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On the Suppression of the Collective Beam-Beam Instability of Long Bunches*

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ABSTRACT

In storage rings with very low frequency of synchrotron oscillations coherent beam-beam instabilities can be suppressed due to the "phase averaging" effect, which recently was described by S. Krishnagopal and R. Siemann for the incoherent beam-beam instability. In this paper using the Vlasov's equation we calculate the form factors, which renormalize beam-beam parameters of coherent beam-beam oscillations. This results in the variations of stability criteria as well as widths of stopbands of unstable modes for bunches with lengths comparable with β-function at the interaction point.

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1. Introduction

Recently R. Siemann and S. Krishnagopal[1] showed that the incoherent beam-beam interaction in circular colliders can be strongly suppressed by the spreading of the interaction region (IR) due to the finite length of the colliding bunches. As after the crossing the IR a particle gets the phase advance of betatron oscillations of the order of:

$$\Delta\mu \simeq 2\,\mathrm{arctg}(\sigma_c/\beta^*) \quad ,$$

the Floquet modulation of its betatron oscillations can strongly affect the powers of beam-beam resonances in the region, where the bunch length σ_c has close value to β^*-the value of β- function at the interaction point (IP). In fact, the effect of the suppression of beam-beam instability for incoherent oscillations is caused by particles crossing the IR at the angle, when the beam-beam kick is canceled. Same arguments indicate that the particles executing the oscillations with large amplitude of synchrotron oscillations $|s - ct| \gg \sigma_c$ will feel the nominal beam-beam kick and thus will be perturbed by the beam-beam resonances of the nominal strength.

Even the simplified consideration based on the calculations within the framework of the so-called rigid bunch model[2] indicates that the same effect can suppress the coherent beam-beam interaction of colliding bunches, provided their lengths differ not very much from β^*. Except for the special conditions, when coherent oscillations with large longitudinal amplitudes are artificially excited, the collective motion of the colliding bunches can be considered as very synchronous. Therefore, one may expect more clear indication of the phase averaging effect in the suppression of coherent beam-beam resonances.

In this paper we present more straightforward calculations confirming this effect in the special case when the frequencies of synchrotron oscillations of the particles in the colliding bunches are considerably low, and thus one can neglect the excitation of coherent synchrobetatron beam-beam oscillations.

2. General Equations

The most general description of coherent beam-beam oscillations can be done using Vlasov's equations. If the action-phase variables $\vec{I}, \vec{\psi}$ are used to describe unperturbed oscillations of individual particles, these equations read:

$$\left(\frac{\partial}{\partial\theta_s} + \vec{\nu}\frac{\partial}{\partial\vec{\psi}}\right) f^{(1)} + \frac{\partial L_{1,2}}{\partial\vec{\psi}} \frac{\partial f_0^{(1)}}{\partial\vec{I}} = 0 \ , \tag{1}$$

$$\left(\frac{\partial}{\partial\theta_s} + \vec{\nu}\frac{\partial}{\partial\vec{\psi}}\right) f^{(2)} + \frac{\partial L_{2,1}}{\partial\vec{\psi}} \frac{\partial f_0^{(2)}}{\partial\vec{I}} = 0 \ . \tag{2}$$

Here $\theta_s = \omega_s t$ and ω_s is the revolution frequency of the synchronous particles of bunches. The Lagrangians $L_{1,2}$ and respectively $L_{2,1}$, describe the interaction of a particle from one bunch with the fields of the counter-moving bunch. For the relativistic case ($\gamma = E/Mc^2 \gg 1$) with identical bunches in a e^+, e^--collider ($e_1 e_2 = -e^2$), one can write $L_{a,b}$ in the form:

$$L_{1,2} = \frac{2Ne^2}{c} U_{1,2} \ ,$$

$$U_{1,2} = \int \frac{d\mathbf{k}}{\pi k^2} \exp(i\mathbf{k}\mathbf{r}_\perp^{(1)}) \rho^{(2)}(\mathbf{k}, \theta) \ , \tag{3}$$

$$\rho^{(2)}(\mathbf{k}, \theta) = \int d^2\mathbf{r}_\perp d^3\mathbf{p} \exp(-i\mathbf{k}\mathbf{r}_\perp) f^{(2)}(\mathbf{r}_\perp, \theta + \theta_s, \mathbf{p}, \theta_s) \ . \tag{4}$$

Here E is the energy of a particle, e is its charge and $k^2 = k_x^2 + k_z^2$.

