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INCLUSIVE SEMILEPTONIC DECAYS OF BOTTOM BARYONS AND MESONS INTO CHARMED AND UNCHARMED FINAL STATES: THE CASE OF INFINITELY HEAVY b AND c QUARKS*

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ABSTRACT

Spectator model decay rates are predicted for inclusive semileptonic decays of the Λ_b baryon and the B meson, in the limit of a very heavy b quark, in lowest order of perturbative QCD. For charmless final states the result is only achieved in a short distance limit, sharing similarities with the deep inelastic scattering situation, and allowing for a parton model interpretation. In all cases these results are expressed in terms of sum rules derived with the help of equal-time, local current algebra.

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1. *Introduction*

Processes involving particles containing heavy quarks, like the b quark, are at present of great interest from both the theoretical and the experimental point of view. Experimentally, there are prospects for detailed study of such processes[1]. And theoretically, Isgur and Wise[2] have recently shown that the analysis of these processes greatly simplifies in the formal limit of $M_b \to \infty$. This occurs because in this limit the heavy quark and the heavy hadron containing it become cannonballs: once set into motion, their velocity is difficult to change. Only perturbative processes such as hard gluon emission or an electroweak interaction can effectively modify the velocity. Therefore the velocity becomes a good quantum number as far as nonperturbative aspects of QCD are concerned[3]. The physics of a semileptonic process in such a scenario is quite simple: the heavy quark acts as a static source of colour, quite analogous to the way the charged nucleus, heavy compared to the electron mass, acts as a source of static electric field in atomic physics. If, by the action of an electroweak current, the heavy quark turns into another heavy quark of a different flavour without change of velocity, it will not make a difference for the light degrees of freedom because the strong interactions are blind to flavour labels: the new heavy quark simply replaces its predecessor. Also, the spin of the heavy quark decouples from the dynamics: the hyperfine, magnetic interaction scales as M^{-1}. The hyperfine splitting thus vanishes in this limit, and the pseudoscalar B and the vector B^* mesons become degenerate in mass. Thus in the $M_b,\ M_c \to \infty$ limit, the heavy-quark velocities become good quantum numbers, and there is a new spin-flavour symmetry as well.

The original applications of the Isgur-Wise method were for "elastic" form factors, such as occur in exclusive decays like $B \to Dl\bar{\nu}$. The emphasis in this paper, however, is to apply these techniques to inclusive decay processes. The goal is to find sum rules for the structure functions that describe the inclusive decay properties, in particular the rates differential only in the dilepton mass and the total mass of the final hadronic state.

It turns out that baryon semileptonic decays to charm are the simplest to discuss. This occurs because the dynamics of the spectator diquark system, which is spinless in this case, is especially simple. Since in the infinite-mass limit the dynamics of the heavy-quark system becomes trivial (that of free fields), and decouples from the spectator-system dynamics, there are no complications of spin which enter the picture. As discussed in more detail in Section 2, for any final state, no matter how complicated, the spin correlations between Λ_b and Λ_c are expected to be the same as for the underlying quarks b and c; the spectator system is uncorrelated with these spin degrees of freedom. Since the structure of each matrix element is simple and known, this is also true for the inclusive sum. Therefore, the basic structure of the differential inclusive width is

$$\frac{d\Gamma}{dq^2\, d\epsilon} = \frac{d\Gamma_0}{dq^2}\, \omega(\epsilon, v.v') \tag{1.1}$$

where $d\Gamma_0/dq^2$ is the parton-level differential width, W is the mass of the final hadron system, q the mass of the final-state dilepton, and

$$W^2 = (M_c + \epsilon)^2$$

The structure of Eqn. (1.1) strongly suggests a sum rule, which indeed exists

$$\int_0^\infty d\epsilon\ \omega(\epsilon, v.v') = 1$$

and which establishes the validity of the spectator picture of semileptonic decays: the total inclusive sum is the same as the total inclusive sum at the quark level.

We derive this sum rule in the next section using old-fashioned current algebra techniques[4]. However, before doing this we discuss the underlying physical ideas, which are analogous to the old quantum-mechanics problem of the fate of an atom when its nucleus is suddenly accelerated and moves off with some velocity. Our sum rule just expresses unitarity: the probabilities of finding the atom in ground plus excited states have to sum to unity.

Important in any sum rule is the estimate of when it converges. In this case we start with Λ_b, say, at rest, and end with Λ_c (and the c-quark residing therein) moving off with velocity v, which may or may not be relativistic. The spectator diquark system must respond. If the velocity v is very low, the probability of excitation of the diquark system will be very small and the elastic channel will dominate. For large v, the fastest emitted particle will have on average its velocity v (better, its Lorentz γ) comparable to that of the c quark. In practice, such as $B \rightarrow D, D^* + X + l + \bar{\nu}$, the maximum value of $\gamma = v.v'$ allowed kinematically is about 1.6, so that excitations of half a Gev or so can be expected, along with a corresponding depletion of the elastic contribution.

For charmless semileptonic decays, there are significant modifications to this picture. First of all the b quark imparts its spin to the light quark, so there are more complications of spin in the inclusive sums. In particular for the baryon decays there are now two structure functions and for meson decays six. In addition, we only find sum rules in the "deep inelastic" limit, when the energy release (in the parent rest frame) $W.v$ is large compared to the natural QCD mass scale of a few hundred Mev. (Of course, we must still limit our consideration to these variables small compared to the heavy-quark masses, at least formally.) At the parton level what is happening is that the b quark decays into a low mass quark of large momentum k. The scale of the invariants is then

$$W^2 \approx (k + p_{slow})^2 \approx 2k.p_{slow}$$

$$W.v \approx (k + p_{slow}).v \approx k.v$$

with p_{slow}, the four-momentum of the spectator system, expected to be not large in the rest frame of the parent. We also see that the scaling variable x

$$x\Lambda = \frac{W^2}{2W.v} = \frac{2k.p_{slow}}{2k.v} = (E - p_{\|})_{slow}$$

has the interpretation of the value of the light cone variable $(E - p_{\|})_{slow}$ of the spectator system as measured relative to the direction of the outgoing fast quark of momentum k. We shall find a parton model interpretation for what the sum rules express. First of all, in the scaling limit, the differential width takes the form

$$\frac{d\Gamma}{dq^2\, dW^2} = \frac{d\Gamma_0}{dq^2}\, \varphi_1(W^2, W.v)$$

where for free fields

$$\varphi_1(W^2, W.v) = \delta(W^2)$$

The sum rule is

$$\int dW^2\, \varphi_1(W^2, W.v) = 1$$

which reveals the structure function in the scaling limit as just the probability of the spectator system to have a given value of x:

$$2(W.v)\Lambda\varphi_1(W^2, W.v) \to f_1(x)$$

Again there is a question of convergence of the sum rule, which is easily answered, given the scaling picture: all the dependence on W^2 and $W.v$ comes through the combination x, and the sum rule converges for x of order unity. This is quite consistent with the expectation that large values of $(E - p_{\|})_{slow}$ are rare.

Only one form factor appears to survive in the scaling limit (for baryon decays); this appears to be analogous to the vanishing of the longitudinal/transverse ratio in conventional deep inelastic phenomena, although there is considerable room for more study of this point.

Similar results hold for meson semileptonic decays. The analysis, though, is somewhat more involved because the spin of the meson is no longer carried by the b quark, and there are spin correlations that need be taken into account: these are naturally incorporated with the help of the trace formalism introduced in Section 5.

The semileptonic decay of the B meson to charm is again the simplest to discuss in the framework of the Isgur-Wise method, since the final state contains a c quark which we consider also heavy. The differential inclusive width looks like Eqn. (1.1), and there is a sum rule for ω which ensures the validity of the spectator picture of the semileptonic decays also in this case.

For charmless decays, the spectator picture is, as before, only achieved in the limit when $W.v$ is large compared to Λ. Here, the scalar form factors are six in number: two of them are reminiscent of the two baryonic form factors and have a similar parton-model interpretation; the rest are shown not to contribute to the total sum.

Since the b quark is heavy, we expect corrections calculable in perturbative QCD. As mentioned before, the velocity of the b quark is a good quantum number as far as nonperturbative low-energy aspects of QCD are concerned and only hard gluon emissions of momentum $k^2 \sim M_b^2$ can effectively modify it: the QCD perturbative expansion in $\alpha_s(k^2)$ can be thus viewed as an expansion in the number of velocity changes of the b quark. Because QCD is asymptotically free, $\alpha_s(k^2) \ll 1$, it makes sense to use the perturbative expansion to compute the corrections. The issue of how these perturbative QCD corrections would affect the sum rules is beyond the scope of the present analysis.

In what follows we first consider the simple baryon decays, and then the meson decays. In Section 2 we lay out the kinematics and formalism for Λ_b decays into charmed and then to charmless final states. In Section 3 we discuss the sum rules for Λ_b semileptonic decay into charmed final states, while Section 4 is devoted to sum rules for charmless semileptonic Λ_b decays. Sections 5 to 7 repeat all this for B-meson decays. Section 5 contains the kinematics and formalism for both charmed and uncharmed final states, Sections 6 and 7 the sum rules for semileptonic decays into charmed and uncharmed final states respectively. Section 8 is devoted to concluding comments.

