

Chromatic Correction in the SLC Bunch Length Compressors*

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The SLC Ring to Linac (RTL) transport lines employ intense bending and strong transverse focusing to produce the momentum compaction needed for bunch length compression prior to S-band acceleration. In the presence of the large rf induced energy spread needed for compression the consequent chromatic effects—*viz.* the variation with energy of residual output dispersion and of the RTL transfer matrix, threaten to destroy the small emittances produced by the damping rings. We report on the tuning methods that have been developed and used to implement the sextupole based chromatic correction scheme.

Transverse chromatic effects for a low emittance beam (*i.e.*, in which geometric aberrations are negligible) are generally characterized by either (a) non-linear energy dependence of the transverse phase space position (dispersive chromaticity), or (b) energy dependence of the transverse linear transfer coefficient (β -tron chromaticity). The leading order chromatic contributions to the equivalent emittance—*i.e.*, including the effect of a β -mismatch assuming it filaments [1]

$$\begin{aligned} \bar{\epsilon} &= \frac{1}{2} \text{tr} \langle \hat{\sigma} \rangle_\delta = \frac{1}{2\beta} \text{tr} \left[\begin{pmatrix} 1 & 0 \\ \alpha & \beta \end{pmatrix} \langle \sigma \rangle_\delta \begin{pmatrix} 1 & \alpha \\ 0 & \beta \end{pmatrix} \right] \\ &= \left\{ 1 + \frac{1}{2} \left[\text{tr} (\hat{R}_6 \hat{R}_6^T) - 2 \det \hat{R}_6 \right] \langle \delta^2 \rangle + O(\delta^4) + \dots \right\} \epsilon \\ &+ \frac{\|\eta\|^2}{2\beta} \langle \delta^2 \rangle + \frac{\|\eta_6\|^2}{2\beta} (\langle \delta^4 \rangle - \langle \delta^2 \rangle^2) + O(\delta^6) + \dots \quad (1) \end{aligned}$$

where $\langle \hat{\sigma} \rangle_\delta$ is the energy averaged transverse beam matrix in the normal coordinates associated with the unique canonical parameters β, α having the periodicity of the linac lattice. The 2×2 matrix $\hat{R}_6 = (1/\sqrt{\beta\beta_0}) \times \begin{pmatrix} 1 & 0 \\ \alpha & \beta \end{pmatrix} R_6 \begin{pmatrix} \beta & 0 \\ -\alpha & 1 \end{pmatrix}_0$, where the elements $[R_6]_{ij} = T_{i6j}$ in TRANSPORT notation [2]. The local dispersion $\equiv \eta\delta + \eta_6\delta^2 + \dots$, $\|\eta\|^2 = \eta^2 + (\beta\eta' + \alpha\eta)^2$, and $\|\eta_6\|^2 = \eta_6^2 + (\beta\eta_6' + \alpha\eta_6)^2$. Since dispersion is negligible at the beginning of the RTL, $\eta_i = R_{i6}$ and $\eta_{6i} = T_{i66}$. $\bar{\epsilon}$ is a superposition of effects that are either simply additive (dispersion and its chromaticity), or multiplicative (β -tron chromaticity) in the initial emittance. The prompt (prior to filamentation) emittance growth and β -mismatch have a more complicated dependence on relative phases via interference terms, and are not simply additive-multiplicative (Fig. 1). The above expression for $\bar{\epsilon}$ fully incorporates the symplectic constraints to the order given and includes contributions that are third order in the transfer map even though third order coefficients do not explicitly appear. A limit of 12.5% emittance growth for an approximately gaussian energy distribution with $\langle \delta^2 \rangle^{\frac{1}{2}} \cong 0.01$, $\epsilon = 6800 \mu\text{m}\text{-}\mu\text{r}$,