In this paper we assume no dispersion at the (IP). Then the unperturbed betatron oscillations of the particles can be described by the following formulae:

$$(x,z) = \sqrt{J\beta(\theta)}\cos(\psi + \chi(\theta)) \,|_{(x,z)}, \quad p_{x,z} = \frac{p_s}{R_0}\frac{d(x,z)}{d\theta} \ ,$$

$$\frac{d\psi_\alpha}{d\theta} \equiv \psi'_\alpha = \nu_{x,z}(J_x, J_z), \quad \chi'_\alpha + \nu_\alpha = \frac{R_0}{\beta_\alpha}, \quad \alpha = x, z \ , \tag{5}$$

$$I_{x,z} = p_s J_{x,z}/2, \quad \theta_{(1,2)} = \pm\theta_s + \varphi_{1,2}, \quad \theta_s = \omega_s t \ ,$$

$$\varphi = \varphi_c \cos(\psi_c), \quad \psi'_c = \nu_c \ . \tag{6}$$

Here the subscript s marks the values describing the synchronous particle; $2\pi R_0$ is the perimeter of its closed orbit; the subscript c marks the variables of synchrotron oscillations. More generally, Eqs. (5) and (6) generate the canonical transformation from variables $(\mathbf{p}, \mathbf{r})$ to the action-phase variables $(\vec{I}, \vec{\psi})$ of the unperturbed oscillations.

Below we shall discuss the stability of small coherent beam-beam oscillations. This problem can be treated using equations which are obtained by the linearization of Eqs. (1) and (2) for small deviation from the unperturbed distribution functions. For the sake of simplicity we assume that such distributions are uniform in betatron phases:

$$f^{(1,2)} = f_0^{(1,2)}(\vec{I}) \ ,$$

while coherent oscillations are described by the nonuniform and nonstationary addition to f_0:

$$f^{(1,2)}(\vec{I},\vec{\psi},\theta) = f_0^{(1,2)}(\vec{I}) + \tilde{f}^{(1,2)}(\vec{I},\vec{\psi},\theta) \ , \tag{7}$$

$$f_0^{(1,2)} \gg \tilde{f}^{(1,2)}.$$

In fact, for colliding bunches such a structure of the distribution function is not so obvious and generally assumes that the working point of the ring is placed outside the stopbands of the incoherent beam-beam instability.

If the contribution from the stationary part of the perturbation ($L^{(0)} = L[f_0]$) is included in the tunes $\vec{\nu}$ and the β-functions of the ring, the linearized system of equations for $\tilde{f}^{(1,2)}$ reads:

$$\left(\frac{\partial}{\partial\theta_s} + \vec{\nu}(\vec{I})\frac{\partial}{\partial\vec{\psi}}\right)\tilde{f}^{(1)} + \frac{\partial\tilde{L}_{1,2}}{\partial\vec{\psi}}\frac{\partial f_0^{(1)}}{\partial\vec{I}} = 0 \ , \tag{8}$$

$$\left(\frac{\partial}{\partial\theta_s} + \vec{\nu}(\vec{I})\frac{\partial}{\partial\vec{\psi}}\right)\tilde{f}^{(2)} + \frac{\partial\tilde{L}_{2,1}}{\partial\vec{\psi}}\frac{\partial f_0^{(2)}}{\partial\vec{I}} = 0 \ , \tag{9}$$

Using here Eq. (3) and the Fourier expansions for solutions:

$$\tilde{f}^{(1,2)}(\vec{I},\vec{\psi},\theta_s) = \sum_{\vec{m}} f_{\vec{m}}^{(1,2)}(\vec{I},\theta_s)\exp(i\vec{m}\vec{\psi}) \ , \tag{10}$$

one can rewrite Eqs. (8) and (9) in the form:

$$\left(\frac{\partial}{\partial\theta_s} + i\vec{m}\vec{\nu}(\vec{I})\right) f_{\vec{m}}^{(1)} = -i\vec{m}\frac{\partial f_0^{(1)}}{\partial\vec{I}}\frac{Ne^2}{c}[\tilde{U}_{1,2}(\theta)]_{\vec{m}} \ , \tag{11}$$

$$\left(\frac{\partial}{\partial\theta_s} + i\vec{m}\vec{\nu}(\vec{I})\right) f_{\vec{m}}^{(2)} = -i\vec{m}\frac{\partial f_0^{(2)}}{\partial\vec{I}}\frac{Ne^2}{c}[\tilde{U}_{2,1}(\theta)]_{\vec{m}} \ , \tag{12}$$

