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HIGHER ORDER EFFECTS IN BEAM-BEAM DEFLECTION*

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I. INTRODUCTION

Beam-beam deflection is a useful tool for beam centering and size measurement in existing and future linear colliders [1]. It is indispensable in the Stanford Linear Collider when beam intensity becomes too strong for conventional wire scans. In future linear colliders beam-beam deflection may be one of the few viable methods from which information can be drawn about beam sizes.

Because of the importance of beam-beam deflection at higher intensity, it is crucial to address the problem of disruption. At low intensity, it is enough to use the rigid deflection formula:

$$\langle\phi\rangle_1 = \frac{-2r_eN_2}{\gamma}\frac{1}{\Delta}(1-e^{-\frac{\Delta^2}{2\Sigma^2}}) \tag{1}$$

where $\Sigma^2 = \sigma_1^2+\sigma_2^2$, ϕ is the deflection angle , r_e the classical electron radius, Δ the impact parameter and σ the transverse RMS beam size. 1 and 2 are beam labels. The limiting cases of (1) are:

$$\langle\phi\rangle_1 = \left(-\frac{r_eN_2}{\gamma}\frac{\Delta}{\Sigma^2}\right) \qquad \Delta \ll 2.23\sigma,$$

$$\langle\phi\rangle_1 = \left(-\frac{2r_eN_2}{\gamma}\frac{1}{\Delta}\right) \qquad \Delta \gg 2.23\sigma. \tag{2}$$

At high intensity, the bunches steer and deform each other considerably. This leads to a nonlinear deviation from the rigid formula. Below we describe some techniques attempted at modeling this effect.

II. LOWEST ORDER ANALYTICAL CALCULATION

We start with the equations relating the beam distribution and deflection of individual particles. For the 2-beam system in figure 1, a formulation of disruption with $\Delta=0$ has been laid out in [2]. The same can be applied here except the absence of cylindrical symmetry:

With the distributions $n_0(x,y,z)$ for both beams, the effect on a particle in beam 1 by beam 2 is

$$\frac{d^2\vec{x_1}}{dt^2} = \frac{-4r_eN_2}{\gamma}n_{L2}(-2t-z_1)\int\frac{\vec{x_1}-\vec{x_2}}{|\vec{x_1}-\vec{x_2}|^2}n_{t_2}(\vec{x_2})d\vec{x_2}$$

$$= \frac{-4r_eN_2}{\gamma}n_{L2}(-2t-z_1)\vec{\nabla}_1 f_2(\vec{x_1}), \tag{3}$$

$$f_2(\vec{x_1}) = \frac{1}{2}\int dx\, dy\, n_{t2}(x,y)\ \ln[(x_1-x)^2+(y_1-y)^2]$$

where $\vec{x}=(x,y), \vec{\nabla}=(\partial/\partial x, \partial/\partial y)$. The above equation is solved to the lowest order and then inverted to the same degree of accuracy to derive the change in the distribution of beam 1:

$$\delta n_{t1}(x_1,y_1,t,z_1)$$
$$= \frac{4N_2r_e}{\gamma}g(t,z_1)[n_{t01}\nabla_1^2 f_2 + (\vec{\nabla}_1 n_{t01})\cdot(\vec{\nabla}_1 f_2)]. \tag{4}$$

The same formula applies to beam 2 except for a different initial distribution offset by Δ. Two terms contribute to the angular change of beam 1 : that caused by the distribution change of beam 2, and that by the change in beam 1 itself. Substituting δn_{t2} for n_{t2} and integrating over time, followed by an ensemble average over beam 1:

$$\begin{aligned}\langle\delta\phi_{1x}\rangle &= \left(\frac{D_2}{\sigma_{z2}}\frac{1}{\sqrt{\pi}}\right)D_1\otimes \\ &\left\{2\sigma_2^2\frac{1}{\Delta}\left[e^{-\frac{\Delta^2}{2(\sigma_1^2+\sigma_2^2)}}-e^{-\frac{\Delta^2}{\sigma_1^2+2\sigma_2^2}}\right]\right. \\ &\left.-\ e^{-\frac{\Delta^2}{2\sigma_2^2}}2\sigma_1^2\int_0^\infty dr\frac{1}{r^2}I_1\left(\frac{r\Delta}{\sigma_2^2}\right)Q(r,\sigma_1,\sigma_2)\right\},\end{aligned}$$