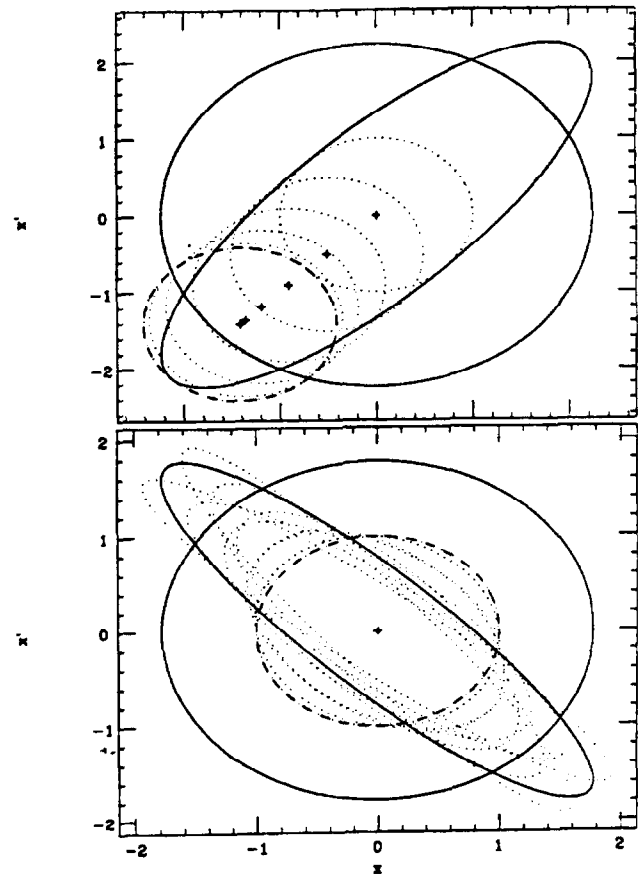


FIG. 1 The matched monochromatic phase space (dashed circle) is smeared by the energy distribution (dotted circles—each corresponding to an energy deviation between $\pm 1\sigma$) into the effective phase space (solid ellipse), which will filament in the linac into the solid circle having area $\pi = \bar{\epsilon}$. (top) dispersive chromaticity, (bottom) β -tron chromaticity.

and $\beta = 3\text{ m}$ entails the tolerances $\|\eta_6\| \lesssim 0.5\text{ m}$, and $\frac{1}{2} \text{tr}(\hat{R}_6 \hat{R}_6^T) - \det \hat{R}_6 \lesssim 1250$. ($\|\eta\| \lesssim 7\text{ mm}$ [3].)

Chromatic Correction Design

If a lattice has at least a two cell structure, $R^{\text{cell}} \neq \pm I$, the net transfer matrix $R = \pm I$, and the β -tron beam envelope has the periodicity of the lattice, then $\hat{R}_{611} = \hat{R}_{622} = 0$ and $\hat{R}_{621} = -\hat{R}_{612}$. Inclusion and setting of one sextupole family per plane is then necessary and sufficient for obtaining $\hat{R}_{6ij} = 0$ [4, 2], thereby correcting all trajectories and thus the β -tron chromaticity for an arbitrary beam. The trajectory correction is necessary to correct the η_6 associated with an arbitrary incoming dispersion function η . If bending is present within the lattice and the dispersion function has the periodicity of the lattice, then at least two cells with $R^{\text{cell}} \neq I$ and net transfer matrix $R = +I$ is

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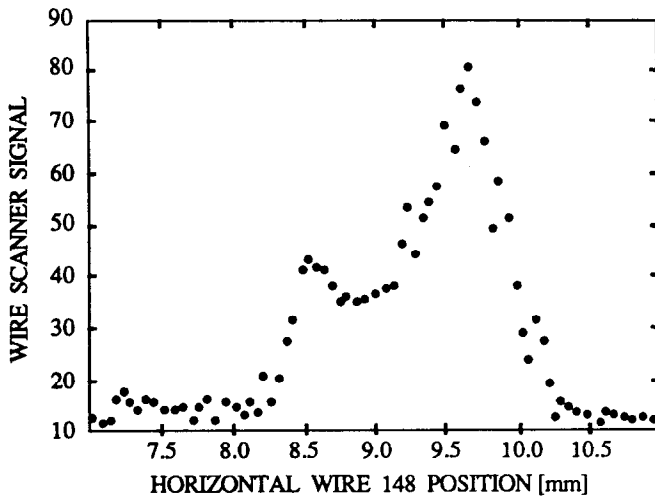


FIG. 2 Horizontal wire beam profile with δ^2 asymmetry.

sufficient for $\eta_{6i} = 0$, independent of sextupole strengths. If sextupoles are present the same condition that $R^{\text{cell}} \neq I$ and $R = +I$, with the further requirement that the number of cells $\neq 3$, is also necessary to remove η_6 contributions originating in T_{ijk} terms for an arbitrary initial η .

The RTLs [5] consist of two two-cell $R = -I$ transformers ('Stage 1'), nested within the first of two DFDDF $R = -I$ cells ('Stage 2'). Stage 1 cells have two identical dipoles and a SF, SD sextupole pair nominally set so that $R_{6ij} = 0$ across each $R = -I$ segment. Since the η -function has the lattice symmetry and the total $R = +I$ the net $\eta_{6i} = 0$. A Stage 2 cell has three non-identical horizontal and two (for e^+) or one (for e^-) vertical dipoles, and a SD-SF-SD-SF pattern of independently powered and rolled sextupoles. Due to the absence of symmetry within the $R = -I$ cell this is not enough degrees of freedom to enforce exact $R_{6ij} = 0$ for both planes, but is sufficient to compensate for the η_6 engendered by the asymmetric incoming η ($\eta_i^{\text{in}} = 0$), and reduce the remaining components to tolerable levels.