Since the phase of synchrotron oscillation ψ_c is a cyclic variable, Eqs. (10–12) generally contain harmonics of the distribution functions in synchrotron phases

f_{m_x,m_z,m_c} describing the coherent synchrotron (or, respectively, synchrobetatron) oscillations of the bunches. The possibility to select such modes as the normal modes of coherent motion and thus to classify solutions using definite multipolarity of synchrotron oscillations m_c is generally determined by the ratio between the rate of time variations of amplitudes of coherent oscillations, which we shall denote here as $Im(\nu)$, and ν_c. For the beam-beam interaction this rate is specified by the so-called beam-beam parameter ξ. For bunches with Gaussian distributions in transverse coordinates:

$$\rho_0(x,z) = \frac{1}{2\pi\sigma_x\sigma_z} \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{z^2}{2\sigma_z^2}\right)$$

these parameters for respectively vertical and radial oscillations are:

$$\xi_z = \frac{N r_0 \beta_z^*}{2\pi\gamma\sigma_z(\sigma_x+\sigma_z)} , \tag{13}$$

$$\xi_x = \frac{N r_0 \beta_x^*}{2\pi\gamma\sigma_x(\sigma_x+\sigma_z)}, \quad r_0 = e^2/Mc^2 . \tag{14}$$

In this paper we shall assume that parameters both of the bunches and of the ring are in the region:

$$Im(\nu) \gg \nu_c , \tag{15}$$

and therefore the synchrotron oscillations of particles can cause the adiabatically slow variations of the parameters of coherent oscillations, but do not classify the collective modes of the beam. For these fast coherent beam-beam oscillations the coupling of synchrotron modes is so strong that it becomes more reliable to describe the coherent oscillations assuming the variables Δp and φ as the integrals of motion. In this case the distribution functions can be presented by the expansions:

$$\tilde{f}^{(1,2)} = \sum_{\vec{m}} f_{\vec{m}}^{(1,2)}(\vec{I}_{\perp}, \Delta p, \varphi, \theta_s) \exp(i\vec{m}_{\perp}\vec{\psi}_{\perp}) \ , \tag{16}$$

describing the bunches with a rigid longitudinal distribution.

Below we shall make the calculations for coherent betatron oscillations. Neglecting for the sake of simplicity the frequency spread of the bunches due to momentum spread and assuming as the unperturbed the distribution functions:

$$f_0^{(1,2)} = F_0(\vec{J}_{\perp})\delta(\Delta p)\rho_0(\varphi) \ , \tag{17}$$

let us write solutions of Eqs. (11) and (12) in the form:

$$f_m^{(1,2)} = \rho_0(\theta \mp \theta_s)\delta(\Delta p)\exp(i\vec{m}_{\perp}\vec{\nu}_{\perp}\theta_s)A_m^{(1,2)}(\vec{I}_{\perp}, \varphi, \theta_s) \ . \tag{18}$$

The substitution of these expressions into Eqs. (11) and (12) as well as making use of the expansions:

$$\exp(ik\sqrt{J\beta}\cos\phi) = \sum_{m=-\infty}^{\infty} J_m(k\sqrt{J\beta})\exp(im\phi) \ ,$$

$$\phi = \psi + \chi(\theta) \ ,$$

where $J_m(x)$ is the Bessel function, yields the system of equations for amplitudes

$A_m^{(1,2)}$:

$$\frac{\partial A_m^{(1)}}{\partial \theta_s} = -i\vec{m}_\perp \frac{\partial F_0}{\partial \vec{J}_\perp} \left(\frac{4Ne^2}{pc}\right) \rho_0(\varphi)\rho_0(2\theta_s+\varphi) \int \frac{d\mathbf{k}}{\pi k^2} Q_m(\vec{J},\theta_s)\times$$

$$\times \exp(i\vec{m}_\perp \vec{\phi}_\perp(\theta_s)) \sum_{\vec{m}'_\perp} \exp(i\vec{m}'_\perp \vec{\phi}_\perp(\theta_s)) \int d\Gamma'_\perp d\varphi Q^*_{m'}(\vec{J}',\theta_s) A^{(2)}_{m'}(\vec{J}',\varphi',\theta_s)\ , \quad (19)$$

$$\frac{\partial A_m^{(2)}}{\partial \theta_s} = -i\vec{m}_\perp \frac{\partial F_0}{\partial \vec{J}_\perp} \left(\frac{4Ne^2}{pc}\right) \rho_0(\varphi)\rho_0(2\theta_s+\varphi) \int \frac{d\mathbf{k}}{\pi k^2} Q_m(\vec{J},\theta_s)\times$$

$$\times \exp(i\vec{m}_\perp \vec{\phi}_\perp(\theta_s)) \sum_{\vec{m}'_\perp} \exp(i\vec{m}'_\perp \vec{\phi}_\perp(\theta_s)) \int d\Gamma'_\perp d\varphi Q^*_{m'}(\vec{J}',\theta_s) A^{(1)}_{m'}(\vec{J}',\varphi',\theta_s)\ , \quad (20)$$

where

$$Q_m(\vec{J},\theta_s) = J_{m_x}(k_x\sqrt{J_x\beta_x(\theta_s)}) J_{m_z}(k_z\sqrt{J_z\beta_z(\theta_s)})\ . \quad (21)$$