2. *The Formalism for Baryon Decays*

The Isgur-Wise results take an especially simple form for the semileptonic decays of the Λ_b. The "elastic" process has the following amplitude[5][6]:

$$\langle \Lambda_c | \, J_\mu \, | \Lambda_b \rangle = \sqrt{\frac{M'M}{4E'E}} \bar{u}(v')\gamma_\mu(1-\gamma_5)u(v) \; F_{el}(v.v') \tag{2.1}$$

where M, v, P and M', v', p' are masses, four-velocities, and momenta of Λ_b and Λ_c, respectively; $F_{el}(v.v')$ is the universal form factor of Isgur and Wise, which does not depend on the masses of the heavy quarks, and for which $F_{el}(1) = 1$. One sees that the spin structure is identical to what exists in the free quark limit. This occurs essentially because the spectator light quark system is a spinless diquark; hence all spin correlations remain within the heavy-quark system. It is evident that the same feature holds for general final states:

$$\langle \Lambda_c X | \, J_\mu \, | \Lambda_b \rangle = \sqrt{\frac{M'M}{4E'E}} \bar{u}(v')\gamma_\mu(1-\gamma_5)u(v) \; F(v, v'; X) \tag{2.2}$$

The simplicity of this result leads to simplicity for the inclusive properties of these decays as well. Let us construct the differential decay width for

$$\Lambda_b \to (\Lambda_c + X)_W + (l + \bar{\nu})_q \tag{2.3}$$

where q and W are the four-momenta of the dilepton system and the hadron system, respectively. We find

$$\frac{d\Gamma}{dW^2 dq^2} = \sum_X \frac{d\Gamma_0}{dq^2} \mid F(v, v'; X) \mid^2 \, \delta(W^2 - (P_X + p')^2) \tag{2.4}$$

where

$$\frac{d\Gamma_0}{dq^2} = \frac{G_F^2 | \, V_{cb} \, |^2}{2} \int \frac{d^3 l}{(2\pi)^3 2 l_0} \frac{d^3 \nu}{(2\pi)^3 2\nu_0} \delta(q^2 - (l+\nu)^2) \delta \left[(P - q - P_X)^2 - M'^2 \right]$$

$$\times\ 2\pi M' \left[Tr\ \frac{(1+\not{v}')}{2}\gamma_\mu(1-\gamma_5)(\frac{1+\not{v}}{2})\gamma_\nu(1-\gamma_5)\right] [Tr\ \not{l}\gamma^\mu(1-\gamma_5)\ \not{\nu}\gamma^\nu(1-\gamma_5)] \tag{2.5}$$

In the "free-field" limit of no form factors and no excitations of inelastic final states we would simply have

$$\frac{d\Gamma}{dW^2 dq^2} \rightarrow \frac{d\Gamma_0}{dq^2}\ \delta(W^2 - M'^2) \tag{2.6}$$

implying that $d\Gamma_0/dq^2$ is essentially just the spectator-model differential width.

In the infinite-mass limit there is additional simplification. We expect for finite v and v' that the excitation energy of hadrons remains finite, i.e.

$$W^2 = (M' + \epsilon)^2 \tag{2.7}$$

with ϵ bounded. Then

$$\frac{d\Gamma}{dq^2 d\epsilon} = \frac{d\Gamma_0}{dq^2} \sum_X |\ F(v, v'; X)\ |^2\ \delta(\epsilon - v'.P_X) \tag{2.8}$$

with this time $d\Gamma_0/dq^2$ really just the spectator-model expression in the limit. This occurs because the dependence on the final state X in Eqn. (2.5) only appears in the delta-function and in $v' = \frac{W-P_X}{M'} \approx \frac{W}{M'}$. P_X can be safely dropped relative to the other momenta P and q which tend to infinity.

We therefore are led to define the invariant structure function

$$\omega(\epsilon, v.v')\ \equiv \sum_X |\ F(v, v'; X)\ |^2\ \delta(\epsilon - v'.P_X) \tag{2.9}$$

It is clear that were there a sum rule for it

$$\int_0^\infty d\epsilon\ \omega(\epsilon, v.v') = 1 = |\ F_{el}(v.v')\ |^2\ +\ \int_0^\infty d\epsilon\ \omega_{inel}(\epsilon, v.v') \tag{2.10}$$

the spectator picture of semileptonic decays would emerge. This is the subject of the next section; it indeed turns out to be true once the contribution of the channel

$\Lambda_b \to (D+Y)_W + (l+\bar{\nu})_q$ is also included. We may note here that this class of final states is separately described by a structure function of spectator form, as in Eqns. (2.8) and (2.9) .

However, the main thrust of this paper has to do with charmless final states. In this case the matrix elements are not quite as simple. Their form in the infinite mass limit is as follows:

$$\langle X | \, J_\mu \, | \Lambda_b \rangle = \sqrt{\frac{M}{2E}} \bar{\psi}(X,v)\gamma_\mu(1-\gamma_5)u(v) \tag{2.11}$$

The information on the final state is in the spinor variable ψ, because the final light quark system now has spin 1/2.

We may again try to construct a structure function for inclusive processes analogous to that in Eqn.(2.9). However this time it transforms as a Dirac matrix:

$$\Phi(W,v) \equiv \sum_X \psi(X,v)\bar{\psi}(X,v)(2\pi)^3 \, \delta^4(P_X - W) \tag{2.12}$$

where $W = P - q$. We have included for later convenience the energy-momentum conserving delta function. (But note that there is one factor 2π missing). Upon summation over all final states X, this matrix must be expressible only in terms of W and v. There is additional simplification coming from the presence of the V-A $(1-\gamma_5)$ factors. The relevant general structure depends upon only two invariant form factors, whose arguments we can take to be W^2 and $W.v$.

$$\Phi = \phi_1 \, \not{W} + \phi_2 \; v.W \; \not{v} + noncontributing \;\; terms \tag{2.13}$$

Note that for a free massless particle in the final state we would have

$$\Phi_{free} = \not{W} \; \delta(W^2) \; \theta(W_0) \tag{2.14}$$

But in any case, in general the differential decay width may be written as (the details are in the appendix; in particular see Eqn. (A.5))

$$\frac{d\Gamma}{dW^2dq^2} = \Omega^\lambda . \left[Tr\ \gamma_\lambda \left(\frac{1-\gamma_5}{2}\right) \Phi(W,v)\right] \tag{2.15}$$

and

$$\Omega^\lambda = \frac{G_F^2 | V_{ub} |^2}{2} \int \frac{d^3l}{(2\pi)^3 2l_0} \frac{d^3\nu}{(2\pi)^3 2\nu_0} \delta(q^2 - (l+\nu)^2)\ \delta\left[W^2 - (P-q)^2\right]$$

$$\times \frac{1}{4}\left[Tr\ \gamma^\lambda \gamma_\mu (1-\gamma_5)(\frac{1+\not{v}}{2})\gamma_\nu(1-\gamma_5)\right] [Tr\ \ \not{l}\gamma^\mu(1-\gamma_5)\ \not{\nu}\gamma^\nu(1-\gamma_5)]\ (2\pi) \tag{2.16}$$

In analogy to the previous case, we may neglect W^2 in the delta-function, thereby allowing explicit evaluation of the remaining expression. After appropriate averaging, the lepton trace must reduce to a multiple of $(q^\mu q^\nu - g^{\mu\nu}q^2)$ allowing the remaining trace to be readily evaluated. The net result has the form

$$\Omega^\lambda = \Omega(P,q)(2\ q.v\ q^\lambda + q^2 v^\lambda) \tag{2.17}$$

Knowing the free-field expression, Eqn. (2.15) , allows us to infer the "spectator" width $d\Gamma_0/dq^2$:

$$\frac{d\Gamma_0}{dq^2} = 2W_\lambda \Omega^\lambda = 2\Omega\left[2\ q.v\ q.W + q^2 v.W\right] \tag{2.18}$$

Consequently

$$\frac{d\Gamma}{dW^2dq^2} = \left(\frac{d\Gamma_0}{dq^2}\right) \frac{Tr\ (2\ q.v\ \not{q} + q^2\ \not{v})(1-\gamma_5)\Phi}{4\left[2\ q.v\ q.W + q^2 v.W\right]} \tag{2.19}$$

This form will be useful in the interpretation of the sum rules found in Section 4.

Before continuing, however, we mention that it is in principle possible to separate the two form factors ϕ_1 and ϕ_2 with help of lepton angular correlation measurements. Explicitly, in the center of mass frame of $l\ \bar{\nu}$ where the $\hat{z}$ axis points along $(-\vec{W})$, the direction of the charged lepton defines the polar angle θ. The rate is proportional to

$$q^2\times\left[\phi_1\left(\frac{\vec{W}^2}{M}\,sin^2\theta+W.v+\eta\mid\vec{W}\mid(v^0-\frac{W^0}{M})cos\theta\right)+\phi_2(v.W)\left(\frac{\vec{W}^2}{M^2}\,sin^2\theta+1\right)\right] \tag{2.20}$$

The above equation neglects the charged lepton mass. For $l^-\ \bar{\nu}$ $(l^+\ \nu)$ the quantity $\eta=1(-1)$. The form factors ϕ_1 and ϕ_2 are thus separated.

3. *Sum Rules for Semileptonic Baryon Decay into Charm*

In the preceding section we have already conjectured the existence of a sum rule for the inclusive structure function ω defined in Eqn.(2.10). It is now time to derive it[6]. To do this we revert to ancient techniques of current algebra. We consider currents of the form

$$J=\bar{c}\Gamma b \qquad J^\dagger=\bar{b}\bar{\Gamma}c \tag{3.1}$$

where Γ is any Dirac matrix and $\bar{\Gamma}=\gamma_0\Gamma^\dagger\gamma_0$. The indices on the V-A currents are dropped for notational convenience. The equal-time commutators are then given by the expression

$$\begin{aligned}[J^\dagger(0,\vec{x}),J(0)]=&\left(\bar{b}\bar{\Gamma}\gamma_0\Gamma b-\bar{c}\Gamma\gamma_0\bar{\Gamma}c\right)\ \delta^3(\vec{x})\\ \equiv&J_3(0)\ \delta^3(\vec{x})\end{aligned} \tag{3.2}$$

where possible QCD corrections (present only for the commutator of space components) are here ignored. We take matrix elements of these commutation relations between Λ_b states of equal, arbitrary but finite velocity v. Upon expansion into intermediate states and Fourier transformation into momentum space, standard manipulation leads to the following expression

$$\sum_n \mid \langle n|\, J(0)\,|\Lambda_b\rangle \mid^2 (2\pi)^3\delta^3(\vec{P}-\vec{p}_n-\vec{q}) - \sum_n \mid \langle n|\, J^\dagger(0)\,|\Lambda_b\rangle \mid^2 (2\pi)^3\delta^3(\vec{P}-\vec{p}_n+\vec{q})$$

$$= \langle \Lambda_b|\, J_3(0)\,|\Lambda_b\rangle = \frac{M}{2E}\bar{u}(v)\bar{\Gamma}\gamma_0\Gamma u(v) \tag{3.3}$$

In addition to $\Lambda_c X$, the first term also picks up a contribution from the inclusive DY channel; in the limit of Isgur and Wise its amplitude has the following structure

$$\langle D(v')Y|\, J(0)\,|\Lambda_b(v)\rangle = \sqrt{\frac{MM'}{4EE'}}\bar{\psi}_Y(v,v';Y)\bar{\mathcal{D}}\Gamma u(v) \tag{3.4}$$

where $\psi_Y(v,v';Y)$ is a spinor (analog to $F(v,v';X)$ of Eqn. (2.2)), and $\mathcal{D}$ is the wave-function of the D meson, as defined in Eqn. (5.2) of Section 5.

