new colliders provided energies up to 44 GeV. There were five detectors at PETRA: CELLO, JADE, MARK J, PLUTO, TASSO; and four at PEP: DELCO, HRS, MAC, and MARK II. These experiments allowed a large number of studies of the production and decay of the τ; many τ properties given in this review are the average values of measurements from these experiments. The higher energies of PETRA and PEP also allowed measurement of the τ lifetime. Three early measurements came from the Mark II (Feldman *et al* 1982) and MAC (Ford *et al* 1982) experiments at PEP and the CELLO experiment (Behrend *et al* 1983) at PETRA.

The 1979–1985 period also saw the beginning of new sets of τ studies at lower energies. The CESR e^+e^- storage ring at Cornell and the rebuilt DORIS II ring at DESY began operating at about 10 GeV where the upsilon resonances and mesons containing bottom quarks are produced. Tau pairs are also produced and so the CLEO experimenters at CESR and the CRYSTAL BALL and ARGUS experimenters at DORIS began to study the τ. In addition, the new Mark III detector at SPEAR began operation and the Mark III experimenters began a new era of τ studies at low energies.

The 1979–1985 period ended with increased interest in the physics of the τ, the increased interest coming from the comparison of new τ measurements with new calculations on τ branching ratios. The new measurements came from the experiments just listed. The new branching ratio calculations came from precise applications of weak interaction decay theory to the τ by Truong (1984) and by Gilman and Rhie (1985). The comparison of measurement with theory led to a mystery in the 1-charged-particle decay modes of the τ, a mystery which is not resolved as of 1991 (section 9.1).

The modern period of τ research, 1985 to the present, is broad and eclectic in experimental techniques and theory. Higher energy τ studies, up to about 65 GeV, have come from the experiments at the TRISTAN e^+e^- storage ring: AMY, TOPAZ and VENUS. A wealth of τ data is coming from the study of Z^0:

$$\begin{aligned} &e^+ + e^- \rightarrow Z^0 \\ &Z^0 \rightarrow \tau^+ + \tau^- \end{aligned} \tag{2.21}$$

The present sources of τ data are the ALEPH, DELPHI, L3, and OPAL experiments using the LEP e^+e^- storage ring at CERN with some early contribution from the Mark II detector at the SLAC e^+e^- linear collider.

In 1985 experiments at the CERN proton-antiproton colliders began to contribute to τ research (Savoy-Navarro 1985) through study of the decay

$$W^- \rightarrow \tau^- + \bar{\nu}_\tau \tag{2.22}$$

the W^- being produced in

$$p + \bar{p} \rightarrow W^- + \text{hadrons} \tag{2.23}$$

This τ production process is valuable for both the study of the $\tau - W - \nu_\tau$ vertex and as a signal for W production (section 4.1).

2.3. Major properties of the τ

The τ mass is

$$m_\tau = 1784.1^{+2.7}_{-3.6} \ \text{MeV/c}^2 \tag{2.24}$$

based mostly on a 1978 study (Bacino *et al* 1978) of the behavior of σ_τ at threshold. Further precise measurements of m_τ are certainly required. Turning to the other static properties of the τ, the electric change is of course equal to the charge on the electron and the τ spin is 1/2. References on the τ and ν_τ spins are Alles and Alles-Borelli (1976), Tsai (1978), Kirkby (1978) and Alles (1979). There are no direct measurements of the τ magnetic moment.

The weak interactions of the τ through the $\tau - W - \nu_\tau$ and $\tau - Z^0 - \tau$ vertices agree with conventional theory (excluding the uncertainties discussed in section 9). This agreement is partly based on measurement and partly on faith in the universality of V-A coupling. For example, the expected V-A structure of the $\tau - W - \nu_\tau$ vertex in τ decay needs a great deal more testing (section 9).

Table 2 lists the τ decay modes with branching fractions greater than about 5%. Here as throughout this article, only the τ^- decay mode is listed. It is always assumed that the charge conjugate τ^+ decay mode has the same branching fraction and other decay properties.

2.4. Major properties of the ν_τ

All our knowledge of the properties of the tau neutrino, ν_τ, is deduced from τ decays. The upper limit on the mass (Albrecht *et al* 1988) is

$$m_{\nu_\tau} < 35 \ \text{MeV/c}^2 \tag{2.25}$$

with a 95% confidence level. This limit comes from studying the decay

$$\tau^- \rightarrow \nu_\tau + 3\pi^- + 2\pi^+ \tag{2.26}$$

and measuring the end point of the mass spectrum of the five-pion system. The ν_τ may have zero mass.

As far as we know the ν_τ is stable, lower limits on the lifetime are discussed in section 8. The ν_τ spin is 1/2 (Alles 1979).

The weak interaction couplings of the ν_τ in the $\tau - W - \nu_\tau$ vertex agree with conventional theory. As with the τ this agreement is based partly on measurements and partly on faith in the universality of V-A coupling.

2.5. *The other known leptons*

The properties of the four other known leptons in comparison with the τ and ν_τ are given in Table 3. We have no answers to basic questions about the existence and masses of these leptons: why are there 3 generations? What rule or law fixes the masses or their ratios? Do the neutrinos have zero or non-zero masses?

3. τ pair production in e^+e^- annihilation

3.1. *General formulas for τ pair production*

The reaction

$$e^+ + e^- \rightarrow \tau^+ + \tau^- \tag{3.1}$$

takes place through the two diagrams in figure 3. The considerations here are restricted (a) to the case in which the e^+ and e^- beams have equal and opposite momentum, hence the laboratory frame is the barycentric frame, and (b) to unpolarized e^+ and e^- beams. The differential cross section is

$$\frac{d\,\sigma_\tau}{d\Omega} = \frac{\alpha^2(\hbar c)^2}{4s}\,(f_{\gamma\gamma} + f_{\gamma z} + f_{zz}) \tag{3.2}$$

The functions $f_{\gamma\gamma}$, $f_{\gamma z}$, and f_{zz} are the contributions from γ exchange, from the interference between γ and Z^0 exchange, and from Z^0 exchange.

Next we have to specify the strengths and structures of the $\tau-\gamma-\tau$ and $\tau-Z^0-\tau$ vertices; always assuming that the $e-\gamma-e$, $e-Z^0-e$, γ propagator and Z^0 propagator obey conventional electroweak theory (Halzen and Martin 1984, Quigg 1983). We immediately take the $\tau-\gamma-\tau$ vertex to have the conventional form

$$V^\mu_{\tau\gamma\tau} = -i\,q\,\gamma^\mu \tag{3.3}$$

where q is the unit of electron charge and γ^μ is a Dirac matrix. Tests of this are discussed in section 3.4.

We allow more flexibility to the $\tau - Z^0 - \tau$ vertex so that structure can be tested quantitatively. We begin with the standard structure for the $e - Z^0 - e$ vertex (Halzen and Martin 1984)

$$V^\mu_{eze} = -i\,\frac{g_{weak}}{\cos\theta_W}\,\gamma^\mu\,(v_e - a_e\gamma^5) \tag{3.4}$$

There are several ways to normalize the vector, v, and axial vector, a, coupling parameters. I use (Ellis and Gaillard 1976)

$$v_e = -1 + 4\sin^2\theta_W \tag{3.5}$$

$$a_e = -1 \tag{3.6}$$

as being easiest to remember. Here θ_W is the Weinberg angle with

$$\sin^2\theta_W = 0.2259 \pm 0.0046 \tag{3.7}$$

In analogy we define the $\tau - Z^0 - \tau$ vertex

$$V^\mu = -i\,\frac{g_{weak,\tau}}{\cos\theta_w}\,\gamma^\mu\,(v_\tau - a_\tau\gamma^5) \tag{3.8}$$

General formulas for $d\sigma_\tau/d\Omega$ are given by Jadach and Was (1989) as functions of the energy, E_{tot}, and of the weak interaction parameters; v_τ, a_τ, θ_W and g_{weak}. I shall not repeat these general formulas here, but shall discuss $d\sigma_\tau/d\Omega$ and σ_τ in five different energy regions. These regions, figure 4, are: from the τ pair production threshold to about 10 GeV, from about 10 GeV to just below the Z^0 resonance, the Z^0 resonance itself, and far above the Z^0 resonance. An overview of the behavior of σ_τ with E_{tot} is given in figure 4.

3.2. *From threshold to about 10 GeV*

The energy region between the τ pair threshold, 3.56 GeV, and about 10 GeV is important for three reasons. First, present τ research uses the CESR, DORIS II, and BEPC storage rings in that region. Second, the proposed tau-charm factories, two-ring e^+e^- colliders specifically designed for τ physics (section 10), will operate between τ threshold and about 5 GeV. Third, the proposed B-factory e^+e^- colliders, designed for bottom quark physics will operate at a barycentric energy of about 10 GeV.

At 10 GeV and below we neglect $f_{\gamma z}$ and f_{zz} in (3.2). Then excluding electromagnetic radiative corrections

$$\frac{d\sigma_\tau}{d\Omega} = \frac{\alpha^2(\hbar c)^2}{4s}\,\beta(2 - \beta^2\sin^2\theta) \tag{3.9}$$

and

$$\sigma_\tau = \frac{4\pi\alpha^2(\hbar c)^2\beta}{3s}\,\frac{(3-\beta^2)}{2} \tag{3.10}$$

for $\sqrt{s} \lesssim 10$ GeV. Numerically, with s in GeV^2.

$$\sigma_\tau = \frac{86.8}{s}\left[\frac{\beta(3-\beta^2)}{2}\right]\ \text{nb} \tag{3.11}$$

In (3.9) θ is the angle between the e^- momentum and the τ^- momentum. The maximum values of σ_τ in this region occurs at $\sqrt{s} = 4.2$ GeV and is 3.5 nb. Once β

approaches 1, σ_τ is proportional to $1/E_{tot}^2$ leading to a rapid decrease in σ_τ as E_{tot} increases (figure 4) in this low energy region.

It is convenient to define

$$R_\tau = \sigma_\tau / \sigma_{point}$$

where

$$\sigma_{point} = \frac{4\pi\alpha^2(\hbar c)^2}{3s} \tag{3.12}$$

For precise comparison of theory and measurement radiative corrections must be applied to the formulas for $d\sigma_\tau/d\Omega$ and σ_τ. In this energy range these corrections (Bonneau and Martin 1971, Schwitters and Strauch 1976) are purely electromagnetic with the contributions being: photon loops in the e^+ and e^- lines, figure 5a; one-photon exchange at the $e - \gamma - e$ vertex, figure 5b; charged particle loops in the γ propagator, figure 5c; and emission of real photons by the e^+ or e^-, figure 5d. At this energy analogous effects at the $\tau - \gamma - \tau$ vertex and on the τ lines are ignored. The first three effects modify the flux of the single virtual photons involved in one-photon exchange as noted by Schwitters and Strauch (1976). These amplitudes contribute in lowest order in α to σ_τ by interfering with the basic amplitude in figure 3a, hence they correct (3.9) and (3.10) by a term of order α. The amplitude in figure 5d contributes squared, hence it also gives an order α correction. The last effect, the emission of a real photon by the e^+ or e^- reduces the barycentric energy at which $e^+ + e^- \to \tau^+ + \tau^-$ occurs. If k is the energy of the emitted photon, the new barycentric energy is given by

$$s' = s - sk/\sqrt{s} \tag{3.13}$$

From Bonneau and Martin (1971) as formulated by Schwitters and Strauch (1976)

$$\begin{aligned}\sigma_{\tau,rad.corr.} = \sigma_\tau(s)\left[1 + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{6} - \frac{17}{13}\right) + \frac{13}{12}\,t\right] \\ + \left[\sigma_\tau(s') - \sigma_\tau(s)\right] \int_0^E \frac{dk}{k}\left(\frac{k}{E}\right)^t \left(1 - \frac{k}{E} + \frac{k^2}{E^2}\right)\end{aligned} \tag{3.14}$$

where

$$t = \frac{2\alpha}{\pi}\left[\ell n\left(\frac{s}{m_e^2}\right) - 1\right]$$
$$E = \sqrt{s}/2$$

In (3.14) the first term increases $\sigma_\tau(s)$ by a factor of order α, but the second term can have a much larger effect if $\sigma_\tau(s')$ is substantially different from $\sigma_\tau(s)$ due to the $1/s$ term in (3.10).

A more general discussion of these radiative corrections is given by Berends and Kleiss (1981) and by Berends and Böhm (1988).

In this section I have discussed only the differential and total cross sections for

$$e^+ + e^- \to \tau^+ + \tau^- \quad ,$$

and furthermore only when the e^+ and e^- beams are unpolarized. However by measuring the angle and energy correlations between the decay products of the τ^+ and τ^-, more extensive investigations can be conducted of the $\tau - \gamma - \tau$ vertex and the τ decay process. Quoting from Tsai (1971) where ℓ can now be replaced by τ:

> "Far above the threshold, the helicities of ℓ^+ and ℓ^- tend to be opposite to each other. Near the threshold the directions of spins of ℓ^+ and ℓ^- prefer to be parallel to each other, and the sum of the two spins prefers to be either parallel or anti-parallel to the direction of the incident electron. Because the parity conservation is violated maximally in the decays of ℓ^+ and ℓ^-, the angular distributions of decay products depend strongly on the spin orientation of the heavy leptons. Since the spins of ℓ^+ and ℓ^- are strongly correlated in the production, we found a strong correlation between the energy-angle distributions of the decay products of ℓ^+ and ℓ^-."

For example if

$$\tau^+ \to \nu_\tau + \pi^+$$
$$\tau^- \to \nu_\tau + \pi^-$$

there are angles and energy correlations between the π^+ and π^- which depend on the properties of the $\tau - \gamma - \tau$ vertex. This area has been extensively studied theoretically by Nelson and his colleagues in a series of papers (Goozovat and Nelson 1991, Nelson 1991, Nelson 1990).

Quoting Goozovat and Nelson (1991)

> "At the $\gamma^* \to \tau^+\tau^-$ vertex, measurements can be made to better establish the vertex's P and C symmetry properties ..., and to test for an unexpected violation of CP/T invariance in $\gamma^* \to \tau^+\tau^-$."

Another area for future research requires longitudinal polarization of the incident e^- or e^+ beams (Lemke 1990).

3.3. At threshold

As E_{tot} decreases towards $2m_\tau$, the τ pair production threshold, σ_τ in (3.10) goes to 0 as β goes to 0. However, it was pointed out by Voloshin (1989) that the formula in (3.10) is modified at threshold by the attractive coulomb force between the τ^+ and τ^-. The cross section σ_τ in (3.10) is to be multiplied by

$$F_c = \frac{\pi\alpha/\beta}{1 - \exp\left(-\pi\alpha/\beta\right)} \tag{3.15}$$

Thus at threshold where $s = 4m_\tau^2$ and $\beta = 0$

$$\sigma_\tau \text{ (threshold)} = \frac{\pi^2 \alpha^3 (\hbar c)^2}{2m_\tau^2} = 0.23 \text{ nb} \tag{3.16}$$

This does not include the radiative corrections of (3.14). Noting that the maximum low energy value of σ_τ is 3.5 nb at 4.2 GeV, one sees that at threshold σ_τ is about 1/10 of its maximum value. This is important for the research in tau physics proposed for the tau-charm factory, section 10. As of this writing, 1991, the formula in (3.15) has not been checked by experiment.

3.4. From about 10 GeV to below the Z^0 resonance

The region from about 10 GeV to 70 GeV was extensively studied in the 1980's by experiments at the PETRA (Wu 1984), PEP (Rückstuhl 1984, Gan 1985), and TRISTAN (Ozaki 1987, Abe 1990) e^+e^- storage rings. From these experiments comes much of our present data on τ decays, data which is used in sections 5 through 7. At this writing, 1991, only the experiments at the TRISTAN ring (Abe 1990) are continuing to collect data.

In this energy region the τ production cross section, σ_τ, is still dominated by $f_{\gamma\gamma}$, the photon exchange process, hence σ_τ is decreasing as $1/s$. As the energy increases Z^0 exchange affects σ_τ through the $f_{\gamma z}$ interference term in (3.2). Thus the first studies of the $\tau - Z^0 - \tau$ vertex were made in the 1980's in this energy region (Barish and Stroynowski 1988). But these studies have been superseded by the more precise measurements of $e^+e^- \to \tau^+\tau^-$ at the Z^0 resonance.

The dominance of $f_{\gamma\gamma}$ and the small effect of $f_{\gamma z}$ allowed extensive searches for deviations from conventional electroweak theory. Two different models were used to parameterize deviations. One model, an old one (Feynman 1949, Drell 1958), allows for modifications of the photon propagator or $\tau - \gamma - \tau$ vertex in the diagram of figure 3a such as

$$\sigma_\tau \text{ (modified)} = \sigma_\tau \, F_\pm^2(s) \tag{3.17a}$$

where

$$F_\pm(s) = 1 \mp \frac{s}{s - \Lambda_\pm^2} \tag{3.17b}$$

The other newer model (Eichten, Lane and Peskin 1983) assumes that the τ and e are composite particles and introduces an effective Lagrangian for a contact interaction between the constituent particles. Thus for a vector-vector interaction

$$L_{eff} = \pm \frac{g^2}{2\Lambda_\pm^{c2}} \, \bar{\psi}_2 \gamma^\mu \psi_2 \bar{\psi}_1 \gamma_\mu \psi_1 \tag{3.18}$$

with $g^2/4\pi$ set equal to 1 to define Λ^c.

No deviations have been found, hence there are only lower limits on the parameters $\Lambda_\pm$ and $\Lambda^c_\pm$. Examples are given in Table 4.

3.5. *At the Z^0 resonance*

At present, 1991, this is an area of active research by the four experiments at the LEP circular e^+e^- collider (Dydak 1991): ALEPH, DELPHI, L3, and OPAL, and there are the early results from the Mark II experiment (Abrams *et al* 1989) at the SLAC linear e^+e^- collider. The SLD experiment at that linear collider is begiinning to acquire data.

At the Z^0 resonance all three terms in (3.19) are important, but the τ mass can be set to 0. Following the treatment of Ellis and Gaillard (1976) in

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2(\hbar c)^2}{4s}\left(f_{\gamma\gamma} + f_{\gamma z} + f_{zz}\right) \tag{3.19}$$

$$f_{\gamma\gamma} = 1 + \cos^2\theta \tag{3.20a}$$

$$f_{\gamma z} = 2\left[v_e v_\tau\left(1+\cos^2\theta\right) + 2a_e a_\tau \cos\theta\right] Re\,\chi \tag{3.20b}$$

$$\begin{aligned} f_{zz} = {} & \left[\left(v_e^2 + a_e^2\right)\left(v_\tau^2 + a_\tau^2\right)\left(1+\cos^2\theta\right) + \right. \\ & \left. 8\, v_e\, v_\tau\, a_e\, a_\tau \cos\theta\right] |\chi|^2 \end{aligned} \tag{3.20c}$$

Here v and a are defined in section 3.1 and

$$\chi = \frac{G_F}{8\sqrt{2}\pi\alpha}\,\frac{sm_z^2}{s - m_z^2 + im_z\Gamma_z} \tag{3.21}$$

where m_z is the Z^0 mass, Γ_z is the Z^0 width, and θ_W is the Weinberg angle. Present values are

$$m_z = 91.161 \pm 0.031 \text{ GeV}$$
$$\Gamma_z = 2.534 \pm 0.027 \text{ GeV}$$

It is convenient to define

$$g = \frac{G_F}{8\sqrt{2}\pi\alpha} = 4.497 \times 10^{-5} \text{ GeV}^{-2} \tag{3.22a}$$

so that

$$\chi = g\,\frac{sm_z^2}{s - m_z^2 + im_z\Gamma} \tag{3.22b}$$

The χ term describes the Z^0 resonant shape without radiative corrections and is symmetric about $\sqrt{s} = m_z$. With radiative corrections σ_τ is smaller at the peak and has an unsymmetric shape. Some of the more accessible references to radiative corrections in the Z^0 region are Ellis and Peccei (1986), Alexander *et al* (1988) and Cahn (1987).

It is convenient to rewrite (3.19) in the form

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2(\hbar c)^2}{4s}\left[A_{sym}(1+\cos^2\theta) + A_{asym}\cos\theta\right] , \qquad (3.23)$$

that is in terms symmetric and asymmetric in $\cos\theta$. Here

$$A_{sym} = 1 + 2\,v_e v_\tau \, Re\chi + (v_e^2 + a_e^2)\,(v_\tau^2 + a_\tau^2)\;|\chi|^2 \qquad (3.24a)$$

$$A_{asym} = +4a_e\, a_\tau \, Re\,\chi + 8v_e v_\tau a_e a_\tau \;|\chi|^2 \qquad (3.24b)$$

In the total cross section there is no contribution from the asymmetric term and

$$\sigma_\tau = \frac{4\pi\alpha^2(\hbar c)^2}{3s} A_{sym} \qquad (3.25)$$

At $\sqrt{s} = m_z$, ignoring the radiative corrections

$$Re\,\chi = 0 \qquad (3.26a)$$

$$|\chi| = \frac{gm_z^3}{\Gamma_z} = 13.4 \qquad (3.26b)$$

$$\sigma_\tau(\sqrt{s} = m_z, \text{no rad.corr.}) = \frac{4\pi\alpha^2(\hbar c)^2}{3m_z^2}\left[1 + (v_e^2 + a_e^2)(v_\tau^2 + a_\tau^2)\left(\frac{gm_z^3}{\Gamma_z}\right)^2\right] \qquad (3.27)$$

With $a_e = -1$, $a_\tau = -1$, the approximations $v_e \approx 0$ $v_\tau \approx 0$, and using (3.26b) the second term in (3.27) is much larger than the first; the Z^0 exchange dominates. Thus

$$\begin{aligned} R_\tau(\sqrt{s} = m_z) &= \frac{g^2 m_z^6}{\Gamma_z^2} \approx 180 \\ \sigma_\tau(\sqrt{s} = m_z, \text{no rad.corr.}) &\approx 1.9 \text{ nb} \end{aligned} \qquad (3.28)$$

Radiative corrections reduce this σ_τ and shift the peak. Approximately (Cahn 1987)

$$\sigma_\tau(Z^0 \text{ peak, rad.corr.}) \approx 1.4 \text{ nb} \qquad (3.29)$$

The asymmetric term in (3.24) allows precise studies of v_τ and a_τ through measurement of the forward-backward asymmetry parameter

$$A_{FB} = (n_F - n_B)/(n_F + n_B) \qquad (3.30)$$

Here n_F and n_B are the number of τ^- with $0 \leq \theta < 90°$ and $90° < \theta \leq 180°$, the forward and backward hemispheres with respect to the e^- direction. From (3.23) with integration over all angles

$$A_{FB} = \frac{3}{8}\, A_{asym}/A_{sym} \tag{3.31}$$

From (3.24) it can be seen that A_{FB} is a complicated function of v_e, a_e, v_τ, a_τ, and θ_W. In the neighborhood of the Z^0 the behavior of A_{FB} as a function of s is approximated by setting $v_e = 0$, $v_\tau = 0$ in (3.24). Then

$$A_{FB} \approx \frac{3\ R_e\chi}{2\,a_e^2\,a_\tau^2\,|\chi|^2} \approx \frac{3(s-m_z^2)}{2\,a_e^2\,a_\tau^2\,gsm_{z^2}} \tag{3.32}$$

Thus A_{FB} changes sign in going through the resonance.