Finally, introducing the momenta:

$$X_m^{(a)}(\vec{J},\theta_s) = \int_0^{2\pi} d\varphi A_m^{(a)}(\vec{J},\varphi,\theta_s), \quad a = 1,2 \quad (22)$$

and assuming the Gaussian linear density distributions in bunches:

$$\rho_0(\varphi) = \frac{1}{\sqrt{2\pi}\sigma_\phi} \exp\left(\frac{-\varphi^2}{2\sigma_\phi^2}\right), \quad \sigma_c = R_0\sigma_\phi\ , \quad (23)$$

one can get for $X_m^{(1,2)}$:

$$\frac{\partial X_m^{(1)}}{\partial \theta_s} = (1/\sqrt{\pi}\sigma_\phi) \exp\left(-\frac{\theta_s^2}{\sigma_\phi^2} + i\vec{m}_\perp \vec{\phi}_\perp(\theta_s)\right) \sum_{\vec{m}'_\perp} \exp(i\vec{m}'_\perp \vec{\phi}_\perp(\theta_s))\times$$

$$\times \int d\Gamma'_\perp K_{m,m'}(\vec{J},\vec{J}',\theta_s) X^{(2)}_{m'}(\vec{J}') \ , \tag{24}$$

$$\frac{\partial X^{(2)}_m}{\partial \theta_s} = (1/\sqrt{\pi}\sigma_\phi) \exp\left(-\frac{\theta_s^2}{\sigma_\phi^2} + i\vec{m}_\perp \vec{\phi}_\perp(\theta_s)\right) \sum_{\vec{m}'_\perp} \exp(i\vec{m}'_\perp \vec{\phi}_\perp(\theta_s)) \times$$

$$\times \int d\Gamma'_\perp K_{m,m'}(\vec{J},\vec{J}',\theta_s) X^{(1)}_{m'}(\vec{J}') \ , \tag{25}$$

where

$$K_{m,m'}(J,J',\theta_s) = -i\vec{m}_\perp \frac{\partial F_0}{\partial \vec{J}_\perp} \left(\frac{2Ne^2}{pc}\right) \int Q_m(\vec{J},\theta_s) Q^*_m(\vec{J}',\theta_s) \frac{d^2k}{\pi k^2} \ . \tag{26}$$

According to these equations, the perturbation of colliding bunches caused by the counter-moving beam is described by a sequence of rather short periodic kicks, which for two bunches colliding at one (IP) are spaced by apart the revolution period T_s. Between collisions both bunches execute free oscillations $f_{\vec{m}}(\theta_s) \sim \exp(-i\vec{m}\vec{\nu}\theta_s)$, but after crossing through the interaction region the amplitudes $X^{(1,2)}_m$get the variations:

$$X^{(1,2)}_m(0+) = X^{(1,2)}_m(0-) + \delta X^{(1,2)}_m,$$

where $X^{(1,2)}_m(0\mp)$ are the amplitudes before and after collision. The values $\delta X^{(1,2)}_m$ determine the corresponding variations in the harmonics $f^{(1,2)}_m$. Then the obvious requirement for eigensolutions:

$$f_{\vec{m}}(0+) = \lambda e^{2\pi i \vec{m}\vec{\nu}} f_{\vec{m}}(0-) \tag{27}$$

yields the system of homogeneous integral equations, which determine both the eigenfunctions $f^{(1,2)}_m$ and the eigenvalues λ of coherent modes.

3. The Factorization

The calculation of the values $\delta X_m^{(1,2)}$ can be significantly simplified if a particular geometry of colliding bunches is assumed. Therefore below we shall consider separately two important cases: when colliding bunches are round ($\sigma_x = \sigma_z; \beta_x^* = \beta_z^*$), and when they are flat ($\sigma_x \gg \sigma_z$).