The first term in Eqn. (3.3) will clearly involve the quantity ω of interest. This can be accomplished by introducing an extra delta-function of energy conservation, which is then integrated over q_0 at fixed $\vec{q}$.

$$\sum_n \mid \langle n|\, J(0)\,|\Lambda_b\rangle \mid^2 (2\pi)^3\delta^3(\vec{P}-\vec{p}_n-\vec{q}) =$$

$$= \int_{-\infty}^{\infty} dq_0 \int \frac{d^3p'}{(2\pi)^3} \sum_X \mid \langle \Lambda_c X|\, J(0)\,|\Lambda_b\rangle \mid^2 (2\pi)^3\delta^4(P-(p'+P_X)-q) +$$

$$\int_{-\infty}^{\infty} dq_0 \int \frac{d^3p'}{(2\pi)^3} \sum_Y \sum_{\mathbf{D}=D,D^*} \mid \langle \mathbf{D}Y|\, J(0)\,|\Lambda_b\rangle \mid^2 (2\pi)^3\delta^4(P-(p'+P_Y)-q) =$$

$$= \int_{-\infty}^{\infty} dq_0 \sum_X \int d^3p' \frac{MM'}{4EE'} \mid \bar{u}(v')\Gamma u(v) \mid^2 \mid F(v,v';X) \mid^2 \delta^4(P-p'-P_X-q) +$$

$$\int_{-\infty}^{\infty} dq_0 \sum_{Y} \sum_{\mathbf{D}=D,D^*} \int d^3p' \frac{MM'}{4EE'} \mid \bar{\psi}_Y \bar{\mathcal{D}}(v')\Gamma u(v) \mid^2 \delta^4(P - p' - P_Y - q) =$$

$$= \int_{-\infty}^{\infty} dq_0 \frac{MM'}{2E} \sum_{X} \mid F(v, v'; X) \mid^2 \delta[(P - P_X - q)^2 - M'^2] \; \bar{u}(v)\bar{\Gamma}(1+ \not{v}')\Gamma u(v) +$$

$$\int_{-\infty}^{\infty} dq_0 \frac{MM'}{2E} \sum_{Y} \sum_{\mathbf{D}=D,D^*} \mid \bar{\psi}_Y \bar{\mathcal{D}}(v')\Gamma u(v) \mid^2 \delta[(P - P_Y - q)^2 - M'^2] \qquad (3.5)$$

The contributions from the D as well as from the the three polarization states of the D^* are denoted by $\sum_{\mathbf{D}=D,D^*}$.

Identifying $P - q = W = p' + P_X$, and expanding out the argument of the delta-function to leading order in M^{-1} gives

$$(P - P_X - q)^2 - M'^2 \cong W^2 - 2W.P_X - M'^2 \cong 2M'(\epsilon - v'.P_X) \qquad (3.6)$$

and similarly for P_Y. Thus

$$\sum_n \mid \langle n \mid J(0) \mid \Lambda_b \rangle \mid^2 (2\pi)^3 \delta^3(\vec{P} - \vec{p}_n - \vec{q}) \cong \frac{M}{4E} \int_{-\infty}^{\infty} dq_0 \; \omega(\epsilon, v.v') \; \bar{u}(v)\bar{\Gamma}(1+ \not{v}')\Gamma u(v) \qquad (3.7)$$

The invariant function ω now includes the contribution from both the $\Lambda_c X$ and the DY channels; ω reads

$$\omega(\epsilon, v.v') \equiv \sum_X \mid F(v, v'; X) \mid^2 \delta(\epsilon - v'.P_X) + \sum_Y \bar{\psi}_Y \Lambda_+(v')\psi_Y \delta(\epsilon - v'.P_Y) \quad (3.8)$$

It is appropriate now to make explicit the relation between the variable q_0 and the invariant variables of intrinsic interest. It is useful to introduce here the variables

$W.v$ and W^2, which will evidently become of central importance when dealing with the charmless final states. We have

$$W.v = P.v - q.v = M - \gamma(q_0 - \vec{q}.\vec{v})$$

$$W^2 = -M^2 + 2MW.v + q^2 = -M^2 + 2MW.v + \gamma^{-2}[(M - W.v) + \gamma\vec{v}.\vec{q}]^2 - \mid \vec{q} \mid^2 \quad (3.9)$$

This elimination of q_0 gives a parabola in $W.v - W^2$ space. We simplify by choosing $\vec{q}$ such that

$$\vec{W}.\vec{v} = (\vec{P} - \vec{q}).\vec{v} = 0 \quad (3.10)$$

and obtain

$$W^2 = W_0^2 - \mid \vec{W} \mid^2 = \frac{(W.v)^2}{\gamma^2} - \mid \vec{W} \mid^2 \quad (3.11)$$

This parabola is shown in Fig. 2.

The important part of the sum will occur for $W^2 \gtrsim M'^2$. In particular the qualitative estimates made in Section 1 imply, for Λ some reasonable multiple of Λ_{QCD} and $v.v' = \gamma$ large

$$W^2 = (M' + \epsilon)^2 = (p' + \sum_i k_i)^2 \cong M'^2 + 2p' \sum_i k_i \sim M'^2 + 2M'\gamma\Lambda$$

Consequently,

$$\epsilon \lesssim (v.v' - 1)\Lambda \quad (3.12)$$

where the subtraction expresses the vanishing of ϵ as $v.v' \longrightarrow 1$. This in turn implies that the shaded region in Fig. 2 is the important one for the sum. Therefore the change in $W.v$ as one crosses the important region of the sum becomes negligible, because

$$\delta(W.v) \cong M'\delta(v'.v) = \frac{\gamma^2}{2(W.v)}\delta W^2 \approx \frac{\gamma^2 M'\epsilon}{(W.v)} \approx \gamma^2 \frac{\epsilon}{v.v'} \quad (3.13)$$

which is $O(1)$, in contrast to $W.v$ which is $O(M')$. This in turn implies that the change in $v.v'$ across the important region is $O(M'^{-1})$ in the Isgur-Wise limit. We find the simple result

$$dW^2 = 2M'd\epsilon = 2W_0 dW_0 = -2W_0 dq_0 \approx -2E'dq_0 \tag{3.14}$$

or

$$dq_0 = -\frac{M'}{E'}d\epsilon \tag{3.15}$$

So everything in the first term of the commutator except the spinor product has essentially the necessary form for the desired sum rule. Since the spinor product has the same structure as for free fields, and the sum rule is true for free fields, we can already anticipate that things will work out satisfactorily. But for free fields the second term in the commutator does contribute.

The second term has to do with z-graph contributions, as illustrated in Fig. 3. They are also described by a structure function $\tilde{\omega}$ although the physical significance of $\tilde{\omega}$ is more obscure, because it is not a cross-section for anything. We may write, with $M'\bar{v}' = (\sqrt{(\vec{P}-\vec{q})^2 + M'^2}, -(\vec{P}-\vec{q}))$

$$\sum_n \mid \langle n|\, J^\dagger(0)\, |\Lambda_b\rangle \mid^2 (2\pi)^3 \delta^3(\vec{P} - \vec{p}_n + \vec{q}) =$$

$$\int_{-\infty}^{\infty} dq_0 \int \frac{d^3p'}{(2\pi)^3} \sum_X \mid \langle \Lambda_b \Lambda_b \bar{\Lambda}_c X|\, J^\dagger(0)\, |\Lambda_b\rangle \mid^2 (2\pi)^3 \delta^4(P + q - (2P + p' + P_X)) \;=$$

$$\int_{-\infty}^{\infty} dq_0 \frac{M}{4E} \sum_X \mid \bar{F}(v, \bar{v}'; X) \mid^2 \delta(\bar{\epsilon} - \bar{v}'.P_X) \;\; \mid \bar{v}(\bar{v}')\Gamma u(v) \mid^2 =$$

$$= \frac{M}{4E} \int_{-\infty}^{\infty} dq_0 \; \tilde{\omega}(\bar{\epsilon}, v.\bar{v}') \; \bar{u}(v)\bar{\Gamma}(\bar{\not{v}}' - 1)\Gamma u(v) \tag{3.16}$$

(Other intermediate states with the same quantum numbers, such as $\Lambda_b\Lambda_b\bar{D}X$, $\Lambda_b B\bar{D}X$, $\Lambda_b B\bar{\Lambda}_c X$, $BB\bar{\Lambda}_c X$, $BB\bar{D}X$, have been omitted, but it can be shown that they contribute additively to $\tilde{\omega}$, as in (3.7)).

Important here is the fact that the spinor structure is just that of free fields, while the remainder again is fixed by invariance considerations. We should also emphasize that, unlike the free-field case, the matrix element can be connected due to gluon exchanges; nothing in what follows is sensitive to that feature.

We also mention that we are neglecting the "vacuum polarization" contributions (Fig. 3),(which also may be connected) because they appear to produce self-cancelling contributions. However, there may be more to learn from a careful study of these contributions.