At $\sqrt{s} = m_z$ we cannot neglect v_e and v_τ since $Re\chi = 0$ at $\sqrt{s} = m_z$, hence

$$A_{FB}(\sqrt{s} = m_z) = \frac{3v_e v_\tau a_e a_\tau |\chi|^2}{1 + (v_e^2 + a_e^2)(v_\tau^2 + a_\tau^2)\ |\chi|^2} \tag{3.33a}$$

Neglecting the 1 in the denominator

$$A_{FB}(\sqrt{s} = m_z) \approx 3\left(\frac{v_e a_e}{v_e^2 + a_e^2}\right)\left(\frac{v_\tau a_\tau}{v_\tau^2 + a_\tau^2}\right) \tag{3.33b}$$

The experiments at LEP (Dydak 1991) find that the measured values of A_{FB} agree with the τ having the conventional values of v_τ and a_τ. Jadach and Was (1989) have given a thorough discussion of the theory and measurement techniques.

As at lower energies (section 3.2) there is much to be investigated at the Z^0 using the correlations between the decay products of the two τ's. Bernabéu, Rius and Pich (1991) have reviewed the subject and give earlier references. Other references are Goozovat and Nelson (1991) and Nelson (1991, 1990).

In all the above discussions, the e^- and e^+ beams were unpolarized, but in the next few years e^+e^- collisions at the Z^0 resonance with longitudinally polarized e^- beams will be available, (Treille 1990, Swartz 1990, Barish and Stroynowski 1988). The longitudinally polarized e^- beam enhances the accuracy of investigations of the $\tau - Z^0 - \tau$ vertex.

3.6. *Above the Z^0 resonance*

Well above the Z^0 resonance χ in (3.22b) simplifies to the constant

$$\chi_\infty = gm_z^2 = .37 \tag{3.34}$$

Then the three f terms in (3.19) are all independent of the energy, and $d\sigma_\tau/d\Omega$ and

σ_τ once more are proportional to $1/s$. Indeed

$$\sigma_\tau = \frac{4\pi\alpha^2(\hbar c)^2}{3s} \; [1 + (v_e^2 + a_e^2)(v_\tau^2 + a_\tau^2) \; \chi_\infty^2] \tag{3.35}$$

Since

$$(v_e^2 + a_e^2)(v_\tau^2 + a_\tau^2)\,\chi_\infty^2 = .14 \tag{3.36}$$

the first term in (3.33) contributes most to the cross section. Thus at very high energies as at low energies, $e^+e^- \rightarrow \tau^+\tau^-$ occurs mainly through the one-photon exchange diagram in figure 3a.

If there are no new processes contributing to $e^+e^- \rightarrow \tau^+\tau^-$ in the TeV energy region

$$\sigma_\tau = \frac{0.1}{s}\ \text{pb} \tag{3.37}$$

with s in TeV^2, a very small cross section compared to that in (3.11), the low energy region, or to the 1.4 nb at the Z^0 peak.

4. $\tau - \nu_\tau$ production in particle decays

4.1. Decay of the W to τ and ν_τ

The decay process

$$W^+ \rightarrow \tau^+ + \nu_\tau \tag{4.1}$$

is used for two purposes. The narrower purpose is to test the $\tau - W - \nu_\tau$ at the mass of the W. The broader purpose (Savoy-Navarro, 1990) is to detect the presence of a W through the $W \rightarrow \tau\nu_\tau$ decay and the subsequent decays

$$\tau \rightarrow \text{1-charged-particle} + \text{neutral-particles} \tag{4.2a}$$

or

$$\tau \rightarrow \text{3-charged-particle} + \text{neutral-particles} \tag{4.2b}$$

The decay width for

$$W^+ \rightarrow \ell^+ + \nu_\ell \tag{4.3}$$

with $\ell = e,\ \mu$, or τ is

$$\Gamma(W^+ \rightarrow \ell^+\nu_\ell) = \frac{G_F m_W^3}{6\sqrt{2}\pi} = 0.23\ \text{GeV} \tag{4.4a}$$

The lepton mass is set to zero here since even for the τ, $(m_\tau/m_W)^2 << 1$. Since the total W width is 2.11 ± 0.03 GeV

$$B(W^+ \to \ell^+ \nu_\ell) = 0.11 \tag{4.4b}$$

We expect $\Gamma(W^+ \to \tau^+\nu_\tau) = \Gamma(W^+ \to e^+\nu_e)$. This has been tested in the UA1 experiment at CERN by Albajar *et al* (1987); they find

$$g_\tau / g_e = 1.01 \pm 0.09 \pm 0.05 \tag{4.5a}$$

as well as

$$g_\mu / g_e = 1.05 \pm 0.07 \pm 0.08 \tag{4.5b}$$

Quoting Albajar *et al* (1987) these are "direct tests of the electron-muon-tau universality of the weak charged couplings" at the mass of the W. Alitti *et al* (1991), the UA2 experiment at CERN, recently reported

$$g_\tau / g_e = 0.997 \pm 0.056 \pm 0.042 \tag{4.5c}$$

A preliminary result from the CDF experiment at Fermilab (Roodman 1991) is

$$g_\tau / g_e = 0.99 \pm 0.07 \tag{4.5d}$$

The use of $W \to \tau\nu_\tau$ to identify W's began with the UA1 experiment at the CERN $\bar{p}p$ collider (Savoy-Navarro 1985). This technique is now being continued by the CDF experiment at the Fermilab $\bar{p}p$ collider (Gladney *et al* 1989). The signature for such events is a single narrow and isolated jet containing one to three charged particles with or without photons. In some samples two charged particles are allowed because the detector cannot accurately count the number of charged particles in the narrow jet of the τ.

4.2. Decay of D mesons to τ and ν_τ

The pure leptonic decay modes of the charged D mesons are

$$\begin{aligned} D^+ &\to e^+ + \nu_e \\ D^+ &\to \mu^+ + \nu_\mu \\ D^+ &\to \tau^+ + \nu_\tau \end{aligned} \tag{4.6}$$

and

$$
\begin{aligned}
D_s^+ &\rightarrow e^+ + \nu_e \\
D_s^+ &\rightarrow \mu^+ + \nu_\mu \\
D_s^+ &\rightarrow \tau^+ + \nu_\tau
\end{aligned}
\tag{4.7}
$$

None of these decays have been seen because the branching fractions are small. The decay width (Kim 1989, Berger, Clavelli and Wright 1983) is given by

$$
\Gamma(D^+ \rightarrow \ell^+ \nu_\ell) = \frac{G_F^2}{8\pi} f_D^2 m_D m_\ell^2 \, |V_{cd}|^2 \left(\frac{1 - m_\ell^2}{m_D^2} \right)^2 \tag{4.8a}
$$

$$
\Gamma(D_s^+ \rightarrow \ell^+ \nu_\ell) = \frac{G_F^2}{8\pi} f_{D_s}^2 m_{D_s} m_\ell^2 \, |V_{cs}|^2 \left(\frac{1 - m_\ell^2}{m_{D_s}^2} \right)^2 \tag{4.8b}
$$

where $\ell = e$, μ, or τ: Here m_D and m_ℓ are the D and ℓ masses; G_F is the Fermi coupling constant. The Cabibbo-Kobayashi-Maskawa matrix elements have the mean values:

$$
\begin{aligned}
V_{cd} &= .221 \\
V_{cs} &= .974
\end{aligned}
\tag{4.9}
$$

The weak decay constants, f_D and f_{D_s}, are proportional to the probability of quark-antiquark annihilation in the meson, figure 6. Calculations of f_D and f_{D_s} depend on the model used for quark dynamics in the meson (Kim 1989); values in the range of 150 to 300 MeV are found. Finally note that the m_ℓ^2 term suppresses the $e\nu_e$ and $\mu\nu_\mu$ modes compared to the $\tau\nu_\tau$ mode.

Thus the $\tau\nu_\tau$ modes are most accessible if the τ can be identified with substantial efficiency. For a numerical example, I assume $f_D = 200\ MeV$ and $f_{D_s} = 200$ MeV. Then

$$
\begin{aligned}
\Gamma\left(D^+ \rightarrow \tau^+ \nu_\tau\right) &= 5.0 \times 10^{-16} \text{ GeV} \\
\Gamma\left(D_s^+ \rightarrow \tau^+ \nu_\tau\right) &= 4.1 \times 10^{-14} \text{ GeV}
\end{aligned}
\tag{4.10}
$$

Using for the lifetimes

$$
\begin{aligned}
\tau_{D^+} &= (1.06 \pm 0.03) \times 10^{-12} \text{ s} \\
\tau_{D_s} &= (4.6 \pm 0.3) \times 10^{-13} \text{ s}
\end{aligned}
\tag{4.11}
$$

the branching ratios, for $f_D = f_{D_s} = 200$ MeV, are

$$
\begin{aligned}
B(D^+ \rightarrow \tau^+ \nu_\tau) &= 0.80 \times 10^{-3} \\
B(D_S^+ \rightarrow \tau^+ \nu_\tau) &= 2.9 \times 10^{-2}
\end{aligned}
\tag{4.12}
$$

Thus $D_s^+ \rightarrow \tau^+ \nu_\tau$ measurements should provide the first determination of the basic parameter in D_s decays, f_{D_s}. This decay and the several percent branching ratio are also crucial to proposals to produce beams of ν_τ's by the processes (section 8.4)

$$p + N \to D_s^+ \ \ldots \tag{4.13a}$$

$$D_s^+ \to \tau^+ + \nu_\tau \tag{4.13b}$$

$$\tau^+ \to \bar{\nu}_\tau + \ \ldots \tag{4.13c}$$

Here p is a proton and N a nucleon. There is an analogous sequence for D_s^-. The mass difference between D_s and τ allows one semileptonic decay mode

$$D_s^+ \to \tau^+ + \nu_\tau + \pi^0 \tag{4.14}$$

which has a very small branching ratio. Therefore the decay in (4.13b) is the crucial decay in this process for making ν_τ beams.

4.3. Decay of B mesons to τ and ν_τ

The physics of the pure leptonic decay

$$B^+ \to \tau^+ + \nu_\tau \tag{4.15}$$

$$B_c^+ \to \tau^+ + \nu_\tau \tag{4.16}$$

is the same as the physics of the pure leptonic decays of D and D_s (section 4.2). Thus the $\tau\nu_\tau$ decay width is given by

$$\Gamma(B^+ \to \tau^+\nu_\tau) = \frac{G_F^2}{8\pi}\, f_B^2\, m_B\, m_\tau^2\, |V_{ub}|^2 \left(1 - \frac{M_\tau^2}{m_B^2}\right)^2 \tag{4.17}$$

V_{ub} is poorly known, Aguilar-Benitez *et al* (1990) give

$$V_{ub} = 0.001 \text{ to } 0.007 \tag{4.18}$$

The B_c decay in (4.16) will have a larger width since $V_{cb} = 0.030$ to 0.058.

The large mass of the B relative to the τ allows substantial decay rates to semileptonic modes

$$\begin{aligned} B^+ &\to \tau^+ + \nu_\tau + \text{ (mesons)}^0 \\ \bar{B}^0 &\to \tau^+ + \nu_\tau + \text{ (mesons)}^- \end{aligned} \tag{4.19}$$

Present studies of these decays do not distinguish the charge of the B, therefore in this discussion the charge of the B is ignored. Following the treatment of Heiliger and

Sehgal (1989) and of Cortes, Pham and Tounsi (1982), an estimate of the total decay width for

$$B \rightarrow \ell^+ + \nu_\ell + \text{ mesons} \tag{4.20}$$

is given by

$$\Gamma(B \rightarrow \ell\nu_\ell X) = \frac{G_F^2 m_b^2}{192\,\pi^3} |V_{bc}|^2 \, I\left(\frac{m_c}{m_b}, \frac{m_\ell}{m_b}, 0\right) \tag{4.21}$$

Here m_b and m_c are the masses of the b and c quarks, and I have ignored a second term multiplied by $|V_{bu}|^2$. $I(x,y,z)$ has a complicated form (Cortes, Pham and Tounsi 1982). But unlike the pure leptonic decays (4.8), there is no multiplicative factor m_ℓ^2, therefore $\Gamma(B \rightarrow e\nu_e X)$ and $\Gamma(B \rightarrow \mu\nu_\mu X)$ are about the same size.

Heiliger and Sehgal (1989) estimate

$$\Gamma(B \rightarrow \tau\nu_\tau X)/\Gamma(B \rightarrow e\nu_e X) \approx 0.3 \tag{4.22}$$

Measurements (Aguilar-Benitez *et al* 1990) give

$$\Gamma(B \rightarrow e\nu_e X) = 0.121 \pm 0.006 \tag{4.23}$$

therefore we expect

$$\Gamma(B \rightarrow \tau\nu_\tau X) \approx 0.04 \tag{4.24}$$

We turn next to individual semileptonic decay modes involving the τ and ν_τ, such as

$$B^+ \rightarrow \tau^+ + \nu_\tau + \bar{D}^0 \tag{4.25a}$$
$$B^+ \rightarrow \tau^+ + \nu_\tau + \bar{D}^{*0} \tag{4.25b}$$

Using the first of these decays as an example and considering the more general case

$$B^+ \rightarrow \ell^+ + \nu_\ell + \bar{D}^0 \quad , \tag{4.26}$$

the decay spectrum in the B rest frame is

$$\frac{d\Gamma(B \rightarrow \ell\nu_\ell D)}{dE_\ell dE_D} = \frac{G_F^2}{4\pi^3} |V_{bc}|^2 \left(|f_+|^2 A + R_e f_+^* f_- B + |f_-|^2 C \right) \tag{4.27}$$

where f_+ and f_- are the two form factors (Heiliger and Sehgal 1989). The coefficients are

$$A = m_D \left[2E_\ell E_v - m_D(W_0 - E_D) + \frac{1}{4}\, m_\ell^2 (W_0 - E_D) - m_\ell^2 E_v \right] ,$$
$$B = m_\ell^2 \left[E_v - \frac{1}{2}(W_0 - E_D) \right] , \; C = \frac{1}{4}\, m_\ell^2 (W_0 - E_D) \tag{4.28}$$
$$W_0 = (m_B^2 + m_D^2 - m_\ell^2)/2m_B$$

where E_i is the energy of particle i.

Since coefficients B and C are proportional to m_ℓ^2, only the $\ell = \tau$ case in (4.26) may have $d^2\Gamma/dE_\ell dE_D$ spectra sensitive to both form factors. In addition, as also discussed by Heiliger and Sehgal (1989), the longitudinal polarization of the τ in (4.25a) is sensitive to both f_+ and f_-. Average values of τ polarization are in turn obtained by studying the decay of the τ. Thus

$$B^+ \rightarrow \tau^+ + \nu_\tau + \bar{D}^0$$

provides a powerful way to study the two form factors in this semileptonic decay. Similar considerations apply to

$$B^+ \rightarrow \tau^+ + \nu_\tau + \bar{D}^{*0}$$

but the spin 1 of the D^{*0} leads to a more complicated matrix element with four form factors.

In this discussion conventional theory has been used. But these semileptonic decays can be used to search for new physics, given sufficiently precise measurements of decay spectra and τ polarization (Kalinowski 1990).

5. General and leptonic decays of the τ

5.1. *General theory of τ decays*

All experimental studies of τ decays, except possibly those discussed in section 9, are explained by the decay process of figure 7a.

$$\tau^- \rightarrow \nu_\tau + W^-_{virtual} \tag{5.1a}$$
$$W^-_{virtual} \rightarrow \text{ final particles} \tag{5.1b}$$

Conventional weak interaction theory (Halzen and Martin 1984, Quigg 1983) dictates completely the form and strength of the $\tau - W - \nu_\tau$ vertex if that vertex is taken to be the same as the $e - W - \nu_e$ vertex, namely

$$V^\mu_{\tau W \nu_\tau} = -i\, \frac{g_{weak,\tau}}{\sqrt{2}}\, \gamma^\mu (1 - \gamma^5) \tag{5.2}$$

This assumes conventional V-A coupling The case of other than V-A coupling is discussed in section 9.7.

The second vertex (5.1b) is straightforward for the pure leptonic decay modes (figure 7b).

$$\tau^- \rightarrow \nu_\tau + e^- + \bar{\nu}_e \tag{5.3a}$$

$$\tau^- \rightarrow \nu_\tau + \mu^- + \bar{\nu}_\mu \tag{5.3b}$$

since a form analogous to (5.2) applies. Therefore, everything about the modes can be calculated: decay widths, angular distributions, momentum spectra. It is more complicated but still straightforward to calculate the properties of the radiative leptonic decays (figure 7c).

$$\tau^- \rightarrow \nu_\tau + e^- + \bar{\nu}_e + \gamma \tag{5.4a}$$

$$\tau^- \rightarrow \nu_\tau + \mu^- + \bar{\nu}_\mu + \gamma \tag{5.4b}$$

as discussed in section 7.1.

At present there is no general method for calculating the decay widths and properties of the semi-leptonic decay modes (figure 7d)

$$\tau^- \rightarrow \nu_\tau + \text{ hadrons} \tag{5.5}$$

The calculation of the behavior of the W-hadron vertex requires understanding the quantum chromodynamics of that vertex, section 6.8. Special methods apply to particular modes such as

$$\begin{aligned} &\tau^- \rightarrow \nu_\tau + \pi^- \\ &\tau^- \rightarrow \nu_\tau + K^- \\ &\tau^- \rightarrow \nu_\tau + \rho^- \end{aligned} \tag{5.6}$$

But in the energy range involved in τ decays there are no precise general methods in quantum chromodynamics.

A rough prediction of the total decay width for semi-leptonic modes is derived from figure 7e in which the strong interactions between quarks are ignored in the calculation, although it is the strong interaction which takes the quark-antiquark pair into hadrons. Then the total semi-leptonic decay width, denoted by Γ_{had}, is

$$\Gamma_{had} = \Gamma(\tau^- \rightarrow \nu_\tau + \bar{u} + d) + \Gamma(\tau^- \rightarrow \nu_\tau + \bar{u} + s) \tag{5.7a}$$

compared to the two pure leptonic decay widths

$$\begin{aligned} \Gamma_e &= \Gamma(\tau^- \rightarrow \nu_\tau + e^- + \bar{\nu}_e) \\ \Gamma_\mu &= \Gamma(\tau^- \rightarrow \nu_\tau + \mu^- + \bar{\nu}_\mu) \end{aligned} \tag{5.7b}$$

Next we make two approximations: set all masses except m_τ to 0 and ignore $\Gamma(\tau^- \to \nu_\tau + \bar{u} + s)$ since

$$\frac{\Gamma(\tau^- \to \nu_\tau + \bar{u} + s}{\Gamma(\tau^- \to \nu_\tau + \bar{u} + d} = \tan^2\theta_c = 0.05 \tag{5.8}$$

Counting the three quark colors

$$\begin{aligned} \Gamma_{had} &= 3\,\Gamma_e \\ \Gamma_\mu &= \Gamma_e \end{aligned} \tag{5.9}$$

Since the branching fraction into mode i is

$$B_i = \Gamma_i/\Gamma_{tot} \tag{5.10}$$

this rough calculation predicts

$$\begin{aligned} B_e &= B_\mu = 20\% \\ B_{had} &= 60\% \end{aligned} \tag{5.11}$$

It is surprising that this is close to the measured values (Aguilar-Benitez *et al* 1990)

$$\begin{aligned} B_e &= (17.7 \pm 0.4)\% \\ B_\mu &= (17.8 \pm 0.4)\%\ , \end{aligned} \tag{5.12}$$

surprising because about half of Γ_{had} comes from the $\tau^- \to \nu_\tau \pi^-$ and $\tau^- \to \nu_\tau \rho^-$ modes. And the physics of these modes is not described by the approximation in (5.7a).

5.2. *Topological branching ratios and techniques for studying τ decays*

Early in the history of τ research, the development of techniques for studying τ decays led to the use of τ decay topologies:

$$\begin{aligned} \tau &\to \text{1-charged-particle + neutral particles} \\ \tau &\to \text{3-charged-particle + neutral particles} \\ \tau &\to \text{5-charged-particle + neutral particles} \end{aligned} \tag{5.13}$$

and so forth. Neutral particles means neutrinos and photons. The corresponding topological branching ratios are designed by B_1, B_3, B_5, ...and have the present values (Aguilar-Benitez *et al* 1990).

$$\begin{aligned} B_1 &= (86.13 \pm 0.33)\% \\ B_3 &= (13.76 \pm 0.32)\% \\ B_5 &= (1.13 \pm 0.27)\% \\ B_7 &< 1.9 \times 10^{-4} \end{aligned} \tag{5.14}$$

Thus 99% of τ decays have 1 or 3 charged particles, and this has a crucial role in the techniques for studying τ decays. In addition, the precise values of B_1 and B_3 have taken on an unexpected importance because of the current problem in understanding B_1 as the sum of many 1-charged-particle decay modes (section 9.1).

The study of τ decays includes three steps: (a) the <u>isolation</u> of

$$e^+ + e^- \to \tau^+ + \tau^- \tag{5.15}$$

from the other e^+e^- reactions; (b) the <u>identification</u> of one or both of the decay modes; and (c) the <u>measurement</u> of the vector momenta of the observed particles in the mode or modes. These steps carried out through a variety of methods are described in detail in most experimental papers on the τ and in some reviews: Barish and Stroynowski (1988), Burchat (1988), Kiesling (1988), Hayes and Perl (1988). I shall give some examples.

The techniques for isolation of $e^+ + e^- \to \tau^+ + \tau^-$, step (a), begin with the observation that these events have charged particle topologies of 1-1, 1-3, 3-3, 1-5, 3-5 and rarely 5-5. Here $m - n$ means one τ decay mode has m-charged-particles and the other τ decay mode has n-charged particles. The τ pair events must be isolated from

$$e^+ + e^- \to \text{hadrons} \tag{5.16}$$

events with the same topologies. A variety of isolation techniques are used. For example, at high energies, above about 10 GeV, the charged particles and photons from one τ decay have momenta roughly opposite in direction to the momenta of the charged particles and photons from the other τ decay. This is less likely for $e^+e^- \to$ hadron events , allowing partial isolation of the τ pair events. The higher the energy the more distinctive the τ pair events and the cleaner the isolation using this distinction.

As another example 1–1, 1–3, and 1–5 τ pair events may be partially isolated from $e^+e^- \to$ hadron events by requiring the 1-charged-particle to be an e or μ and to have a momentum roughly opposite in direction to the momenta of the other charged particles. This isolation criterion depends upon the rarity of $e^+e^- \to$ hadron events with a separated e or μ. But some $e^+e^- \to$ hadron events can mimic this τ pair behavior, such as

$$e^+ + e^- \to D^+ + D^- \tag{5.17}$$

with a semi-leptonic decay of one of the D's:

$$D \to e \text{ or } \mu + \text{ neutral particles} \tag{5.18}$$

Other reactions from which $e^+e^- \to \tau^+\tau^-$ must be isolated are lepton pair production

$$e^+ + e^- \to e^+ + e^- \tag{5.18a}$$
$$e^+ + e^- \to \mu^+ + \mu^- \quad ; \tag{5.18b}$$

radiative lepton pair production

$$e^+ + e^- \to e^+ + e^- + n\gamma \quad , n \geq 1 \tag{5.19a}$$
$$e^+ + e^- \to \mu^+ + \mu^- + n\gamma \quad , n \geq 1\,, \tag{5.19b}$$

and four-lepton production.

$$e^+ + e^- \to e^+ + e^- + e^+ + e^- \tag{5.20a}$$
$$e^+ + e^- \to e^+ + e^- + \mu^+ + \mu^- \tag{5.20b}$$

The processes in (5.20) are most easily confused with $e^+e^- \to \tau^+\tau^-$ when one e^+e^- pair is not observed in the detector. Even the process

$$e^+ + e^- \to e^+ + e^- + \tau^+\tau^- \tag{5.20c}$$

can be confused with $e^+e^- \to \tau^+\tau^-$.