Since we have assumed that the lengths of bunches σ_c and β^* can have the comparable values, in these calculations we have to take into account that in the close vicinity of the interaction point one has:

$$\beta_\alpha(s) = \beta_\alpha^* + \frac{s^2}{\beta_\alpha^*}, \quad s = R_0\theta \ , \tag{28}$$

and thus, the corresponding phase advance of betatron oscillations can be large enough:

$$\phi_\alpha(s) = \mathrm{arctg}(\frac{s}{\beta_\alpha^*}) \ . \tag{29}$$

Let us first discuss the case, when the bunches are round ($\sigma_x = \sigma_z; \beta_z^* = \beta_x^*$). Taking into account that the nontrivial behaviour of the amplitudes $X_m^{(1,2)}$ is associated with the tuning of the working point towards nonlinear resonances $\vec{m}\vec{\nu} = n$, and assuming $\xi \ll 1$, one can rewrite, for instance, Eq. (24) in the form:

$$\frac{\partial X_m^{(1)}}{\partial \theta_s} = (1/\sqrt{\pi}\sigma_\phi) \exp\left(-\frac{\theta_s^2}{\sigma_\phi^2} + i2\vec{m}_\perp \vec{\phi}_\perp(\theta_s)\right) \int d\Gamma'_\perp K_{m,m}(\vec{J}, \vec{J}', \theta_s) X_m^{(2)}(\vec{J}') \ . \tag{30}$$

If $\beta_x(s) = \beta_z(s) = \beta(s)$, using Eqs. (21, 26) and the substitutions $k_x \to k_x\sqrt{\beta(s)}$ and $k_z \to k_z\sqrt{\beta(s)}$ one can easily verify that the kernel $K_{m,m}$ does not depend on

θ_s. Then, the integration of Eq. (30) over θ_s yields:

$$\delta X_m^{(1)} = Y_m(\sigma_c/\beta^*) \int d\Gamma'_{\perp} K_{m,m}(\vec{J}, \vec{J'}) X_m^{(2)}(\vec{J'}) \ , \tag{31}$$

where

$$Y_m(\zeta) = \int_{-\infty}^{\infty} \frac{du}{\sqrt{\pi}} \exp[-u^2 + 2i(m_x + m_z)\,\mathrm{arctg}(u/\zeta)], \quad \zeta = \beta^*/\sigma_c \ . \tag{32}$$

Except for the factor $Y_m(\zeta)$, Eq. (31) coincides with that which had been previously obtained, for instance in the papers[3–5] to describe coherent oscillations of colliding bunches of zero length. This and the complementary equation for $\delta X_m^{(2)}$ yield the system of integral equations, which enable one to calculate the stability criteria, coherent tune shifts, widths of stopbands and other important parameters of oscillations. For bunches of finite length the dependencies of these parameters on σ_c is described by the function $Y_m(\zeta)$. Since this dependence can be factored out of the kernel of the integral equation (31), the influence of the expansion of the interaction region as well as Floquet modulation is equivalent to the renormalization of the beam-beam parameter:

$$\xi_{eff} = Y_m(\zeta)\xi \ ,$$

and provided the factor $Y_m(\zeta)$ is small, describes the suppression of coherent beam-beam resonances. As one more interesting feature of this modification of ξ we have to mention the following. Since the position of the stopbands relative to particular resonance $\nu = n/m$ is determined by the sign of the production[4]:

$$\xi_{eff}\Delta = \xi Y(\zeta)\Delta \le 0 \quad , \Delta = \nu - n/m \quad ,$$

in the regions of ζ, where $Y(\zeta) < 0$, the stopbands occur above the resonant values of ν.

The asymptotes of $Y_m(\zeta)$ can be easily calculated for extremely short or extremely long bunches. For short bunches $\zeta \gg 1$ one can use the expansion:

$$\mathrm{arctg}(u) \simeq u, \quad u \ll 1 \ ,$$

when the calculation of the integral in Eq. (32) yields the result of the paper[1]:

$$Y_m(\zeta) = \exp(-q^2/\zeta^2) \simeq 1 - (q/\zeta)^2, \quad q = m_x + m_z \ , \tag{33}$$

It indicates weak suppression of coherent beam-beam resonances when the interaction region is only slightly extended due to finite bunch length.

The inverse asymptote of $Y_m(\zeta)$ can be obtained by rewriting this function in the form:

$$Y_m(\zeta) = \int\limits_{-\infty}^{\infty} \frac{du}{\sqrt{\pi}} e^{-u^2} T_q \left(\frac{\zeta}{\sqrt{\zeta^2 + u^2}} \right) \ , \tag{34}$$

where $T_q(x)$ are the Chebyshev polynomials. Then using the relationship $T_q(0) = (-1)^q$, one can easily find that for round, long bunches

$$Y_q(\zeta) \to (-1)^q, \quad \zeta \ll 1 \ , \tag{35}$$

the infinite extension of the interaction region of the bunches does not suppress the coherent beam-beam instability[2].