The z-graph contributions can again be expected to contribute for $W^2 \gtrsim M'^2$, near the free field locale. And again qualitative arguments as given in Section 1 easily show that the reflection of the shaded region around the horizontal axis is where the z-graph sum should saturate. With this inference, we are ready to combine the two pieces, obtaining

$$\frac{M}{4M'E}\int dq_0\ \omega(\epsilon, v.v')\bar{u}(v)\bar{\Gamma}[\gamma_0\sqrt{M'^2+(\vec{P}-\vec{q})^2}-\vec{\gamma}.(\vec{P}-\vec{q})+M']\Gamma u(v)$$

$$+\frac{M}{4M'E}\int dq_0\ \tilde{\omega}(\bar{\epsilon}, v.\bar{v}')\bar{u}(v)\bar{\Gamma}[\gamma_0\sqrt{M'^2+(\vec{P}-\vec{q})^2}+\vec{\gamma}.(\vec{P}-\vec{q})-M']\Gamma u(v)$$

$$=\frac{M}{2E}\bar{u}(v)\bar{\Gamma}\gamma_0\Gamma u(v) \tag{3.17}$$

Here we carefully display the spinor-product factors, insensitive to the value of ϵ and observe there are, for general $\vec{v}$, two independent kinematical structures. Therefore the coefficient of each satisfies a sum rule, leading to

$$\frac{E'}{2M'}\left(\int dq_0\ \omega(\epsilon, v.v')+\int dq_0\ \tilde{\omega}(\bar{\epsilon}, v.\bar{v}')\right)=1$$

$$\frac{E'}{2M'}\left(\int dq_0\ \omega(\epsilon, v.v')-\int dq_0\ \tilde{\omega}(\bar{\epsilon}, v.\bar{v}')\right)=0 \tag{3.18}$$

and, finally, using Eqn. (3.15) ,

$$\int_0^\infty d\epsilon\ \omega(\epsilon, v.v') = 1$$

$$\int_0^\infty d\bar{\epsilon}\ \tilde{\omega}(\bar{\epsilon}, v.\bar{v}') = 1 \tag{3.19}$$

This is the desired result.

4. *Sum Rules for the Baryonic Charmless Semileptonic Decays*

The problem of deriving sum rules for the main case of interest, the decays of Λ_b into noncharmed, ordinary hadrons, is both similar and different from what was encountered in the previous section. The similarity lies in the use of equal-time commutation relations in a way completely analogous to what was done there. The main difference can be seen in Fig. 2, considered in the limit of small M'. Under those circumstances, the simplifications encountered in the previous case no longer occur. The important regions of parameter space for the sum no longer occur for fixed $v.v'$. And for small $v.v'$ (which in Fig. 2 corresponds to small values of W^2), there is not the clean separation of z-graph contributions from the direct contributions of interest.

These obstacles seem possible to be overcome only by a limitation of goals, namely looking for sum rules for the situation when the invariant parameter $W.v$ is large compared to the natural scale Λ (with Λ some reasonable multiple of Λ_{QCD}), but of course small compared to the heavy-quark mass M. This is a short-distance, parton-model limit, and we shall find many similarities with the corresponding situation in the classical case of deep-inelastic scattering.

We begin as in the previous section with the expression for the equal time commutator of currents, with the c-quark replaced, for simplicity here, with an

s-quark*. The procedure is then completely analogous to the previous case, until we reach Eqn. (3.3) , at which point we write down, for the direct term

$$\sum_n |\langle n|\, J(0)\,|\Lambda_b\rangle\,|^2\,(2\pi)^3\delta^3(\vec{P}-\vec{p}_n-\vec{q}) =$$

$$\int dq_0\,\sum_X |\langle X|\, J(0)\,|\Lambda_b\rangle\,|^2\,(2\pi)^3\delta^4(P-P_X-q) =$$

$$= \frac{M}{2E}\int dq_0\;\bar{u}(v)\bar{\Gamma}\Phi(W,v)\Gamma u(v) \tag{4.1}$$

where again $W = P - q = P_X$, and Φ is defined in Eqn. (2.12)

The kinematic analysis for the z-graph term again follows similar lines, so that the expression for the sum rules becomes, in hopefully self-evident notation,

$$\frac{M}{2E}\int dq_0\;\bar{u}(v)\bar{\Gamma}\Phi(W,v)\Gamma u(v)\;+\frac{M}{2E}\int dq_0\;\bar{u}(v)\bar{\Gamma}\bar{\Phi}(\bar{W},v)\Gamma u(v) =$$

$$= \frac{M}{2E}\bar{u}(v)\bar{\Gamma}\gamma_0\Gamma u(v) \tag{4.2}$$

with $\bar{W} = P - q = -P_X$; therefore $\bar{W}_0 \leq 0$. It is useful to check that for free fields, the sum rule works. Recalling the free-field limit for Φ, Eqn. (2.14) , we have

$$\begin{aligned}
\int dq_0\;\Phi(W,v) &= \int dW_0\;(\not{W}+M')\delta(W^2-M'^2)\\
&= \frac{1}{2}\left[\gamma_0 - \frac{\vec{\gamma}.(\vec{P}-\vec{q})-M'}{\sqrt{(\vec{P}-\vec{q})^2+M'^2}}\right]\\
\int dq_0\;\tilde{\Phi}(\bar{W},v) &= \frac{1}{2}\left[\gamma_0 + \frac{\vec{\gamma}.(\vec{P}-\vec{q})-M'}{\sqrt{(\vec{P}-\vec{q})^2+M'^2}}\right]
\end{aligned} \tag{4.3}$$

which gives us the correct sum of Eqn. (4.2) . To consider the general case, we first of all apply the reasoning given in the introduction, Section 1, that argues

* Were we to choose the case of interest $\bar{u}\gamma_\mu(1-\gamma_5)b$ we would have to consider the action of the currents on the spectator diquark. This point, which appears superficially not to create any real problems, is currently under study.

that the important values of W^2 and $W.v$ in this case are for

$$W^2 \lesssim 2\Lambda W.v \tag{4.4}$$

Also, in the limit of large W^2 and $W.v$ we see that again $W.v$ may be taken as essentially constant while summing over W^2. From

$$\frac{W.v}{\gamma} = W_0 - \vec{v}.\vec{W} = \sqrt{\mid \vec{W} \mid^2 + W^2} - \vec{v}.\vec{W} = \sqrt{\mid \vec{W} \mid^2 + 2\Lambda x(W.v)} - \vec{v}.\vec{W}$$

$$\simeq \mid \vec{W} \mid \left(1 + \frac{\Lambda x \gamma}{\mid \vec{W} \mid}\right)\left(1 - \frac{\vec{v}.\vec{W}}{\mid \vec{W} \mid}\right) \tag{4.5}$$

we see that the variation in $W.v$ is indeed small as x varies, say, from zero to one.

This approximation is further supported by the expectation that in this limit we may have scaling behavior of the structure functions φ_i. This follows from general dimensional arguments as well as the more detailed reasoning presented in Section 1. When $\Gamma = \gamma_\mu(1 - \gamma_5)$ we write

$$\Phi = \not{W}\varphi_1 + \not{v}\, W.v\varphi_2 + noncontributing \quad terms \tag{4.6}$$

with

$$2\Lambda W.v\varphi_i = f_i(x) \tag{4.7}$$

and the scaling variable

$$x = \frac{W^2}{2\Lambda W.v} \tag{4.8}$$

We are now ready to insert these expressions into the sum rule Eqn (4.2). We need to change integration variables from q_0 to W^2; the connection is

$$dq_0 = -dW_0 = -\frac{dW^2}{2 \mid W_0 \mid} \tag{4.9}$$

We then have

$$\int \frac{dW^2}{2 \mid W_0 \mid} \bar{u}(v)\bar{\Gamma} \left[(\not{W}\varphi_1 + W.v \not{v}\varphi_2) - (\bar{\not{W}}\tilde{\varphi}_1 + \bar{W}.v \not{v}\tilde{\varphi}_2) \right] \Gamma u(v) =$$

$$= \bar{u}(v)\bar{\Gamma}\gamma_0\Gamma u(v) \tag{4.10}$$

The extraction of the sum rules is analogous to the previous case in Section 3. As the γ_μ are linearly independent, it follows that

$$\frac{1}{2}\int dW^2 \left[(\varphi_1 + \tilde{\varphi}_1) + \gamma^2(\varphi_2 + \tilde{\varphi}_2) - \gamma^2 \frac{\vec{v}.\vec{W}}{\mid W_0 \mid}(\varphi_2 - \tilde{\varphi}_2) \right] = 1$$

$$\frac{1}{2}\vec{W}\int dW^2 \frac{1}{\mid W_0 \mid}(\varphi_1 - \tilde{\varphi}_1) + \frac{1}{2}\vec{v}\gamma^2 \int dW^2 \left[(\varphi_2 + \tilde{\varphi}_2) - \frac{\vec{v}.\vec{W}}{\mid W_0 \mid}(\varphi_2 - \tilde{\varphi}_2) \right] = 0 \tag{4.11}$$

$\vec{v}$ and $\vec{W}$ are arbitrary vectors. The only way the previous identities can hold in any generic frame is by demanding that

$$\frac{1}{2}\int dW^2 (\varphi_1 + \tilde{\varphi}_1) = 1, \qquad \frac{1}{2}\int dW^2 \frac{\mid \vec{W} \mid}{\mid W_0 \mid}(\varphi_1 - \tilde{\varphi}_1) = 0$$

$$\frac{1}{2}\int dW^2 (\varphi_2 + \tilde{\varphi}_2) = 0, \qquad \frac{1}{2}\int dW^2 \frac{\mid \vec{W} \mid}{\mid W_0 \mid}(\varphi_2 - \tilde{\varphi}_2) = 0 \tag{4.12}$$

Combining these equations one finds

$$\frac{1}{2}\int dW^2 \left[\left(1 + \frac{\mid \vec{W} \mid}{\sqrt{W^2 + \mid \vec{W} \mid^2}} \right) \varphi_1 + \left(1 - \frac{\mid \vec{W} \mid}{\sqrt{W^2 + \mid \vec{W} \mid^2}} \right) \tilde{\varphi}_1 \right] = 1 \tag{4.13}$$

and similar combinations involving φ_1, $\tilde{\varphi}_1$ and φ_2, $\tilde{\varphi}_2$. If we assume that φ_i and $\tilde{\varphi}_i$ tend to zero fast enough as W^2 gets large, in the regime where $\mid \vec{W} \mid^2 \gg \Lambda^2$ we find

$$\int dW^2 \varphi_1 = \int dW^2 \tilde{\varphi}_1 = 1$$

$$\int dW^2 \varphi_2 = \int dW^2 \tilde{\varphi}_2 = 0 \tag{4.14}$$

This is the main result of this section.