Since no isolation technique is perfect, two corrections must be applied to the sample of isolated τ pair events: the correction for the non-selection of some τ pair events, and the correction for the inclusion of non-τ-pair events in the sample. The fraction of lost τ pair events ranges from very small, 5 or 10%, to very large, 90 or 95%, depending on whether the isolation criteria are very loose or very light or in between. The τ pair sample must be corrected for this loss, not only in the number of events but also in the distributions of the properties of the sample. For example an isolation criterion which requires that the momenta of the charged particles in a 1-1 topology be roughly opposite leads to a bias in the distribution of the angle between the momenta. The corrections for isolation loss depend on the properties of the experimental apparatus as well as on the properties of τ decays. In most τ research these corrections have to be calculated through a complicated computer simulation of the properties of the apparatus and computer processing of artificial τ pair events through the simulated apparatus. These procedures almost always require Monte Carlo methods.

There is also a complicated correction for the inclusion in the τ pair sample of events from other processes such as $e^+e^- \rightarrow$ hadron, $e^+e^- \rightarrow e^+e^-$, $e^+e^- \rightarrow \mu^+\mu^-$. Again both the number of isolated τ pair events and the distributions of their properties must be corrected for the contamination by these background events. And again the corrections depend on the properties of the apparatus and the properties of the contaminating events. But knowing the properties of the contaminating events can itself be a problem when those events come from $e^+e^- \rightarrow$ hadrons and are themselves poorly understood. One of the great strengths of the tau-charm factory concept (section 10) for τ research is that it provides a direct experimental way to determine the number and properties of contaminating events.

In steps (b) and (c) the identification of a τ decay mode requires the establishment of the momenta and nature of the charged particles: are they e, μ, π, or K? Identification also requires finding the photons, if any, in the event and measuring their momenta. These photons come from

$$\pi^0 \rightarrow \gamma + \gamma \quad , \tag{5.21}$$

or from η decays such as

$$\begin{aligned} &\eta \rightarrow \gamma + \gamma \\ &\eta \rightarrow \pi^+ + \pi^- + \gamma \quad , \end{aligned} \tag{5.22}$$

or from radiative production of τ pairs

$$e^+ + e^- \rightarrow \tau^+ + \tau^- + \gamma \quad , \tag{5.23}$$

or from radiative τ decays such as (5.4). Therefore, if there are photons, identification of the decay mode requires that the π^0's and η's be reconstructed and their momenta determined.

The precision of decay mode identification and measurement is dependent upon the properties of the apparatus. Charged particles may be misidentified or may be impossible to identify. Photons may not be detected, especially low energy photons. Two photons may be detected as one photon because their flight paths are very close to each other. Therefore, corrections must be applied to the results of the identification and measurement of the decay modes. Once again these corrections almost always required computer programs which simulate the apparatus and the processing of artificial τ pair events through these programs using Monte Carlo methods.

I have outlined the steps and the problems in the experimental study of τ decays so that the reader understands qualitatively the state of our present knowledge of these decays. As described in this section, in section 6, and in section 9.1, we know a great deal about many decay modes, particularly those with branching fractions above 5%, table 3. But experimentally, we know little or nothing about complex modes such as

$$\tau^- \to \nu_\tau + \pi^- + n\pi^0 \quad , \quad n > 2 \tag{5.24}$$

And we even have uncertainties about the branching ratio for

$$\tau^- \to \nu_\tau + \pi^- + 2\pi^0 \tag{5.25}$$

in spite of the average measured value being. (7.5±0.9)% (Aguilar-Benitez *et al* 1990).

5.3. The pure leptonic decay modes

The average measured branching ratios for the pure leptonic modes are

$$B_e = \; B(\tau^- \to \nu_\tau \, e^- \bar{\nu}_e) = (17.7 \pm 0.4)\% \tag{5.26a}$$
$$B_\mu = \; B(\tau^- \to \nu_\tau \, \mu^- \bar{\nu}_\mu) = (17.8 \pm 0.4)\% \tag{5.26b}$$

Conventional weak interaction theory with a V-A current at the $\tau - W - \nu_\tau$ vertex gives the decay width (Tsai 1971, Marciano and Sirlin 1988, Wu 1990a) for $\ell = e$ or μ.

$$\Gamma_\ell = \Gamma(\tau^- \to \nu_\tau \ell^- \bar{\nu}_\ell) \; = \; \frac{G_F^2 \, m_\tau^5}{192\,\pi^3} \, F_\ell(y) \, F_W \, F_{rad} \tag{5.27a}$$
$$F_\ell(y) = 1 - 8y + 8y^3 - y^4 - 12y^2 \, lny \tag{5.27b}$$
$$y = m_\ell^2 / m_\tau^2$$
$$F_W = 1 + \frac{3}{5} \, \frac{m_\tau^2}{m_W^2} \tag{5.27c}$$
$$F_{rad} = 1 - \frac{\alpha_\tau}{2\pi} \left(\pi^2 - \frac{25}{4} \right) \tag{5.27d}$$

This assumes all neutrinos are massless. The first two terms on the right side of (5.27a) are derived in Tsai (1971), the first term by itself is derived for $\mu^- \to \nu_\mu + e^- + \bar{\nu}_e$ in most particle physics textbooks. F_ℓ is the correction for the mass of the ℓ.

$$\begin{aligned} F_e &= 1.0000 \\ F_\mu &= 0.973 \end{aligned} \tag{5.28a}$$

F_W is the correction for the finite W mass:

$$F_W = 1.0003 \tag{5.28b}$$

Finally F_{rad} is the electromagnetic radiative correction:

$$F_{rad} = 0.9957 \tag{5.28c}$$

using $\alpha_\tau = 1/133.3$ as defined by Marciano and Sirlin (1988). The factor F_{rad} includes virtual photon corrections, the emission of real photons, and the emission of light-fermion pairs. For example, $\Gamma(\tau^- \to \nu_\tau\, \mu^- \bar{\nu}_\mu)$ in (5.27a) gives the width for the set of decays:

$$\begin{aligned}
&\tau^- \to \nu_\tau + \mu^- + \bar{\nu} \\
&\tau^- \to \nu_\tau + \mu^- + \bar{\nu}_\mu + \gamma \\
&\tau^- \to \nu_\tau + \mu^- + \bar{\nu}_\mu + \gamma + \gamma \\
&\tau^- \to \nu_\tau + \mu^- + \bar{\nu}_\mu + e^+ + e^- \\
&\vdots \qquad\qquad \vdots
\end{aligned} \tag{5.29}$$

Thus Γ in (5.27a) is what Marciano and Sirlin (1988) call the radiative inclusive decay width. Radiative decays are further discussed in section 7.1.

In (5.27a)

$$\frac{G_F^2\, m_\tau^5}{192\pi^3} = 4.13 \times 10^{-13} \ (1^{+0.008}_{-0.010}) \ \text{GeV} \tag{5.30}$$

where the error comes from the uncertainty in m_τ

$$m_\tau = 1784.1^{+2.7}_{-3.6} \ \text{GeV/c}^2 \tag{5.31}$$

From (5.27a)

$$\Gamma_e = \Gamma(\tau^- \to \nu_\tau\, e^- \bar{\nu}_e) = 4.11 \times 10^{-13} \ (1^{+0.008}_{-0.010}) \ \text{GeV} \tag{5.32}$$

We had no feeling for the significance of a width of 10^{-13} GeV, therefore it is useful to define the partial lifetime.

$$T(\tau^- \to \nu_\tau\, e^- \bar{\nu}_e) = \hbar/\Gamma(\tau^- \to \nu_\tau\, e^- \bar{\nu}_e) = 1.594 \times 10^{-12} \ (1^{-0.0010}_{+0.0008}) \ \text{s} \tag{5.33}$$

If the τ decayed only through this mode, this would be its lifetime; every additional decay mode shortens the life.

Returning to (5.27) and using the definition of the branching ratio for mode i, $B_i = \Gamma_i/\Gamma_{tot}$

$$B_\mu/B_e = \Gamma_\mu/\Gamma_e = 0.973 \tag{5.34}$$

From (5.26) the ratio of measured B_μ to B_e is

$$(B_\mu/B_e)_{meas} = 1.006 \pm 0.032 \tag{5.35}$$

Thus within the errors, measurements argue with the predicted value of B_μ/B_e.

In (5.27) the neutrino masses are set to zero. If ν_τ is given a non-zero mass and m_ℓ is maintained non-zero, the formula becomes complicated (Shrock 1981, Gomez-Cadenas and Gonzalez-Garcia 1989, Stoker *et al* 1989). However, if $m_\ell = 0$ and $m_{\nu_\tau} \neq 0$ $\Gamma(\tau^- \rightarrow \nu_\tau \ell^- \bar{\nu}_\ell)$ has again a simple form (Bjorken and Llewellyn Smith 1973, Stoker *et al* 1989). Indeed it is the same as (5.27a) with y in (5.27b) redefined as

$$y = m_{\nu_\tau}^2/m_\tau^2 \tag{5.36}$$

5.4. *The τ lifetime*

As just discussed, conventional weak interaction theory and the assumption of a pure V-A current at the $\ell - W - \nu_\ell$ vertices allows an exact calculations of Γ_e and Γ_μ. Combining these calculated widths with the measured branching ratios B_e and B_μ leads to predictions for the τ lifetime, T_τ:

$$T_\tau(\ell) = \frac{\hbar}{\Gamma_{tot}} = \frac{\hbar B_\ell}{\Gamma_\ell}\,, \quad \ell = e \text{ or } \mu \tag{5.37}$$

Incidentally, if F_W and F_{rad} are ignored in (5.27a) one can use the very well measured μ lifetime.

$$T_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} = 2.197 \times 10^{-6} \text{ s} \tag{5.38}$$

to rewrite (5.37) as

$$T_\tau = T_\mu \left(\frac{m_\mu}{m_\tau}\right)^5 B_\ell \tag{5.39}$$

which is a convenient mnemonic.

For a thorough analysis it is better to go back to (5.37), using the values of Γ_e and Γ_μ from (5.27) and measured values of B_e and B_μ. As discussed by Hayes and Perl

(1988) it is necessary to take account of correlated errors in some joint measurements of B_e and B_μ. They calculate

$$T_\tau\,(predict) = (2.874 \pm 0.042) \times 10^{-13}\ \mathrm{s} \tag{5.40}$$

The techniques for measuring T_τ are discussed by Barish and Stroynowski (1988) and by Jaros (1984). The techniques are difficult and the possibility of undetected bias is ever present. The average value is (Aguilar-Benitez *et al* 1990)

$$T_\tau\ (\mathrm{meas}) \;=\; (3.03 \pm 0.08) \times 10^{-13}\ \mathrm{s} \tag{5.41}$$

This measured value is close to but not "right on" the prediction of (5.40) and there has been much speculation as to the significance of the difference

$$T_\tau\ (\mathrm{meas}) - T_\tau\ (pred)\ = (0.16 \pm 0.09) \times 10^{-13}\ \mathrm{s} \tag{5.42}$$

which is about 2 standard deviations. This is discussed in section 9.2.

5.5. Momentum spectra in pure leptonic decays

In the decay

$$\tau^- \to \nu_\tau + \ell^- + \bar{\nu}_\ell \;\;,\; \ell = e, \mu \tag{5.43}$$

the momentum spectrum of the ℓ in the rest frame of an *unpolarized* τ is (Tsai 1971)

$$\frac{dn}{dp_\ell}\,\alpha\; p_\ell^2 \left[3m_\tau - 4E_\ell - \frac{2m_\ell^2}{E_\ell} + \frac{3m_\ell^2}{m_\tau}\right] \quad , \tag{5.44}$$

using the V-A vertex in (5.2). To get a feeling for this spectrum it is convenient to set $m_\ell = 0$ and define

$$x = 2p_\ell/m_\tau \quad , \quad 0 \le x \le 1 \tag{5.45}$$

Then the normalized spectrum in the rest frame is

$$\frac{dn}{dx} = 2x^2(3 - 2x) \tag{5.46}$$

with $dn/dx = 0$ at $x = 0$ and the spectrum peak at the maximum $p_\ell = m_\tau/2$

If the τ has a velocity $\beta = v/c$ in the laboratory frame, as $\beta \to 1$, the spectrum peak shifts towards the $x = 0$ point of the spectrum. In the limit $\beta = 1$ and for the τ unpolarized the laboratory momentum spectrum of the ℓ is given by

$$\frac{dn}{dx_{lab}} = \frac{1}{3}\,(4x_{lab}^3 - 9x_{lab}^2 + 5) \tag{5.47}$$

where

$$x_{lab} = p_{\ell,lab}/p_{\ell,lab}(max)$$

In (5.47) $dn/dx_{lab} = 0$ at $x_{lab} = 1$ and the spectrum peak is at $p_{\ell,lab} = 0$!

The significance of (5.47) is that at high energies, $E_{beam} >> m_\tau$, the momentum spectrum of the e or μ in pure leptonic decays is dominated by the Lorentz boost. Sensitive statistics of those spectrum, intended to test the V-A vertex in (5.2), are best done at low energies, preferably close to threshold.

Existing studies of these spectra agree with the V-A vertex in (5.2), but there is a great deal more to be investigated; hence this discussion is continued in section 9.7.

6. Hadronic decays of the τ

6.1. Introduction and examples: $\tau^- \to \nu_\tau \pi^-, \tau^- \to \nu_\tau K^-,\ \tau^- \to \nu_\tau \rho^-$

A full understanding of the branching ratios of the various τ decay modes requires that theory successfully predict the measured branching ratios. In this section we test that understanding by comparing prediction with measurement. We shall see that where we can make a prediction the comparison is successful. But the comparisons are incomplete because, as I discussed in section 5.1, there is no general and precise method for calculating the branching ratio or other properties of decay modes containing hadrons

$$\tau^- \to \nu_\tau + \text{ (hadrons)}^- \tag{6.1}$$

There are however special methods for some modes, general rules for the quantum numbers of the hadronic state and a general method for formulating the problem. Before discussing the general rules and formulation (section 6.2) and surveying the special methods (sections 6.3-6.6), I will give three examples of decay width calculations to give the reader insight into the physics of the hadronic decay modes.

In the decay

$$\tau^- \to \nu_\tau + \pi^- \tag{6.2}$$

the $W-\pi$ vertex, figure 8a, involves strong as well as weak interactions; hence a calculation from first principles requires a low energy quantum chromodynamic calculation which is beyond present skills. The same $W-\pi$ vertex occurs in

$$\pi^- \to \mu^- + \bar{\nu}_\mu \tag{6.3}$$

as shown in figure 8b. For many years we have buried our lack of skills in strong interaction calculations by an empirical constant f_π in the π decay width formula (Aguilar-Benitez *et al* 1990)

$$\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu) \ = \ \frac{G_F^2 \cos^2\theta_c f_\pi^2 m_\pi m_\mu^2}{8\pi} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 F_{rad}(\pi) \tag{6.4}$$

Here

$$F_{rad}(\pi) = 1 - 1.35\, \frac{\alpha}{2\pi} \tag{6.5}$$

is the radiative correction and θ_c is the Cabibbo angle with

$$\cos\,\theta_c \;=\; .975$$

From the π lifetime and (6.4)

$$f_\pi = 131.74 \pm 0.15 \text{ MeV} \tag{6.6}$$

In terms of f_π the $\tau^- \rightarrow \nu_\tau\,\pi^-$ decay width is (Tsai 1971).

$$\Gamma(\tau^- \rightarrow \nu_\tau\,\pi^-) = \frac{G_F^2 f_\pi^2 \cos^2\theta_c\, m_\tau^3}{16\pi}\left[1 - \frac{m_\pi^2}{m_\tau^2}\right]^2 \tag{6.7}$$

In (6.7) and in all other formula for the decay widths in this section I ignore the finite W mass and radiative corrections to Γ. The radiative corrections are of order $\alpha/2\pi$, less than 1%, and can be ignored for most uses of present τ decay data. But as the precision of τ decay measurements improves, radiative correction such as $F_{rad,\ell}$ in (5.27a) will be required.

It is convenient to relate the individual hadronic decay widths to what I call the base $\tau^- \rightarrow \nu_\tau\, e^-\bar{\nu}_e$ decay width

$$\Gamma_{e\,0} = \frac{G_F^2 m_\tau^5}{192\pi^3} \tag{6.8}$$

And ignoring radiative corrections I define the ratio of branching ratios:

$$r(\tau^- \rightarrow \nu_\tau\,\pi) = \frac{B(\tau^- \rightarrow \nu_\tau\,\pi^-)}{B(\tau^- \rightarrow \nu_\tau\, e^-\bar{\nu}_e)} \;=\; \frac{\Gamma_\pi}{\Gamma_{e\,0}} \;=\; \frac{12\pi^2 f_\pi^2 \cos^2\theta_c}{m_\tau^2}\left[1 - \frac{m_\pi^2}{m_\tau^2}\right]^2 \tag{6.9}$$

Using (6.6)

$$r(\tau^- \rightarrow \nu_\tau\,\pi^-,\ \mathit{predict}) = 0.606 \tag{6.10}$$

where *predict* means predicted.

The average measured value of $B(\tau^- \rightarrow \nu_\tau\,\pi^-)$ is (Aguilar-Benitez *et al* 1990):

$$B(\tau^- \rightarrow \nu_\tau\pi^-) = (11.0 \pm 0.5)\% \tag{6.11}$$

Using (5.26a)

$$r(\tau^- \rightarrow \nu_\tau\pi^-,\ \text{meas}) \;=\; 0.62 \pm 0.03 \tag{6.12}$$

which agrees with (6.10).

The decay width for

$$\tau^- \rightarrow \nu_\tau + K^- \tag{6.13}$$

is obtained in an analogous way using $\Gamma(K^- \rightarrow \mu^- \bar{\nu}_\mu)$ to derive (Aguilar-Benitez *et al* 1990)

$$f_K = 160.6 \pm 1.4 \text{ MeV} \tag{6.14}$$

And as in (6.7)

$$\Gamma(\tau^- \rightarrow \nu_\tau \pi^-) = \frac{G_F^2 f_K^2 \sin^2\theta_c m_\tau^3}{16\pi}\left[1 - \frac{m_K^2}{m_\tau^2}\right]^2 \tag{6.15}$$

but

$$\sin\theta_c = .221 \tag{6.16}$$

appears instead of $\cos\theta_c$. This gives the prediction

$$\frac{B(\tau^- \rightarrow \nu_\tau K^-)}{B(\tau^- \rightarrow \nu_\tau \pi^-)}_{predict} = \tan^2\theta_c \left(\frac{f_K}{f_\pi}\right)^2 \left[\frac{m_\tau^2 - m_K^2}{m_\tau^2 - m_\pi^2}\right]^2 = 0.071 \tag{6.17}$$

Since the average measured value (Aguilar-Benitez *et al* 1990) is

$$B(\tau^- \rightarrow \nu_\tau K^-) = (0.68 \pm 0.19)\% \tag{6.18}$$

The measured ratio of branching ratios is

$$\left(\frac{B(\tau^- \rightarrow \nu_\tau K^-}{B(\tau^- \rightarrow \nu_\tau \pi^-}\right)_{meas} = 0.062 \pm 0.017 \tag{6.19}$$

which agrees with (6.17).

The third example in this section is the method used by Tsai (1971) to calculate Γ for

$$\tau^- \rightarrow \nu_\tau + \rho^- \tag{6.20}$$

Now the problem is the $W - \rho$ vertex. The method is to relate the $W - \rho$ vertex to the $\gamma - \rho$ vertex in the decay

$$\rho^0 \rightarrow e^+ + e^- \tag{6.21}$$

using the conserved vector current (CVC) principle. Quoting Tsai (1971)

"CVC is equivalent to the statement that the coupling of W to ρ is obtainable from the $\gamma\rho$ coupling by replacing e in the latter by $\sqrt{2}g\cos\theta_c$ where $g^2/M_w^2 = G\sqrt{2}$."

Tsai (1971) then gives the approximate formulas

$$\Gamma(\tau^- \to \nu_\tau \rho^-) = \frac{G_F^2 \cos\theta_c^2 m_\tau^3 m_\rho^2}{64\pi^2}\left[1 - \frac{m_\rho^2}{m_\tau}\right]^2 \left[1 + \frac{2m_\rho^2}{m_\tau^2}\right] \tag{6.22}$$

$$r(\tau^- \to \nu_\tau \rho^-, predict) = \frac{3\pi\cos^2\theta_c m_\rho^2}{m_\tau^2}\left[1 - \frac{m_\rho^2}{m_\tau^2}\right]^2 \left[1 + \frac{2m_\rho^2}{m_{\tau^2}}\right] \tag{6.23}$$

$$= 1.51$$

Using the average measured value

$$B(\tau^- \to \nu_\tau \rho^-) = (22.7 \pm 0.8)\% \tag{6.24}$$

and (5.26a)

$$r(\tau^- \to \nu_\tau \rho^-,\ meas) = 1.28 \pm 0.05 \tag{6.25}$$

A more precise calculation of $\Gamma(\tau^- \to \nu_\tau \rho^-)$ is described in section 6.4, hence I won't comment on this comparison.

Before leaving these examples, it is useful to notice that the phase space factor $(1 - m_h^2/m_\tau^2)^2$ appears in the π, K, and ρ decay mode widths. The additional phase space factor $(1 + 2m_h^2/m_\tau^2)$ appears in the ρ decay mode width (6.22) because the ρ has spin 1 compared to spin 0 for the π and K.

Smith (1991) has given a general review of hadronic decay modes.

6.2. *General formulation of hadronic decay widths*

A general formulation of hadronic decay widths was given by Tsai (1971) and although this paper is twenty years old, it is still the clearest exposition of this formulation. Let h^- in

$$\tau^- \to \nu_\tau + h^- \tag{6.26}$$

represent a particular hadronic final state such as ρ^- or $\pi^- + 3\pi^0$ or $2\pi^- + \pi^+ + \pi^0$. Then $\Gamma(\tau^- \to \nu_\tau h^-)$ has the general form

$$\Gamma(\tau^- \to \nu_\tau h^-) = \frac{G_F^2 m_\tau^3}{32\pi^2} \int_0^{m_\tau^2} dq^2 \left(1 - \frac{q^2}{m_\tau^2}\right)^2$$

$$\times \left\{ \cos^2\theta_c \left[\left(1 + 2\frac{q^2}{m_\tau^2}\right)(v_1(h^-, q^2) + a_1(h^-, q^2)) + a_0(h^-, q^2)\right] \right. \tag{6.27a}$$

$$\left. + \sin^2\theta_c \left[\left(1 + 2\frac{q^2}{m_\tau^2}\right)(v_1^s(h^-, q^2) + a_1^s(h^-, q^2)) + v_0^s(h^-, q^2) + a_0^s(h^-, q^2)\right]\right\}$$

Here q^2 is the square of the invariant mass of the h^- system. The v's and a's, called spectral functions, are different for every different h^-. They are continuous functions of q^2 except in the special cases $h^- = \pi^-$ and $h^- = K^-$ when they are delta functions.