The behaviour of $Y_m(\zeta)$ for a wider region can not be predicted analytically. The inspection of the results of numerical calculations of the integral in Eq. (34) (see in Fig. 1) indicates that coherent resonances, especially of higher orders, can be significantly suppressed for round bunches with lengths $\sigma_c \sim \beta^*$. On the other hand, as the condition (15) must be valid, the value of the suppression factor must be limited from the below. Using, for instance, resuts of the paper[4] for the estimation of the value of the maximum increment for the mode with the given combination (m_x, m_z) one can rewrite (15) in the form:

$$|Y(\zeta)| \gg \frac{\nu_c}{4\xi}(m_x^2 + m_z^2).$$

This condition valids as better as smaller is the frequency of synchrotron oscillations $\xi \gg \nu_c$, but still can be violated for oscillations with multipole numbers

$$|m_{x,z}| \sim 2\sqrt{\xi/\nu_c} \quad .$$

Let us now consider in more details the case, when colliding bunches are flat $(\sigma_x \gg \sigma_z)$. If this also assumes $\epsilon_x \gg \epsilon_z$ and therefore $\beta_x^* \simeq \beta_z^*$, the calculations can be simplified since one can neglect the radial modulation of the beam-beam force. In this case one has $\xi_z \gg \xi_x$, and thus can expect that only resonances of vertical oscillations $m_z \nu_z = n$ are important. Then the kernel $K_{m,m}$ in Eqs. (24, 25) can be written in the form:

$$K_{m,m'}(J, J', \theta_s) = -i\vec{m}_z \frac{\partial F_0}{\partial J_z} \left(\frac{2Ne^2}{pc}\right) \int Q_m(\vec{J}, \theta_s) Q_m^*(\vec{J'}, \theta_s) \frac{dk_z}{\pi k_z^2} \,. \qquad (36)$$

After the obvious substitutions the equation for variations of amplitudes $\delta X_m^{(1,2)}$ can be written in the form similar to Eq. (31), but with different factor $Y_m(\zeta)$:

$$Y_m^f(\zeta) = \int_{-\infty}^{\infty} \frac{du}{\sqrt{\pi}} \exp[-u^2 + 2im_z \operatorname{arctg}(u/\zeta)] \sqrt{1 + \frac{u^2}{\zeta^2}} \,. \qquad (37)$$

This function has the same asymptote as $Y_m(\zeta)$ in Eq. (32) for short bunches $(\zeta \gg 1)$, but diverges like $Y_m^f \sim 1/\zeta$, when $\zeta \to 0$. This difference in the behaviour of the form factors Y_m and Y_m^f describes the mismatching between the beam-beam kick and the β-function, which takes place for flat colliding bunches. As can be seen from Fig. 2, within the intermediate region of parameters ($\sigma_c \sim \beta^*$), the extension of the interaction region generally suppresses coherent resonances, especially of higher orders.

If on the contrary the bunches are flat due to big difference in vertical and horizontal β-functions ($\beta_z \ll \beta_x$), it may happen that the parameters of bunches and of the ring will satisfy condition:

$$\sigma_c \simeq \beta_z^* \ll \beta_x^* \quad . \tag{38}$$

It is obvious that in this region the radial beam-beam resonancies will not be suppressed. For instance, this can be proved making the calcullations for radial oscillations assuming the model distribution function[4]:

$$F_0(I_x, I_z) \;=\; \delta(I_z)\frac{exp(-I_x/I_0)}{(2\pi)^2 I_0} \quad , \tag{39}$$

when the kernel of Eq. (31) gets the form:

$$im_x\delta(I_z)\left(\frac{2Ne^2}{pc\epsilon_0}\right) exp(-J_x/2\epsilon_0) \int\limits_{-\infty}^{\infty} J_{m_x}(u\sqrt{J_x})J_{m_x}(u\sqrt{J_x'})\frac{du}{u} \quad , \tag{40}$$

$$I_0 \;=\; p\epsilon_0 \quad , \sigma_c \ll \beta_x^* \quad .$$

which has exactly the same form as the kernel, calculated for a short bunch. This means that in the region (38) the widths of stopbands as well as increments of unstable radial coherent beam-beam oscillations will be determined by the nominal value of ξ_x. Hence, the limitations on the ring performance due to coherent beam-beam effects in this case will be weak, provided