Let us now show that φ_2 vanishes in the limit, and that φ_1 is positive definite. This follows from the structure of Φ as defined in Eqn. (2.12) . For any n^μ timelike with $n^0 \geq 0$,

$$Tr(\not{n}\ \Phi\ \frac{1+\gamma_5}{2}) \geq 0 \tag{4.15}$$

Indeed, $Tr(\not{n}\ \Phi\ \frac{1+\gamma_5}{2}) = \sum_X (\psi_L)^\dagger\ \gamma_0\ \not{n}\ \psi_L (2\pi)^3 \delta^4(P_X - W)$, with $\psi_L = \frac{1-\gamma_5}{2}\ \psi$; the eigenvalues of $\gamma_0\ \not{n}$ are $\mid n^0 \mid \pm \mid \vec{n} \mid$, both positive, thus leading to (4.15). Taking $n^\mu = W^\mu, v^\mu$ (4.15) leads to, respectively,

$$W^2 \varphi_1 + (v.W)^2 \varphi_2 \geq 0$$

$$\varphi_1 + \varphi_2 \geq 0 \tag{4.16}$$

Since $(v.W)^2 \gg W^2$ the first equation implies that $\varphi_2 \gtrsim 0$, which combined with the vanishing sum rule for φ_2 tells us that $\varphi_2 = 0$. The second one then allows to conclude that φ_1 is positive. The only assumption is that $f_1(x) = 2\Lambda W.v \varphi_1$ is a smooth function of order unity.

These are the expected results from the parton-model point of view. The main result is for φ_1. In the convergence region, $\frac{dW^2}{2\Lambda W.v} \simeq dx$. Therefore, in scaling language, the sum rule is

$$\int_0^\infty dx f_1(x) = 1 \tag{4.17}$$

and can be anticipated to converge for x of order unity.

The inclusive differential decay rate for the semileptonic Λ_b decay into uncharmed final states now reads

$$\frac{d\Gamma}{dq^2\ dW^2} \cong \frac{d\Gamma_0}{dq^2}\ \varphi_1(W^2, v.W) \tag{4.18}$$

and the spectator form for $d\Gamma/dq^2$ is apparent.

It is worth mentioning that in the scaling limit , $W.v \gg \Lambda$, the form factor φ_1 is expected to be negligible for small W^2, $W^2 \lesssim \Lambda^2$, because only a few exclusive channels are available. This is also the case for all form factors of Section 7, which describe semileptonic B decay into uncharmed final states.

5. *The Formalism for the Meson Decays*

The original Isgur-Wise development was applied to relate "elastic" form factors of decays such as $B \rightarrow D\ l\bar{\nu}$ and $B \rightarrow D^*\ l\bar{\nu}$. Here the analysis becomes slightly more cumbersome because, unlike for the baryons, the spin of the meson is no longer carried by the heavy quark and spin correlations enter the game. The new flavour-spin symmetry that arises when b and c quarks are heavy can be implemented in a 4x4 Dirac matrix formalism[7] which proves very convenient in coping with the inclusive decays. The amplitude reads as follows

$$\langle DX|\ J_\mu\ |B\rangle = \sqrt{\frac{MM'}{4EE'}}\ Tr\ \left(\bar{\mathcal{D}}\mathcal{J}_\mu\mathcal{B}\rho(v,v';X)\right) \tag{5.1}$$

where M, E (M', E') are the mass and the energy of the B (D) meson and the trace is on the Dirac matrices,

$$\begin{aligned} \mathcal{D} &= \Lambda_+(v') \qquad for\ 0^-\ D \\ \mathcal{D} &= \gamma_5\ \not{\epsilon}\ \Lambda_+(v') \qquad for\ 1^-\ D^* \end{aligned} \tag{5.2}$$

and similarly for $\mathcal{B}$. (For antiparticles write Λ_- instead of Λ_+). ϵ_μ is the polarization vector of the D^*, for which $v'.\epsilon = 0$ and $\epsilon^2 = -1$. The non-trivial dynamics of the light spectators is contained in the matrix $\rho(v, v'; X)$, which only depends on the heavy meson kinematics through their velocities. $\mathcal{J}_\mu$ is the Dirac matrix of the current, while $\Lambda_\pm(v) = (1 \pm \not{v})/2$ and $\bar{\mathcal{D}} = \gamma^0 \mathcal{D}^\dagger \gamma^0$.

The form of ρ is especially simple in the case of "elastic" semileptonic transitions of B to D or D^*. It can depend upon initial and final velocities in the allowed invariant combinations $\not{v}$, $\not{v}'$, $(\not{v}\ \not{v}')$, but these can be eliminated with the use of the Dirac projection operators $\Lambda_+(v), \Lambda_+(v')$ residing on $\mathcal{B}$ and $\bar{\mathcal{D}}$. Its form is, therefore, restricted to be proportional to the Dirac unit matrix

$$\rho(v.v') = \ \rho_{el}(v.v')\ I \tag{5.3}$$

$\rho_{el}(v.v')$ being the universal function of Isgur and Wise for the meson decays, which also satifies $\rho_{el}(1) = 1$.

The inclusive B decay has also a simple form, after summing over the contributions of the D and the three polarizations of the D^*

$$B \to (D\ or\ D^*\ + X)_W\ + (l + \bar{\nu})_q \tag{5.4}$$

To see this use the following identity (see (A.2))

$$\sum_{D,D^*} Tr\left(\bar{\mathcal{D}}\gamma_\nu(1-\gamma_5)\mathcal{B}\rho(v,v';X)\right) Tr\left(\bar{\rho}(v,v',X)\bar{\mathcal{B}}\gamma_\mu(1-\gamma_5)\mathcal{D}\right) =$$

$$= 2Tr\left(\gamma_\mu(1-\gamma_5)\Lambda_+(v')\gamma_\nu(1-\gamma_5)\Lambda_+(v)\rho\Lambda_+(v')\bar{\rho}\Lambda_+(v)\right) \tag{5.5}$$

The structure of ρ is dictated by the strong interactions which are parity conserving. After appropriate averaging over the final states X, as in (3.8), one has

$$\Lambda_+(v)\left(\sum_X \rho\Lambda_+(v')\bar{\rho}\ \delta(\epsilon - v'.P_X)\right)\Lambda_+(v) = \Lambda_+(v)(A(\epsilon, v.v') + B(\epsilon, v.v')\ \not{v}')\Lambda_+(v)$$

$$= \Lambda_+(v)(A + v.v'B) = \Lambda_+(v)\omega(\epsilon, v.v') \tag{5.6}$$

where ω can be written as

$$\omega(\epsilon, v.v') = \frac{1}{2}\sum_X \; Tr \;\; (\Lambda_+(v)\rho\Lambda_+(v')\bar{\rho}) \;\; \delta(\epsilon - v'.P_X) \tag{5.7}$$

The structure of the differential decay rate is the same as for the Λ_b decay

$$\frac{d\Gamma}{dq^2 \; d\epsilon} = \frac{d\Gamma_0}{dq^2} \; \omega(\epsilon, v.v') \tag{5.8}$$

There is also a sum rule for $\omega(\epsilon, v.v')$, and the "free" spectator decay rate emerges again for the inclusive process. The next section is devoted to this subject.

The last case to consider is semileptonic $\bar{B}^0$ decay with charmless final states. The symmetry of Isgur and Wise is not as useful in reducing the number of invariant form factors as in the previous examples: their number is six (only four actually contribute to the decay rate). Nevertheless, we shall still keep the trace formalism introduced at the begining of this section since it provides an easy way to compare the results to those for the Λ_b and the "free" spectator model. We thus write for the hadronic matrix element, as before,

$$\langle X| \, J_\mu \, |B_d\rangle = \sqrt{\frac{M}{2E}} \; Tr \, (\mathcal{J}_\mu \mathcal{B} \varphi(v; X)) \tag{5.9}$$

The structure function for the inclusive process

$$\mathcal{W}_{\mu\nu}(v, W) \equiv \sum_X \;\; (2\pi)^3 \; \delta^4(P_X - W) \, \langle B_d| \, J_\mu^\dagger \, |X\rangle \, \langle X| \, J_\nu \, |B_d\rangle \tag{5.10}$$

can be written as (see (A.6))

$$\mathcal{W}_{\mu\nu} = \frac{M}{4E} \; Tr \left(\gamma^\lambda \gamma_\nu (1 - \gamma_5) \Lambda_+(v) \gamma_\mu (1 - \gamma_5)\right) \; Tr \left(\gamma_\lambda \frac{1 - \gamma_5}{2} \Phi(v, W)\right)$$

$$+\frac{M}{4E}\ Tr\left(\gamma^\lambda\gamma_\nu(1-\gamma_5)\frac{\gamma^\alpha}{2}\gamma_\mu(1-\gamma_5)\right)Tr\left(\gamma_\lambda\frac{1-\gamma_5}{2}\Psi_\alpha(v,W)\right) \quad (5.11)$$

with

$$\begin{aligned}\Phi(v,W) &\equiv \frac{1}{2}\sum_X (2\pi)^3\ \delta^4(P_X-W)\ \overline{\varphi}(v;X)\Lambda_+(v)\varphi(v;X)\\ \Psi_\alpha(v,W) &\equiv \frac{1}{2}\sum_X (2\pi)^3\ \delta^4(P_X-W)\ \overline{\varphi}(v;X)\Lambda_+(v)\gamma_5\gamma_\alpha\Lambda_+(v)\varphi(v;X)\ .\end{aligned} \quad (5.12)$$

The general structure of Φ and Ψ_α is as follows

$$\Phi(v,W) = \phi_1\ \not{W} + (v.W)\ \not{v}\phi_2 + noncontributing\ \ terms$$

$$\Psi_\alpha(v,W) = [(v.W)\psi_1(\gamma_\alpha - v_\alpha\ \not{v}) + ((v.W)\ \not{v}\psi_2 + \not{W}\psi_3)\,(W_\alpha - (v.W)v_\alpha)]\,\gamma_5$$

$$+\psi_4\ \epsilon_{\alpha\sigma\eta\rho}\gamma^\sigma W^\eta v^\rho + noncontributing\ \ terms\ . \quad (5.13)$$

The number of form factors is still six, and no reduction has occurred.Each form factor is a real function of W^2 and $v.W$.