It is useful to understand the parts of (6.27a). The terms $(1 - q^2/m_\tau^2)^2$ and $(1 + 2q^2/m_\tau^2)$ are the same phase space factors that appeared in the last section's equations. The v and the a refer to spectral functions connected to the Lorentz vector and axial vector parts of the weak charged current, the so-called "V" and "A" parts. It is confusing, but there are also spin J parts of v and a denoted by the subscripts on v and a: 1 means $J = 1$ and 0 means $J = 0$. Finally, the state h^- may be non-strange or strange, the latter is denoted by the superscript s. The Cabibbo angle terms, $\cos^2\theta_c$ and $\sin^2\theta_c$, are explicitly separated out of the spectral functions.

In general a Lorentz vector or a Lorentz axial vector current may have $J = 0$ and $J = 1$. But the conserved vector current principle requires $v_0 = 0$. Note v_0^s need not be 0.

For a particular h^- only some or perhaps one of the spectral functions is non-zero. For example, if $h^- = \rho^-$ with $J = 1$ the only non-strange, $J = 1$ spectral functions are v_1 and a_1. As discussed in the next section G-parity conservation sets $a_1 = 0$, leaving only $v_1 \neq 0$. Since the ρ is a resonance of mass m_ρ and width Γ_ρ, v_1 has the form

$$v_1(\rho^-, q^2) = \frac{C}{(m_\rho^2 - q^2)^2 + \Gamma_\rho^2 m_{\rho^2}} \tag{6.28}$$

where C is a constant.

At present there is no general way to calculate a particular v or a spectral function. But there is hope of calculating the spectral function summed over all individual states h_i^-

$$\begin{aligned} v_1(\text{all } h^-, q^2) &= \sum_i v_1(h_i^-, q^2) \\ a_1(\text{all } h^-, q^2) &= \sum_i a_i(h_i^-, q^2) \end{aligned} \tag{6.29a}$$

and so forth. Inserting these in (6.27) gives

$$\Gamma_{had} = \sum_i \Gamma(\tau^- \to \nu_\tau h_i^-) \tag{6.29b}$$

This is discussed in section 6.7.

6.3. *Application of charge conjugation and isospin conservation to hadronic decay modes*

The spectral functions describe the differential width of a hadronic decay mode as a function of the hadronic mass, $\sqrt{q^2}$, and they put some restrictions on the quantum numbers of the mode. Further restrictions on non-strange decay modes are obtained by applying charge conjugation and isospin conservation. This is valuable since almost all the hadronic width, Γ_{had}, comes from non-strange decay modes.

As in other parts of particle physics, it is convenient to use the G-parity concept which combines charge conjugation and isospin conservation. The weak charged current has the following properties

$$\begin{aligned} \text{Isospin} : & \ I = 1 \text{ for vector and axial vector currents} \\ G-\text{parity} : & \ G = +1 \text{ for vector current} \\ & \ G = -1 \text{ for axial vector current} \\ \text{Spin-parity} : & \ J^P = 1^- \text{ for vector current} \\ & \ J^P = 0^-,\ 1^+ \text{ for axial vector current} \end{aligned} \tag{6.30}$$

The G-parity assignment opposite to that in (6.30) corresponds to a so-called second class current, the decay width is then suppressed by a factor of 10^{-4} to 10^{-6} as discussed in section 9.9.

It is straightforward to apply the G and J^P requirements to the non-strange hadrons which are produced in τ decay:

$$\begin{aligned} \pi : G = -1\ ,\ J^P = 0^- \\ \eta : G = +1\ ,\ J^P = 0^- \\ \rho : G = +1\ ,\ J^P = 1^- \\ \omega : G = -1\ ,\ J^P = 1^- \end{aligned}$$

and so forth. For example in $\tau^- \to \nu_\tau \pi^-$ the π with $G = -1,\ J^P = 0^-$ is produced through the axial vector current decay. Conversely, the decay $\tau^- \to \nu_\tau \rho^-$ occurs through the vector current since $G = +1$. However the decay

$$\tau^- \to \nu_\tau + \pi^- + \eta \tag{6.31}$$

is forbidden since $G(\pi\eta) = -1$ requires an axial vector current with $J^P = 0^-$ or 1^+. But for$J = 0$ $P(\pi\eta) = +1$ and for $J = 1$ $P(\pi\eta) = -1$.

In a decay with n π's

$$\tau^- \to \nu_\tau + (n\ \pi)^- \tag{6.32}$$

$G = (-1)^n$. Hence the vector current produces states with an even number of π's, the axial vector current produces states with an odd number of π's.

Isospin conservation is also used to derive inequalities between different hadronic decay modes with the same I (Gilman and Rhie 1985). Consider for example the 3π modes

$$\tau^- \to \nu_\tau + \pi^- + \pi^0 + \pi^0 \tag{6.33a}$$
$$\tau^- \to \nu_\tau + \pi^- + \pi^+ + \pi^- \tag{6.33b}$$

with I=1. Gilman and Rhie (1985) show

$$\frac{\Gamma(\tau^- \to \nu_\tau \pi^- \pi^0 \pi^0)}{\Gamma(\tau^- \to \nu_\tau \pi^- \pi^0 \pi^0) + \Gamma(\tau^- \to \nu_\tau \pi^- \pi^+ \pi^-)} \leq \frac{1}{2} \tag{6.34a}$$

Hence

$$B(\tau^- \to \nu_\tau \pi^- \pi^0 \pi^0) \leq B(\tau^- \nu_\tau \pi^- \pi^+ \pi^-) \tag{6.34b}$$

6.4. Vector hadronic decay states

If the hadronic decay state, h, has the vector property $J^P = 1^-$, the conserved vector current principle gives a relation between $v_1(\tau^- \to \nu_\tau h^-,\ q^2)$ in (6.27) and the cross section for the isotopic spin 1 h^0 states in

$$e^+ + e^- \to h^0 \tag{6.35}$$

For example, if h^- is

$$\tau^- \to \nu_\tau + \rho^- \tag{6.36a}$$

the h^0 state is the $I = 1$ part of

$$e^+ + e^- \to \rho^0 \tag{6.36b}$$

If the h^- states are

$$\begin{aligned} \tau^- &\to \nu_\tau + \pi^- + \pi^0 + \pi^0 + \pi^0 \\ \tau^- &\to \nu_\tau + \pi^- + \pi^+ + \pi^- + \pi^0 \end{aligned} \tag{6.37a}$$

the h^0 states are the $I = 1$ part of

$$
\begin{aligned}
e^+ + e^- &\to \pi^- + \pi^+ + \pi^- + \pi^+ \\
e^+ + e^- &\to \pi^- + \pi^+ + \pi^0 + \pi^0
\end{aligned}
\tag{6.37b}
$$

The general formula is

$$
v_1(\tau^- \to \nu_\tau h^-,\ q^2) = \frac{q^2}{4\pi^2\alpha^2} \sum_i C_i\ \sigma_{I=1}\,(e^+e^- \to h_i^0,\ q^2) \tag{6.38a}
$$

Insight into the meaning of the v's is given by using (3.12) with $\hbar = c = 1$ to rewrite (6.38a) as

$$
v_1(\tau^- \to \nu_\tau h^-, q^2) = \frac{1}{3\pi} \sum_i C_i\ \frac{\sigma_{I=1}(e^+e^- \to h_i^0, q^2)}{\sigma_{point}} \tag{6.38b}
$$

Thus v is proportional to the ratio $\sigma_{I=1}/\sigma_{point}$. Here q^2 is the square of the invariant mass of the h^- and h^0 systems and the C_i are Clebsch-Gordon coefficients. Discussions and applications are given by Tsai (1971), Gilman and Rhie (1985).

The formula for the example in (6.36) is

$$
v_1(\tau^- \to \nu_\tau \rho^-,\ q^2) = \frac{q^2}{4\pi^2\alpha^2}\ \sigma_{I=1}\ (e^+e^- \to \rho^0,\ q^2) \tag{6.39}
$$

Then

$$
\begin{aligned}
\Gamma(\tau^- \to \nu_\tau \rho^-) = \frac{G_F^2 \cos^2\theta_c m_\tau^3}{96\pi^3} &\int_0^{m_\tau^2} dq^2 \left[1 - \frac{q^2}{m_\tau^2}\right]^2 \left[1 + 2\frac{q^2}{m_\tau^2}\right] \\
&\times R_{I=1}(e^+e^- \to \rho^0, q^2)
\end{aligned}
\tag{6.40}
$$

Kühn and Santamaría (1990) have carried out this calculation in careful detail and find

$$
R(\tau^- \to \nu_\tau \rho^-, predict) = 1.32 \pm 0.05
$$

which is in good agreement with (6.25). This calculation uses the $\sigma(e^+e^- \to \rho^0)$ measurements of Barkov *et al* (1985).

Smith (1991) has reviewed the comparison of measurement with theory for the 4-pion vector current decays

$$
\tau^- \to \nu_\tau + 2\pi^- + \pi^+ + \pi^0 \tag{6.41a}
$$

$$
\tau^- \to \nu_\tau + \pi^- + 3\pi^0 \tag{6.41b}
$$

The average measured branching fraction for (6.41a) is (Smith 1991)

$$B(\tau- \longrightarrow \nu_\tau 2\pi^-\pi^+\pi^0) = 5.1 \pm 0.04\% \tag{6.41c}$$

but at present there are no published reliable measurements of $B(\tau^- \longrightarrow \nu_\tau\pi^-3\pi^0)$. The spectral function for (6.41c) is:

$$\begin{aligned} v_1(\tau^- \longrightarrow \nu_\tau\, 2\pi^-\pi^+\pi^0,\ q^2) = \frac{q^2}{4\pi^2\alpha^2}\Big[\frac{1}{2}\,\sigma_{I=1}\,(e^+e^- \longrightarrow 2\pi^-2\pi^+,\, q^2) \\ + \sigma_{I=1}\,(e^+e^- \longrightarrow \pi^-\pi^+2\pi^0,\, q^2)\Big] \end{aligned} \tag{6.42}$$

Smith (1991) gives the ratio of branching ratios

$$\begin{aligned} r(\tau^- \longrightarrow \nu_\tau 2\pi^-\pi^+\pi^0,\ predict) &= 0.275^{+0.03}_{-0.09} \\ r(\tau^- \longrightarrow \nu_\tau 2\pi^-\pi^+\pi^0,\ meas) &= 0.285 \pm 0.023 \end{aligned}$$

showing good agreement.

Four sets of resonances have been found in the

$$\tau^- \longrightarrow \nu_\tau + 2\pi^- + \pi^+ + \pi^0$$

decay mode (Albrecht *et al* 1991, Albrecht *et al* 1987a):

$$\begin{array}{ll} 30 \pm 4\% & \rho^0\pi^-\pi^0 \\ 26 \pm 5\% & \rho^-\pi^-\pi^+ \\ 10 \pm 5\% & \rho^+\pi^-\pi^- \\ 33 \pm 5\% & \omega\pi^- \end{array} \tag{6.43}$$

6.5. Axial vector hadronic decay states

We see from sections 6.1 and 6.4 that we have a precise way to calculate Γ for

$$\tau^- \longrightarrow \nu_\tau + \pi^-$$

and we have a general and sometimes precise way to calculate Γ for

$$\tau^- \longrightarrow \nu_\tau + (n\pi)^-\ ,\ n \text{ even}$$

But we do not have a general way to calculate Γ for

$$\tau^- \longrightarrow \nu_\tau + (n\pi)^-\ ,\ n \text{ odd}$$

because these states come through the axial vector current with no equivalent of the CVC principle. This is particularly unfortunate for the 3π states

$$\tau^- \longrightarrow \nu_\tau + \pi^- + \pi^0 + \pi^0 \tag{6.44a}$$

$$\tau^- \to \nu_\tau + \pi^- + \pi^+ + \pi^- \tag{6.44b}$$

because they have substantial branching ratios: $(7.5 \pm 0.9)\%$ for (6.44a) and $(6.7 \pm 0.6)\%$ for (6.44b).

These 3π states are important as discussed in section 9.1, therefore some comments. It is difficult to study the $\tau^- \to \nu_\tau \pi^- \, 2\pi^0$ state because the $2\pi^0$'s must be reconstructed from 4γ's and there is contamination from other decay modes. Hence almost all our knowledge of the dynamics of the 3π states comes from $\tau^- \to \nu_\tau 2\pi^- \pi^+$; recent experimental studies are Rückstuhl *et al* (1986), Schmidke *et al* (1986), and Albrecht *et al* (1986). These experiments show the mass spectrum of the 3π system has its peak in the 1200 to 1300 MeV region and that the J^P of the 3π system is mostly 1^+. This is consistent with the 3π decay mode going mostly through the $a_1(1270)$ resonance.

$$\tau^- \to \nu_\tau + a_1^-(1270) \to \nu_\tau + (3\pi)^- \tag{6.45}$$

Smith (1991) has reviewed the experiments.

In principle it is possible to calculate $\Gamma(\tau^- \to \nu_\tau (3\pi)^-)$ from the properties of the $a_1(1270)$ resonance, but in practice there are too many uncertainties. Indeed the modern interest is to use the measured behavior of the 3π system in $\tau^- \to \nu_\tau 2\pi^- \pi^+$ to calculate the properties of the $a_1(1270)$. The literature on this subject includes Tsai (1971), Pham, Roiesnel, and Truong (1978), Kawamoto and Sanda (1978), Bowler (1986), Törnqvist (1987) and Kühn and Santamaría (1990).

Albrecht *et al* (1990) have used the decay mode

$$\tau^- \to \nu_\tau + 2\pi^- + \pi^+$$

to measure for the first time parity violation in τ decays. Using the theory of Kühn and Wagner (1984) Albrecht *et al* (1990) have shown

> "... the ν_τ to be a left-handed particle and the $\bar{\nu}_\tau$ to be right-handed both with a significance of more than three standard deviations ..."

The theory for the 5π decay modes

$$\tau^- \to \nu_\tau + \pi^- + 4\pi^0 \tag{6.46a}$$

$$\tau^- \to \nu_\tau + \pi^- + \pi^+ + \pi^- + 2\pi^0 \tag{6.46b}$$

$$\tau^- \to \nu_\tau + \pi^- + \pi^+ + \pi^- + \pi^+ + \pi^- \tag{6.46c}$$

is still more difficult and uncertain in application. (Pham, Roiesnel, and Truong 1978). The best that can be done at present (Gilman and Rhie 1985) is to measure $\Gamma(\tau^- \to \nu_\tau 3\pi^- 2\pi^+)$ and then use isotopic spin conservation to set upper bounds on the decay widths for (6.46a) and (6.46b).

6.6. *Strange hadronic decay states*

A hodgepodge of methods (Kiesling 1988, Barish and Stroynowski 1988, Gilman and Rhie 1985, Oneda 1987) are used to calculate the decay widths of the strange hadronic decay modes.

$$\begin{aligned} &\tau^- \to \nu_\tau + K^- \\ &\tau^- \to \nu_\tau + K^{*-}(890) \\ &\tau^- \to \nu_\tau + (K\pi\pi)^- \end{aligned} \tag{6.47}$$

and so forth. The example of $\Gamma(\tau^- \to \nu_\tau K^-)$ was given in (6.15).

At present there is rough agreement between the calculations and the measurements. The inclusion of a $\sin^2\theta_c$ factor in the width calculation insures a small width and the measured widths have relatively large errors, $\Gamma(\tau^- \to \nu_\tau K^-) = (0.68 \pm 0.19)\%$ for example.

6.7. *The quantum chromodynamics of hadronic τ decays*

In the past five years there has been increasing interest in the application of quantum chromodynamics to the hadronic decays of the τ. (Pich 1989, Pich and Narison 1988, Braaten 1989, Braaten 1988, Pumplin 1990, Chýla, Kataev and Larin 1991, Gorishny, Kataev and Larin 1991). Pich (1989) gives a review and additional references.

Of special interest is the theory of the total hadronic decay width.

$$\Gamma_{had} = \sum_i \Gamma(\tau^- \to \nu_\tau h_i^-) \tag{6.48a}$$

from (6.29b). Defining

$$r_{had} = \Gamma_{had}/\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e) \quad , \tag{6.48b}$$

in a seminal paper Braaten (1988) argued that r_{had} could be calculated using a perturbative QCD approximation since nonperturbative corrections could be small. The general formula (Pich 1989) is

$$r_{had} = 3\left[F_{had,rad}\left(1 + \sum_{n>0} c_n \alpha_s^n(m_\tau) + C\right)\right] \tag{6.49}$$

Here

$$F_{had,rad} = 1.0215 \pm 0.0050$$

is an electroweak radiative correction and

$$\alpha_s(m_\tau) = constant/\ell n(m_\tau/\Lambda_\tau) \tag{6.50}$$

is the strong coupling constant at the τ mass. In (6.49) the $\sum_n$ term is a perturbative

QCD expansion and it is hoped that $n \leq 3$ is sufficient. The final term C contains mostly the non-perturbative QCD contributions to r_{had}. The square bracket in (6.49) is thus the correction to the crude calculation of (5.11) which gave $r_{had} = B_{had}/B_e = 3$.

The present measured value of r_{had} from (5.12) is

$$r_{had}(meas) = 3.64 \pm 0.11 \tag{6.51a}$$

Pich (1989) gives a range of theoretical values for r_{had}

$$r_{had}(predict) = 3.4 \text{ to } 4.0 \tag{6.51b}$$

depending on: (a) the value of Λ_τ in the 100–300 MeV range in (6.50) and (b) uncertainties in α_s^3 contributions to (6.49). Chýla, Kataev and Larin (1991) have given a recent discussion.

6.8. Overview of hadronic τ decays

Looking back over this section we see successful comparisons of predictions with measurement for hadronic branching ratios. But this does not mean that there exists a usable and precise theory of hadronic τ decays. All the predictions are made by connecting $\Gamma(\tau^- \to \nu_\tau h^-)$ with another experimental measurement, not by making a fundamental calculation using quantum chromodynamics. Indeed in the future, theoretical work will probably go in the opposite direction: precise experimental studies of τ hadronic decay modes will be used to develop a usable and precise quantum chromodynamic theory of the W-hadron vertex in the 1 GeV region.

There are large numbers of future measurements to be made on the larger multiplicity hadronic modes, particularly those with three or more neutral hadrons (section 9.6).

7. Miscellaneous decay modes of the τ

In this section I discuss two classes of τ decay modes which are not in sections 5 and 6: radiative decays and decays forbidden by lepton number conservation.

7.1. Radiative τ decays

In any τ decay one or more photons may be emitted. Since the decay width is multiplied by a factor of order $\alpha = 1/137$ for each emitted photon, experimental study is limited to one photon radiative decays for the present and the near future. Indeed, as noted below, even such studies are difficult.

We have two interests in the decays

$$\tau^- \to \nu_\tau + \ell^- + \bar{\nu}_e + \gamma \; ; \; \ell = e, \; \mu \tag{7.1}$$

$$\tau^- \to \nu_\tau + \text{ (hadrons)}^- + \gamma \tag{7.2}$$

First, a precise measurement of the branching ratio of a decay

$$\tau^- \rightarrow \nu_\tau + a + b + \ldots \tag{7.3}$$

requires that we understand experimentally and theoretically how we have treated the radiative decay

$$\tau^- \rightarrow \nu_\tau + a + b + \ldots + \gamma \tag{7.4}$$

Experimentally, if the γ is not observed or has very low energy the decay in (7.4) will be counted in (7.3); if the γ has high energy the decay may not fit the criteria for (7.3) and may not be counted. The observed branching ratio for (7.3) must be corrected for the uncounted events.

The second interest in radiative τ decays is that they provide another probe into the physics of τ decay, testing for unconventional aspects.

The leptonic radiative decays (7.1) have been thoroughly discussed by Wu (1990a) and in connection with radiative corrections by Marciano and Sirlin (1988). Insight into the photon energy spectrum is provided by transcribing a formula given for radiative decay of the muon by Kinoshita and Sirlin (1959).

$$\begin{aligned} \frac{d\Gamma(\tau^- \rightarrow \nu_\tau \ell^- \bar{\nu}_e \gamma)}{dy} &= \Gamma(\tau^- \rightarrow \nu_\tau \ell^- \bar{\nu}_e)\, \frac{\alpha}{3\pi}\, (1-y) \\ \times \left\{ \left[\frac{3}{y} - 2(1-y)^2\right] \left[2\ell n\, \frac{m_\tau}{m_\ell} - \frac{17}{6} + \ell n(1-y)\right] \right. &- \left. \frac{1}{12}\,(1-y)(22-13y) \right\} \end{aligned} \tag{7.5}$$

Here $y = 2E_\gamma/m_\tau$, E_γ is the γ energy in the τ rest system, and $\ell = e$ or μ. For $y \lesssim .5$ a rough approximation is

$$\frac{d\Gamma(\tau^- \rightarrow \nu_\tau \ell^- \bar{\nu}_e \gamma)}{dy} = \Gamma(\tau^- \nu_\tau \ell^- \bar{\nu}_e)\, \frac{1-y}{y}\, \left[\frac{\alpha}{\pi}\left(2\ell n\, \frac{m_\tau}{m_\ell} - \frac{17}{6}\right)\right] \tag{7.6}$$

which shows the characteristic $1/y$ bremsstrahlung spectrum. The numerical factor in the square bracket is 0.031 for $\ell = e$ and 0.0065 for $\ell = \mu$.

There has been only one experimental study of τ radiative decays, that carried out by Wu (1990a) and Wu *et al* (1990b) on

$$\tau^- \rightarrow \nu_\tau + \mu^- + \bar{\nu}_\mu + \gamma \tag{7.7}$$

The purpose was to see if there are anomalous photons associated with τ decays, an example of the second interest in radiative τ decays. It was a difficult measurement using present data and techniques. For example, the radiative decay in (7.7) must be separated from radiative τ pair production

$$e^+ + e^- \rightarrow \tau^+ + \tau^- + \gamma \tag{7.8}$$

and subsequent decay of the τ's. Wu *et al* (1990b) found that the behavior of the radiative decay in (7.7) was consistent with theory.

Concluding this section, radiative hadronic decays such as

$$\tau^- \rightarrow \nu_\tau + \pi^- + \gamma \tag{7.9a}$$

$$\tau^- \rightarrow \nu_\tau + \rho^- + \gamma \tag{7.9b}$$

provide interesting ways to examine the hadronic vector and axial vector currents. There is an extensive theoretical literature on radiative hadronic τ decays and on radiative corrections to hadronic τ decays (Queijeiro and Garcia 1988, Dominguez and Sola 1988, Banerjee 1986, Garcia and Rivera Rebolledo 1981, Kim and Resnick 1980). But there are no experimental studies of radiative hadronic decays of the τ. Improved techniques are required.