$$Q(r,\sigma_1,\sigma_2) = \left[e^{-\frac{r^2}{2\sigma_2^2}} - 2e^{-\frac{r^2}{2\Sigma^2}} + e^{-\frac{r^2}{2C^2}}\right], \tag{5}$$

$$C^2 = \left(\frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+2\sigma_2^2}\right),\ D_1 = \frac{r_eN_1}{\gamma}\frac{\sigma_{z1}}{\sigma_1^2},\ D_2 = \frac{r_eN_2}{\gamma}\frac{\sigma_{z2}}{\sigma_2^2}$$

where I_1 is the Bessel function. The remaining integral is well behaved although no closed form can be found.

The contribution to $<\delta\phi_{1x}>$ due to the change in beam 1 itself is equal to (5) with the following substitutions: *interchanging* σ_1 and σ_2 and *replacing* D_1 by D_2.

The total angular change is plotted in figure 2 with nominal SLC parameters ($\sigma_{1,2}=2\mu$m, $D_{1,2}=0.1$, $\sigma_{z1,z2}=1$

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mm). It modifies the rigid deflection formula by roughly 0.8% near $\Delta = 0$.

This method takes into account the realistic distribution and does not rely on transverse symmetry. It can be iterated to obtain progressively better results. The algebra however is formidable.

III. RIGID TWO-DISK MODEL

To focus on the nonlinear nature of the problem, we developed a conceptual model to elucidate the disruption effects at different Δ as depicted in Figure 3. The longitudinal distributions have been compressed into two δ-function peaks $2\sigma_z$ apart, each carrying a transverse Gaussian distribution with half of the total charge. The whole process of bunch crossing is concentrated in three steps corresponding to the coincidences of the "disks". At each crossing the rigid deflection formula for transverse Gaussian distributions is used to calculate the kick on each disk, which in turn is used to propagate the disk to the next crossing point. The kicks at each step are compounded towards the end. In the following D is as defined in Section II, x' is the average deflection angle.

Small impact parameter - Suppression

In this case after the 2-disk crossing is complete as in Figure 3, the compounded kick received by beam 1 is

$$x' = -\frac{1}{2}\frac{r_e N}{\gamma\sigma_\perp}\frac{\Delta}{\sigma_\perp}\left(1 - \frac{1}{4}D\right). \tag{6}$$

Thus the effect of disruption is a suppression of the rigid deflection result (2). This can be understood since at small Δ the deflection force decreases with Δ. As disruption effect pulls the two beams closer, the deflection is reduced.

Large impact parameter - Enhancement

In the regime where the two beams are far apart transversely, we can use the second formula in (2) and get:

$$x' = -\frac{2r_e N}{\gamma\sigma_\perp}\left(\frac{\sigma_\perp}{\Delta}\right)\left[1 + D\left(\frac{\sigma_\perp}{\Delta}\right)^2\right]. \tag{7}$$

Thus the net effect is an enhancement for large Δ.

Near maximum deflection - Shift of the peak

Disruption shifts the deflection peak which can serve as a useful signature. We can calculate this from the expansion of equation (1) around the peak ($\Delta \approx 2.23\sigma$). This is then used to calculate the shift by disruption:

$$\frac{\{\text{Shift of peak}\}}{\sigma} = 0.3190D. \tag{8}$$

These results agree with that of Section II.

IV. SEMI-RIGID TWO-DISK MODEL

The two-disk model is generalized to include changes in the second moment as well as a continuous treatment over the whole range of Δ. This is achieved via a program combining the analytical expression for single particle deflection and multiparticle tracking over a continuous range of impact parameters. The longitudinal distributions are again compressed into two δ-disks. The transverse distributions are however flexible by taking on a Gaussian distribution of particles, each allowed to move independently. The kick a particle receives from a Gaussian bunch is given by

$$\Delta\phi_{x,y} = -\frac{2r_e N}{\gamma}\frac{\Delta_{x,y}}{\Delta^2}(1 - e^{-\frac{\Delta^2}{2\sigma^2}}). \tag{9}$$

Each particle is propagated independently between crossings. Before the next kick is applied, the transverse RMS value as well as the centroid shift is calculated and substituted into (9) to obtain the next kick for each particle.