$$\xi_x \ll (\xi_z)_{eff} \ll 1 \quad . \tag{41}$$

4. Discussion

The calculations, which have been presented above, definitely indicate the possibility of significant suppression of the fast coherent beam-beam instability due to the "phase averaging" effect. In the case of dipole oscillation these results are in general agreement with the prediction of simplified calculations based on the rigid bunch model[2]. For multipole coherent beam-beam oscillations the same suppression effect takes place too. However, since the sign of the suppression factor depends on the value of ζ, the positions of the stopbands relative the resonant values $\nu = n/m$ generally can differ for oscillations of different multipolarity. Let us also note that for two dimensional coherent oscillations the suppression factor depends on the partial multipole numbers in the combination $m_x + m_z$. Therefore, it may happen that the instability of oscillations with higher, but close m_x and m_z will be suppressed less than that with lower, but more different m_x and m_z.

The suppression of the instability of coherent beam-beam oscillations in the region $\sigma_c \simeq \beta^*$ can offer some new possibilities for the realization of schemes, which previously were supposed to be limited by the coherent beam-beam instability. As two widely known examples we can mention here the scheme with 4-beam compensated colliding beams; and asymmetric B factories having low and high energy rings of the different perimeter.

In low energy rings like coming Phi-factories, when the parameters are not limited by the synchrotron radiation of bunches, the round cross sections of colliding bunches can be realized rather easily[6]. For this geometry due to $\beta_x^* = \beta_z^*$ one can expect the strong suppression of the instability for both vertical and horizontal betatron coherent oscillations, if $\xi_x \simeq \xi_z$. Together with the suppression of coherent beam-beam instability along the main coupling resonance $\nu_x = \nu_z$[4] this possibility makes the round beam geometry as the most attractive.

In the high energy rings like coming B-factories, due to the problems with the synchrotron radiation background, the flat bunches present the most favorable geometry (see, for instance in[7]). In this case, provided $\beta_x^* \gg \beta_z^*$, in order to suppress coherent beam-beam instability, the beam-beam parameter for horizontal oscillations must be significantly decreased. Let us underline that this limitation on the beam and ring parameters is specific for colliders with flat bunches (B-factories, Phi-factories, *etc*).

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Figure Captions

Fig. 1 The dependence of the form factor Y_m on the ratio ζ; round beams; a: $q = 1$, b: $q = 2$, c: $q = 3$, d: $q = 4$.

Fig. 2 The same as in Fig. 1, but for flat beams.