7.2. Search for τ decays violating lepton conservation

Since the early days of τ research, there have been searches (Hayes *et al* 1982) for decays which violate the conservation of τ lepton number. Examples of such proposed decays are

$$\begin{aligned}
&\tau^- \rightarrow e^- + \gamma \\
&\tau^- \rightarrow \mu^- + \gamma \\
&\tau^- \rightarrow e^- + \pi^0 \\
&\tau^- \rightarrow \mu^- + \pi^0 \\
&\tau^- \rightarrow e^- + e^+ + e^- \\
&\tau^- \rightarrow e^- + \mu^+ + \mu^-
\end{aligned} \tag{7.10}$$

and so forth. The interest is the same as searches for lepton number non-conservation in decays such as $\mu^- \rightarrow e^- + \gamma$ and $K^0 \rightarrow \mu^\pm + e^\mp$: the desire to find connections between the leptons and the desire to break out of the standard model of elementary particle physics.

No violations of τ lepton number conservation have been found. Table 5 from Aguilar-Benitez *et al* (1990) gives the upper limits on the branching ratios. Most of these limits are from Albrecht *et al* (1987b), Keh *et al* (1988) and Hayes *et al* (1982). Searches for these modes are straightforward because all the particles in the final state can be detected, and the mass of the τ reconstructed if there indeed was such a τ decay. Then in a sample of 10^n τ pairs with one identified τ decay, the upper limit is of order 3×10^{-n} if an event with the unconventional decay is not found.

There is a class of more difficult searches for τ lepton number nonconservation, in that class one of the particles in the final state cannot be detected. For example, Baltrusaitis *et al* (1985) have carried out an interesting but null search for a hypothetical light Goldstone boson G by looking for the decays

$$\begin{aligned} \tau^- &\rightarrow e^- + G \\ \tau^- &\rightarrow \mu^- + G \end{aligned} \tag{7.11}$$

The boson G is weakly interacting, hence not detected. There are serious backgrounds to the search for the decays in (7.11), namely

$$\begin{aligned} \tau^- &\rightarrow \nu_\tau + e^- + \bar{\nu}_e \\ \tau^- &\rightarrow \nu_\tau + \mu^- + \bar{\nu}_\mu \end{aligned} \tag{7.12}$$

as well as misidentification of the π in

$$\pi^- \rightarrow \nu_\tau + \pi^- \tag{7.13}$$

as an e or a μ.

Thus at present all experimental studies of τ decay agree with the conservation of τ lepton number, but in the future it will be possible to conduct much more sensitive searches. Discussions of unconventional theories which include τ lepton number nonconservation have been given by Masiero (1990), Romão, Rius and Valle (1991), and Heusch (1989a).

8. The tau neutrino

8.1. Introduction

A 1991 reviewer of our knowledge of the geography of the tau neutrino or of any neutrino is caught between two widely separated lands. The land of what we know is sparsely settled: everything we know about neutrinos is consistent with their being massless, spin 1/2 particles obeying conventional weak interaction theory. The land of speculation is thickly settled with hypotheses about non-zero neutrino masses, oscillations among types of neutrinos, neutrinos as dark matter, and so forth. My choice for this review is to stay close to the land of what we know about ν_τ. General reviews of neutrino properties and possibilities are Boehm and Vogel (1987), Langacker (1988), Valle (1989), and for astrophysical and cosmological aspects, Kolb, Schramm and Turner (1989) and Bahcall (1989).

8.2. Mass of ν_τ

As already noted in section 2.4, the 95% confidence upper limit on m_{ν_τ} is

$$m_{\nu_\tau} < 35 \text{ MeV/c}^2 \tag{8.1}$$

obtained by Albrecht *et al* (1988) from the endpoint of the decay

$$\tau^- \to \nu_\tau + 3\pi^- + 2\pi^+ \tag{8.2}$$

This method can be used to search for ν_τ masses at the several MeV/c^2 level (Gomez-Cadenas *et al* 1990) given a sufficient large sample of τ pairs, precise measurements of the particle momenta in (8.2) and of m_τ, and a good knowledge of the contamination of the τ decay sample by events from $e^+e^- \to$ hadrons. That knowledge requires that the τ sample be acquired near the $e^+e^- \to \tau^+\tau^-$ threshold using a tau-charm factory e^+e^- collider (section 10).

The decay mode (Gomez-Cadenas, Gonzales-Garcia and Pich 1990)

$$\tau^- \to \nu_\tau + K^- + K^+ + \pi^- \tag{8.3}$$

and the decay mode (Gomez-Cadenas and Gonzales-Garcia 1989, Mendel *et al* 1986)

$$\tau^- \to \nu_\tau + e^- + \bar{\nu}_e \tag{8.4}$$

can also be used to look for a small m_{ν_τ}. But these modes have less sensitivity than (8.2).

At present no one knows how to use direct measurements of τ decay to search for an m_{ν_τ} value below 1 MeV/c^2. More stringent limits have been derived from astrophysical and cosmological considerations (Kolb and Turner 1990, Harari and Nir 1987). For example, as discussed by Kolb and Turner (1990), the Cowsik-McClelland cosmological relic bound (Cowsik and McClelland 1972) on a stable light neutrino is

$$m_\nu \leq 92 \; eV/c^2 \tag{8.5}$$

As another example Grifols and Massó (1990) argue that the behavior of the supernova SN1987A limits the Dirac mass of ν_τ to

$$\begin{aligned} m_{\nu_\tau} &\lesssim 14 \text{ keV/c}^2 \\ m_{\nu_\tau} &\geq 34 \text{ keV/c}^2 \end{aligned} \tag{8.6}$$

within a factor of 3. Other references on this subject are Raffelt and Seckel (1988) and Gaemers, Gandhi, and Lattimer (1989).

If m_{ν_τ} is non-zero and if ν_τ mixes with another neutrino then detection of the neutrino mixing or oscillations can give us m_{ν_τ} (section 8.5).

8.3. Other properties of ν_τ

The ν_τ spin of 1/2 was established in the early days of τ research (Alles 1979, Kirkby 1979) from the properties of τ decay modes.

The behavior of the ν_τ in the $\tau - W - \nu_\tau$ is consistent with conventional weak interaction theory but, as discussed in section 9.7, there is a great deal more to be learned about this vertex.

Limits on possible unconventional behavior of the $\nu_\tau - Z^0 - \nu_\tau$ vertex are set by our experimental knowledge of the invisible Z^0 decay width $\Gamma(Z^0,\text{ invisible})$:

$$\Gamma(Z^0,\text{ invisible}) = \Gamma(Z^0 \rightarrow \nu_e\bar{\nu}_e) + \Gamma(Z^0 \rightarrow \nu_\mu\bar{\nu}_\mu) + \Gamma(Z^0 \rightarrow \nu_\tau\bar{\nu}_\tau) \tag{8.7}$$

The average measured value of the width (Dydak 1991) corresponds to

$$N_\nu = 2.89 \pm 0.1 \tag{8.8}$$

types of massless or near massless neutrinos. The uncertainty in N_ν relative to 3 sets an upper limit on $\Gamma(Z^0 \rightarrow \nu_\tau\bar{\nu}_\tau|$ and hence on an anomalous $\nu_\tau - Z^0 - \nu_\tau$ coupling. An example has been given by Rizzo (1990).

Finally, turning to the question of the stability of ν_τ, all we can say is that there is no evidence that it is unstable. The ν_τ might be stable because it has zero mass, or it might be stable even with non-zero mass. If the ν_τ is unstable, its decay may not be observable by present techniques. This would be the case, for example, if the decay process were

$$\nu_\tau \rightarrow \nu_x + \bar{\nu}_x + \nu_y \tag{8.9}$$

where ν_x and ν_y are not ν_τ. If the ν_τ decays to one or more detectable particles such as

$$\nu_\tau \rightarrow \gamma + \nu_x \tag{8.10a}$$
$$\nu_\tau \rightarrow e^+ + e^- + \nu_x \tag{8.10b}$$

then we can calculate a crude lower limit on the ν_τ lifetime, τ_{ν_τ}. Consider low energy studies of τ decays using $e^+e^- \rightarrow \tau^+\tau^-$, take the ν_τ momentum to be of the order of 1 GeV/c, and assume that ν_τ decays of the types in (8.10) would have been seen by now in detectors which have dimensions of order 1 m. Since the time dilation is p/m

for $m << p$, the lower limit on T_{ν_τ} is

$$(T_{\nu_\tau}/m_{\nu_\tau})_{lower\ limit} \approx 1\ s/(\mathrm{eV}/\mathrm{c}^2) \tag{8.11}$$

The theory of radiative decays of massive neutrinos (8.10a) has been discussed by Roos (1987). General discussions of the theory of neutrino decay are given by Böehm and Vogel (1987), Langacker (1988), and Valle (1989).

Limits on T_{ν_τ} from astrophysical and cosmological limits are given by Kolb and Turner (1990). An interesting limit on T_{ν_τ} has been derived from observations on the supernova explosion SN 1987a. Oberauer, Hagner, and von Feilitzsch (1989) find

$$(T_{\nu_\tau}/m_{\nu_\tau}) > 3.3 \times 10^{14} s/(eV/c^2) \tag{8.12}$$

if $m_{\nu_\tau} < 20$ eV/c^2. Similar limits are given by Kolb and Turner (1989) and Chupp *et al* (1989).

8.4. Proposed ν_τ interaction experiments

As yet there are no experiments on the interaction of the ν_τ with matter. The study of ν_τ interactions would be directed first to the weak charged current reaction

$$\nu_\tau + N \rightarrow \tau^- + \text{hadrons} \tag{8.12}$$

where N is a nucleon. Eventually the weak neutral current reaction

$$\nu_\tau + N \rightarrow \nu_\tau + \text{hadrons} \tag{8.13}$$

and the weak leptonic reaction

$$\nu_\tau + e^- \rightarrow \nu_\tau + e^- \tag{8.14}$$

might be studied. However, at present just studying (8.12) is very difficult because: (a) it is necessary to produce a neutrino beam with sufficient ν_τ intensity and (b) it is difficult to identify the $\nu_\tau - N$ interaction.

The best known method for producing a neutrino beam containing ν_τ's begins with the reactions

$$p + N \rightarrow D_s^{\pm} + \text{hadrons} \tag{8.15a}$$
$$p + N \rightarrow B^{\pm 0} + \text{hadrons} \tag{8.15b}$$

Here N means p, n or nucleus. These reactions are followed by the meson decays (sections 4.2, 4.3)

$$\begin{aligned} D_s^- &\rightarrow \tau^- + \bar{\nu}_\tau \\ D_s^+ &\rightarrow \tau^+ + \nu_\tau \end{aligned} \tag{8.16a}$$

$$\begin{aligned} B^{\pm 0} &\rightarrow \tau^- + \bar{\nu}_\tau + \text{ hadrons} \\ B^{\pm 0} &\rightarrow \tau^+ + \nu_\tau + \text{ hadrons} \end{aligned} \tag{8.16b}$$

and then the τ decays

$$\begin{aligned} \tau^- &\rightarrow \nu_\tau + \text{ other particles} \\ \tau^+ &\rightarrow \bar{\nu}_\tau + \text{other particles} \end{aligned} \tag{8.17}$$

This beam of ν_τ's and $\bar{\nu}_\tau$'s would also contain the other neutrinos: ν_e, $\bar{\nu}_e$, ν_μ, $\bar{\nu}_\mu$. Indeed there would be as many or more non-τ neutrinos than τ neutrinos.

The reactions

$$\begin{aligned} \nu_\tau + N &\rightarrow \tau^- + \text{ hadrons} \\ \bar{\nu}_\tau + N &\rightarrow \tau^+ + \text{ hadrons} \end{aligned} \tag{8.18}$$

would then be studied using a neutrino interaction detector with properties which allowed separation of (8.18) from non-ν_τ reactions such as

$$\begin{aligned} \nu_e + N &\rightarrow e^- + \text{ hadrons} \\ \nu_e + N &\rightarrow \nu_e + \text{ hadrons} \end{aligned} \tag{8.19}$$

and so forth.

One bubble chamber experiment (Talebzadeh *et al* 1987) used this method with 400 GeV protons interacting in a Cu target and beam dump. No ν_τ or $\bar{\nu}_\tau$ interactions were found, but the upper limit was consistent with the expected rate of such interactions assuming conventional weak interaction theory.

There have been studies for ν_τ interaction experiments using external proton beams from the Fermilab Tevatron (Hafen *et al* 1980, Asratyan *et al* 1980) and from the CERN SPS (Myatt 1983). But there have not been any experiments.

As discussed by De Rújula and Rückl (1984), Isaev and Tsarev (1989), Winter *et al* (1989), and Foverre (1990) the higher energies of future proton accelerators and proton-proton colliders brings two substantial benefits. First the cross section for the D_s and B production reactions (8.15) increase with energy. Second, the principle proposed method for detecting

$$\nu_\tau + N \rightarrow \tau^- + \text{ hadrons}$$

and

$$\bar{\nu}_\tau + N \rightarrow \tau^+ + \text{hadrons}$$

uses the spatial separation between the primary ν_τ or $\bar{\nu}_\tau$ interaction vertex and the secondary decay vertex of the τ^- or τ^+. The larger the initial proton energy in (8.15) the larger the average ν_τ and $\bar{\nu}_\tau$ energies, and hence the larger the separation between the vertices. The authors referenced at the beginning of this paragraph discuss proposed ν_τ interaction experiments, calculating expected event rates. There are two methods for accomplishing the ν_τ and $\bar{\nu}_\tau$ production (8.15, 8.16, 8.17): an external proton beam interacting with nucleons in a beam dump or proton-proton collisions in a collider. Three future accelerators are considered: the Accelerator and Storage Complex at Serpukhov (UNK), the Superconducting Super Collider (SSC) and the Large Hadron Collider (LHC).

8.5. ν_τ mixing and oscillation

At present, June 1991, there is no confirmed evidence for the mixing of the τ neutrino with any other neutrino. The theory of neutrino mixing and oscillation is recounted well by Böehm and Vogel (1987).

The present upper limits on $\nu_e \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_\tau$ mixing come from the oscillation search experiment of Ushida *et al* (1986), figure 9. A general review has been given by Eichler (1987). There are proposals to FNAL and to CERN for more sensitive searches for $\nu_e \rightarrow \nu_\tau$ and $\nu_\mu - \nu_\tau$ oscillations: Kodama *et al* (1990), Armenise *et al* (1990), and Astier *et al* (1991). An interesting discussion has been given by Frekers (1991) on searching for $\nu_\mu \rightarrow \nu_\tau$ oscillations using the KAON 30 GeV proton accelerator proposed for the TRIUMF laboratory.

The τ neutrino may be connected with the possible existence of a neutrino with a mass of about 17 keV/c^2, which I designate here by ν_{17}. Starting with the work of Simpson (1985) there has been some indications that the ν_{17} is produced in about 1% of the beta decays of the nuclei ^3H, ^{14}C, ^{35}S, and perhaps other nuclei. (Hime and Jelley 1991, Sur *et al* 1991). However at present there are also contradictory experiments which do not observe the ν_{17}, for example Böehm *et al* (1991). If the ν_{17} exists there are three hypotheses. The ν_{17} might be the ν_μ; the ν_{17} might be the ν_τ; or the ν_{17} might be a neutrino which has unconventionally small coupling to the Z^0 and hence does not contribute significantly to the invisible width of the Z^0 (8.7). The limits on $\nu_e - \nu_\mu$ oscillations give an upper limit on $\nu_e - \nu_\mu$ mixing considerably below the roughly 1% mixing of $\nu_e - \nu_\pi$ given by Hime and Jelley (1991) and by Sur *et al* ; (1991). Thus if the ν_{17} exists, it is the ν_τ and the ν_τ has a mass about 17 keV/c^2; or the ν_{17} does not couple like a conventional neutrino to the Z^0. In addition, if the ν_{17} is the ν_τ, $\nu_e - \nu_\tau$ oscillations should eventually be detected with approximately 1% mixing. All this depends upon whether or not the existence of the ν_{17} is confirmed.

8.6. *Speculations about ν_τ*

I conclude this chapter on the ν_τ by reminding the reader about some other speculations about ν_τ; all these speculations require a non-zero mass ν_τ. Valle (1989) has given a thorough discussion of the implications of a non-zero neutrino mass.

One class of speculations consider if the ν_τ could be the proposed dark matter of the universe (Harari 1989, Bergström and Rubinstein 1991, McKay and Ralston 1988, Langacker 1988, Giudice 1990, Giudice 1991). For example Harari (1989) has discussed the possibility that m_{ν_τ} lies in the range of 15–65 eV/c^2, and the use of $\nu_\mu - \nu_\tau$ oscillations to detect such a mass.

Another class of speculations concerns the possibility of a non-zero magnetic moment for the ν_τ. Calculations using the standard model (Lee and Schrock 1977, Marciano and Sanda 1977) give the magnetic moment

$$\mu_{\nu_\tau} = \frac{3eG_F m_{\nu_\tau}\hbar}{8\pi^2\sqrt{2}c} \tag{8.20}$$

if the ν_τ is a Dirac neutrino with non-zero mass m_{ν_τ}. In terms of the Bohr magnetron

$$\mu_B = \frac{e\hbar}{2m_e c} \tag{8.21a}$$

$$\mu_{\nu_\tau} = \kappa_{\nu_\tau}\mu_B \tag{8.21b}$$

Then (8.20) gives

$$\kappa_{\nu_\tau} = \frac{3G_F m_{\nu_\tau} m_e}{4\pi^2\sqrt{2}} \tag{8.22}$$

using (8.1)

$$m_{\nu_\tau} < 35 \text{ MeV/c}^2$$

$$\kappa_{\nu_\tau} \leq 1.1 \times 10^{-11} \tag{8.23}$$

from standard model calculations. There are two questions: (a) what are the present experimental or deduced upper limits on κ_{ν_τ} and (b) how large might κ_{ν_τ} be in unconventional theories of the τ and ν_τ. I take up the first question here.

Present limits are:

$$\kappa_{\nu_\tau} \lesssim 4 \times 10^{-6} \tag{8.24}$$

from the upper limits on the cross section for

$$e^+ + e^- \to \nu_\tau + \bar{\nu}_\tau + \gamma \tag{8.25}$$

as calculated by Grotch and Robinett (1988) and Deshpande and Sarma (1991). A similar limit was found by Rizzo (1990) using the total measured cross section for

$$e^+ + e^- \to Z^0 \to \nu_\ell + \bar{\nu}_\ell \;\; , \quad \ell = e, \mu, \; \tau \tag{8.26}$$

Depending upon assumptions about m_{ν_τ} there are also astrophysical upper limits on κ_{ν_τ} (Fukugita and Yazaki (1987), Barbieri, Mohapatra and Yanagida (1988))

9. Future research areas in τ physics

In this chapter I discuss areas in tau physics which need to be studied in future experiments. Three of these areas concern not-understood or not-settled observations in existing data: comparison of individual and topological one-charged particle branching fractions (section 9.1); comparison of the τ lifetime with the purely leptonic branching fractions (section 9.2); and the observation of additional particles produced in association with Z^0 decay to τ pairs (section 9.3). These questions may be settled in the next few years.

In any case there are a great many areas in τ physics which require longer statistics and greater precision than is available in existing or soon-to-be-acquired data. These areas are discussed in in sections 9.4 – 9.13.

9.1. Comparison of individual and topological one-charged particle branching fractions

Since 1985 there has been a problem in understanding how the one-charged particle topological branching ratio (section 5.2)

$$B_1 = (86.13 \pm 0.33)\% \tag{9.1}$$

is composed of the individual one-charged particle branching fractions such as

$$\begin{aligned}
&\tau^- \to \nu_\tau + e^- + \bar{\nu}_e \\
&\tau^- \to \nu_\tau + \mu^- + \bar{\nu}_\mu \\
&\tau^- \to \nu_\tau + \pi^- \\
&\tau^- \to \nu_\tau + K^- \\
&\tau^- \to \nu_\tau + \rho^- \\
&\tau^- \to \nu_\tau + \pi^- + n\pi^0 \;\; , \;\; n > 1
\end{aligned} \tag{9.2}$$

If one uses the directly measured average values or upper limits on branching ratios as given mostly by Aguilar-Benitez *et al* (1990), the second column of Table 6, there is no problem in meeting the requirement.

$$\sum_i B_{1i} \;=\; B_1 \tag{9.3}$$

Here B_{1i} is the branching fraction for the i^{th} one-charged particle decay mode.

But as discussed by Truong (1984), Gilman and Rhie (1985) and Gilman (1987), the relatively large upper limits on the complex decay modes can be reduced by using conventional theory and other data. There are four methods:

(*a*) Conservation of strong isospin gives upper limits on modes such as $\nu_\tau \pi^- 2\pi^0$ and $\nu_\tau \pi^- 4\pi^0$ using the better measured $\nu_\tau 2\pi^- \pi^+$ and $\nu_\tau 3\pi^- 2\pi^+$ modes.

(*b*) Modes containing the η can be studied using the $\eta \rightarrow \pi^+\pi^-\pi^0$ decay.

(*c*) The conserved vector current principle relates $B(\tau^- \rightarrow \nu_\tau \pi^- 3\pi^0)$ to the cross section for $e^+e^- \rightarrow (4\pi)^0$

(*d*) The decay $\tau^- \rightarrow \nu_\tau \pi^- \eta$ is expected to proceed through a second class weak current.

The third column in Table 6 gives the result of adding the better measured branching fractions with the improved upper limits on the poorly measured or unmeasured branching fractions. The $\sum_i B_{1i}$ is now about 5% smaller than B_1.

In the past half decade there have been numerous discussions of the possible significance of this apparent discrepancy. If the discrepancy is real then there are unexpected or unexpectedly large one-charge particle decay modes which contribute to B_1 but which have not been detected in studies of individual decay modes. Statistical studies of the significance of the apparent discrepancy have been done by Hayes and Perl (1988) and by Hayes, Perl, and Efron (1989).

The CELLO Collaboration (Behrend *et al* 1990) has made a special effort to measure uniformly the individual B_{1i}'s and B_1. Their results are given in the fourth column of Table 6. There is no discrepancy between $\sum_i B_{1i}$ and B_1 in their work. This has been discussed in detail by Kiesling (1989). An earlier study by Burchat *et al* (1987) also used a uniform analysis method.

At this time, July 1991, the issue is not resolved. Is there no problem at all, as illustrated by the results of Behrend *et al* (1990)? Or, is there a discrepancy of the magnitude shown in the third column of Table 6? A crucial number is $B(\tau^- \rightarrow \nu_\tau \pi^- 2\pi^0)$ since the conservation of strong isospin requires (Gilman and Rhie 1985)

$$B(\tau^- \rightarrow \nu_\tau \pi^- 2\pi^0) \leq B(\tau^- \rightarrow \nu_\tau 2\pi^- \pi^+) \tag{9.4}$$

From Table 6 the world average value of $B(\tau^- \rightarrow \nu_\tau \pi^- 2\pi^0)$ is $7.5 \pm 0.9)\%$ whereas Behrend *et al* (1990) gives $(10.0 \pm 1.5 \pm 1.1)\%$. A preliminary measurement from the ALEPH experiment at LEP (Zhang 1990) also gives $(10.0 \pm 0.5 \pm 0.5)\%$, but the Crystal Ball Collaboration has recently reported (Antreasyan 1990) $(5.7 \pm 0.5^{+1.7}_{-1.0})\%$.