The differential decay rate may be written as

$$\frac{d\Gamma}{dq^2\ dW^2} = \Omega^\lambda\ Tr\left(\gamma_\lambda\frac{1-\gamma_5}{2}\Phi(v,W)\right) + \Omega^{\lambda\alpha}Tr\left(\gamma_\lambda\frac{1-\gamma_5}{2}\Psi_\alpha(v,W)\right) \quad (5.14)$$

with

$$\Omega^{\lambda\alpha} = \frac{G_F^2|\ V_{ub}\ |^2}{2}\int\frac{d^3l}{(2\pi)^3 2l_0}\frac{d^3\nu}{(2\pi)^3 2\nu_0}\delta(q^2-(l+\nu)^2)\ \delta\left[W^2-(P-q)^2\right]$$

$$\times\frac{1}{4}\left[Tr\ \gamma^\lambda\gamma_\mu(1-\gamma_5)\frac{\gamma_\alpha}{2}\gamma_\nu(1-\gamma_5)\right][Tr\ \ \not{l}\gamma^\mu(1-\gamma_5)\ \not{\nu}\gamma^\nu(1-\gamma_5)]\ (2\pi) \quad (5.15)$$

and

$$\Omega^\lambda = \Omega^{\lambda\alpha}v_\alpha \quad (5.16)$$

(Ω_λ is the same as defined in Section 2, Eqn.(2.16)). The projector $(q_\mu q_\nu - q^2 g_{\mu\nu})$

coming from the lepton pair integration allows again to write

$$\Omega_{\lambda\alpha} = \Omega(P,q)\ (2\ q_\lambda q_\alpha + q^2 g_{\lambda\alpha}) \tag{5.17}$$

Finally,

$$\frac{d\Gamma}{dW^2dq^2} = \left(\frac{d\Gamma_0}{dq^2}\right)\ \frac{Tr\ \left((2\ q.v\ \not q + q^2\ \not v)(1-\gamma_5)\Phi + (2\ \not q\ q^\alpha + q^2\ \gamma^\alpha)(1-\gamma_5)\Psi_\alpha\right)}{4\,[2\ q.v\ q.W + q^2 v.W]} \tag{5.18}$$

The term Ψ_α is not present in the baryon structure function. We will see, however, that it will not contribute to $d\Gamma/dq^2$ after integrating over W^2 in the kinematic region where $v.W \gg \Lambda$.

6. *Sum Rules for Semileptonic Meson Decay into Charm*

The derivation of a sum rule for the inclusive structure function ω of the B meson charmed decay[8] (as defined in Eqn. (5.8)) can now be carried through. It follows closely the derivation of Section 3 for the baryons and we refer to that section for the details. An analogous expression to (3.3) is obtained in this case

$$\sum_n |\ \langle n|\, J(0)\, |B\rangle\ |^2\ (2\pi)^3\delta^3(\vec{P}-\vec{p}_n-\vec{q}) - \sum_n |\ \langle n|\, J^\dagger(0)\, |B\rangle\ |^2\ (2\pi)^3\delta^3(\vec{P}-\vec{p}_n+\vec{q})$$

$$= \langle B|\, J_3(0)\, |B\rangle = \frac{M}{2E} Tr\left(\bar{\Gamma}\gamma_0\Gamma\Lambda_+(v)\right) \tag{6.1}$$

The current matrix element is given by the trace formula (5.1). The by now familiar manipulations give, for the direct graph

$$\sum_n |\ \langle n|\, J(0)\, |B\rangle\ |^2\ (2\pi)^3\delta^3(\vec{P}-\vec{p}_n-\vec{q}) =$$

$$\int_{-\infty}^{\infty} dq_0 \int \frac{d^3p'}{(2\pi)^3} \sum_X \sum_{\mathbf{D}=D,D^*} |\ \langle DX|\, J(0)\, |B\rangle\ |^2\ (2\pi)^3\delta^4(P-(p'+P_X)-q) =$$

$$\int_{-\infty}^{\infty} dq_0 \int d^3p' \frac{MM'}{4EE'} \sum_X \sum_{\mathbf{D}=D,D^*} Tr(\bar{\mathcal{D}}\Gamma\mathcal{B}\rho)Tr(\bar{\rho}\bar{\mathcal{B}}\bar{\Gamma}\mathcal{D})\delta^4(P - p' - P_X - q) =$$

$$\int_{-\infty}^{\infty} dq_0 \frac{MM'}{2E} Tr(\bar{\Gamma}\Lambda_+(v')\Gamma\Lambda_+(v)) \sum_X Tr(\Lambda_+(v)\rho\Lambda_+(v')\bar{\rho})\ \delta[(P - P_X - q)^2 - M'^2]$$

$$\cong \frac{M}{2E} \int_{-\infty}^{\infty} dq_0\ \omega(\epsilon, v.v')\ Tr(\bar{\Gamma}\Lambda_+(v')\Gamma\Lambda_+(v)) \tag{6.2}$$

For the z-graph one finds

$$\sum_n |\ \langle n|\, J^\dagger(0)\, |B\rangle\ |^2\ (2\pi)^3\delta^3(\vec{P} - \vec{p}_n + \vec{q}) =$$

$$\int_{-\infty}^{\infty} dq_0 \int \frac{d^3p'}{(2\pi)^3} \sum_X \sum_{\bar{\mathbf{D}}=\bar{D},\bar{D}^*} |\ \langle BB\bar{\mathbf{D}}X|\, J(0)\, |B\rangle\ |^2\ (2\pi)^3\delta^4(P - (2P + p' + P_X) + q)$$

$$\cong \frac{M}{2E} \int_{-\infty}^{\infty} dq_0\ \tilde{\omega}(\bar{\epsilon}, v.\bar{v}')\ Tr(\bar{\Gamma}\Lambda_-(\bar{v}')\Gamma\Lambda_+(v)) \tag{6.3}$$

(States like $\Lambda_c Y$ which also have the same quantum numbers can contribute as intermediate states, and should in principle be taken into account. It is not difficult to realize that upon summing over Y and the polarizations of the Λ_c they give a contribution which adds up to $\omega(\epsilon, v.v')$). Combining the two pieces, the direct and the z-graph contributions, yields

$$\frac{M}{4M'E} \int dq_0\ \omega(\epsilon, v.v') Tr\left(\Gamma\Lambda_+(v)\bar{\Gamma}\left[M' + \gamma_0\sqrt{M'^2 + (\vec{P} - \vec{q})^2} - \vec{\gamma}.(\vec{P} - \vec{q})\right]\right)$$

$$-\frac{M}{4M'E} \int dq_0\ \tilde{\omega}(\bar{\epsilon}, v.\bar{v}') Tr\left(\Gamma\Lambda_+(v)\bar{\Gamma}\left[M' - \gamma_0\sqrt{M'^2 + (\vec{P} - \vec{q})^2} - \vec{\gamma}.(\vec{P} - \vec{q})\right]\right)$$

$$= \frac{M}{2E} Tr \left(\Gamma \Lambda_+(v) \bar{\Gamma} \gamma_0 \right) \tag{6.4}$$

which leads to

$$\frac{E'}{2M'} \left(\int dq_0 \ \omega(\epsilon, v.v') + \int dq_0 \ \tilde{\omega}(\bar{\epsilon}, v.\bar{v}') \right) = 1$$

$$\frac{E'}{2M'} \left(\int dq_0 \ \omega(\epsilon, v.v') - \int dq_0 \ \tilde{\omega}(\bar{\epsilon}, v.\bar{v}') \right) = 0 \tag{6.5}$$

and finally,

$$\int_0^\infty d\epsilon \ \omega(\epsilon, v.v') = 1$$

$$\int_0^\infty d\bar{\epsilon} \ \tilde{\omega}(\bar{\epsilon}, v.\bar{v}') = 1 \tag{6.6}$$

This is again the desired result.

Splitting up the sum rule into elastic and inelastic contributions, it reads

$$1 = \frac{1}{2}(1 + v.v') \mid \rho_{el}(v.v') \mid^2 + \int_0^\infty d\epsilon \ \omega_{inel}(\epsilon, v.v') \tag{6.7}$$

Note the presence of the kinematic factor $\frac{1}{2}(1 + v.v')$ multiplying the elastic contribution, not present for the baryons.

7. *Sum Rules for the Charmless Semileptonic Decays of the Heavy Mesons*

This section is devoted to the derivation of the sum rules for the decay of the $\bar{B}^0$ meson into noncharmed hadrons. The analysis is once again verbatim to that of Section 4 for the baryons, the only technical complication being the appearance of a larger number of form factors. Let us in this section specialize to V-A currents (see Eqns. (3.1) and (3.2)), i.e.

$$J_\mu = \bar{u}\Gamma_\mu b\ , \quad (J_3)_{\mu\nu} = \bar{b}\Gamma_\mu\gamma_0\Gamma_\nu b - \bar{u}\Gamma_\nu\gamma_0\Gamma_\mu u$$

where $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$. The starting point is again an expression which follows from the equal-time commutator of these currents:

$$\sum_n \langle B|\, J^\dagger_\mu(0)\,|n\rangle\,\langle n|\,J_\nu(0)\,|B\rangle\,(2\pi)^3\delta^3(\vec{P}-\vec{p}_n-\vec{q})-$$

$$\sum_n \langle B|\, J_\nu(0)\,|n\rangle\,\langle n|\,J^\dagger_\mu(0)\,|B\rangle\,(2\pi)^3\delta^3(\vec{P}-\vec{p}_n+\vec{q}) =$$

$$= \langle B|\,\Gamma_\mu\gamma_0\Gamma_\nu\,|B\rangle = \frac{M}{2E}\,Tr(\Gamma_\nu\Lambda_+(v)\Gamma_\mu\gamma_0) \tag{7.1}$$

