Comment on

"Feynman's Proof of the Maxwell Equations"

by F. J. Dyson, [Am. J. Phys 58, 209–211 (1990)]

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**Comment on "Feynman's Proof of the Maxwell Equations,"
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Although Dyson notes in his article[1] that when Feynman constructed the "proof" over forty years ago he was certainly aware of the fact that *if* one assumes a Lagrangian or Hamiltonian formalism the "proof" is trivial, most of the comments I have seen[2-5] amount to little more than elaborations of this fact. Indeed, they go little beyond an earlier discussion[6] covering essentially this same point of which they do not seem to be aware. Hojman and Shepley's paper[7] is interesting because they show that, while the commutation relations allow one to construct a Lagrangian, it is not unique. Dombey's comment[8] is interesting because he notes that if the *sources* can be described by *div* $\mathbf{E} = 4\pi\rho$, *curl* $\mathbf{H} = 4\pi\mathbf{j}$, then one obtains Levy-Leblond's "Galilean Electrodynamics" with no explicit limiting velocity.[9,10] This removes the paradox mentioned by Dyson of having a Galilean invariant set of equations lead to a Lorentz invariant theory; if one follows the route suggested by Dombey, one must be willing to abandon the concept of "displacement currents."

None of these comments or papers address the fact that, as Dyson states quite clearly, "In 1948 [Feynman]...was still doubting all the accepted dogmas of quantum mechanics. He was exploring possible alternatives to the standard theory. His purpose was to discover a new theory, not to reinvent the old one. ...His proof of the Maxwell equations was a demonstration that his program had failed."

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This comment is directed to Feynman's original objective. We claim that, in an alternative context now available to us, his proof could play a significant role in pointing toward a new theory.

According to Dyson,[11] one alternative theory Feynman was exploring at the time was a discrete *Zitterbewegung* model for the motion of a single electron in 1+1 dimensions[12] where the "random walk" uses *imaginary* step lengths,

$$x_{i+1}(t_{i+1}) = x_i(t_i) \pm i\epsilon h/mc; \qquad t_{i+1} = t_i + \epsilon h/mc^2 . \quad (1)$$

As elaborated by Jacobson and Schulman,[13] this model does allow one to compute the solution of the free particle Dirac equation in 1+1 dimensions as the limit of an infinite number of steps and $\epsilon \to 0$. The *ad hoc* introduction of the *imaginary* step length seems to be motivated simply by the desire to reproduce the Hamiltonian theory, and does not in itself suggest "new physics."

We have developed an alternative approach to the derivation of the Dirac Equation[14,15] in which the steps are real and fixed at the h/mc length, the trajectory in space-time is the same as in the Feynman model with $\epsilon = 1$, but the number of *paths* which must be counted is larger than in the Feynman model, because we allow a (still finite and discrete) background time scale that allows time to advance but the particle not to move. Spin conservation for time-like connectivity between events and particle number conservation for space-like connectivity allows us to define amplitudes with an algebraic sign as the

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difference between classes of paths and eliminate the need for imaginary steps. Our exact combinatorial result, which invokes the path counting developed by McGoveran for the transport operator in the context of the ordering operator calculus,[16] can be approximated by the usual continuum wave function whenever the boundary conditions allow for a large but *unknown* number of bends in the trajectories. The usual commutation relations between position and momentum can be derived at the discrete level using finite difference operators.

A physical theory constructed on these foundations[17] using *bit-string* representations (ordered sequences of 0's and 1's which combine by *exclusive or*) allows us to arrive in due course at a relativistic finite number quantum particle theory that describes the basic scattering processes of the standard model of quarks and leptons. More significant for our current purpose is the fact that the time evolution of the system is not given by a Hamiltonian operator but by *Program Universe*. The universe generated by this algorithm has N bit-strings of length S. This universe grows by picking, arbitrarily and independently, two bit-strings and combining them by XOR (addition, mod 2). If the result is *not* the null string (not all 0's), we adjoin the new string to the universe ($N \rightarrow N + 1$), and pick again. If the result *is* the null string, we pick a single bit, arbitrarily and independently for each string, and concatenate it to the growing end of that string ($S \rightarrow S + 1$); then we pick again. Single particle "trajectories" of the type needed to specify the discrete and finite *Zitterbewegung* model

mentioned above are sub-collections of bit-strings selected from this universe to meet the appropriate boundary conditions.

We conclude that the two most important ingredients needed for the Feynman proof of the Maxwell equations—time evolution and commutation relations arrived at without using a Lagrangian or Hamiltonian formalism—have already been established in discrete physics. It is also possible to show that we can define mass ratios directly from relativistic deBroglie wave interference, and derive our discrete version of relativistic 3-momentum conservation. Then Newton's second law (Dyson's Eq. (1)) becomes a *definition* of force, as in Mach. Dyson's Eqs. (4), (5), and (6) then follow from the same algebraic steps that Dyson presents.

In the light of Dombey's remark, we still need to arrive at the inhomogeneous Maxwell Eqs. (7) and (8) starting from our theory. Here we note that massless quanta have no structure in our theory, other than a small number of bits used to insure charge conservation, CPT invariance and the like; the "space-time structure" of these massless quanta is represented simply by a string of 1's (forward light cone) or a string of 0's (backward light cone). Additional structure is, necessarily, *context dependent.* This requires us to view the "Maxwell fields" as a simple way to interpolate mathematically between the motion of charges and currents in the sources and sinks. Thus our theory meets Dombey's consistency requirement, and we can adopt Feynman's proof as providing a well-defined "correspondence limit"

for a *finite and discrete relativistic particle quantum mechanics* in the classical Maxwell equations.

Details of our contention will be presented elsewhere.

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