Lattice Errors

Anomalous sextupole fields $\Delta(1/f_2)$ will contribute $\|\eta_6\| = \frac{1}{2}\sqrt{\beta\beta_0}\eta^2|\Delta(1/f_2)|$ and $\Delta R_6 = -R \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} [\eta\Delta(1/f_2)]R$. Quadrupole errors (including orbit errors in sextupoles) affect chromaticity both directly and by their effect on the dispersion—in the RTLs the large dispersion makes the latter the dominant effect. A shift $\Delta\eta$ produces

$$\Delta\eta_{6i} = (R_{6ij} + 2T_{ijk}\eta_k)\Delta\eta_j + T_{ijk}\Delta\eta_j\Delta\eta_k \quad (2)$$

$$\Delta R_{6ij} = 2T_{ilk}(\Delta\eta_k)R_{lj} \quad (3)$$

(Dipoles do likewise.) From the known anomalous η at the entrance to the linac, anomalous η_6 several times tolerance may be expected.

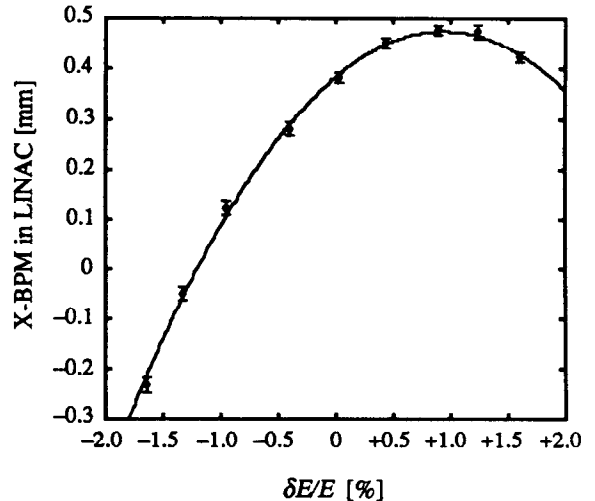


FIG. 3 Linac horizontal BPM data fit to a quadratic.

Tuning Scheme

Controls for on-line tuning have been developed using the Stage 1 sextupoles. Since $\text{tr}(\hat{R}_6\hat{R}_6^T) - 2\det\hat{R}_6 = (\hat{R}_{611} - \hat{R}_{622})^2 + (\hat{R}_{621} + \hat{R}_{612})^2$ two controls, *viz.*, $\hat{R}_{611} - \hat{R}_{622}$ and $\hat{R}_{621} + \hat{R}_{612}$ —corresponding to energy dependent terms in β and α , are generically necessary and sufficient to compensate the beam envelope β -tron chromaticity. Equal but opposite polarity excitations of the SFs in cells 1 and 3, and independently in cells 2 and 4, will generate η_6 of any phase and a range of amplitudes while keeping R_6 constant. Equal excitations of the same pairs, and also of the corresponding SDs, will on the other hand keep η_6 constant while varying the four combinations of the R_6 elements that enter into the x and y emittances.

The latter, β -tron chromaticity tuning, can best be done by shifting the phase and amplitude of the bunch compressor rf to minimize the energy spread in the beam but vary the energy centroid. Beam profile measurements with an array of wire scanners can then be used to find the settings of the four 'knobs' that minimize the energy centroid dependence of the shape of phase space.

Measurement Technique and Tuning in Practice

Although commonplace for electrons, profile measurements of the e^+ beam entering the linac have been difficult in the past because inserted phosphorescent screens disturb the e^- beam and stop e^+ production. Thin screens were used with some success, but the first (appalling) clear look at the e^+ beam profile was obtained after four wire scanners [6] were recently installed (Fig. 2). The fact that the skew in the distribution vanished when the energy spread was reduced indicated a chromatic effect—in particular, an even function of the energy deviation in the transfer map since the energy distribution is symmetric.

Measurements of the dispersive chromaticity are made by varying the RTL beam energy in steps comparable to the nominal RTL energy spread and correlating transverse