Figure 4 shows such a calculation where the rigid deflection formula (1), the deflection of rigid 2-disks and that including second moment changes are compared. The effect of the second moment counteracts that due to the rigid 2-disk model, especially at small Δ, where the pinching of the beams enhances the deflection the most.

V. TRACKING RESULTS

Tracking has been employed to simulate the disruption effect in the realistic SLC environment. In some cases the accuracy is limited by the computer capacity we could muster. In the simulation each beam has 20000 particles meshed into a 32×32 grid transversely and 100 compartments longitudinally. Simulation was carried out for different disruption parameters D and different optical conditions defined by the divergence parameter A given in [2]: $A = (\sigma_z/\beta^*)$, which is a measure of the inherent divergence with β^* being the lattice beta at the collision point. Figures 5(a) and (b) show tracking results for different D and A, with $D = 0.1, A = 0.05$ corresponding to the SLC running condition. The simulation becomes difficult as Δ increases and cylindrical symmetry is thus less exact.

VI. CONCLUSION

We demonstrated different approaches in addressing disruption effects in beam-beam deflection. Short of an analytical scheme which encompasses all essential features of disruption at non-zero Δ, we settle for methods which have different emphases on the problem. The results are consistent to a large degree. Extensions of these techniques, in particular the semi-rigid disks and multi-particle tracking, are being worked on for improved understanding of this phenomenon.

REFERENCES

[1] W. Koska et al, Nucl.Inst.Meth. A286:32, 1990
R. Erickson, SLAC-PUB-4479, November 1987

[2] P. Chen and K. Yokoya, Physical Review D 38, 9871988

[3] Y. Chao and P. Chen, SLAC-PUB 5221,1990

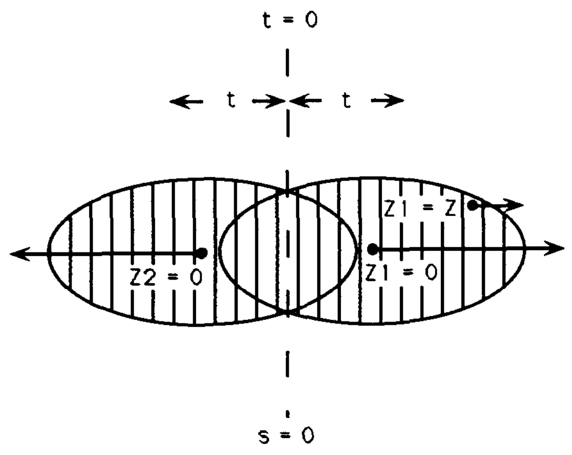

Figure 1. 2-beam system in longitudinal dimension

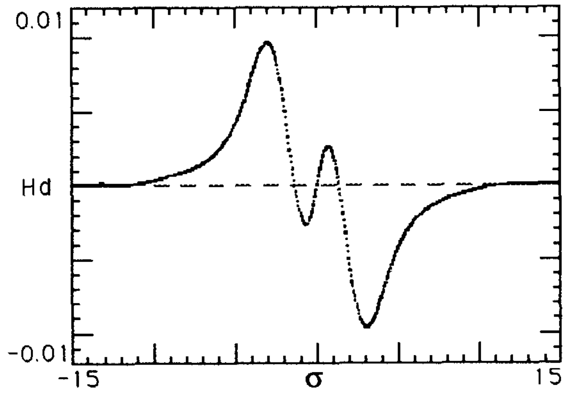

Figure 2. Net effect of disruption from analytic calc.
Hd = (Dσt) / (2 σz)