9.2. Comparison of τ lifetime with leptonic branching fractions

In section 5.4, I noted that the average measured τ lifetime T_τ is larger by about 2 standard deviations than that calculated from B_e and B_μ. Although this is not a statistically significant difference, there has been speculation about the possible meaning of such a difference. One speculation is that G_F for the $\tau - W - \nu_\tau$ vertex is smaller than G_F for the $e - W - \nu_e$ and $\mu - W - \nu_\mu$ vertices, violating $e - \mu - \tau$ universality. Another speculation (Shin and Silverman 1988, Rajpoot 1989) is that there are two τ neutrinos, $\nu_{1\tau}$ and $\nu_{2\tau}$, with

$$\nu_\tau = \cos\theta_\tau \nu_{1\tau} + \sin\theta_\tau \nu_{2\tau} \tag{9.5a}$$

$$m_{\nu_{1\tau}} \approx 0 \tag{9.5b}$$

$$m_{\nu_{2\tau}} > m_\tau \tag{9.5c}$$

Then $G_F^2 \sin^2\theta_\tau$ appears in all decay widths and T_τ is longer by the factor $1/\sin^2\theta_\tau$.

It will take more precise measurements of B_e, B_μ, and T_τ to decide whether or not there is a real discrepancy.

9.3. *Additional particles produced in Z^0 decays to τ pairs?*

In June 1991 Decamp *et al* (1991) described a possible excess of events consisting of

$$Z^0 \to \tau^+ + \tau^- + x^+ + x^- \tag{9.6}$$

where the x^+x^- are e^+e^- pairs, $\mu^+\mu^-$ pairs, or $\pi^+\pi^-$ pairs. This possible excess was observed in data obtained with the ALEPH experiment at LEP. The masses of the x^+x^- pairs are in the range of 0.18 to 1.82 GeV/c^2. At present I know of no explanation for this excess either within the standard model or as speculation outside the standard model. Decamp *et al* (1991) state that the probability that a statistical fluctuation produced this excess "can be as large as 10^{-2}". We must wait for more data from this and other experiments.

9.4. *Precise measurements of B_e, B_μ, B_π, and B_ρ*

The measured values of B_e, B_μ, B_π, and B_ρ have fractional errors

$$0.02 \lesssim \Delta B_i/B_i \lesssim 0.04 \tag{9.7}$$

These are average measured values, the individual experiments have larger fractional errors; in addition we don't understand how to average the systematic errors over the experiments (Hayes and Perl 1988). Thus for at least some of these modes the $\Delta B/B$ may be 0.05.

It will be valuable to compare more precise measured values of these branching fractions with predicted values: B_e and B_μ from weak interaction theory (section 5.3), B_π and B_ρ from that theory and other data (section 6.1). The goal is a fractional error of

$$\Delta B_i/B_i \approx 0.005 \quad , \tag{9.8}$$

an improvement by a factor of 10. The n measurements of B_e, B_μ, B_π, and B_ρ with a precision of $\Delta B_i/B_i \approx 0.005$ can be compared with the theoretical predictions for B_μ/B_e, $1 - B_\mu/B_e$, B_π/B_e and B_ρ/B_e. Gomez-Cadenas, Heusch and Seiden (1989), Tsai (1989a), Tsai (1989b), Heusch (1989b) and others have discussed how such precise studies can uncover new physics such as a Higgs-like particle or a leptoquark.

To obtain the precision of (9.8) it is necessary to use the method due to Gomez-Cadenas, Heusch and Seiden (1989), collecting the τ pair sample at a few MeV above τ pair threshold using the mode

$$\tau^- \to \nu_\tau + \pi^- \tag{9.9}$$

to tag the τ pairs. Since the τ's are produced almost at rest the π is almost monochromatic in energy. This combined with efficient $e - \pi$, $\mu - \pi$, and $K - \pi$ separation gives very clean τ pair selection. Backgrounds from $e^+e^- \to$ hadrons will be measured directly by going below τ threshold. A tau-charm factory e^+e^- collider (section 10) is required for this measurement.

9.5. Precise measurement of Cabibbo-suppressed decay modes.

The errors are large on present measurements of the Cabibbo-suppressed decay modes such as:

$$\begin{aligned} &\tau^- \to \nu_\tau + K^- \\ &\tau^- \to \nu_\tau + K^*(892)^- \\ &\tau^- \to \nu_\tau + K^*(1430)^- \\ &\tau^- \to \nu_\tau + K^- + n\pi^0 \ , \ n > 1 \end{aligned} \tag{9.10}$$

Substantial improvement in $\Delta B_i/B_i$ are possible leading to precise studies of the vertex $W_{virtual} \to$ hadrons with $S = \pm 1$.

9.6. Untangling multiple π^0 and η decay modes.

As discussed in section 9.1, we have scanty information on τ decay modes with three or more neutral mesons: π^0's and η such as:

$$\begin{aligned} &\tau^- \to \nu_\tau + \pi^- + n\pi^0 \ , \ n > 2 \\ &\tau^- \to \nu_\tau + \pi^- + \eta + n\pi^0 \ , \ n \geq 0 \end{aligned} \tag{9.11}$$

The untangling and study of these decay modes requires that τ pair data be acquired at low E_{tot} where γ's from π^0 and η's are well separated in angle so that the π^0's and η's can be efficiently reconstructed. Furthermore the backgrounds from $e^+e^- \to$ hadrons must be measured directly. In addition a special detector is required with close-to-4π γ detection efficiency even for low energy γ's. These requirements can only be jointly met at a tau-charm factory (Seiden 1989, Kirkby 1989b, Gan 1989).

9.7. Full study of dynamics of $\tau^- \to \nu_\tau e^- \bar{\nu}_e$ and $\tau^- \to \nu_\tau \mu^- \bar{\nu}_\mu$

As discussed briefly in sections 5.3 and 5.5, in the standard model the matrix element for the decay

$$\tau^- \to \nu_\tau + e^- + \bar{\nu}_e \tag{9.12}$$

has the form

$$M = \frac{G}{\sqrt{2}} \left[\bar{u}_e \gamma^\mu (1-\gamma_5) v_{\bar{\nu}_e}\right] \left[\bar{u}_{\nu_\tau} \gamma_\mu (1-\gamma_5) u_\tau\right]$$

where the u's and v's are Dirac spinors of particle and antiparticles. If we want to allow some deviation in the $\tau - W - \nu_\tau$ vertex from the standard model then we write

$$M = \frac{G}{\sqrt{2}} \left[\bar{u}_e \gamma^\mu (1-\gamma_5) v_{\bar{\nu}_e}\right] \left[\bar{u}_{\nu_\tau} \gamma_\mu (v_\tau + a_\tau \gamma_5) u_\tau\right] \tag{9.13}$$

This leads to a formula for the e energy spectrum known since the first theoretical studies of μ decay:

$$\begin{aligned} \frac{d\Gamma_e}{\Gamma_e dx} &= 4\left[3(x^2 - x^3) + 2\rho\left(\frac{4}{3}x^3 - x^2\right)\right] \\ \rho &= \frac{3}{4}\,\frac{(v_\tau - a_\tau)^2}{(v_\tau - a_\tau)^2 + (v_\tau + a_\tau)^2} \end{aligned} \tag{9.14}$$

Here $x = 2p_e/m_\tau$, p_e is the electron momentum, ρ is the Michel parameter, and the ν_e, ν_τ, and e masses have been set to zero. This is a generalization of (5.46). The same formula holds for

$$\tau^- \to \nu_\tau + \mu^- + \bar{\nu}_\mu$$

with $m_\mu = 0$.

In the standard model $v_\tau = 1$, $a_\tau = -1$ and $\rho = 0.75$. Stroynowski (1990a) gives the average measured values

$$\begin{aligned} \rho_e &= 0.705 \pm 0.041 \\ \rho_\mu &= 0.763 \pm 0.051 \\ \rho_e \text{ and } \rho_\mu &= 0.727 \pm 0.033 \end{aligned} \tag{9.15}$$

to be compared with the 0.75 value. So far, so good.

But as discussed in a beautiful paper by Fetscher (1990), the $\tau - W - \nu_\tau$ vertex can be much more general than allowed by (9.13). Indeed, this has been known for μ decay for four decades (Scheck 1978) and was discussed for τ decay in the 1970's. In (9.13)

$$(v_\tau + a_\tau\gamma_5) = \left(\frac{v_\tau - a_\tau}{2}\right)(1-\gamma_5) + \left(\frac{v_\tau + a_\tau}{2}\right)(1+\gamma_5) \qquad (9.16a)$$

Also use the notation

$$\begin{aligned} \frac{1}{2}\,(1-\gamma_5)u &= u_L \\ \frac{1}{2}\,(1+\gamma_5)u &= u_R \end{aligned} \qquad (9.16b)$$

to denote left-handed (L) and right-handed (R) spinors. Then (9.13) is rewritten

$$\begin{aligned} M = \frac{2G}{\sqrt{2}}\Big\{&(v_\tau - a_\tau)\left[\bar{u}_{eL}\gamma^\mu v_{\bar{\nu}_e}\right]\left[\bar{u}_{\nu_\tau}\gamma_\mu u_{\tau L}\right] \\ +\ &(\nu_\tau + a_\tau)\left[\bar{u}_{eL}\gamma^\mu v_{\bar{\nu}_e}\right]\left[\bar{u}_{\nu_\tau}\gamma_\mu u_{\tau R}\right]\Big\} \end{aligned} \qquad (9.17)$$

This is now easily generalized. Let $\ell = e$ or μ and let the $\ell - W - \nu_\ell$ vertex also be non-standard with $1 - \gamma_5 \rightarrow v_\ell + a_\ell\gamma_5$. Then, following Fetscher (1990) and Mursula and Scheck (1985) define

$$\begin{aligned} g^V_{LL} &= (v_\ell - a_\ell)(\nu_\tau - a_\tau)/4 \\ g^V_{LR} &= (v_\ell - a_\ell)(v_\tau + a_\tau)/4 \end{aligned} \qquad (9.18)$$

and so forth with the superscript V denoting the vector γ^μ coupling. Then (9.17) is more generally

$$M = \frac{4G}{\sqrt{2}} \sum_{ij} g^V_{ij}\ \left[\bar{u}_{\ell i}\gamma^\mu v_{\bar{\nu}_\ell}\right]\left[\bar{u}_{\nu_\tau}\gamma_\mu u_{\tau_j}\right] \qquad (9.19)$$

with $i = L,\ R$ and $j = L,\ R$. The final generalization adds scalar and tensor coupling with γ^μ in (9.19) replaced by 1 and $\sigma^{\mu\nu} = i\left[\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right]/2$ respectively. Denoting the coupling operators by Γ^S, Γ^V, and Γ^T.

$$M = \frac{4G}{\sqrt{2}} \sum_{\substack{i,j=L,R \\ N=S,V,T}} g^N_{ij}\ < \bar{u}_{\ell i}\,|\Gamma^N|\, v_{\bar{\nu}_\ell} > < \bar{u}_{\nu_\tau}\,|\Gamma_N|\, u_{\tau j} > \qquad (9.20)$$

Of the 12 g^N_{ij}'s, g^T_{LL} and g^T_{RR} are identically zero. Since the g^N_{ij}'s can be complex, there are 19 independent parameters ignoring an overall phase. This is in contrast to the standard model where

$$g_{LL}^{V} = 1 \tag{9.21a}$$

$$\text{all other } g_{ij}^{N} = 0 \tag{9.21b}$$

In μ decay

$$\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e \tag{9.22}$$

a tremendous amount of work has been done to set upper limits on (9.18b) (Fetscher, Gerber and Johnson 1986). As discussed by Fetscher (1990), Pich (1990a) and others a great deal of work needs to be done to carry out similar investigations of the τ leptonic decays. Evidence in $\tau \rightarrow \nu_\tau \ell^- \bar{\nu}_e$ for any $g_{ij}^N \neq 0$ except g_{LL}^V means the discovery of new physics. The detailed study of τ leptonic decays will make use of the correlated spins of the τ's produced in pairs through correlations of the momenta and angles of the e's and μ's. Many of these studies are best carried out close to the τ pair threshold, which can be done at a tau-charm factory. As noted by Stroynowski (1990b) in a private communication, it is even possible at a tau-charm factory to study the sequence

$$\tau^- \rightarrow \nu_\tau + \mu^- + \bar{\nu}_\mu$$
$$\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$$

so that the polarization of the μ^- is measured, an important aspect of studying the g_{ij}^N's.

9.8. *Detailed study of 5 and 7-charged particle decay modes*

As noted in section 6 we know very little about τ decay modes with 5 and 7-charged particles; indeed we only have an upper limit on the total branching fraction of the latter modes. We need low energy experiments with large statistics, the low energy would allow sorting out of the charged and neutral particles.

9.9. *Study of rare decay modes*

Some hadronic decay modes will have small branching fractions because of large multiplicity, for example

$$B(\tau^- \rightarrow \nu_\tau\ 4\pi^- 3\pi^+ n\pi^0\,,\ n \geq 0) \leq 1.9 \times 10^{-4}\ ,$$

or because the modes have moderate multiplicity but include K's or η's. At present we don't expect any unusual physics to be associated with such modes as long as they obey the first-class hadronic decay current rules (section 6.3) (Tsai 1971, Barish and Stroynowski 1988, Burchat 1988, Pich 1990a). Namely, for non-strange hadronic states the G-parity is $G = +1$ for the weak vector current and $G = -1$ for the weak axial vector current.

On the other hand, second-class weak currents have

$$\text{Vector:}\ G = -1\ ,\quad J^P = 1^- \tag{9.23a}$$

$$\text{Axial vector:}\ G = +1\ ,\quad J^P = 0^-,\, 1^+ \tag{9.23b}$$

Decays with such properties have never been seen in nuclear or elementary particle physics because they have very small branching fractions. The τ offers the best possibility to observe decays through the second class current. Possibilities are (Leroy and Pestieau 1978, Pich 1987, Zachos and Meurice 1987)

$$\tau^- \rightarrow \nu_\tau + \pi^- + \eta \tag{9.24a}$$

$$\tau^- \rightarrow \nu_\tau + b_1\,(1235) \tag{9.24b}$$

The b_1 (1235) has $G = +1,\ J^P = 1^+$ so (9.24b) obeys (9.23b).

In the standard model, second-class current decays do not occur if one ignores the electromagnetic corrections to isospin symmetry in the strong interaction. Therefore there are two interests in observing and studying second-class current decays. First, what is the strength of a second-class current decay due to the electromagnetic correction, that is a decay within the standard model? Second, are there second-class current decays whose properties cannot be explained by the standard model? Interesting discussions are given by Berger and Lipkin (1987) and by Bramon, Narison and Pich (1987).

There are two ways to estimate the strength of a second-class current decay due to the electromagnetic correction. That correction introduces the fine structure constant α in a second-class current decay amplitude. Then

$$K^2 = \frac{\Gamma(\text{second-class current})}{\Gamma(\text{first-class current})} \sim \alpha^2 \sim 10^{-4} \tag{9.25}$$

Alternately, the second-class current decay may be thought of as due to the difference of the d quark and u quark current masses: $\Delta m = m_d - m_u \sim 1$ MeV.

$$K^2 = \frac{\Gamma\ (\text{second-class current})}{\Gamma\ (\text{first-class current})} \sim \left(\frac{m_d - m_u}{m_\pi}\right) \sim 10^{-4} \tag{9.26}$$

More generally the range of such crude estimates is

$$10^{-3} \lesssim K^2 \leq 10^{-5} \tag{9.27}$$

Thus for the second-class current decay

$$\tau^- \rightarrow \nu_\tau + \pi^+ + \eta$$

the estimates are (Pich 1987, Zachos and Meurice 1987)

$$B(\tau^- \rightarrow \nu_\tau \pi^- \eta) \sim K^2 \, B(\tau^- \rightarrow \nu_\tau \pi^-) \sim 10^{-4} \text{ to } 10^{-6} \tag{9.28}$$

The observation and study of such a small decay mode requires: large statistics, good control of errors, and direct knowledge of backgrounds.

Another class of rare decay modes, the radiative decay modes such as

$$\begin{aligned} \tau^- &\rightarrow \nu_\tau + e^- + \bar{\nu}_e + \gamma \\ \tau^- &\rightarrow \nu_\tau + \mu^- + \bar{\nu}_u + \gamma \\ \tau^- &\rightarrow \nu_\tau + \pi^- + \gamma \\ \tau^- &\rightarrow \nu_\tau + \rho^- + \gamma \end{aligned} \tag{9.29}$$

were discussed in section 7.1.

9.10. Study of electromagnetic moments of the τ

I consider first the magnetic dipole moment. As summarized by Barish and Stroynowski (1988), the τ magnetic dipole moment is given by

$$\mu_\tau = g_\tau \, \frac{e\hbar}{2m_\tau c} \, \frac{1}{2} \tag{9.30}$$

where g_τ is the gyromagnetic ratio and the final 1/2 is the τ spin. Since g_τ is equal to 2 in lowest order for a Dirac particle, it is usual to define:

$$a_\tau = (g_\tau - 2)/2 \tag{9.31}$$

Then

$$\mu_\tau(\text{anom}) = a_\tau \, \frac{e\hbar}{2m_\tau c} \tag{9.32}$$

is the anomalous magnetic moment. Quantum electrodynamics and the other parts of the standard model predict

$$a_\tau = \frac{\alpha}{2\pi} + \sum_{n>1} c_n \alpha^n + C \tag{9.33}$$

where the c_n's are due to higher order QED corrections and the C is due to weak and hadronic interaction corrections. Samuel, Li and Mendel (1990) give a recent calculation of μ_τ (anom). A profound goal in τ research is to measure μ_τ (anom), but at present no one knows how to measure it in high precision experiments as has been carried out for the e and μ.

There are a number of low precision methods for investigating μ_τ (anom). Silverman and Shaw (1983) first suggested looking for anomalous behavior in the $d\sigma/d\Omega$ for

$$e^+ + e^- \rightarrow \tau^+ + \tau^- \tag{9.34}$$

They found

$$a_\tau \leq 0.02 \tag{9.35}$$

Note that (9.33) gives $a_\tau \approx \alpha/2\pi \approx 10^{-3}$. See also Domokos *et al* (1985). Laursen, Samuel and Sen (1984) suggested the use of the radiation zeros concept to study the radiative decays

$$\tau^- \rightarrow \nu_\tau + \ell^- + \bar{\nu}_\ell + \gamma \ ; \ \ell = e, \mu \tag{9.36}$$

Grifols and Méndez (1991) used data on

$$e^+ + e^- \rightarrow Z^0 \rightarrow \tau^+ + \tau^- + \gamma \tag{9.37}$$

to set the limit

$$a_\tau \leq 0.11 \tag{9.38}$$

This upper limit on a_τ is at $q^2 = 0$, (9.35) was at $q^2 \sim 1000$ GeV^2. Thus there is a long way to go in investigating a_τ (Silverman 1989).

The question of the τ electric dipole, ϵ_τ, moment has been discussed by Hoogeveen and Stodolsky (1988), del Aguila and Sher (1990) and by Grifols and Méndez (1991). Conventional theory requires $\epsilon_\tau = 0$. Present upper limits are (del Aguila and Sher 1990)

$$\epsilon_\tau \leq 1.4 \times 10^{-16} \ e \text{ cm} \tag{9.39a}$$

from the $do/d\Omega$ of (9.34); and (Grifols and Méndez 1991)

$$\epsilon_\tau \leq 6 \times 10^{-16} \ e \text{ cm} \tag{9.39b}$$

from (9.37). Bernreuther and Nachtmann (1989) have discussed search methods for a τ electric dipole moment at a tau-charm factory.

9.11. Searching for τ lepton number nonconservation

In section 7.2 I discussed searches, so far null, for τ lepton number nonconservation in τ decays, and the reader is referred to that section.

Another area where such nonconservation may appear is in e^+e^- annihilation:

$$e^+ + e^- \to \tau^\pm + x^\mp \tag{9.40}$$

where x is not a τ and not a τ^* (section 9.12). Gomez-Cadenas *et al* (1991) looked for

$$e^+ + e^- \to \tau^\pm + e^\mp \tag{9.41a}$$

and

$$e^+ + e^- \to \tau^\pm + \mu^\mp \tag{9.41b}$$

at $E_{tot} = 29$ GeV. Their null result has the 95% C.L. upper limits.

$$\sigma(e^+e^- \to \tau^\pm e^\mp)/\sigma_{point} \leq 1.2 \times 10^{-3} \tag{9.42a}$$
$$\sigma(e^+e^- \to \tau^\pm \mu^\mp)/\sigma_{point} \leq 4.1 < 10^{-3} \tag{9.42b}$$

where σ_{point} is given by (3.12).

Akrawy *et al* (1991) looked for the reactions in (9.41) at the Z^0, again with a null result. Their 95% C.L. upper limits in terms of Z^0 branching ratios are

$$B(Z^0 \to \tau^\pm e^\mp) \leq 7.2 \times 10^{-5} \tag{9.43a}$$
$$B(Z^0 \to \tau^\pm \mu^\mp) \leq 35 \times 10^{-5} \tag{9.43b}$$

To compare with (9.42) I define

$$B(Z^0 \to \ell^+\ell^-) = 0.033 \quad \ell = e,\ \mu, \text{ or } \tau$$

then

$$B(Z^0 \to \tau^\pm e^\mp)/B(Z^0 \to \ell^+\ell^-) \leq 2.2 \times 10^{-3} \tag{9.44a}$$
$$B(Z^0 \to \tau^\pm \mu^\mp)/B(Z^0 \to \ell^+\ell^-) \leq 11. \times 10^{-3} \tag{9.44b}$$

Future searches can be extended with larger statistics or to higher energies. Theories which can lead to (9.40) are described by Bigi *et al* (1986), Bernabéu *et al* (1987), Bernabéu and Santamaría (1987), Rizzo (1988), Eilam and Rizzo (1987), Valle (1989) and Levine (1987).

9.12. *Searching for excited τ's*

There is a long history of searches for excited leptons:

$$\begin{aligned} e^{*-} &\to e^- + \gamma \\ \mu^{*-} &\to \mu^- + \gamma \\ \tau^{*-} &\to \tau^- + \gamma \end{aligned} \tag{9.45}$$

So far, none have been found. The present best lower limits on the mass of a possible τ^* come from experiments at LEP (Akrawy *et al* 1990, Adeva *et al* 1990, Decamp *et al* 1990). Assuming the process

$$e^+ + e^- \to Z^0 \to \tau^{*+} + \tau^{-*} \to \tau^+ + \tau^- + \gamma + \gamma \tag{9.46}$$

the lower limit on m_{τ^*} is

$$m_{\tau^*} \gtrsim 45 \text{ GeV/c}^2$$

The sensitivities of the processes

$$e^+ + e^- \to Z^0 \to \tau^{*\pm} + \tau^{\mp} \to \tau^+ + \tau^- + \gamma \tag{9.47a}$$

and

$$e^+ + e^- \to \gamma_{virtual} \to \tau^{*\pm} + \tau^{\mp} \to \tau^+ + \tau^- + \gamma \tag{9.47b}$$

depend on the strengths of the $\tau^*\tau Z$ and $\tau^*\tau\gamma$ coupling. The experiments just referenced have searched with null results up to m_{τ^*} masses of about 89 GeV/c^2.