Inserting the hadronic matrix element given in Eqn.(5.9), yields (see (A.3), and (5.12) for the definition of Φ and Ψ_α)

$$\sum_n \langle B|\, J^\dagger_\mu(0)\,|n\rangle\,\langle n|\,J_\nu(0)\,|B\rangle\,(2\pi)^3\delta^3(\vec{P}-\vec{p}_n-\vec{q}) =$$

$$\int_{-\infty}^{\infty} dq_0\,\sum_X \frac{M}{2E}\,Tr(\bar{\varphi}\bar{\mathcal{B}}\Gamma_\mu)\;Tr(\Gamma_\nu\mathcal{B}\varphi)\;(2\pi)^3\delta^4(P-P_X-q) =$$

$$\int_{-\infty}^{\infty} dq_0\,\sum_X \frac{M}{2E}\,Tr(\bar{\varphi}\Lambda_+(v)\Gamma_\mu)\;Tr(\Gamma_\nu\Lambda_+(v)\varphi)\;(2\pi)^3\delta^4(P-P_X-q) =$$

$$= \int_{-\infty}^{\infty} dq_0 \, \frac{M}{2E} \, [Tr(\Gamma_\nu \Lambda_+(v)\Gamma_\mu \Phi) + Tr(\Gamma_\nu \Lambda_-(v)\gamma^\alpha \Lambda_+(v)\Gamma_\mu \Psi_\alpha)] \tag{7.2}$$

The analysis of the z-graph term can be done along similar lines; the expression for the sum rules becomes

$$\frac{M}{2E} \int dq_0 \, [Tr(\Gamma_\nu \Lambda_+(v)\Gamma_\mu \Phi(W,v)) + Tr(\Gamma_\nu \Lambda_-(v)\gamma^\alpha \Lambda_+(v)\Gamma_\mu \Psi_\alpha(W,v))]$$

$$-\frac{M}{2E} \int dq_0 \left[Tr(\Gamma_\nu \Lambda_+(v)\Gamma_\mu \tilde{\Phi}(\bar{W},v)) + Tr(\Gamma_\nu \Lambda_-(v)\gamma^\alpha \Lambda_+(v)\Gamma_\mu \tilde{\Psi}_\alpha(\bar{W},v))\right] =$$

$$= \frac{M}{2E} \, Tr(\Gamma_\nu \Lambda_+(v)\Gamma_\mu \gamma_0) \tag{7.3}$$

The different tensor structure of the various terms involved again implies the splitting of the sum rule into two, with the following identifications

$$\int dq_0 \, Tr(\Gamma_\nu \Lambda_+(v)\Gamma_\mu \Phi(W,v)) - \int dq_0 \, Tr(\Gamma_\nu \Lambda_+(v)\Gamma_\mu \tilde{\Phi}(\bar{W},v)) = Tr(\Gamma_\nu \Lambda_+(v)\Gamma_\mu \gamma_0)$$

$$\int dq_0 \, Tr(\Gamma_\nu \Lambda_-(v)\gamma^\alpha \Lambda_+(v)\Gamma_\mu \Psi_\alpha(W,v)) - \int dq_0 \, Tr(\Gamma_\nu \Lambda_-(v)\gamma^\alpha \Lambda_+(v)\Gamma_\mu \tilde{\Psi}_\alpha(\bar{W},v)) = 0 \tag{7.4}$$

The first one is the same expression as the baryonic sum rule of Section 4, and we are thus immediately led to conclude

$$\int dW^2 \phi_1 = \int dW^2 \tilde{\phi}_1 = 1$$

$$\int dW^2 \phi_2 = \int dW^2 \tilde{\phi}_2 = 0 \tag{7.5}$$

in the situation where $v.W$ is large compared to Λ. Furthermore, since the inequality $Tr(\not{n}\Phi\frac{1+\gamma_5}{2}) \geq 0$ still holds, we recover in this limit $\phi_2 = 0$ and $\phi_1 \geq 0$, in accordance with the parton-model predictions.

The second sum rule of Eqn. (7.4) is a new feature of the charmless meson decays. Let us now show that the integral over dW^2 of the new invariant form factors ψ_I also vanishes. For this purpose rewrite it as

$$Tr(\Gamma_\nu \Lambda_-(v)\gamma^\alpha \Lambda_+(v)\Gamma_\mu \gamma^\beta) \int \frac{dW^2}{\mid W_0 \mid}(T_{\alpha\beta} - \tilde{T}_{\alpha\beta}) = 0 \tag{7.6}$$

with

$$T_{\alpha\beta} = v.W(g_{\alpha\beta} - v_\alpha v_\beta)\, \psi_1 + \left(v.W v_\beta \psi_2 + W_\beta \psi_3\right)(W_\alpha - v.W v_\alpha) + \psi_4 \epsilon_{\alpha\beta\eta\rho} W^\eta v^\rho \tag{7.7}$$

Eqn. (7.6) implies

$$\int \frac{dW^2}{\mid W_0 \mid}(T_{\alpha\beta} - \tilde{T}_{\alpha\beta}) = 0 \tag{7.8}$$

As in Section 4, to derive the sum rules $\vec{W}$ is held fixed, $\mid \vec{W} \mid \gg \Lambda$; in the region $W^2 \lesssim 2\Lambda v.W$ we have $\frac{|\vec{W}|}{|W_0|} \to 1$.

The coefficients of $g_{\alpha\beta}$, $\epsilon_{\alpha\beta\eta\rho}$ and of the spatial components $v_i W_j$ and $W_i v_j$ must vanish identically since they cannot cancel against anything. This yields

$$\int \frac{dW^2}{\mid W_0 \mid}\left((v.W)\psi_I(v,W) - (v.\bar{W})\tilde{\psi}_I(v,\bar{W})\right) = 0 \quad for \quad I = 1,2,3$$

$$\int \frac{dW^2}{\mid W_0 \mid}\left((W^\rho v^\eta - W^\eta v^\rho)\psi_4(v,W) - (\bar{W}^\rho v^\eta - \bar{W}^\eta v^\rho)\tilde{\psi}_4(v,\bar{W})\right) = 0 \tag{7.9}$$

From the first one,

$$\int dW^2 \left((\psi_I + \tilde{\psi}_I) - \frac{\vec{v}.\vec{W}}{\mid W_0 \mid}(\psi_I - \tilde{\psi}_I)\right) = 0 \tag{7.10}$$

The component $\eta = 0, \rho = i$ of the second one gives

$$\int dW^2 \left(\vec{v}\,(\psi_4 + \tilde{\psi}_4) - \frac{\vec{W}}{\mid W_0 \mid}(\psi_4 - \tilde{\psi}_4)\right) = 0 \tag{7.11}$$

Since $\vec{v}$ and $\vec{W}$ are independent, we conclude in both cases that

$$\int dW^2(\psi_I + \tilde{\psi}_I) = \int dW^2(\psi_I - \tilde{\psi}_I) = 0$$

The desired result follows

$$\int dW^2 \psi_I = \int dW^2 \tilde{\psi}_I = 0 \qquad I = 1, 2, 3, 4 \tag{7.12}$$

Similarly, from the coefficients of $W_\alpha W_\beta$, it follows

$$\int dW^2 \ (v.W)\psi_3 = 0$$

The decay rate of Eqn. (5.18) in the kinematical region $\Lambda \ll v.W \ll M$ reads

$$\frac{d\Gamma}{dW^2 \ dq^2} = \frac{d\Gamma_0}{dq^2}\left(\phi_1 + \psi_1 - \frac{1}{3}(v.W)\psi_3\right) \tag{7.13}$$

The sum rules derived in this section ensure once again that upon integration over W^2 the differential decay rate is that of the spectator model.

8. *Conclusions*

In all the cases considered, we find a sum rule which expresses the content of the spectator picture of semi-leptonic decays. The main result is essentially

$$\int dW^2 \, \frac{d\Gamma}{dq^2\, dW^2} = \frac{d\Gamma_0}{dq^2}$$

as expressed more precisely in Eqns. (1.1), (4.18), (7.13).

For the case of charmless final states, this result only holds in a "scaling" limit when $W.v$, the energy release to hadrons in the parent rest frame is large.

Even in the Isgur-Wise limit, most of the uncharmed processes we have considered involve in general more than one structure function $\phi(W^2, W.v)$. The Λ_b and the B semileptonic decays into charmed final states have a unique structure function each. For Λ_b into uncharmed final states there are two structure functions, while for B into uncharmed final states there are six. Nevertheless, in all cases there is a "principal structure function" which carries the sum, anagolous to F_2 for classical deep-inelastic processes. Thus the results seem to be essentially universal despite the differences in the technology.

This is also the case with another complication, namely the presence of, say, charmed baryons in B-meson decay final states or charmed mesons in Λ_b decay final states. The formalism has been extended to handle this, yet the result retains its simple form.

For charmed final states, the contribution of inelastic final states to the sum rule is moderate. At the no-recoil value of momentum transfer $q^2 = (M_B - M_D)^2$, where the "elastic" Isgur-Wise function is normalized to unity, the inelastic contributions vanish. At the maximum recoil, when $q^2 = 0$, we may expect the elastic contribution to have decreased by about a factor two, so that inelastic final states $B \to D^*\pi l\nu$, $D^{**}l\nu$, etc. must make up the difference. Some evidence that this is the case comes from observation of $B \to D^{**}\pi$, $D^*\pi\pi$, etc. which, assuming factorization, provides some measure of the importance of these contributions when $q^2 = m_\pi^2$.

However, the main thrust of this paper has been the consideration of the sum rules for final states containing no charm. In this case, we found the generic result for $v.W \gg \Lambda$

$$\frac{d\Gamma}{dq^2\, dW^2} \cong \frac{d\Gamma_0}{dq^2}\, \phi(W^2, v.W)$$

with the sum rule for the principal structure function

$$\int dW^2\, \phi(W^2, v.W) = 1$$

The kinematics of the process, along with an estimate of the range of W^2 required for convergence, strongly suggests a scaling behaviour for ϕ

$$2\Lambda v.W \phi(W^2, v.W) \longrightarrow f(\frac{W^2}{2\Lambda v.W})$$

with

$$\int_0^\infty dx\, f(x) = 1$$

The variable x was interpreted as the value of $(E - p_{\parallel})$ of the spectator-quark system in the rest frame of the parent, with the z axis chosen along the direction of the dilepton recoil. An important issue is the determination not only of the area under the curve $f(x)$ (this is given by the sumrule), but also the shape. Were the shape under control, reasonable estimates of exclusive decays, e.g. the experimentally important ones $B \longrightarrow \pi l \nu$ or $\rho l \nu$, would be within reach using the idea of semilocal duality. However, such considerations lie outside the scope of this paper.

Another important issue beyond the scope of this paper is the role of perturbative-QCD corrections. Unlike the previous problem, this one appears to be accessible theoretically.