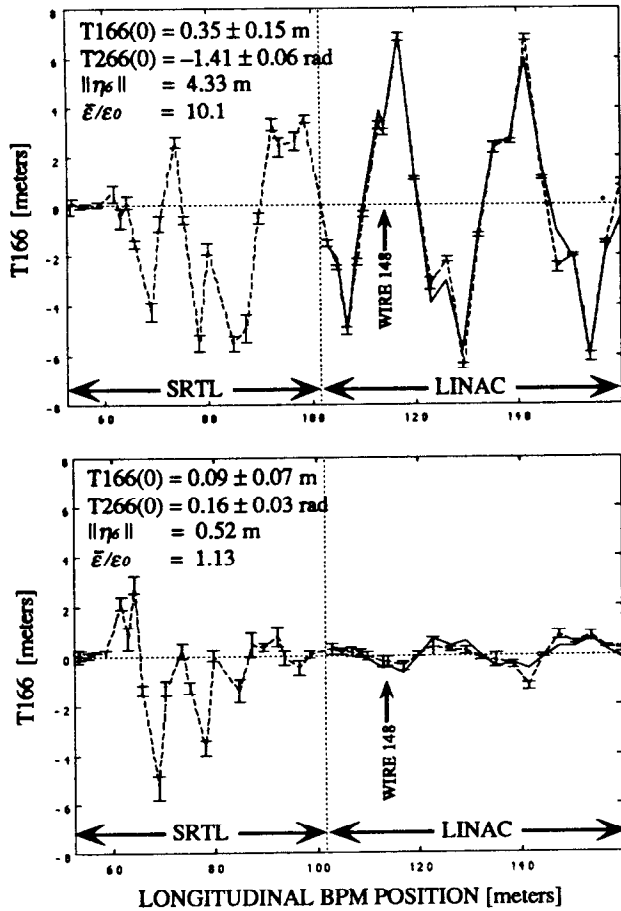


FIG. 4 Measured T_{166} in the e^+ (South) RTL and beginning of the linac. The dashed line connects the measured points—the solid line is the propagated initial conditions fit at the linac input. (top) before and (bottom) after, sextupole tuning.

Beam Position Monitor (BPM) readings with these energy variations. A parabola is fit to data at the a^{th} BPM and the linear and quadratic fit coefficients are identified as the first and second order dispersion at this BPM

$$x^a(\delta) = x^a(0) + R_{16}^a \delta + T_{166}^a \delta^2 \quad (4)$$

(The offset is discarded.) The energy variation δ is accomplished by rf amplitude and phase settings of the RTL bunch compressor cavity. The phase is chosen to minimize beam energy spread (the crest the rf wave), and the amplitude is varied through ~ 9 points from $+1.5$ to -1.5 times the RTL beam rms energy spread ($\cong 1\%$). Fig. 3 shows a representative fit to horizontal BPM data for the e^+ beam entering the linac.

Such fits are performed for the ~ 25 RTL BPMs and the first ~ 20 BPMs in the linac in x and y . The horizontal linear dispersion within the RTL itself is so large that the RTL BPMs enter a range $x \gtrsim 8$ mm of non-linear response—so their usefulness is limited to measuring the large RTL horizontal linear dispersion. However the linac BPMs are well within their linear range and produce accurate higher order dispersion determinations. Dispersion (both first and second order) propagates as a free β -tron oscillation down the linac, the initial conditions of which are

found by fitting the ~ 20 linac dispersion measurements (for both x and y) to a free β -tron oscillation (Fig. 4):

$$R_{16}^a = R_{1i} R_{i6}^0 \quad \text{and} \quad T_{166}^a = R_{1i} T_{i66}^0 \quad (5)$$

Here R_{16}^a and T_{166}^a are the previous parabolic fit results at BPM a , R_{1i} are the modeled transfer matrix elements from the beginning of the linac to the a^{th} BPM, and the R_{i6}^0 and T_{i66}^0 coefficients are the fitted initial conditions of the oscillation. This is also done for the vertical plane.

The sextupole ‘knobs’ are then set according to their calculated calibrations to cancel the measured anomalous η_{6i} . Fig. 4 bottom shows a subsequent measurement verifying that the oscillation amplitude has been essentially brought within tolerance—beam profile wire observations made at the same time confirm the absence of the asymmetric tail in Fig. 2.

Conclusion

We have succeeded in using the lattice symmetries and the predicted sensitivities to sextupole strength changes to tune the deleterious chromatic aberrations to acceptably small levels in the SLC Ring to Linac transport lines.

1. W. Spence, unpublished.
2. K. L. Brown and R. Servranckx, *Physics of Particle Accelerators: BNL/SUNY Summer School*, American Institute of Physics, 1984. $i, j, \dots = 1, \dots, 4$ label components in transverse phase space; repeated indices are summed. Juxtaposed transfer coefficients, taken from right to left, correspond to successive segments of beamline.
3. J. T. Seeman, *et al.*, *XIV Intl. Conf. on High Energy Accelerators*, Tsukuba, Japan, 1988.
4. K. L. Brown, *IEEE Trans. Nucl. Sci.* NS-26 (1979) 3490; D. C. Carey, *Nucl. Inst. Meth.* 189 (1981) 365; S. Kheifets, *et al.*, *2nd ICFA Advanced Beam Dynamics Workshop*, Lugano, Switzerland, 1988.
5. T. H. Fieguth and J. J. Murray, *XIIth Intl. Conf. on High Energy Accelerators*, Fermilab, 1983, p. 401.
6. M. C. Ross, *et al.*, “Wire Scanners for Beam Size and Emittance Measurement at the SLC”, these proceedings.