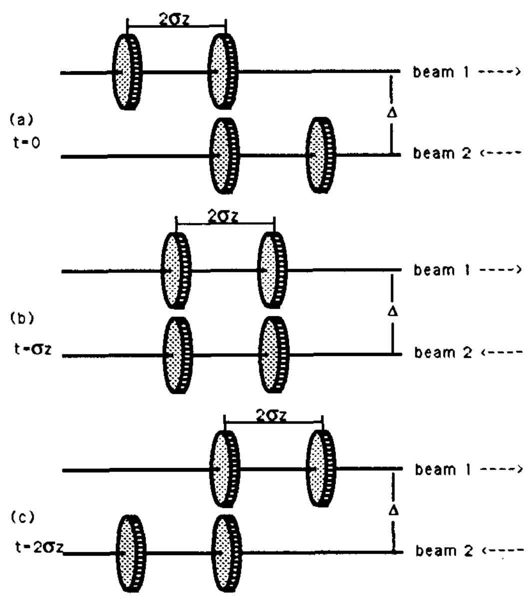

Figure 3. The conceptual 2-disk model

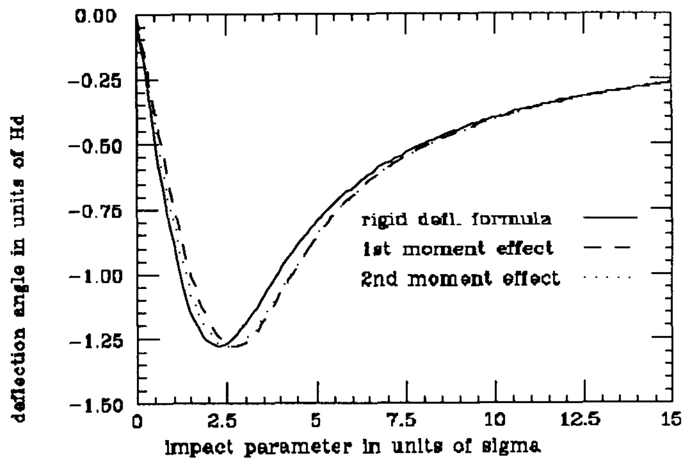

Figure 4. zero, 1st and 2nd moment disruption effect

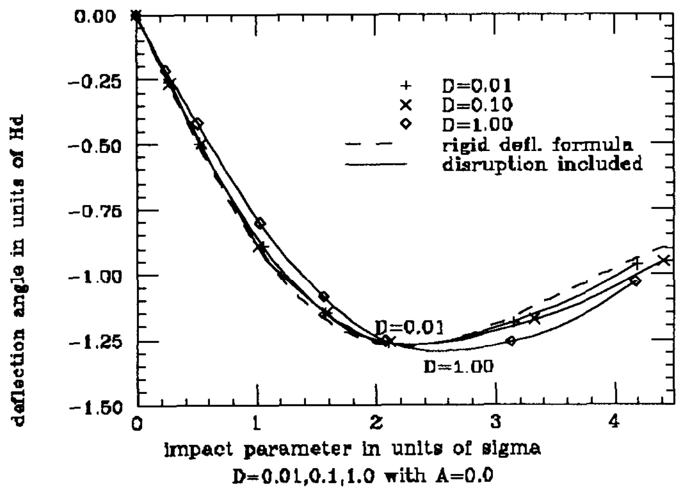

Figure 5(a).Tracking result with A=0.0

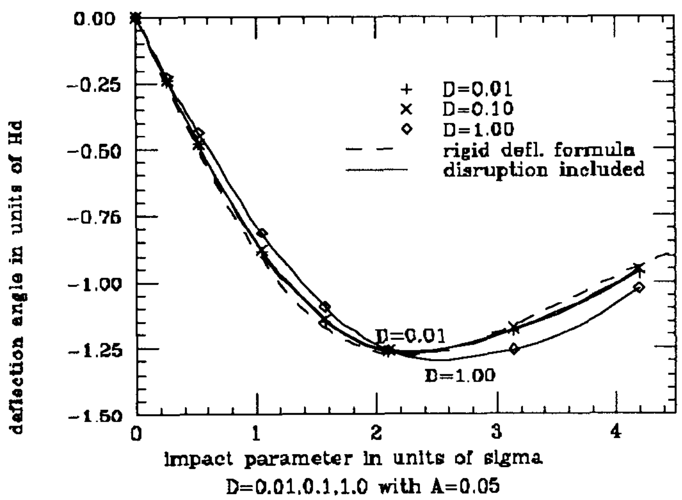

Figure 5(b).Tracking result with A=0.05