We also mention that there exist other applications of this sum rule approach. A notable one, now under consideration[9], is the class of Penguin-induced decays which are controlled by the subprocesses $b \longrightarrow s\gamma$ and/or $b \longrightarrow s l^+ l^-$.

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APPENDIX A

This appendix is devoted to some expressions that have been used to deal with products of traces of Dirac matrices and to re-shuffle the matrices inside the traces. The following expression has been used as the basis for them all:

$$Tr(X\Lambda_+(v))Tr(Y\Lambda_+(v)) =$$

$$= \frac{1}{2}Tr(X\Lambda_+(v)Y\Lambda_+(v)) - \frac{1}{2}Tr(X\Lambda_+(v)\gamma^\alpha\gamma_5\Lambda_+(v)Y\Lambda_+(v)\gamma_\alpha\gamma_5\Lambda_+(v)) \quad \text{(A.1)}$$

where X, Y are 4×4 matrices.

i) Using (A.1), the definition of $\mathcal{B}$ in Eqn. (5.12), and

$$\sum_{B,B^*} \bar{\mathcal{B}}\gamma_5\gamma_\alpha\mathcal{B} = 0$$

it follows that

$$\sum_{B,B^*} Tr(\bar{\mathcal{B}}X)Tr(\mathcal{B}Y) = 2Tr(\Lambda_+(v)X\Lambda_+(v)Y) \quad \text{(A.2)}$$

ii)

$$Tr(\Gamma_\nu\Lambda_+(v)\varphi)\ Tr(\bar{\varphi}\Lambda_+(v)\Gamma_\mu) = \frac{1}{2}\ Tr(\Gamma_\nu\Lambda_+(v)\Gamma_\mu\bar{\varphi}\Lambda_+(v)\varphi)$$

$$-\frac{1}{2}\ Tr(\Gamma_\nu\Lambda_+(v)\gamma_5\gamma^\alpha\Lambda_+(v)\Gamma_\mu\bar{\varphi}\Lambda_+(v)\gamma_5\gamma_\alpha\Lambda_+(v)\varphi) \quad \text{(A.3)}$$

We also have

iii)

$$(1-\gamma_5)\ \not{\phi} = \frac{1}{2}(1-\gamma_5)\gamma_\lambda\ Tr(\gamma^\lambda \frac{1-\gamma_5}{2}\ \not{\phi}) \tag{A.4}$$

In expressions like $Tr(\Gamma_\nu \Lambda_+(v)\Gamma_\mu \phi)$, the surviving contribution is the part of ϕ proportional to $(1-\gamma_5)\gamma_\alpha$. Therefore

$$Tr(\Gamma_\nu \Lambda_+(v)\Gamma_\mu \phi) = \frac{1}{2}Tr(\gamma^\lambda \Gamma_\nu \Lambda_+(v)\Gamma_\mu)\ Tr(\gamma_\lambda \frac{1-\gamma_5}{2}\phi) \tag{A.5}$$

Using

$$(1-\gamma_5)\Lambda_-(v)\gamma^\alpha \Lambda_+(v)(1+\gamma_5) = \frac{1}{2}(1-\gamma_5)(\gamma^\alpha - v^\alpha\ \not{v})(1+\gamma_5)$$

it follows from (A.3) and (A.5) that

$$Tr(\Gamma_\nu \Lambda_+(v)\varphi)\ Tr(\bar{\varphi}\Lambda_+(v)\Gamma_\mu) = \frac{1}{4}\ Tr(\gamma^\lambda \Gamma_\nu \Lambda_+(v)\Gamma_\mu)\ Tr(\gamma_\lambda \frac{1-\gamma_5}{2}\bar{\varphi}\Lambda_+(v)\varphi) +$$

$$+\frac{1}{4}\ Tr(\gamma^\lambda \Gamma_\nu \frac{\gamma_\alpha}{2}\Gamma_\mu)\ Tr(\gamma_\lambda \frac{1-\gamma_5}{2}\bar{\varphi}\Lambda_+(v)\gamma_5\gamma_\alpha \Lambda_+(v)\varphi) \tag{A.6}$$

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FIGURE CAPTIONS

1) The inlusive semileptonic decay of the Λ_b baryon into charm.

2) Parabolas of fixed q_0: each one is labeled by $\mid \vec{W} \mid^2$ and γ (we consider $\vec{v}.\vec{W} = 0$). The shaded region is the important physical one, where the sumrule converges.

3) a) Direct graph. b) z-graph. c) Vacuum polarization.

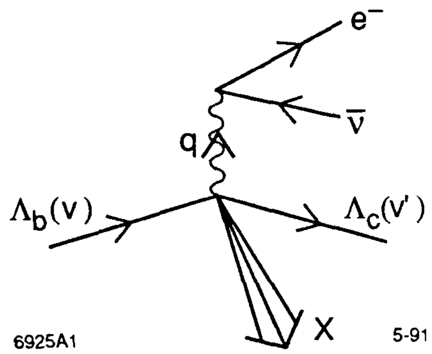

Fig. 1

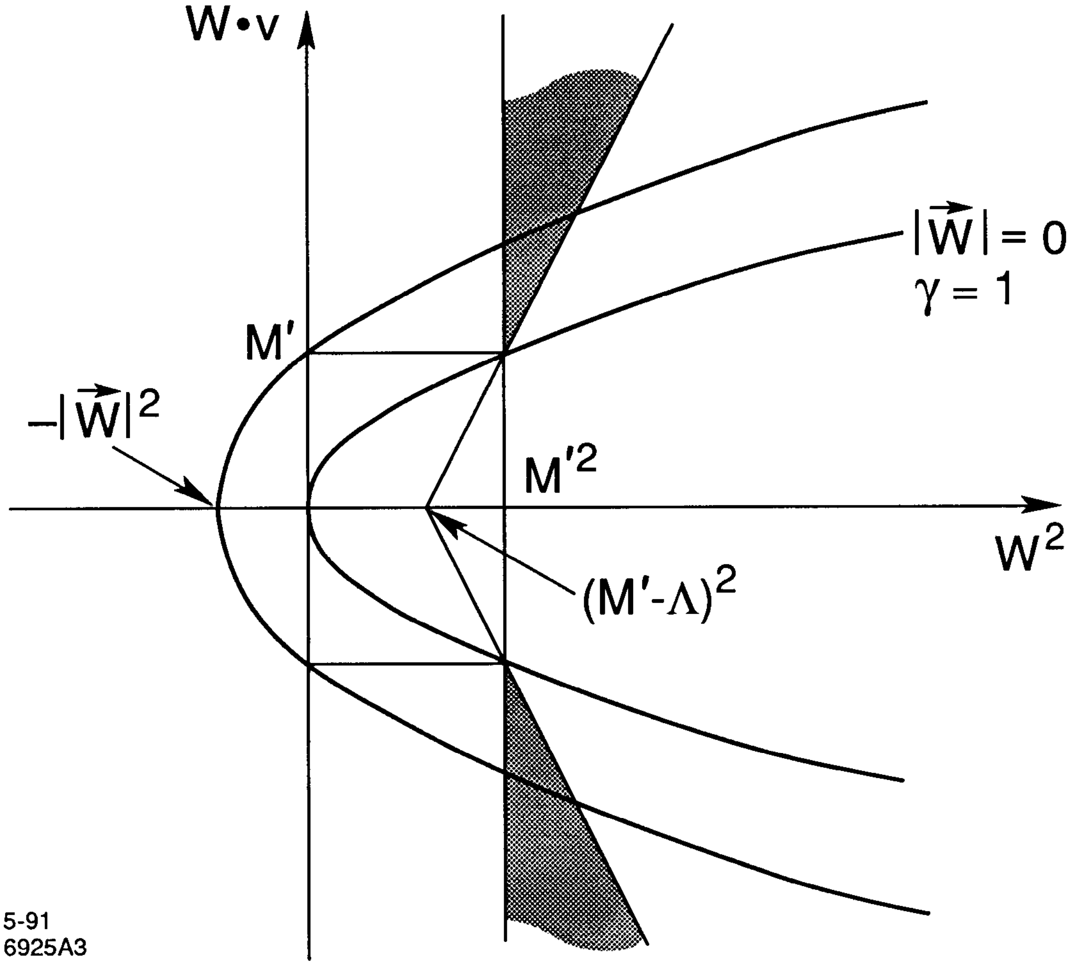

Fig. 2

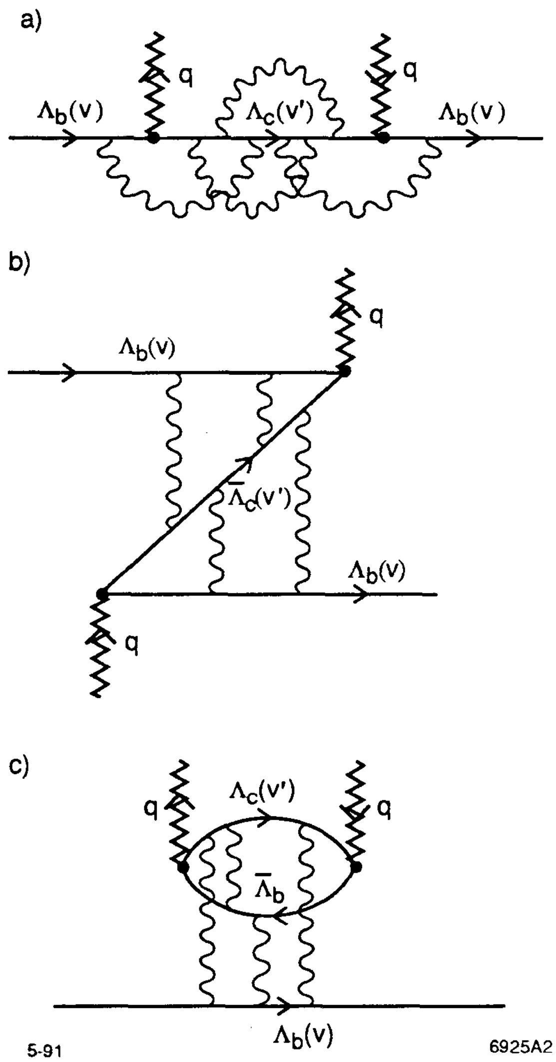

Fig. 3