9.13. *Can τ^-–nucleus or $\tau^+\tau^-$ atoms be studied?*

At present there are no experimental methods for studying the two kinds of τ atoms which have been discussed in the literature. First, in analogy to the μ^-– nucleus atom, Strobel and Wills (1983) have discussed τ^-– nucleus atoms, considering nuclei from ${}^1_1$H to ${}^{24}_{12}Mg$. They calculate lifetimes and x-ray transition energies for 2P to 1S and 3D to 2P transactions. The lifetimes are small compared to the τ lifetime, therefore if sufficient τ^--nucleus atoms could be formed the x-rays could be detected. The capture of τ^- in nuclei has recently been discussed by Ching and Oset (1991).

The second type of atom is $\tau^+\tau^-$ in analogy to positronium which is e^+e^-. $\tau^+\tau^-$ is sometimes called tauonium. It was first discussed by Moffat (1975) and later by Avilez, Montemayor and Moreno (1978) and Avilez, Ley-Koo, and Moreno (1979). These authors discuss the atomic physics of $\tau^+\tau^-$ and the possibilities of studying that physics by forming $\tau^+\tau^-$ atoms in e^+e^- annihilation. There are experimental difficulties due to (a) relatively long lifetimes for atomic transition and (b) the small *integrated* cross section for

$$e^+ + e^- \rightarrow (\tau^+\tau^-)_{atom} \tag{9.48}$$

Yndurain (1991) has considered the physics of the atomic hyperfine structure in the $\tau^+\tau^-$ atom.

10. The tau-charm factory concept and design

10.1. The tau-charm factory concept

Tau physics research has been carried out at e^+e^- colliders ranging in energy from about 4 GeV, somewhat above the τ-pair production threshold, to about 92 GeV, the Z^0 resonance (section 2.2). And as the energy of the LEP e^+e^- collider is raised towards 200 GeV, τ physics research will continue into that energy range. But there has never been an e^+e^- collider specifically designed and built to do τ physics research.

Such an e^+e^- collider, called a tau-charm factory, was proposed by Kirkby in 1987 (Kirkby 1987, Kirkby 1989a). The first technical design of a tau-charm factory collider was made by Jowett in 1987 (Jowett 1987, Jowett 1988, Jowett 1989).

At a tau-charm factory the τ-pairs would be produced for study at low energies in the vicinity of the τ-pair production threshold. The collider would have a high luminosity:

$$L_{max} \approx 10^{33}\ \mathrm{cm}^{-2}\ s^{-1} \tag{10.1}$$

As illustrated in figure 10, there are three important advantages in doing τ research in this low energy region:

(i) The most serious and difficult hadronic backgrounds in τ pair data come from D, D_s and B meson production and decay. By obtaining the τ pair data below the ψ'' energy, these backgrounds are avoided.

(ii) Below the ψ' energy, the nature of the $e^+e^- \rightarrow$ hadrons continuum changes very slowly with energy as the energy goes below the $e^+e^- \rightarrow \tau^+\tau^-$ threshold. Therefore for τ data obtained below the ψ' energy, the hadronic contamination can be directly measured by operating the collider just below the τ threshold.

(iii) At τ threshold $\sigma_\tau = 0.23$ nb and 2 MeV above threshold $\sigma_\tau = 0.4$ nb (section 3.3). Thus τ pair data can be obtained with the τ's produced almost at rest, an important condition for some τ studies (Gomez-Cadenas, Heusch, and Seiden 1989).

At the three principal operating energies for τ research (figure 10) the τ-pair production rate is:

$$\begin{aligned}
&3.57 \text{ GeV (just above threshold)}: \quad 0.5\times 10^7 \quad \tau \text{ pairs/year}\\
&3.67 \text{ GeV (just below the } \psi'): \quad 2.4\times 10^7 \quad \tau \text{ pairs/year}\\
&3.5 \text{ GeV (maximum } \sigma_\tau): \quad 3.5\times 10^7 \quad \tau \text{ pairs/year}
\end{aligned} \tag{10.2}$$

This is based on $L = 10^{33}$ cm^{-2} s^{-1} s/year. Thus τ research at a tau-charm factory would be carried out with very large statistics, minimum backgrounds from non-τ pair events, and the ability to directly measure remaining backgrounds. There is also a tremendous amount of charm quark physics to do at a tau-charm factory as described in the Proceedings of the Tau-Charm Factory Workshop (Beers, 1989) and by Schindler (1989), Schindler (1990a), Schindler (1990b) and Kirkby (1989a). The principal operating points are the $c\bar{c}$ resonances

$$\begin{aligned}
&J/\psi \text{ at } E_{tot} = 3.10\\
&\psi' \text{ at} E_{tot} = 3.69
\end{aligned}$$

and the D pair production points

$$\begin{aligned}
&D^+D^- \text{ at the } \psi'',\ E_{tot} = 3.77 \text{ GeV}\\
&D^0\bar{D}^0 \text{ at the } \psi'',\ E_{tot} = 3.77 \text{ GeV}\\
&D_s\bar{D}_s \text{ at } E_{tot} = 4.03 \text{ GeV}\\
&D_s\bar{D}_s^* \text{at } E_{tot} = 4.14 \text{ GeV}
\end{aligned}$$

10.2. Tau-charm factory collider design

Jowett (1987, 1988, 1989) worked out the basic design for a tau-charm factory collider which would have the four required properties:

$$\begin{aligned}
&\text{(i) } 3.0 \le E_{tot} \lesssim 5.0 \text{ GeV}\\
&\text{(ii) } L_{max} \approx 10^{33} \text{ cm}^{-2} \text{ s}^{-1}\\
&\text{(iii) Highly reliable operation}\\
&\text{(iv) } \Delta E_{tot} \sim \text{few MeV}
\end{aligned} \tag{10.3}$$

The high luminosity is obtained by two basic design concepts. The first design concept requires 20 to 30 bunches per beam, each bunch having about 1.6×10^{11} particles. To avoid the luminosity-limiting effects of so many bunch-bunch interactions, the bunches are only allowed to collide at one or two points. Since the ring circumference is too small to permit the beams to be separated at all the other crossing points; the collider has separate e^+ and e^- rings, figure 11.

The second design concept which yields high luminosity is tight focussing of the bunches at the interaction point. For example, in the recent design of Baconnier *et al* (1990) (figure 12), at the interaction point

$$
\begin{aligned}
\beta_x^* &= 20 \text{ cm} \\
\beta_y^* &= 1 \text{ cm} \\
\sigma_x^* &= 280\ \mu\text{m} \\
\sigma_y^* &= 14\ \mu\text{m} \\
\sigma_z &= 6\ mm
\end{aligned}
\tag{10.4}
$$

After the original work of Jowett further design work was carried out at the 1989 Tau-Charm Factory Workshop (Beers 1989). This group confirmed that $L_{max} \approx 10^{33}$ cm^{-2} s^{-1} was feasible with present technology (Brown, Fieguth, and Jowett 1989).

A separate conceptual design was carried out by Gonichon, Le Duff, Mouton and Travier (1990). This report discusses the accelerator physics in very useful detail, for example comparing flat beams with round beams.

Another conceptual design based more closely on the original Jowett design was prepared by Barish *et al* (1990). Both this design and the Gonichon *et al* design attained $L_{max} \approx 10^{33}$ cm^{-2} s^{-1}.

Danilov *et al* (1990) have also discussed tau-charm factory design.

The most recent conceptual design (Baconnier, 1990) was carried out by physicists from CERN, LAL in France, and CIEMAT in Spain. Figure 12 shows the schematic design. The ring circumference is 360 m and $L_{max} \approx 10^{33}$ cm^{-2} s^{-1}. The high intensity e^+ and e^- injector consists of a linear accelerator followed by a booster synchrotron. The injector would also be use for a separate synchrotron radiation ring.

Summarizing, the basic principles of collider design for a tau-charm factory are:

(i) Separate e^+ and e^- rings.

(ii) One or two interaction regions.

(iii) Tight focussing of the bunches at the interaction points.

(iv) Multiple bunches, about 20 to 30 in each ring.

(v) Rings have a large radius to keep synchrotron radiation moderate and allow a conventional beam pipe.

(vi) Substantial RF overvoltage and low beam pipe impedance to produce short bunches.

(vii) Feedback systems to control multibunch instabilities.

(viii) A high intensity e^+ and e^- injector to maintain luminosity by "top-off" of the circulating bunches.

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Table 1. Experimenters on Proposal SP-2 to use the Mark I detector at SPEAR, the experiment in which the tau was discovered.

Spokesman:	R. R. Larsen	
Experimenters:	Name	Group and Distribution
	A.M. Boyarski	Group C – SLAC
	J. Dakin	Group E – SLAC
	G. Feldman	Group E – SLAC
	G.E. Fischer	Group C – SLAC
	D. Fryberger	Group EFD – SLAC
	H.L. Lynch	Group C – SLAC
	F. Martin	Group E – SLAC
	M.L. Perl	Group E – SLAC
	J.R. Rees	Group C – SLAC
	B. Richter	Group C – SLAC
	R.F. Schwitters	Group C – SLAC
	G. S. Abrams	LBL – UC Berkeley
	W. Chinowsly	LBL – UC Berkeley
	C. E. Friedberg	LBL – UC Berkeley
	G. Goldhaber	LBL – UC Berkeley
	R. J. Hollebeek	LBL – UC Berkeley
	J. A. Kadyk	LBL – UC Berkeley
	G. H. Trilling	LBL – UC Berkeley
	J. S. Whitaker	LBL – UC Berkeley
	J. E. Zipse	LBL – UC Berkeley

Table 2. τ decay modes with branching fractions greater than 5% from Aguilar-Benitez *et al* (1990).

Decay Mode	Average values of measured branching fractions in %
$\tau^- \to \nu_\tau + e^- + \bar{\nu}_e$	17.7 ± 0.4
$\tau^- \to \nu_\tau + \mu^- + \bar{\nu}_\mu$	17.8 ± 0.4
$\tau^- \to \nu_\tau + \pi^-$	11.0 ± 0.5
$\tau^- \to \nu_\tau + \rho^-$	22.7 ± 0.8
$\tau^- \to \nu_\tau + \pi^- + \pi^0 + \pi^0$	7.5 ± 0.9
$\tau^- \to \nu_\tau + \pi^- + \pi^+ + \pi^-$	7.1 ± 0.6

Table 3. Properties of the known leptons (Aguilar-Benitez *et al* 1990).

Lepton	Mass	Lifetime	Lifetime/mass for neutrinos
e	0.511 MeV	$> 2 \times 10^{22}$ years	
ν_e	< 17 eV (95% CL)		> 300 s/eV (90% CL)
μ	105.7 MeV	2.197×10^{6} s	
ν_μ	< 0.27 MeV (90% CL)		> 0.11 s/eV (90% CL)
τ	$1784.^{+2.7}_{-3.6}$ MeV	$3.03 \pm 0.08 \times 10^{-13}$ s	
ν_τ	< 35 MeV (95% CL)		

Table 4. Examples of 95% C. L. lower limits on $\Lambda_\pm$ and $\Lambda^c{}_\pm$ for $e^+e^- \to \tau^-\tau^+$. The $\Lambda^c{}_\pm$ limits are for the vector-vector interaction in (3.18).

Reference	Λ_+(GeV)	Λ_-(GeV)	Λ^c_+(TeV)	Λ^c_-(TeV)
Bartel *et al* (1986)	285	210	4.1	5.7
Adeva *et al* (1986)	235	205		
Behrend *et al* (1989)	318	231		

Table 5. Upper limits on branching ratios for forbidden decay modes of τ with 90% CL (Aguilar-Benitez 1990).

Mode	Upper Limit on Branching Ratio
$\mu^-\gamma$	5.5×10^{-4}
$e^-\gamma$	2.0×10^{-4}
$\mu^-\pi^0$	8.2×10^{-4}
$e^-\pi^0$	1.4×10^{-4}
$\mu^-\mu^+\mu^-$	2.9×10^{-5}
$e^-\mu^+\mu^-$	3.3×10^{-5}
$\mu^-e^+e^-$	3.3×10^{-5}
$e^-e^+e^-$	3.8×10^{-5}
μ^-K^0	1.0×10^{-3}
e^-K^0	1.3×10^{-3}
$\mu^-\rho^0$	3.8×10^{-5}
$e^-\rho^0$	3.9×10^{-5}
$e^-\pi^+\pi^-$	4.2×10^{-5}
$e^+\pi^-\pi^-$	6.3×10^{-5}
$\mu^-\pi^+\pi^-$	4.0×10^{-5}
$\mu^+\pi^-\pi^-$	6.3×10^{-5}
$e^-\pi^+K^-$	4.2×10^{-5}
$e^+\pi^-K^-$	1.2×10^{-4}
$\mu^-\pi^+K^-$	1.2×10^{-4}
$\mu^+\pi^-K^-$	1.2×10^{-4}
$e^-K^*(892)^0$	5.4×10^{-5}
$\mu^-K^*(892)^0$	5.9×10^{-5}
$e^+\mu^-\mu^-$	3.8×10^{-5}
$\mu^+e^-e^-$	3.8×10^{-5}
$e^-\eta$	2.4×10^{-4}

Table 6. Comparison of individual and topological one-charged particle branching fractions. See text for significance of the second through fourth columns.

Decay mode	Branching fraction in %		
	World average Aguilar-Benitez *et al* (1990)	World average with upper limits from theory and other data marked with*	Behrend *et al* (1990)
$e^-\bar{\nu}_e\nu_\tau$	17.7 ± 0.4	17.7 ± 0.4	18.4 ± 0.9
$\mu^-\bar{\nu}_\mu\nu_\tau$	17.8 ± 0.4	17.8 ± 0.4	17.7 ± 0.9
$\rho^-\nu_\tau$	22.7 ± 0.8	22.7 ± 0.8	22.2 ± 1.7
$\pi^-\nu_\tau$	11.0 ± 0.5	11.0 ± 0.5	11.1 ± 1.0
$K^-\nu_\tau + K^{*-}\nu_\tau$	2.1 ± 0.3	2.1 ± 0.3	2.1 ± 0.4
$\pi^-2\pi^0\nu_\tau$	7.5 ± 0.9	$\leq 7.1 \pm 0.6^{*(i)}$	10.0 ± 1.9
$\pi^-n\,had^0\nu_\tau, n > 2$	$7.2 \pm 2.0^{(ii)}$	$\leq 2.2^{*(iii)}$	$4.0 \pm 2.3^{(iv)}$
Sum of above	86.0 ± 2.5	80.6 ± 1.3	$85.5 \pm 2.6^{(v)}$
B_1	86.1 ± 0.3	86.1 ± 0.3	84.9 ± 0.5

$^{(i)}$ Calculated using isospin conservation and $B(\nu_\tau 2\pi^-\pi^+) = (7.1 \pm 0.6)\%$.
$^{(ii)}$ Calculated from $B(hadron^- \geq 2\pi^0\nu_\tau) - B(\pi^-2\pi^0\nu_\tau)$ with corrected errors added linearly, (Aguilar-Benitez *et al* 1990)
$^{(iii)}$ From Gilman and Rhie (1985) and Gilman (1987)
$^{(iv)}$ Calculated from $B(hadron > 2\gamma\nu_\tau) - B(\pi^-2\pi^0\nu_\tau)$, (Behrend *et al* 1990)
$^{(v)}$ Error calculated using $B(hadron > 2\gamma\nu_\tau) = (14.0 \pm 1.3)\%$ (Behrend *et al* 1990) as sum of bottom two branching fractions.

Figure Captions

Fig. 1. A search for heavy leptons by Bernardini *et al* (1973). The caption read, "The expected number of $(\mu^\pm e^\mp)$ pairs vs. m_{HL} for two types of universal weak couplings of the heavy leptons. The dashed lines indicate the 95% confidence levels for m_{HL}. a) HL universally coupled with ordinary leptons and hadrons, b) HL universally coupled with ordinary leptons."

Fig. 2. One of the $e\mu$ events which led Perl *et al* (1975) to discover the τ. The μ moves upward through the muon detector tower and the e moves downward. The numbers 13 and 113 give the relative amounts of electromagnetic shower energy deposited by the μ and e. The six square dots show the positions of longitudinal support posts of the magnetostrictive spark chamber used for tracking.

Fig. 3. The reaction $e^+e^- \to \tau^+\tau^-$ occurs through the processes in (a) and (b).

Fig. 4. The behavior of σ_τ as a function of E_{tot} from threshold to 1 TeV. Above 96 GeV, the curve is based on conventional theory since there are as yet no measurements of σ_τ above 96 GeV.

Fig. 5. Electromagnetic radiative corrections to $e^+e^- \to \tau^+\tau^-$ at low energies.

Fig. 6. Diagram for D^+ or D_s^+ decay to $\ell^+\nu_\ell$ where $\ell = e,\ \mu$, or τ.

Fig. 7. Diagrams for τ decays: (a) general, (b) pure leptonic, (c) radiative leptonic, (d) semi-leptonic also called hadronic, (e) crude free quark model for semi-leptonic.

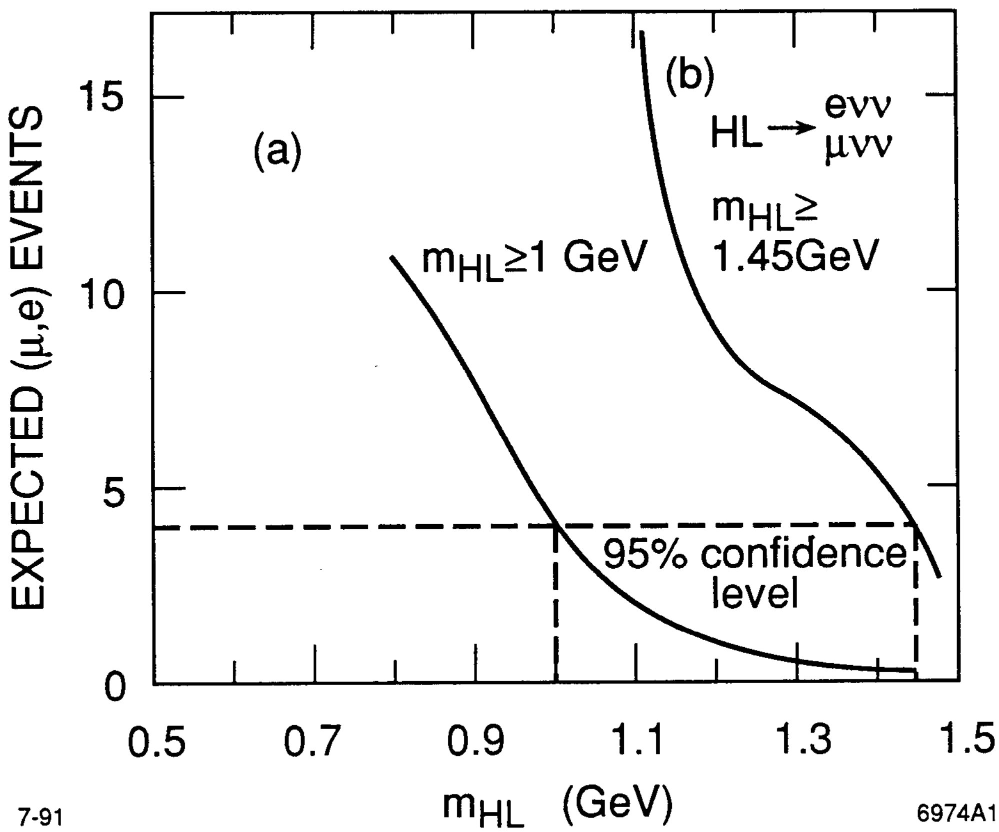
Fig. 8. Diagram for (a) $\tau^- \to \nu_\tau\pi^-$ and (b) $\pi^- \to \mu^-\bar{\nu}_\mu$.

Fig. 9. The 90% C.L. limits for $\nu_\mu \to \nu_\tau$ and $\nu_e \to \nu_\tau$ oscillations from Ushida *et al* (1986).

Fig. 10. $\sigma_{\tau\tau}$, $R_{\tau\tau}$, and $R_{hadrons}$ for the main part of the tau-charm factory energy range.

Fig. 11. Schematic design of a tau-charm factory collider from Jowett (1988).

Fig. 12. Schematic representation of the injector, collider, and synchrotron light source from Baconnier (1990). The collider may be designed to allow a second interaction area to be installed later.