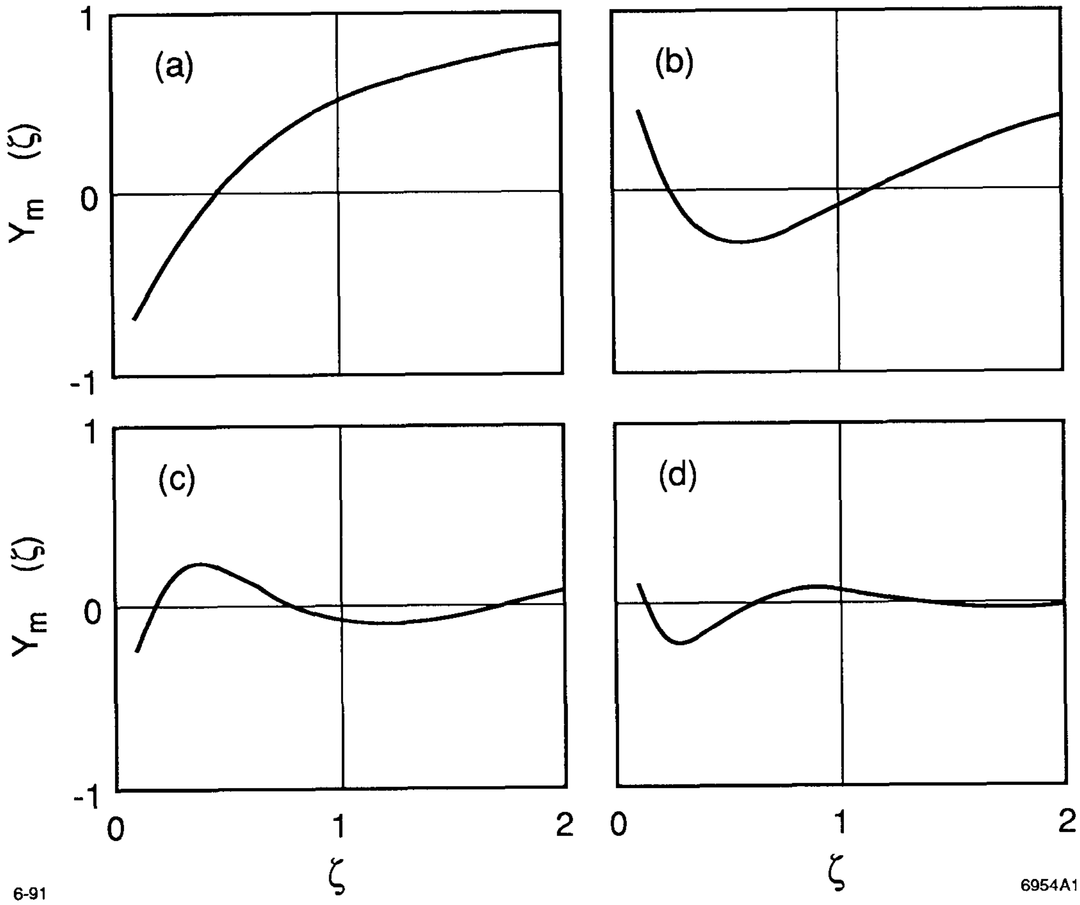

Fig. 1

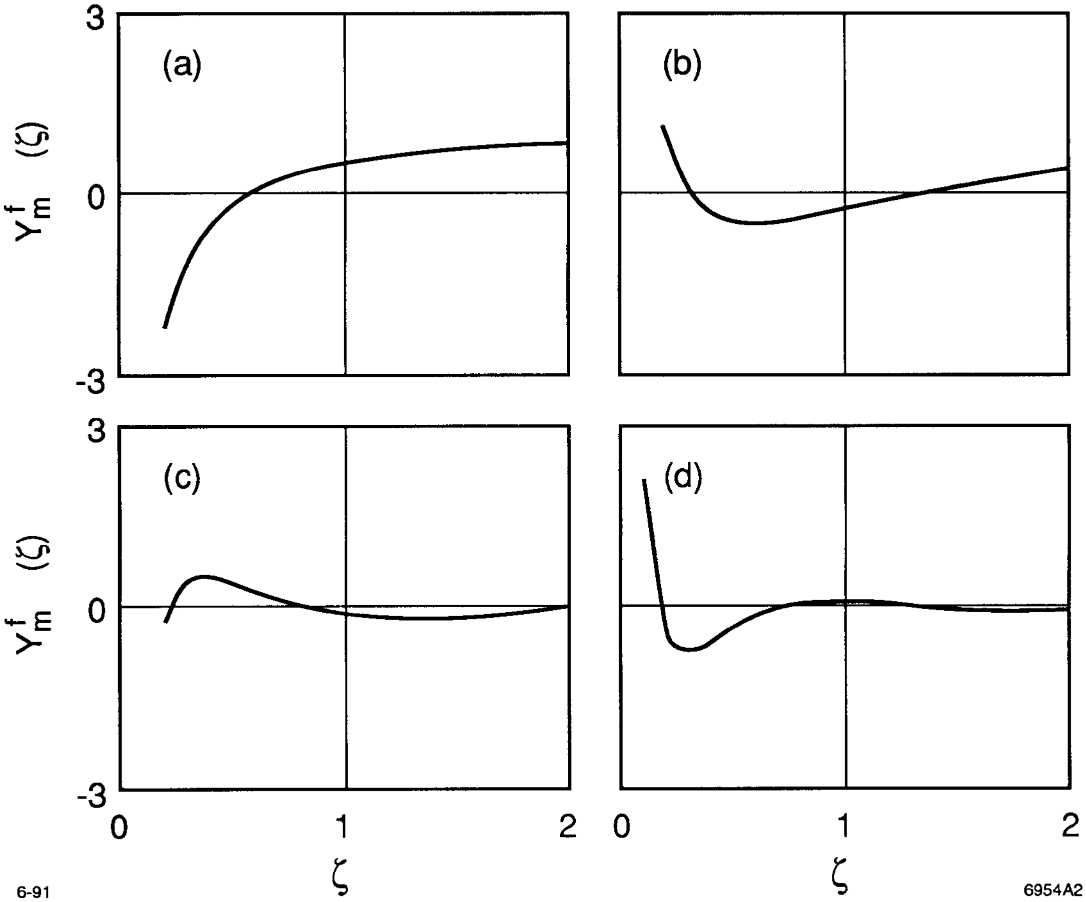

Fig. 2