(a)
(b)
HL→ evv μvv
m_{HL}≥1 GeV
m_{HL}≥ 1.45GeV
95% confidence level
EXPECTED (μ,e) EVENTS
0
5
10
15
0.5
0.7
0.9
1.1
1.3
1.5
m_{HL} (GeV)
7-91
6974A1

Fig. 1

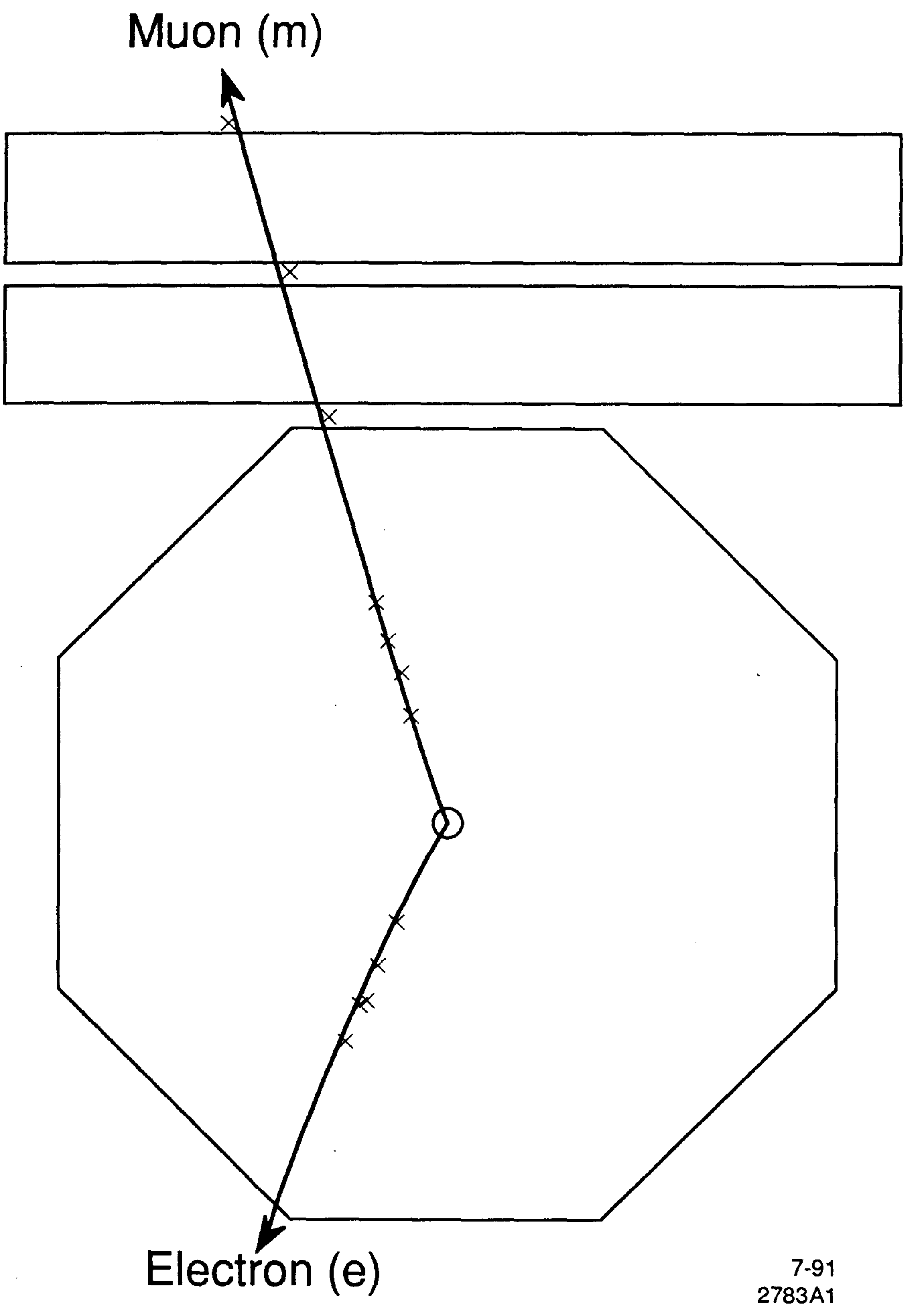


Fig. 2

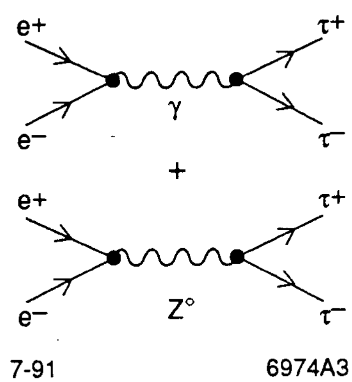


Fig. 3

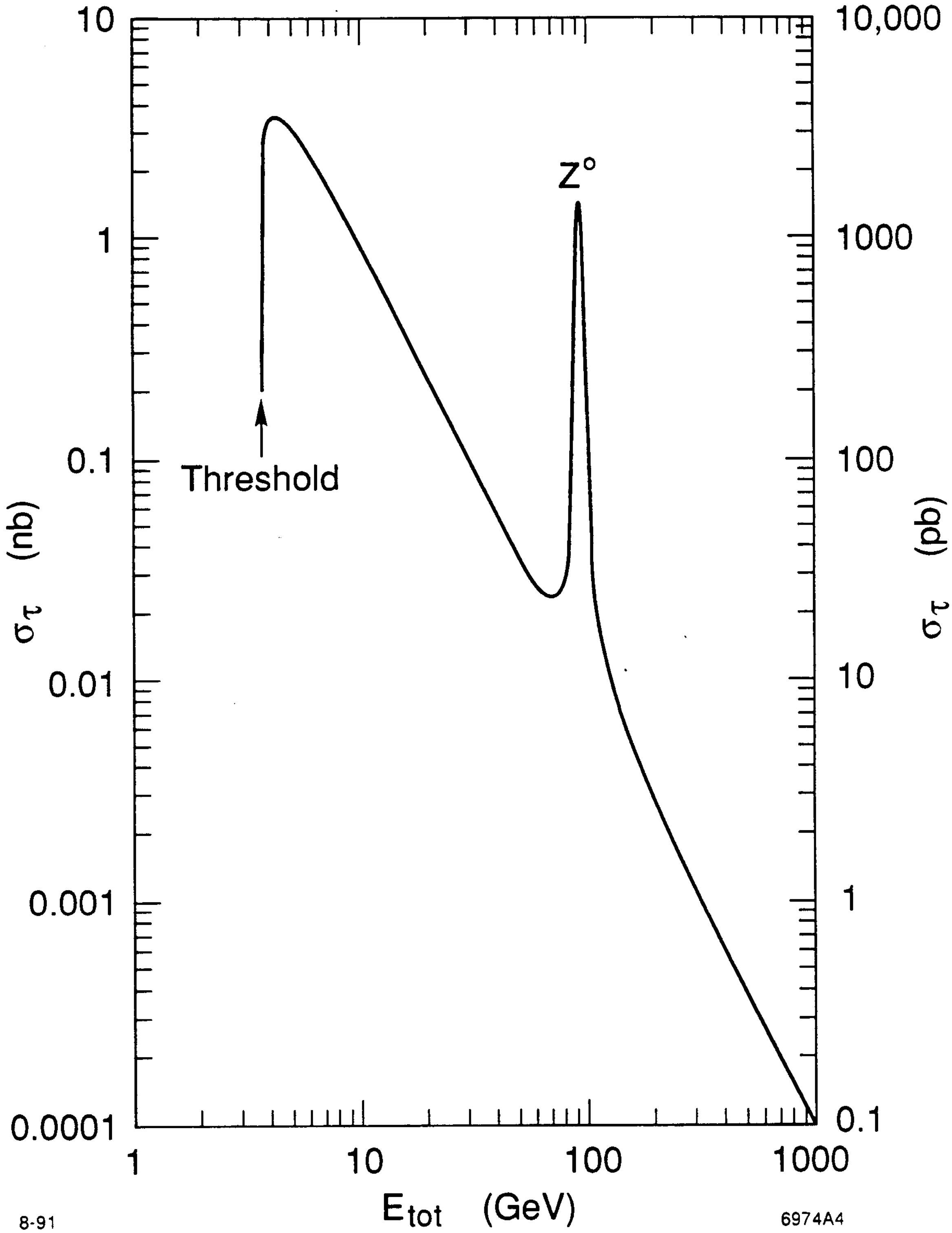

10
1
0.1
0.01
0.001
0.0001
10,000
1000
100
10
1
0.1
Z°
Threshold
σ_τ (nb)
σ_τ (pb)
1
10
100
1000
E_tot (GeV)
8-91
6974A4

Fig. 4

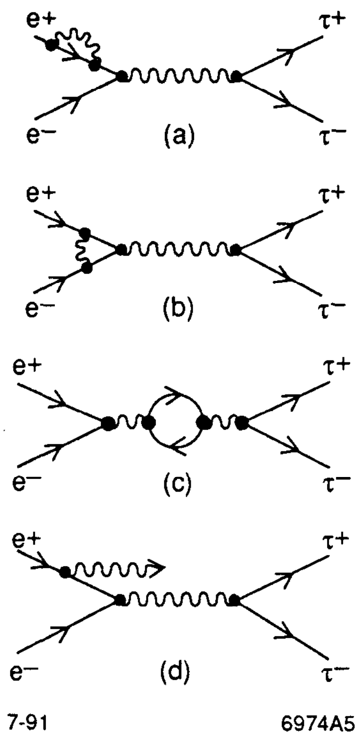


Fig. 5

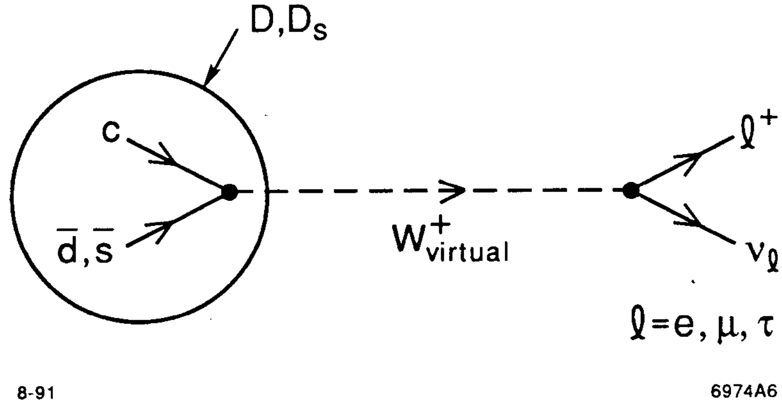


Fig. 6

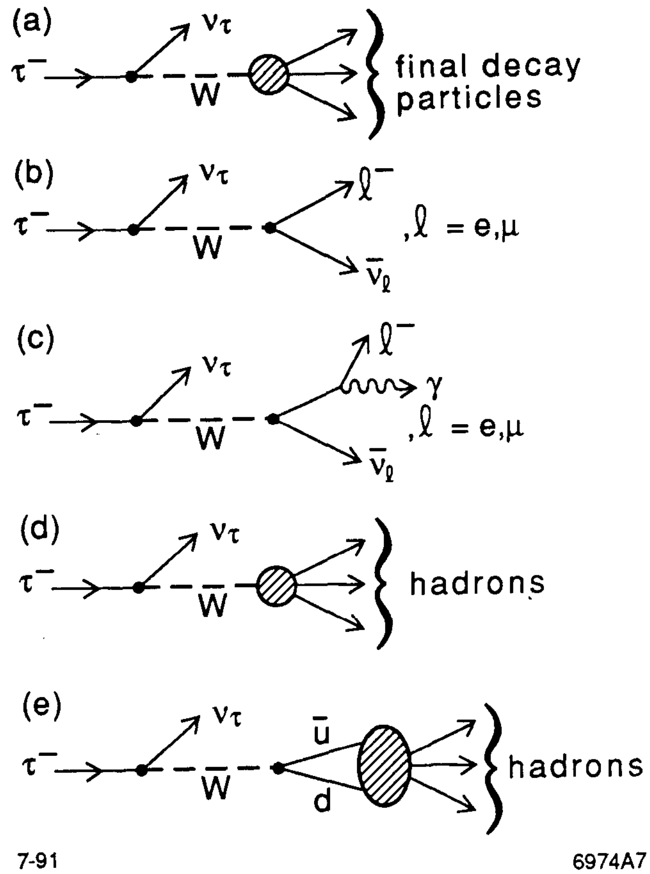

(a)
τ^-
ν_τ
W
final decay particles
(b)
τ^-
ν_τ
W
ℓ^-
$, \ell = e, \mu$
$\bar{\nu}_\ell$
(c)
τ^-
ν_τ
W
ℓ^-
γ
$, \ell = e, \mu$
$\bar{\nu}_\ell$
(d)
τ^-
ν_τ
W
hadrons
(e)
τ^-
ν_τ
W
$\bar{u}$
d
hadrons
7-91
6974A7

Fig. 7

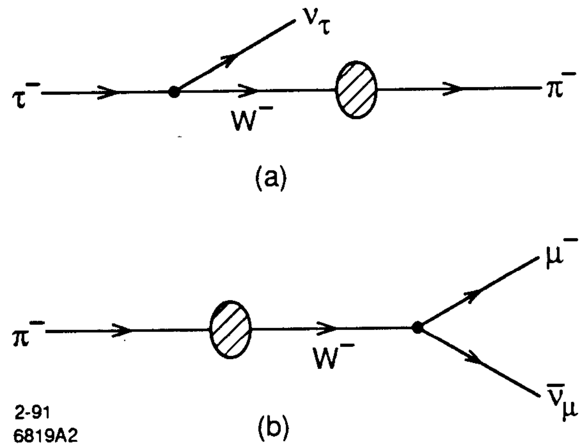


Fig. 8

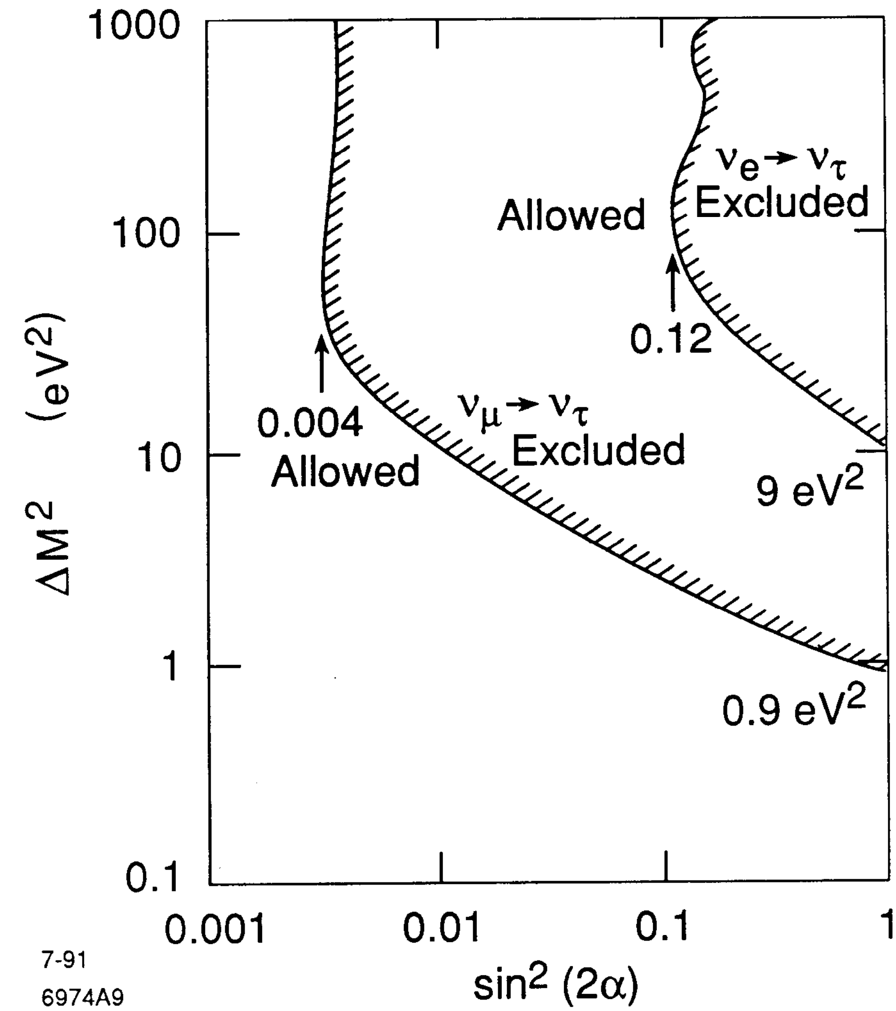
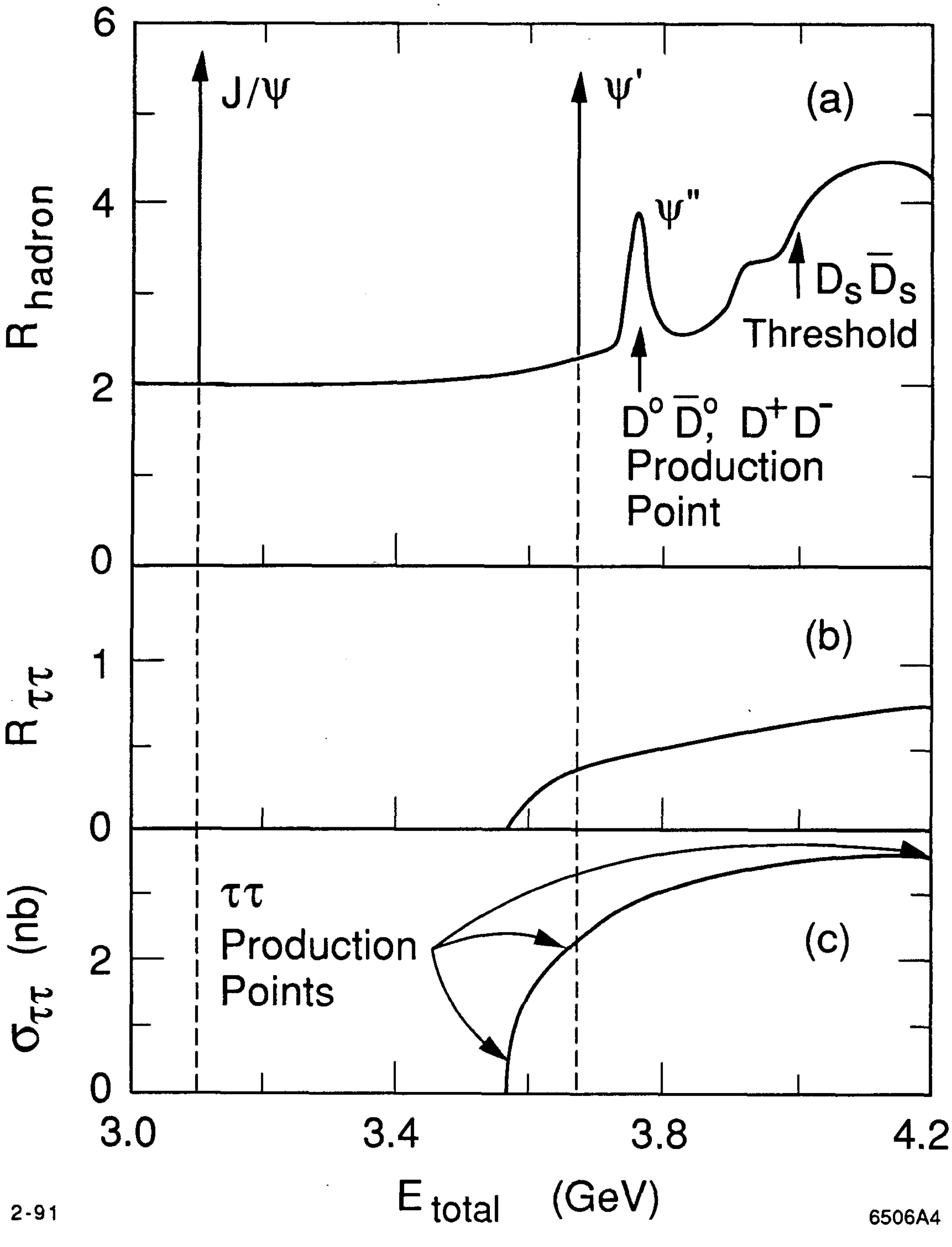

1000
100
10
1
0.1
ΔM^2 (eV^2)
0.001
0.01
0.1
1
$\sin^2(2\alpha)$
Allowed
$\nu_e \rightarrow \nu_\tau$
Excluded
0.12
0.004
Allowed
$\nu_\mu \rightarrow \nu_\tau$
Excluded
9 eV^2
0.9 eV^2
7-91
6974A9

Fig. 9



J/ψ
ψ'
(a)
ψ"
$D_s \bar{D}_s$
Threshold
$D^o \bar{D}^o$, $D^+ D^-$
Production
Point
(b)
ττ
Production
Points
(c)
R_{hadron}
$R_{\tau\tau}$
$\sigma_{\tau\tau}$ (nb)
E_{total} (GeV)
3.0
3.4
3.8
4.2
2-91
6506A4

Fig. 10

τ – Charm Factory

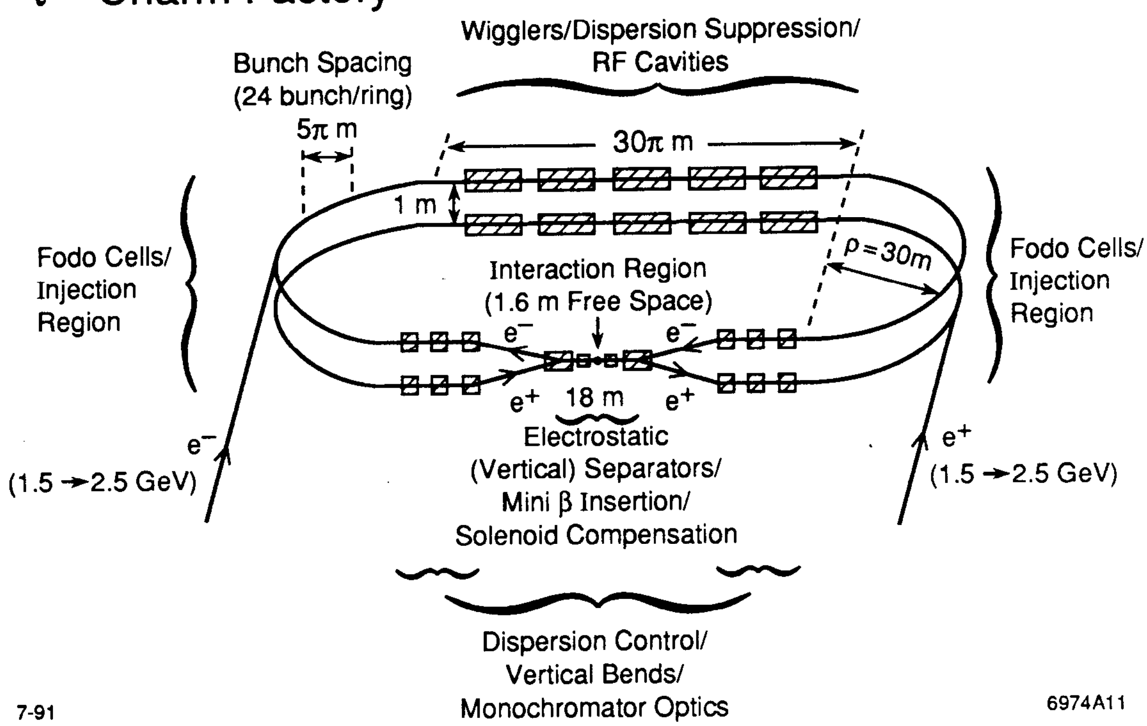


Fig. 11

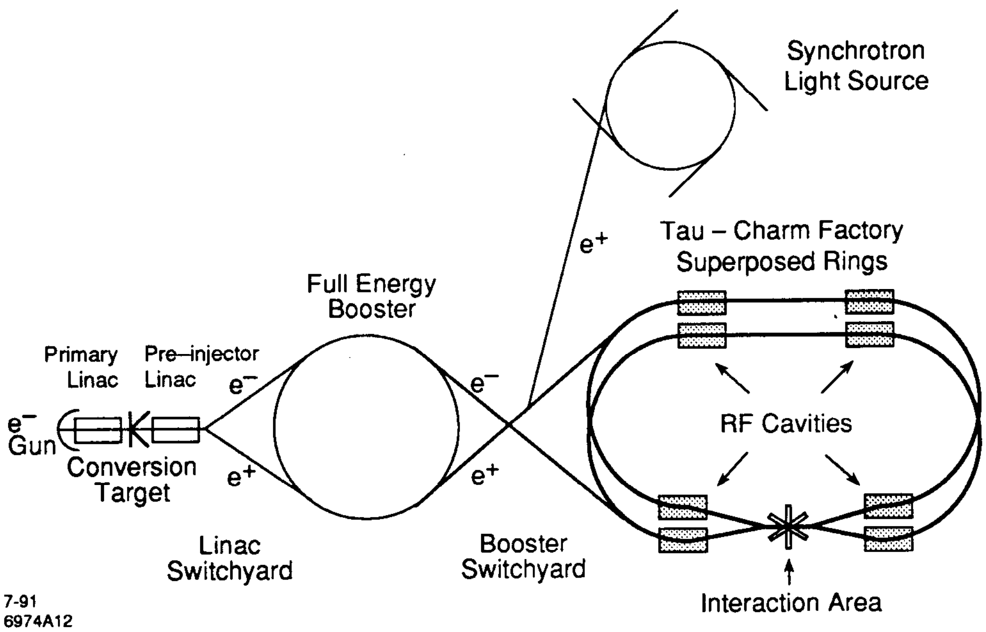

Synchrotron
Light Source
e+
Tau – Charm Factory
Superposed Rings
Full Energy
Booster
Primary
Linac
Pre–injector
Linac
e−
e−
RF Cavities
e−
Gun
Conversion
Target
e+
e+
Linac
Switchyard
Booster
Switchyard
Interaction Area
7-91
6974A12

Fig. 12